Introduction to LASER
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Lecture - 04
Atomic Lineshape Function, g(v)

Welcome to this MOOC on LASERs, Introduction to LASERs. So, today we will introduce the concept of Atomic Lineshape Function.

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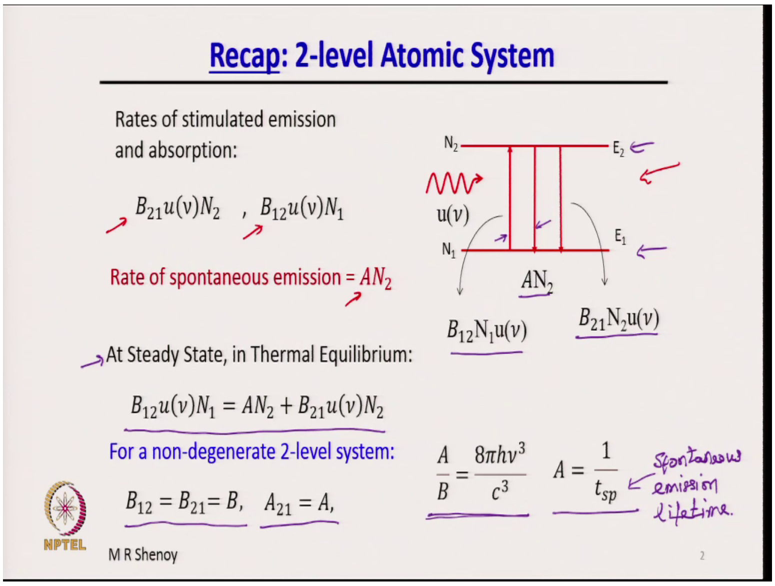

A very quick recap, in the last lecture we had seen 2-level atomic system; where we had seen that this is a 2-level atomic system here; a 2-level atomic system which where the interaction is between a ground state and one of the excited states, which we call level energy level E 2.

And we have seen that the emission and absorption, the processes of emission and absorption; so, this is indicated here.

This process here is representing absorption, that is an atom absorbing a photon making an upward transition and the rate of emission is given by this; B 12 into N 1, N 1 is the number of atoms per unit volume in at level 1 and u nu is the spectral energy density; B 12 is the Einstein coefficient.

The emission comprises of two components here one, that is spontaneous emission and stimulated emission. Stimulated emission is proportional to the rate of stimulated emission is proportional to the spectral density of radiation present. Therefore, at steady state we have seen that at study state the number of upward transitions must be equal to the number of downward transitions.

And we have written this therefore, we have this equation and for a non-degenerate system after equating this with the black body radiation, we have got the Einstein relations A by B is equal to 8 pi h nu cube by c cube and A is equal to 1 by t sp, where t sp is the spontaneous emission lifetime; so, spontaneous emission lifetime. So, we have discussed the method spontaneous emission lifetime.

And for a non-degenerate 2-level system we have B 12 is equal to B 21 equal to B and because there is only one A coefficient, we call A 21 as A.

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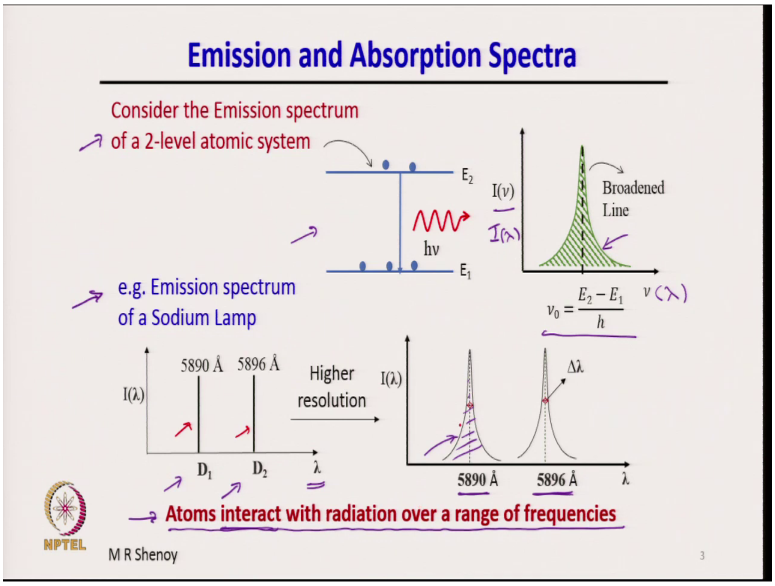

Now, let us look at this process of emission and absorption more carefully. If we consider the emission spectrum of an atomic spectrum, atomic system say a 2-level atomic system; then what we observe is not if we were to go by this picture here then we should have got if we plot frequency versus intensity at that frequency that is the spectral spectrum I nu versus nu or I of lambda versus lambda, one and the same.

We can write I of lambda versus lambda. This is called the spectrum. The emission spectrum, if we observe the emission spectrum, we get radiation over a range of frequencies; what is shown is a spectrum here over a range of frequencies. But we would expect if there were only 2 atomic energy levels at energy E 1 and E 2, we should have got radiation at one frequency nu 0 corresponding to E 2 minus E 1 by h.

But what we actually observe is a spectrum with a certain width a range of frequencies, emission takes place over a range of frequencies. For example, if we look at the emission of a sodium lamp, sodium lamp we widely use in undergraduate laboratories. So, we know that we it has two lines spectral lines called the D 1 D 2 lines. These are easily separable using a grating spectrometer.

But if we observe; so, normally we observe it as two closely separated yellow lines in the spectrum. But, if we observe it with higher resolution that is if we enlarge the scale in lambda then we see that even each one of these lines are comprised of a spectrum that is it is spread over a range of wavelengths. So, what is plotted is lambda versus I of lambda, but centered one line is centered at 5890 angstrom.

And the other one at approximately 5896 angstrom which we call as the wavelength of the D 1 D 2 lines shown here with a separation of 6 angstrom; the point I am making is if you observe a spectral line of emission from any atomic system it shows a finite range of wavelengths, emission over a finite range of wavelengths. And therefore, we conclude that atoms interact with radiation over a range of frequencies, interact here refers to either emission or absorption.

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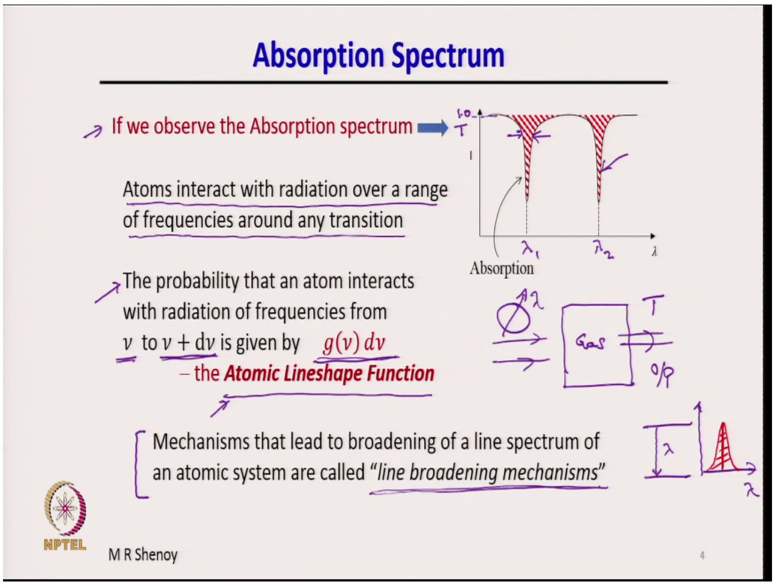

Now, let us look at the absorption spectrum similarly, if we look at the absorption spectrum of a gas or of a material then in the transmission spectrum; so, what is shown here is transmission. So, as I already mentioned in one of the earlier classes, you have an atomic system. So, there is an atomic system and there is a broadband source, there is a broadband source.

Broadband source means we indicate this as a variable lambda source. So, we can vary the spectrum, or it is a broadband source. This comprises of a range of wavelengths passing through this gas, let us say this is a gas and then at the output if we observe the transmitted light. So, this is T standing for transmission, transmission at the output; then if we plot the transmitted intensity as a function of wavelength it will be.

So, this maxima is around 1. So, maximum transmission is 1, but we see a dip characteristic of the atomic system. What is shown is the absorption spec spectrum in which we see transmission dips. So, dips in the transmitted spectrum, the dip is because the wavelengths let us say these are wavelengths lambda 1 and lambda 2, then the atomic system resonantly absorbs around wavelengths lambda 1 and lambda 2.

But again the point to be noted is it is a range of wavelengths, it is a range of wavelengths over which absorption takes place which means we again have the same conclusion that atoms interact with radiation over a range of frequencies around any transition.

Therefore, the probability that an atom interacts; so, atom interacts with radiation over a range of frequencies and therefore, the probability that an atom interacts with radiation of frequencies from nu to nu plus d nu is given by a function called g nu d nu which is called the atomic lineshape function.

And, we will discuss about this function g nu, this tells us this function tells us the strength of interaction. The function tells us g nu tells us the strength of interaction at the frequency nu and that is called an atomic lineshape function. So, the atomic lineshape function maybe I will show in the next slide. But the mechanisms, this is a topic we will discuss in detail a little later. And, the mechanisms that lead to broadening of a line spectrum of an atomic system are called line broadening mechanisms.

This topic we will discuss in detail. So, what this statement means is if you have an a 2-level atomic system then ideally, I should have expected one line. If I plot lambda versus intensity, then corresponding to the wavelength here or frequency corresponding to this energy separation I should have got a line. So, let me just use a different color and I should have got a line like this and that is called a line spectrum.

I showed you in the previous slide here D 1 D 2 we showed as lines. So, this is called line spectrum, but when we observed this under higher resolution the line spectrum showed a finite range of wavelengths, the emission over a finite range of wavelengths. So, the same way this is a line spectrum, but there are mechanisms which will lead to broadening of this line spectrum into a range of frequencies like this.

And, these mechanisms are called line broadening mechanisms, we will discuss this. Now, it is these line broadening mechanisms which are responsible for an atomic lineshape function.

The atomic lineshape function gives us the strength of interaction at any given frequency nu. And, what it means is g nu d nu, the range of frequencies, the atomic the strength of interaction between nu and nu plus d nu is given by g nu d nu. It will become more clear in the next slide. So, let me go to the next slide here.

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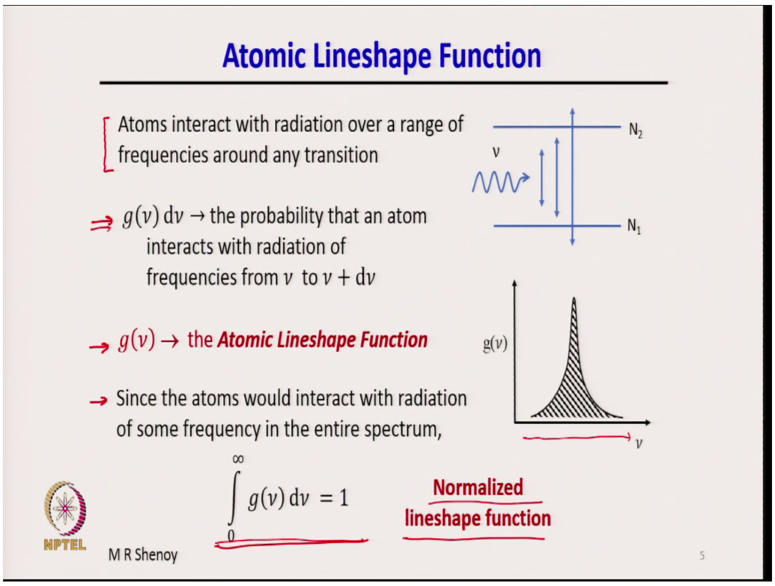

So, the atomic lineshape function. Again, I am just repeating to (Refer Time 11:16) the point, that atoms interact with radiation or a range of frequencies around any transition. g nu d nu represents the probability that an atom interacts with radiation of frequencies from nu to nu plus d nu and such a function g nu is called the atomic lineshape function.

Since, the atoms would interact with radiation of some frequency in the entire spectrum, that is if you take the entire spectrum from 0 to infinity; there will be some frequency with which the atoms will interact because, it has a certain energy difference. And therefore, if we integrate this 0 to infinity, the total probability must come out to be 1.

Please see g nu d nu again here is the probability that an atom interacts with radiation of frequencies from nu to nu plus d nu. And therefore, the probability integrated over the entire frequency spectrum has to be 1 and this is called the normalized lineshape function. This is the definition of normalized lineshape function. The g nu function which is called the atomic lineshape function is defined as integral 0 to infinity g nu d nu is equal to 1.

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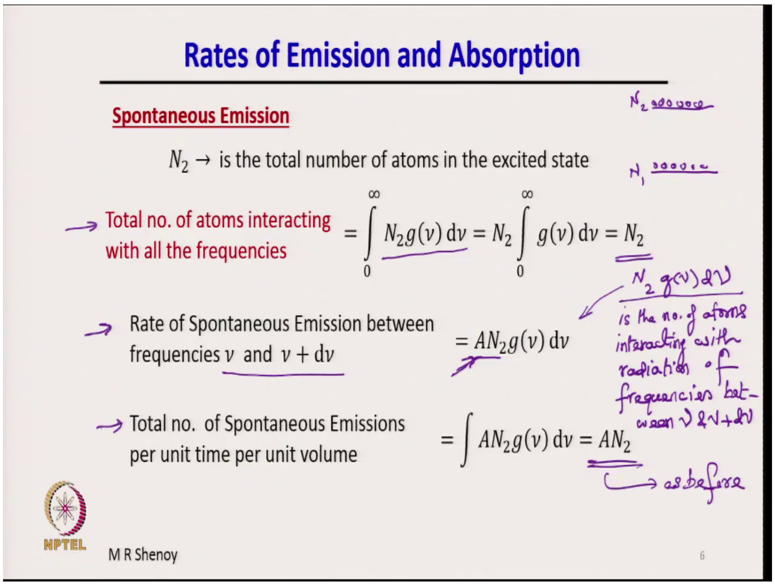

Now, let us look at this look at the rate of emission and absorption, we had seen the rates of emission and absorption earlier in the first slide here. So, we have in the last class we had seen the rates of emission and absorption here and we had got taken these as the expressions for emission and absorption and of course, spontaneous emission here. But, now with the inclusion of the concept of an atomic lineshape function, the expressions get modified.

So, let us see the new expressions for the rates of emission and absorption ok. So, if N 2 is the total number of atoms in the excited state. So, we are looking at the same 2-level atomic system, 1 second. So, we are looking at the 2-level atomic system with N 1 and N 2, the number of atoms. If N 2 is the total number of atoms here per unit volume and N 1 is the total number of atoms here.

Then N 2 into g nu d nu by definition of g nu, it means N 2 into g nu d nu is the number of atoms number of atoms always per unit volume atoms interacting with; interacting with radiation of frequency of frequencies between nu and nu plus d nu. So, this is the number of atoms, if N 2 is the total number of atoms.

Because, g nu d nu represents the probability that an atom interacts with radiation of frequency between nu and nu plus d nu, N 2 into g nu d nu will give the total number of atoms per unit volume interacting with frequencies between nu and nu plus d nu. And therefore, if we integrate this N 2 into g nu d nu from 0 to infinity, N 2 is a constant; so, N 2 comes out and 0 to infinity g nu d nu is 1 and therefore, this is equal to N 2.

So, the total it is consistent now, total number of atoms interacting with all the frequencies is the same N 2. And therefore, the rate of why do we discuss this? The rate of spontaneous emission between frequencies nu and nu plus d nu is A times, earlier we had the expression just A times N 2. But now, we are writing between nu and nu plus d nu A times N 2 into g nu d nu because the number is this now.

And similarly, that therefore, the total number of spontaneous emissions per unit volume is given by this is equal to A times N 2 as before this is what we had in the last class, as before.

The total number remains the same, but over a range of frequencies the fractional number of atoms interacting is given by this expression here, after we introduced the lineshape function.

The line shape function had to be introduced to take care of the practical observations that the emission and absorption spectra are spread over a range of frequencies.

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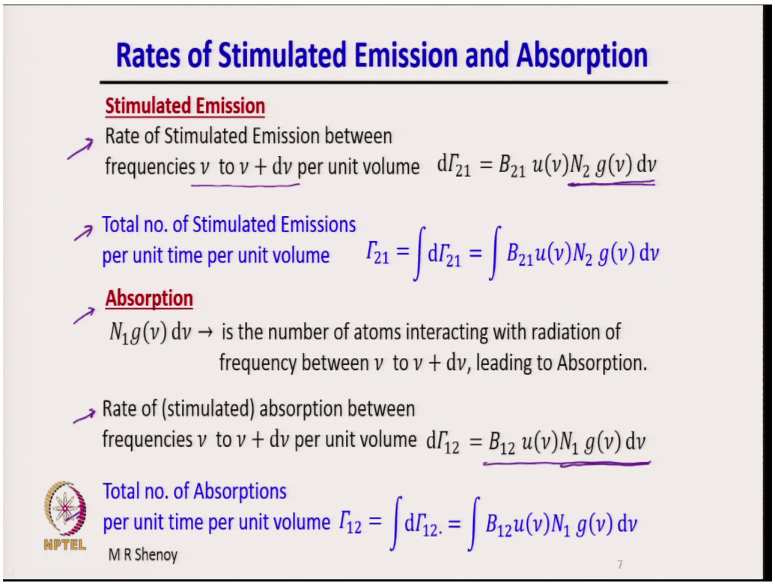

Now, the rate of stimulated emission and absorption; so, exactly like the spontaneous emission, the rate of stimulated emission between frequencies nu to nu plus d nu per unit volume; we call it as d gamma 21 is given by B 21 into u nu into N 2 g nu d nu. Please see earlier we had just N 2, now all N 2s are replaced by N 2 g nu d nu, because this is the number of atoms interacting between nu and nu plus d nu.

And therefore, the total number of spontaneous emissions per unit volume is gamma 21 is equal to integral d gamma 21 is equal to integral B 21 u nu N 2 g nu d nu. Similarly, for absorption, this is for emission. So, similarly for absorption N 1 into g nu d nu is the number of atoms interacting with radiation of frequency between nu and nu plus d nu which leads to an absorption.

And therefore, the rate of stimulated absorption, all absorptions are stimulated. So, rate of stimulated absorption between frequencies nu to nu plus d nu is given by this expression here. Again, as before you see that earlier we had just N 1, now we have N 1 into g nu d nu. And therefore, the total number of absorptions is given by the last expression gamma 12 is equal to integral.

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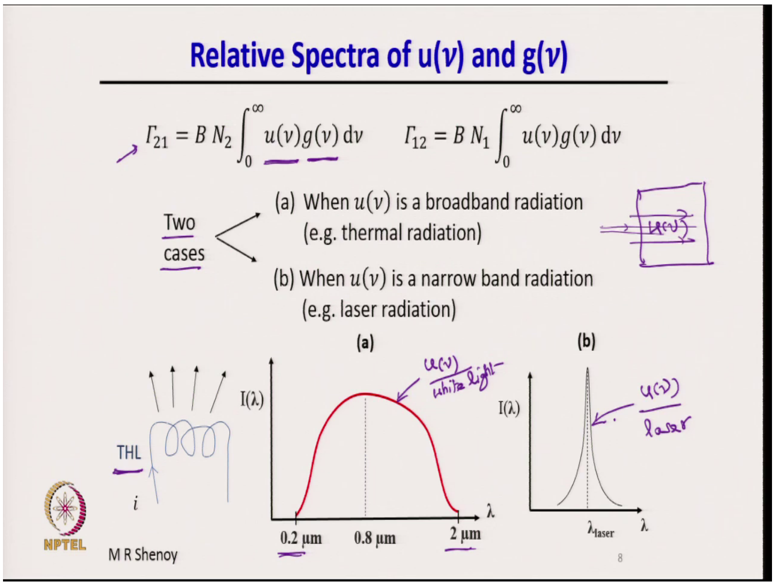

Now, we want to integrate this, while integrating again we take care of two practical issues and that is what we will discuss here; gamma 21 is given by this. B and N 2 where constant so, we have taken it out so; u nu g nu d nu.

Now, what is u nu? u nu is the spectrum of the radiation which is present in the atomic system or in the medium. What is g nu? g nu is the strength of interaction of the atomic system at a given frequency nu. Now, there are two cases, we separate this into two extreme cases; so, that we can understand it more clearly.

When u nu is a broadband radiation; for example, in the atomic system here u inside you have u nu, the radiation which is present here is a broadband spectrum. I have already introduced this word broadband, broadband means over a wide range of wavelength or frequencies; I of lambda is.

For example, if you take THL, THL is Tungsten Halogen Lamp, the normal tungsten halogen lamp where a filament tungsten filament is heated, and this gives out almost a white light like source. And the spectrum, the output varies from approximately from 0.2 micrometer to 2 micrometer with a peak around 0.8 micrometer.

That is 200 nanometer to 2000 nanometer with a peak around 0. 800 nanometer that is in the near infrared. So, this is a wide spectrum of u nu. In the first case u nu could be a wide spectrum like a have a wide spectrum. In a second case when u nu is a narrow band radiation for example, if in this medium if the laser radiation were to propagate. So, if you were to put laser radiation passing through this, then as we know laser is highly monochromatic and if you see the spectrum then it will be very narrow.

This is also u nu; the second case u nu, but this is for example, for a laser. This is for a laser and this is for a white light or an LED, a broadband source; so, white light. So, if we categorize these then we can simplify the integration and the discussion.

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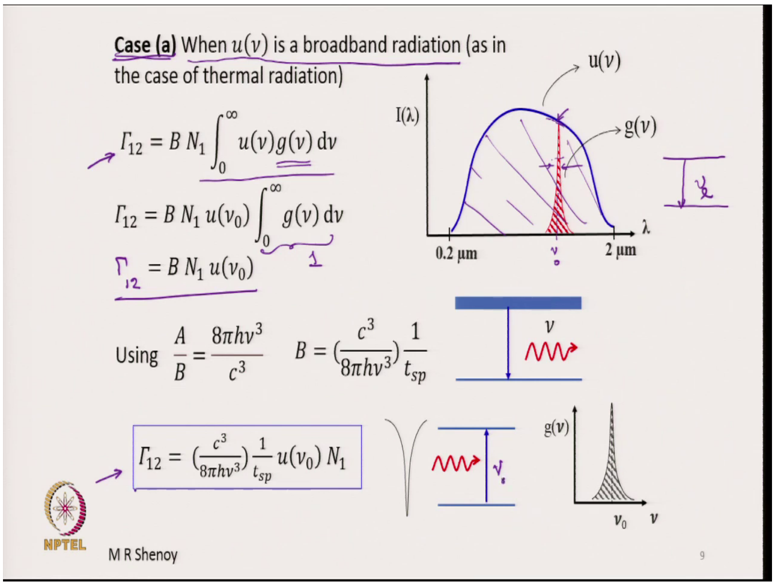

So, let us see this first case 1: when u nu is a broadband radiation, as in the case of thermal radiation. So, gamma 21 12 is equal to B this is absorption, rate of absorption per unit volume is B times N 1 into integral 0 to infinity u nu d g nu. If we see this figure here, this tells us the spectrum; for example, this is u nu and this is g nu. Why this is g nu is so narrow?

Because, it is the line shape function characterizing one transition which is centered around the frequency of the laser or nu 0, around the central frequency corresponding to this energy, energy difference. And therefore, it is a narrow spectrum, g nu is a narrow spectrum, and this is a wide spectrum.

So, in the range of integration, if you look at this integral in the range of integration; so, 0 to infinity which means all of this in the whole range of integration here, the second function g nu is non-zero over a very small range; its non-zero, everywhere else it is 0. And therefore, we can integrate this function by assuming u nu to be constant because therefore, this integral will really run over a very although it is 0 to infinity, the integral will run effectively over a small frequency range.

And over that small frequency range maybe from here to here, we can assume that this u nu changes very little. And therefore, u nu can be taken as a constant and taken outside the integral around nu equal to nu 0, nu 0 is the central frequency. The value of nu 0 corresponding to that if we take u nu outside then we can integrate g nu d nu and integral 0 to infinity g nu d nu is 1; so, this is 1.

And therefore, we have gamma 12 is equal to B N 1 into u nu 0 and using A by B is equal to 8 pi h nu cube by c cube and we can substitute for B. And, we have this expression gamma 12 is equal to c cube by 8 pi h nu cube into 1 by t sp into u of nu 0 into N 1. So, this gives us rate of absorption, exactly like this; so, it is the same thing illustrated here to say that g nu is a very narrow function centered around. So, what I have shown here. So, this is the central frequency around nu 0 and therefore, the rate of absorption is given by this expression.

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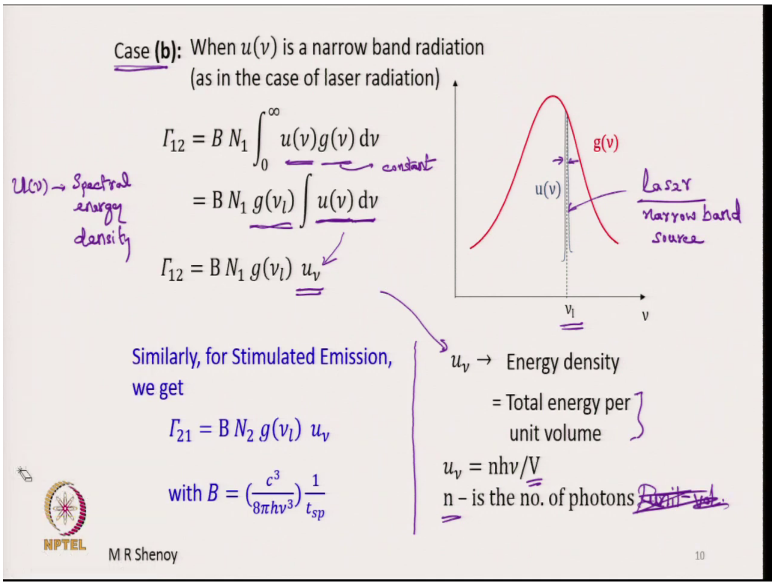

Now, let us see the rate of absorption for the second case, this is the first case. In the second case, if you use a u nu corresponding to a laser radiation. For example: if the u nu was corresponding to a laser then we know that laser is highly monochromatic. And therefore, the spectral width of u nu is now a very narrow band source.

So, this is a narrow band source, narrow band or highly monochromatic source, narrow band source whereas, g nu is the one which is characterizing the transition with a certain finite range which is measurable. We can easily see this g nu in all the emission and absorption spectra.

And, in this case when u nu is much narrower, we can follow the same procedure; we can in the range of integration. Please see u nu will be non-zero only over a small range and therefore, over that range we can assume g nu to be constant. So, we assume this to be constant over the range of integration that is the effective range of integration.

And therefore, we take it out the value of g nu at the laser frequency nu l. So, this is centered around nu l, the laser frequency. We take it out and then we have integral u nu d nu and integral u nu d nu is called u nu the energy density. What is u of nu? Is the spectral energy density that is per unit spectra, unit spectrum and if you integrate over the entire spectrum, then what you get is energy density.

Again, just distinguish u nu is the spectral energy density, spectral energy density and u nu is the total energy density u nu, that is why it is integrated over all the spectrum. So, it is shown here again; u nu is energy density which means total energy per unit volume. If n is the number of photons, this is beside, if n is the number of photons in the medium then u nu is equal to n into h nu by V; number of photons per unit volume per unit volume.

So, if n is the number of photons per unit volume then u nu is n into h nu, energy of 1 photon is h nu, n is the number of photons per unit number of photons in the medium because volume is here, u nu is there. So, this is not, one second let me erase that. So, is the number of photons, then u nu is equal to n h nu by V. So, this is the rate of the rate of absorption, in the case of b. This is a practical case in laser physics, because we deal with the laser radiation in the medium.

We deal with interaction of a laser radiation or a monochromatic radiation in the medium and therefore, this is more relevant to us. And, with respect to case b, similarly for stimulated emission we get gamma 21 is equal to N 2 into g nu l into u nu, instead of N 1 we now have N 2. So, we have the same expression, similar expression for gamma 21 equal to B times N 2 into g nu l into u nu with B is equal to this here.

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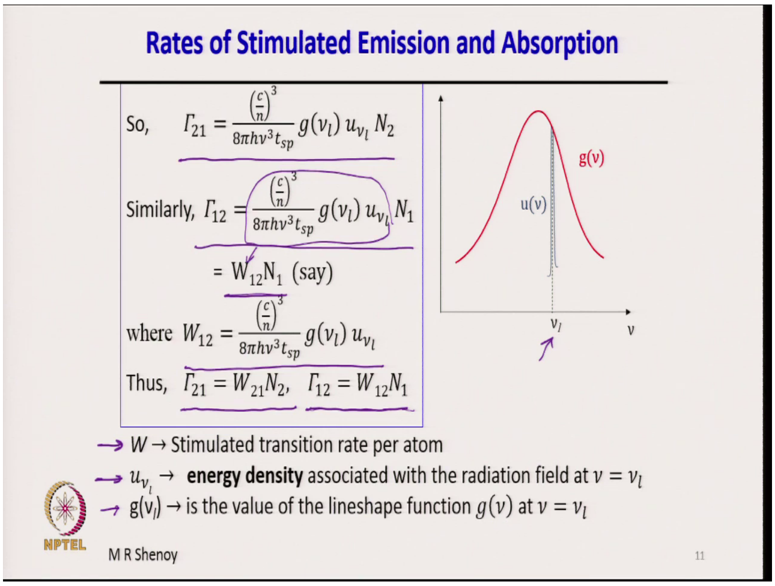

Therefore, to summarize this discussion of rates of stimulated emission and absorption which now we have upgraded with the inclusion of the concept of the atomic lineshape function is given by gamma 21 is equal to. So, these are the expressions which we will use subsequently to determine the amplification. So, gamma 21 is the rate of emission and gamma 12 is the rate of absorption in a non-degenerate atomic system.

You can see that it is almost it is identical except N 2 and N 1. So, this if we call as W 12 into N 1, where this entire term which is here is designated as W 12. So, W oh it is here W 12 then we can write gamma 21 rate of stimulated emission is equal to W 21 into N 2 and rate of absorption is equal to W 12 into N 1; where W is called the stimulated transition rate per atom.

So, it is per atom; therefore, when you multiply by the number of atoms you get the stimulated transition rate, stimulated emission or absorption; u nu is the energy density associated with the radiation field at nu is equal to nu l. So, here and g nu l because in the expression also there is g nu l, g nu l is the value of the lineshape function g nu at nu is equal to nu l. So, with these expressions, we will continue and find out an expression for amplification in a medium by stimulated emission.

Thank you.