Introduction to LASER
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Lecture - 05
Amplification by Stimulated Emission

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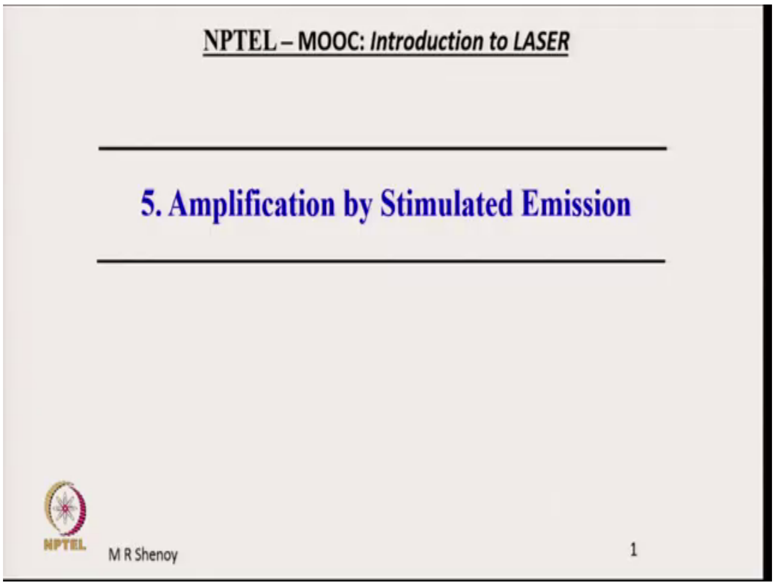

Welcome to this MOOC on LASERS. In this lecture we will discuss Amplification by Stimulated Emission. A very quick recap of what we discussed in the last lecture.

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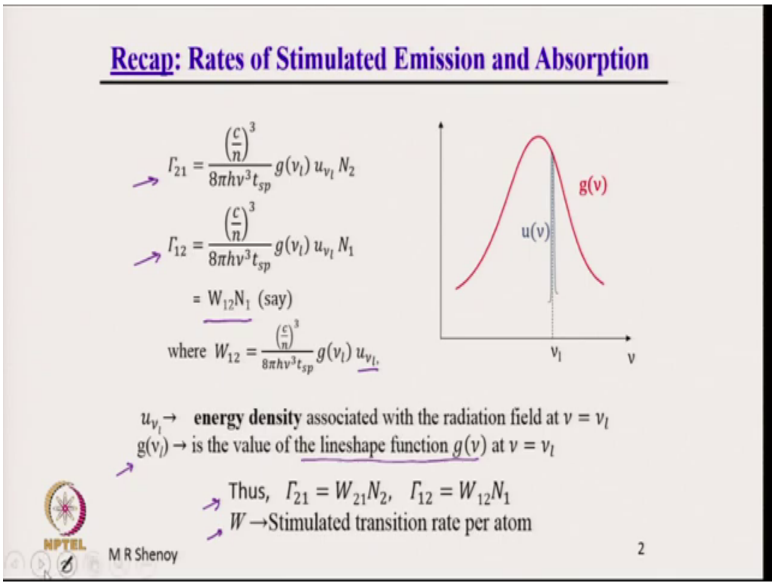

In the last lecture we discussed the rates of stimulated emission and absorption and we have got expressions for the rate of stimulated emission and rate of stimulated absorption. So, the expression gamma 21 is the rate of stimulated emission; which means number of stimulated emissions per unit time per unit volume and gamma 12 is the number of absorptions per unit time per unit volume.

So, it can be written for example, gamma 12 can be written as W 12 into N 1; where W 12 is all the rest of this term except N 1. So, W 12 is this. Where u nu the u nu here is the energy density associated with the radiation field at nu is equal to nu l and g nu l is the value of the line shape function, g nu at nu is equal to nu l.

And therefore, we had the expression that gamma 12 gamma 21 that is the rate of stimulated emission is equal to W 21 into N 2 and gamma 12 is equal to W 12 into N 1, where W is the stimulated transition rate. For non-degenerate systems where wherever sigma e that is emissions cross section is equal to absorption cross section W 12 will be equal to W 21. We will discuss these as we go further.

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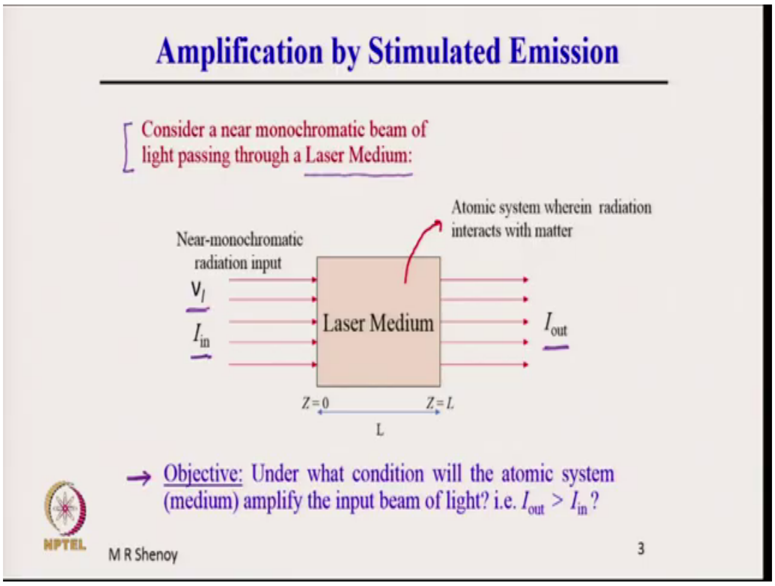

Now, consider a near monochromatic beam of light passing through a laser medium. Laser medium here refers to a medium which can amplify if pumped suitably, a near monochromatic radiation input of frequency nu l. So, this is nu l, I in is the intensity of the incident radiation passing through the laser medium.

If l is the length of the medium and if I out represents the output intensity that is intensity of the output beam, then our objective would be to find out under what conditions. So, this will be our objective in today's lecture under what condition will the atomic system amplify the input beam of light; that is I out is greater than I in. So, we will see by stimulated emission under what condition we can get amplification.

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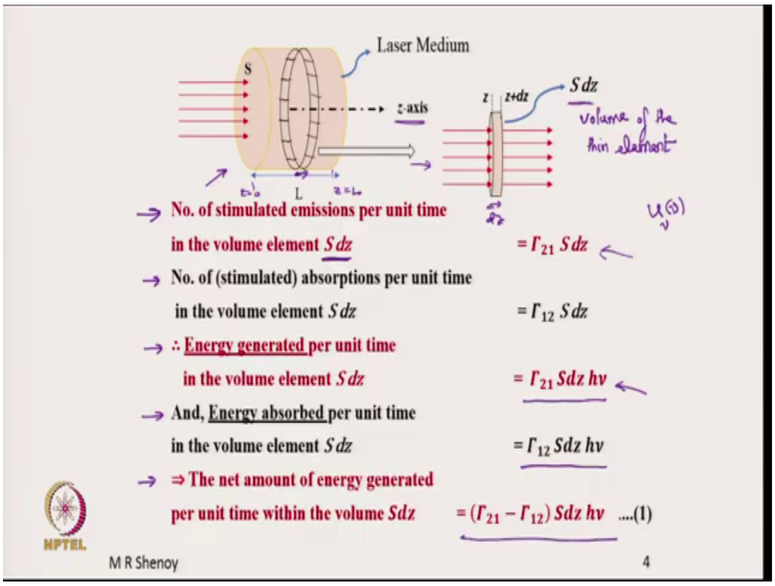

Now, consider a laser medium here which is in the form of a cylindrical rod. So, here we are showing a laser medium in the form of a cylindrical rod of circular cross section. S is the area of cross section. If we now consider a small infinitesimal thickness a small slice of this laser medium of thickness dz which is now shown here; so, it is shown here that this is the thickness of the slice dz here at some distance z.

So, this is z equal to 0. The incident beam is propagating in the z direction. So, this is the z axis and this end incident point is z equal to 0 and this point is z is equal to l. So, we consider a thin slice of this laser rod or laser medium of thickness dz at some z therefore, the volume of the this element is s times d z, S is the cross section. So, cross sectional area multiplied by d z is the thickness will give us the volume.

So, this is the volume of the element; so, volume of the thin slice or element S dz. Now, the number of stimulations, stimulated emissions per unit time in the volume element S dz is given by gamma 21 into S dz because by definition gamma 21 is the rate of stimulated emissions which means number of stimulated emissions per unit time per unit volume.

Therefore, in the volume element of volume S dz the rate of the number of stimulated emissions is gamma 21 into S dz. In the same element the number of stimulated absorptions per unit time in the volume elements S dz because, when in the presence of a radiation of density u nu.

So, we have a radius ratio of density u nu at the frequency nu then, the number of there will be both emissions and absorptions stimulated emissions and absorptions; therefore, the number of absorptions per unit time in the volume element S dz is given by gamma 12 into S dz.

Therefore, the net energy generated every stimulated emission gives out one photon every absorption takes away one photon therefore, the net energy generated per unit time in the volume element S dz will be equal to so, this is gamma 21 into S dz into h nu is the energy generated because, please see this is the number of photons emitted and therefore, each photon is of energy h nu.

Therefore, the energy generated in this volume element S dz is gamma 21 into S dz into h nu. Similarly, the energy absorbed will be equal to gamma 12 into S dz into h nu because every absorption takes away one photon of energy h nu. Therefore, the net amount of energy generated per unit time within the volume element S dz is given by this minus this that is gamma 21 minus gamma 12 into S dz h nu.

So, simply from the definitions of rate of emissions and absorption we have written that the net amount of energy generated within this volume is given by this much.

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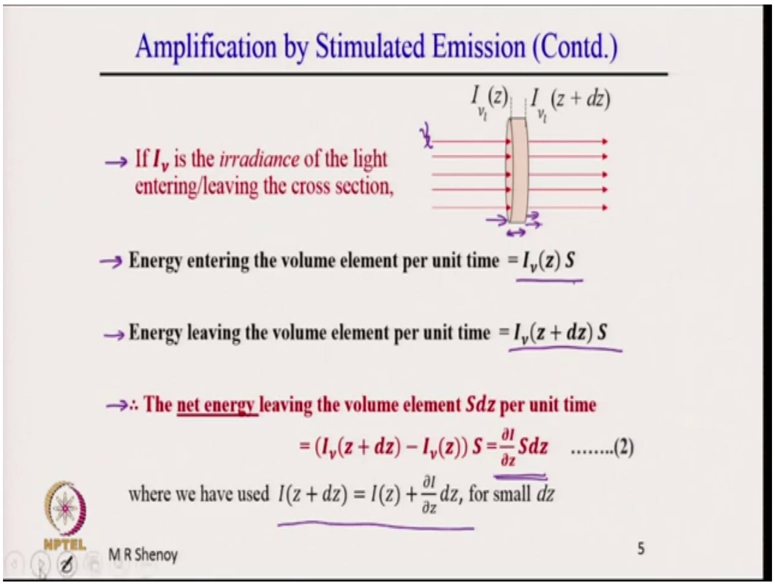

Now, let us go further suppose if I nu was the irradiance of the light entering or leaving the cross section; so, if it is I nu is the intensity here irradiance that is power per unit area is intensity and therefore, energy per unit time is the power; power per unit area is the intensity. So, the intensity of the beam at the input end is I nu of z. So, we are considering nu l as the frequency of the input laser beam or input monochromatic beam.

We have considered a near monochromatic beam entering this medium. And therefore, I nu of z is the intensity at the input of this slice and at the output of the slides the intensity we have designated as I nu into of z plus dz. Therefore, the energy entering the volume element per unit time is I nu of z into S because, energy per unit time is the power; power is intensity multiplied by the area because definition of intensity is power per unit area.

Therefore, energy is power per unit time and therefore, we have; so, power is energy per unit time therefore, the energy entering the volume element per unit time is given by I nu of z into S. The energy leaving the volume element that is at this end energy leaving out of this element is I nu of z plus dz into S; therefore, the net energy leaving the volume element S dz per unit time is this minus this.

So, I nu of z plus d z minus I nu of z into S which we can write as d delta I by delta z into S dz; where we have used because this dz is the volume element the thickness of the volume element is very small infinitesimal small we can write I of z plus dz is equal to I of z into dI by dz into dz. So, delta I by delta z into dz for small dz and therefore, using this we have this net energy leaving the volume element as delta I by delta z into S d z.

Now, in the previous slide here we have the net amount of energy generated is given by this expression in terms of the rates of emission and absorption. Now, we are saying that it is also equal to delta I by delta z into S dz provided I is the intensity.

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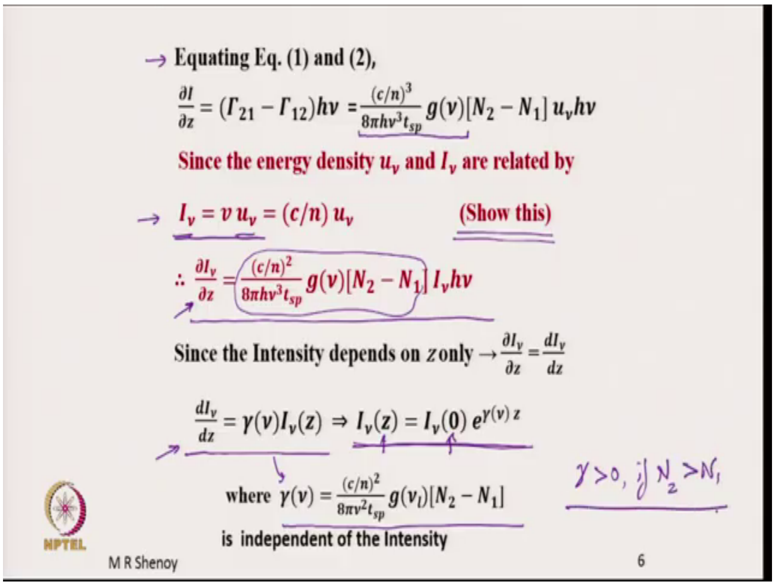

And therefore, equating equations 1 and 2 we have delta I by delta z is equal to gamma 21 minus gamma 12 into h nu, the S dz common S d z cancels. So, there is a S d z which is common in both the equations equation 1 and 2 and therefore, we have delta I by delta z is equal to gamma 21 minus gamma 12 into h nu.

And gamma was W into N. So, this term which is here is W that is the rate per atom into N2 minus N1 into u nu h nu and therefore, since the energy density u nu and I nu are related by I nu is equal to v times u nu; this is v velocity which can be written as c by n into u nu. So, I have left this as show this that the intensity and energy density are related I nu and u nu are related through this relation.

So, making use of this for u nu if we substitute I nu by v; so, we have substituted I nu by v we get this expression. For u nu we have substituted I nu by v v is c by n and therefore, 1 c by n cancels here and therefore, we are left with c by n whole square divided by 8 pi h nu cube t s p into g nu into N 2 minus N 1 into this. So, this h nu will also cancel with this and therefore, we will have in the denominator 8 pi nu square. So, since the intensity depends only on z this partial derivative delta i by delta z can be written as d I by d z or we have an expression d I nu by d z is equal to gamma of nu into I nu, where gamma of nu is all of this.

So, this here with h nu removed from here gamma of nu is c by n square into 8 pi nu square into t s p g nu into N2 minus N1. Once we have d I nu by d z is equal to gamma I of nu into I nu of z we can bring I nu of z to this side in the denominator and integrate and then we get I nu of z is equal to I nu of 0 into e power gamma of nu into z.

What it means is the intensity at any value of z in the medium is equal to the input intensity I nu of 0 here z equal to 0 input intensity into e to the power of gamma of nu into z; where gamma of nu is given by this expression and is independent of the intensity. It is actually not independent we will discuss about this a little later, but at the moment I have assumed it as independent of the intensity.

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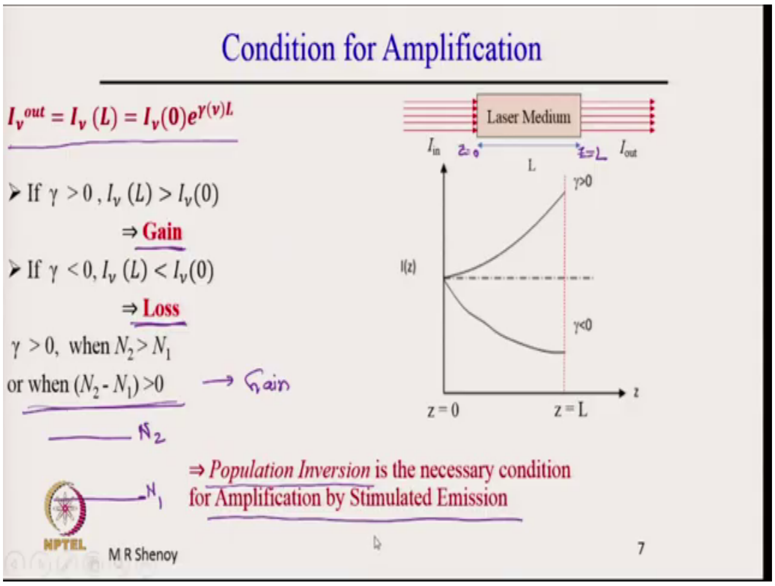

Therefore, I out so, I out here is equal to I nu of l. So, this is the input z is equal to 0 is here; z is equal to 0 and this is z is equal to l and therefore, I nu I out is the intensity output here is equal to I nu of l is equal to I nu of 0 input intensity into e to the power of gamma nu into l.

If gamma is greater than 0, then I nu of l is greater than I nu of 0 that is the output is greater than the input if gamma is greater than 0 that is gamma is positive; that means, there is gain in the medium. If the output is more than the input; that means, there is gain in the medium. If gamma is less than 0 that is, if gamma is negative then we have output less than input or it means there is loss.

When would be gamma greater than 0? Gamma is greater than 0 when N 2 is greater than N 1 or when N 2 minus N 1 is greater than 0. Please look at this expression. So, gamma of nu is here all the rest are positive quantities c n frequency time and g nu of l. So, all are this is a probability and therefore, all are positive quantities; only N2 minus N 1 could be positive or negative depending on whether N 1 is greater than N 2 or N 2 is greater than N 1.

Therefore, gamma would be greater than 0 so, gamma would be greater than 0 if, N 2 is greater than N. So, this is what we have; we have discussed here that when, when N 2 minus N 1 is greater than 0 then, we have gain; so, gamma is greater than 0.

What is N 2 minus N 1? N 2; what is N 2 and what is N 1? Please recall that if I consider the 2 levels N 2 is the number of atoms per unit volume in the excited state and N 1 is the number of atoms in the ground state and normally at thermal equilibrium N 2 is much much less than N 1.

If N 2 can be made greater than N 1 by some mechanism then, we call that situation as population inversion and population inversion therefore, is the necessary condition for amplification by stimulated emission; population inversion is the necessary condition for amplification by stimulated emission. Therefore, a lot of effort or lot of discussion would go into how to achieve population immersion in a given atomic system; whether it is possible at all.

In every medium you may not be able to get population inversion; there are some mediums which we call as laser medium, when we say laser medium its a medium in which we can achieve population inversion by suitably pumping or exciting the atomic system alright.

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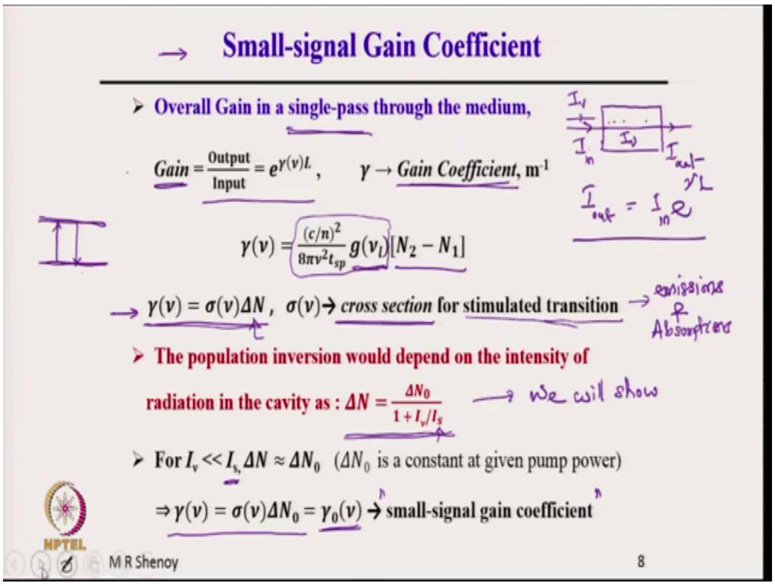

Let us proceed further when we write that the overall gain in a single pass through the medium. So, this is a word usually used single pass gain; single pass gain means you have a medium here of certain length, a certain input beam of some intensity I in is entering the medium and it passes through the medium once this is called single pass and you get the I out.

There are situations where you can make the laser beam pass through the gain medium multiple times and to distinguish this multiple pass we normally call it as a single pass gain. Single pass gain means passing through the gain medium once along its length.

So, the single pass gain refers to therefore, I out is equal to I in into e to the power of gamma into L, where gamma is the gain coefficient. And therefore, output by input that is I out by I in is e to the power of gamma L; that is what is written here e to the power of gamma L and gamma is a function of frequency because gamma contains g g here that is the line shape for value of the line shape function. So, that will be frequency dependent and therefore, gamma is a frequency dependent quantity.

So, gamma is gain coefficient in meter inverse. So, if you are, please see the distinction one is gain another is gain coefficient. Gain coefficient refers to this coefficient gamma which appears in the exponent whereas, gain is output intensity by input intensity or output power by input power of the beam which is equal to e to the power of gamma L this will be a number dimensionless number whereas this is units of length inverse.

So, gamma of nu is written as sigma of nu into delta N. So, this is delta N and all the rest of it here is called sigma of N or its called the cross section for stimulated transition sigma of nu is called the cross section. You can see that its dimension is area because this is per unit volume, this is per unit length and therefore, this must be of dimension of area. So, sigma of nu is called cross section for stimulated transition.

The population inversion actually would depend on the intensity of the radiation in the cavity as delta N is equal to delta N 0 into 1 plus I nu by I s; this result we will show. So, we will show a little later using the rate equations, we will show that delta N is equal it can be written as delta N 0 into 1 plus I nu by I s, where I s is a parameter called the saturation intensity.

We will explain this in a minute and I nu is the intensity of the input radiation and delta N 0 is a constant for a given pumping power all of this will be discussed in detail.

But at the moment we just wanted to point out that this delta N appearing in the gain expression for gain is not actually a constant, but it is to be it is intensity dependent; it depends on the intensity and therefore, delta N can be written as delta N 0 by 1 plus I nu by I s only when the incident radiation.

So, this is the incident radiation whose intensity is I nu or in the medium when the intensity is I nu which is much less than I s the intensity of the beam in the medium when it is so long as it is much less than I s called saturation intensity.

We will as I mentioned we will show this and we will discuss this terms in detail, but just looking at the expression we right now want to call it because, when we say gain coefficient here it reads small signal gain coefficient title say small signal gain coefficient it is a single pass we know what is a single pass gain and what is small signal gain.

Small signal gain coefficient means whenever the intensity of the radiation in the medium is much smaller than in a intensity parameter called I s saturation intensity, what is this we will see in detail then delta N is nearly equal to delta N 0.

Because I nu is much less than I s means this term in the denominator is neglected compared to 1 and therefore, delta N is nearly equal to delta N 0. Delta N 0 is a constant at a given pump power and therefore, we can write gamma of nu is equal to sigma of nu into delta N 0, where gamma 0 this we designate as gamma 0 is called the small signal gain coefficient; small signal gain coefficient.

Two parameters introduced here one single pass gain and small signal gain coefficient. Single pass gain means the gain that is acquired in passing through in one pass through the gain medium which is the laser medium that is called single pass gain. Small signal gain coefficient means for relatively smaller intensities of the signal.

Signal is the one which passes through that is the I nu which we have designated the input beam at a frequency nu if I nu the intensity of the signal beam is much smaller than a saturation intensity; saturation intensity is characteristic of the medium and that we will discuss in detail.

But if the signal intensity is smaller much smaller than the saturation intensity the gain coefficient gamma is a constant and therefore, the intensity through the medium will build up exponentially as e to the power of gamma into l, gamma into z exponentially it will build up.

So, that coefficient is called small signal gain coefficient. Whenever if it is mentioned that it is a small signal gain coefficient it means you simply use the expression I nu of z is equal to I 0 that is input e to the power gamma z that is the meaning alright.

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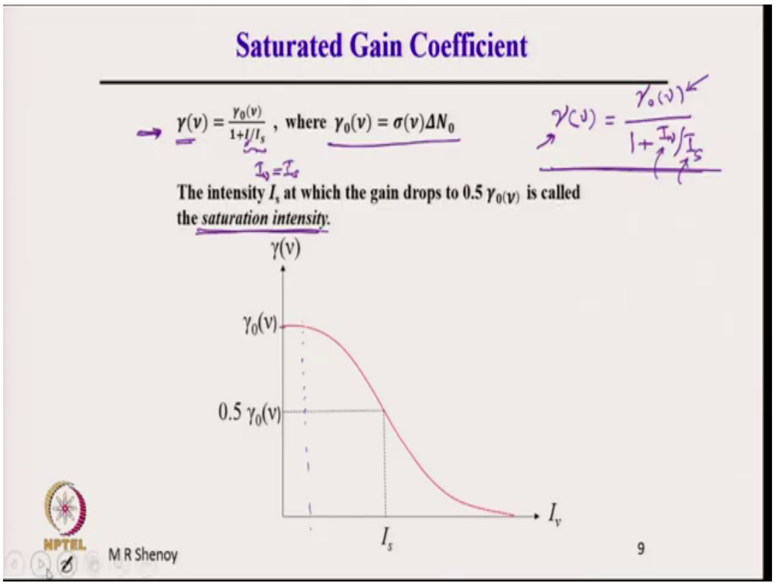

Let us go further and here it is therefore, using this expression I have repeatedly mentioned that, this we will show a bit later. Gamma of nu is equal to gamma 0 divided by 1 plus I by I s, where gamma 0 is the small signal gain coefficient. Therefore, what is this? If we plot gamma of nu gamma of nu as a function of intensity I nu here, so; this is I nu. So, that should be an I nu here. So, gamma as a function of I nu then as for small values of I nu this term is negligible and gamma of nu is equal to gamma 0.

So, for small values you can see up to about this the gain coefficient is almost constant, but as the intensity increases, the second term in the denominator starts becoming significant and therefore, the gain coefficient starts decreasing the intensity i nu at which the gain drops to half of its value is I s. So, the definition of saturation intensity here is the intensity at which so, the intensity I nu is equal to I s when I nu equal to I s some value I s at which the gain drops to half of its value is called the saturation intensity ok.

So, we have introduced one more term that is saturation intensity and the expression for gain coefficient given here let me write it here. So, gamma of nu is equal to gamma 0 of nu into 1 plus I nu divided by I s; this expression is called the saturation gain coefficient. This gain is called the saturation gain coefficient and this coefficient here is called the small signal gain coefficient.

Gamma of nu given by this expression is called the saturated gain coefficient. Saturated gain coefficient does not mean it is one value its value will depend on the intensity I nu. I s is constant for a given medium at a given power at a given pumping power. So, this is the saturated gain coefficient expression for saturated gain coefficient ok.

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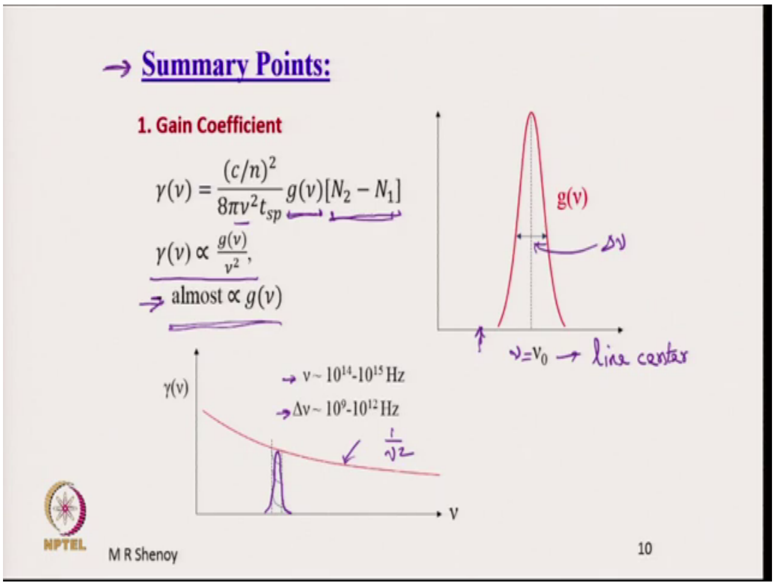

So, there are some I will just summarize these points. These are important points which we have just discussed. First the gain coefficient gamma of nu is given by such an expression and therefore, the first thing that you note is that the gain coefficient is proportional to N 2 minus N 1, first of all N 2 minus N 1 has to be positive which means N 2 must be greater than N 1 which is a situation called population inversion.

Second it is proportional almost proportional to this g of nu which is the line shape function the value of the line shape function at a frequency nu what it means is if you come outside. So, this is g nu if you are somewhere here there is no gain because g nu has come down to 0 and the gain coefficient is maximum at nu is equal to nu 0. This frequency is called the line center so, line center. Line center is the frequency where g of nu is maximum or the peak of g of nu because g of nu is maximum at the line center.

So, it is proportional to g of nu and naturally it is maximum at the line center it is also inversely proportional to nu square because rest of the parameters are constants for the given medium c n 8 pi and t s p only there is a frequency dependent term is nu square and therefore, strictly speaking gamma of nu is proportional to g nu by nu square.

But, I have written here that it is almost proportional to g nu; that is because the nu the frequency nu here is the frequency of light there is a optical radiation typically 10 to the power of 14 into 10 to the power of 15 Hertz. And delta nu here refers to the bandwidth over which gain is present that is g of nu. So, delta nu is this full width at half maximum this is what we are designated as delta nu.

So, the bandwidth that is the frequency range over which g nu is finite is very small 10 to the power of 9 to 10 to the power of 12 compared to this at least 100 smaller which means if you plot nu square and g nu as a function like this then, over the range where g nu is finite if I were to plot this is 1 by nu square right; 1 by x square type of variation 1 by nu square and this is here is the g nu variation.

Because, the bandwidth is much smaller than the frequency nu itself which means over the range delta nu, where g nu is finite there is very little variation in 1 by nu square because its going very slowly because the frequency difference is very very small its a very very narrow range of frequencies.

And therefore, 1 by nu square changes very little and therefore, this is almost means this is 1 by nu square is almost constant and therefore, it is almost proportional to g. Why am I taking so much emphasis on this because the gain coefficient is directly proportional to the atomic line shape function.

And therefore, it is very important to know the atomic line shape function of a laser medium if we want to know the gain spectrum and therefore, in the next lecture, we will discuss in detail the origin that is what are the mechanisms which are responsible for the line shape function and the variation g nu, and that is why these line broadening mechanisms become very important because the amplification bandwidth itself is determined by g nu function; that is the normalized line shape function.

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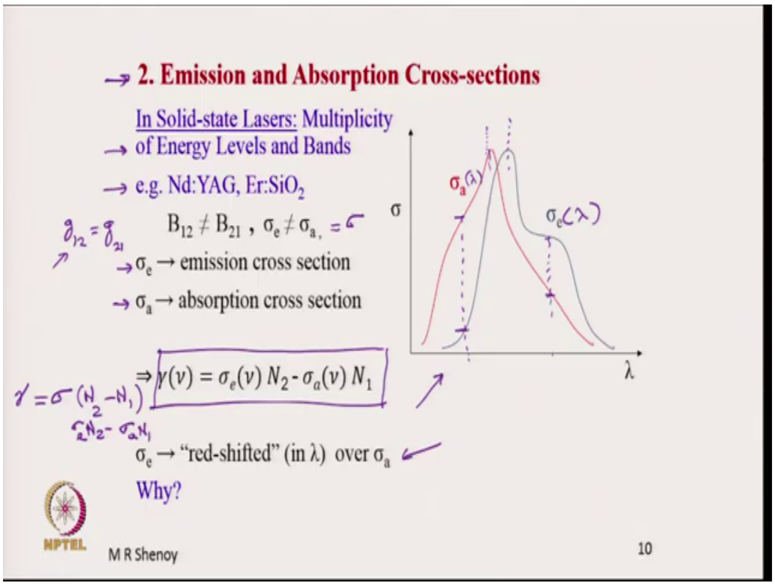

That is the first point the second point that we had seen. So, I am just summarizing the important points which we have seen is the emission and absorption cross section. We had in our calculations we had taken 1 sigma if I just take you back here; let me go back right up to this here.

In writing this gamma 21 which contained we have assumed that gamma 21 and gamma here we can see the expression here please see. So, this is this expression here which we called as sigma let us see where is sigma this is the cross section sigma sigma is the cross section.

Gamma of nu is equal to we had written sigma into N 2 and sigma into N1. We have written 1 sigma and called it as cross section for stimulated transition; simulated transition here refers to emission and absorption. Stimulated transitions are emissions and absorptions absorption.

So, we have used 1 sigma for both stimulated transitions that is if we take a non-degenerate 2 level system like this then the cross section for upward transition; the cross section for upward transition is the same as the cross section for downward transition.

Because this is a non-degenerate 2 level system; however, this is not true when we go to certain solids. So, that is what we are discussing now. In solid state lasers it is not 2 levels there are multiplicity of energy levels and bands corresponding to 1 energy value. There are multiplicity of energy levels for example, if you take a Neodymium YAG or Erbium doped silica these are fiber lasers; we will have B 12 is not equal to B B 21; B 12 was equal to B 21. For non-degenerate system when g 12 was is equal to g 21 that is the degeneracy factor.

But otherwise, B 12 is not equal to B 21; that means, sigma e is not equal to sigma a. We had assumed so far that the emission cross section and absorption cross section are same and we simply used one parameter that is sigma and said that this is the cross section.

But, now we are saying that in most of the solid state lasers where the energy levels are not discrete non degenerate levels we have sigma e different from sigma a the emission cross section sigma e is different from the absorption cross section and we designate it as sigma e and sigma a.

In that case we will get the expression for gamma of nu as sigma e of nu into N 2 because please see gamma was equal to sigma into delta N delta N is N 2 minus N 1. We had a common sigma, but what we are and this N 2 terms came because of stimulated emission and N 1 came for stimulated absorption if sigma is not the same for emission and absorption then we will have sigma e into N 2 here and sigma a into N 1.

So, that is what is written here that the expression for gain is sigma e of nu into N 2 minus sigma a into N 1. So, what is shown in this diagram is typical emission and absorption cross section for a solid state laser or a solid state laser medium.

That the absorption the emission cross section is always red shifted red shifted in lambda over sigma a red shifted means what it is shifted towards longer wavelengths you can see the peaks here. So, the peak of this and the peak of this it is shifted towards longer wavelength the curve corresponding to sigma e of lambda.

So, this is emission cross section and this is absorption cross section the plot of emission cross section versus absorption cross section looks like this. They are different the important point here these are qualitatively shown, but the important point is they are different in other words in sum for some wavelength if you are here then sigma e is this value sigma a is this value.

But if you are here for example, then sigma a is smaller and sigma e is larger and therefore, the gain expression is this. So, whether this total term whether this is positive or negative it is not sufficient that N 2 is greater than N 1 will provide gain that is the important point.

If sigma e is equal to sigma a is equal to sigma then we had written that N 2 should be greater than N 1 population inversion is the necessary condition, but if sigma e is not equal to sigma a then N 2 greater than N 1 is not sufficient it is sigma e into N 2 that should be greater than sigma a into N 1 to have gain in the media alright. So, there is a small question here why it is red shifted. So, please I would request you to please think about it.

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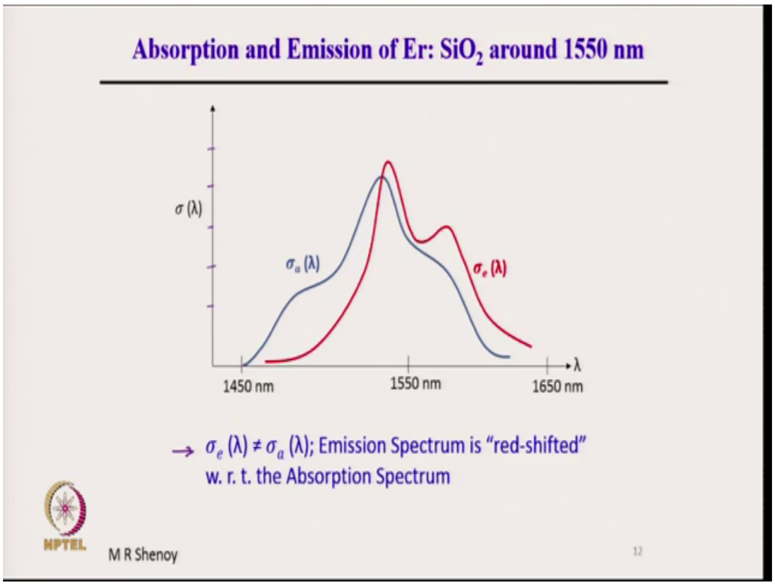

So, here I have shown a nearly correct graph for the absorption and emission of Erbium doped silica. So, you can see that this is sigma a there is the absorption spectrum the typically this varies for Erbium doped silica that is Erbium doped fibers are silica fibers and therefore, we will study later on erbium rare earth doped fiber amplifiers and lasers and erbium doped optical fiber lasers are very very important.

And here you can see that the absorption cross section is different from emission cross section I have not put values here these are typically 10 to the power of minus 22 minus 23 minus 24 centimeter square is the typical numbers for sigma and the wavelength over which you may be aware that the erbium laser is lazing around 1550 nanometer which is the low loss window of optical fiber and qualitatively shown the emission cross section and absorption cross section.

Again the important point is sigma e is not equal to sigma a of lambda the emission spectrum is red shifted with respect to the absorption spectrum.

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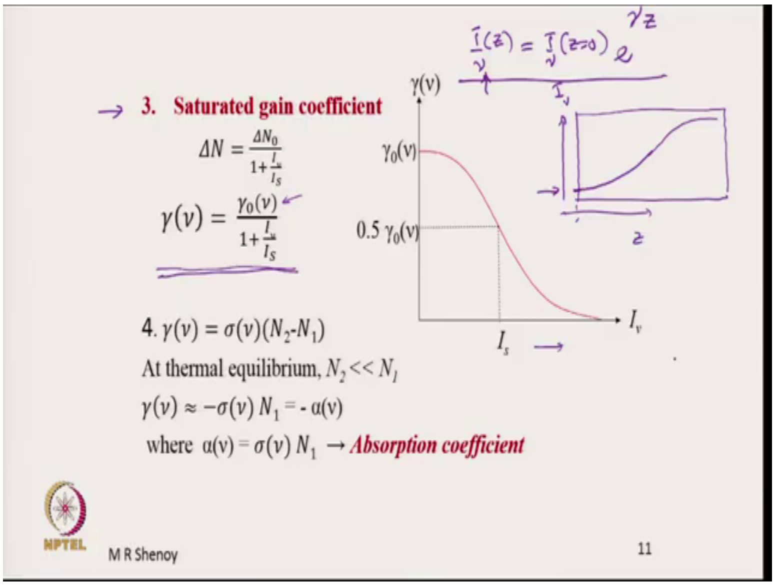

So, finally, the third point and the important point is this is called the saturated gain coefficient gamma 0 is called the small signal gain coefficient and gamma of nu given by this expression is called the saturated gain coefficient and the variation of the gain.

The important point here is that if the intensity of the radiation inside the laser medium I am deliberately showing slightly longer variation let us say the intensity is very small when at the input. So, this axis is intensity I nu and this axis is z z is equal to 0 is the input intensity level is very small, then in the medium the intensity starts building exponentially.

But, when the intensity increases the gain coefficient starts decreasing that is what this graph shows and therefore, the rate at which the intensity builds reduces and finally, it will reduce to an extent that there will be no further gain if the gain coefficient comes down to 0; that means, e to the power of gamma into z is 1 and therefore, the intensity would become constant and that is the importance of this saturated gain coefficient in a single pass or whenever the intensity becomes more that is approaches I s the gain coefficient starts dropping down.

The expression I repeat this again that the expression I of z is equal to I nu at z is equal to 0 that is the input into e to the power of gamma z is true whenever I nu at all values is much smaller than I s the saturated intensity. Otherwise this exponential dependence will change it would become subsequently linear and then it is a non-linear variation with z if the medium is sufficiently long.

We come to here I will stop and then in the next lecture we will take up the line broadening mechanisms which are responsible to the shape of g of nu which will determine the bandwidth of the laser amplifier.

Thank you.