Introduction to LASER
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Lecture – 03
The Einstein Coefficients

Welcome to this MOOC on Introduction to LASERS. Now, we are discussing interaction of radiation with matter. So, today, we will discuss about emission and absorption and The Einstein Coefficients.

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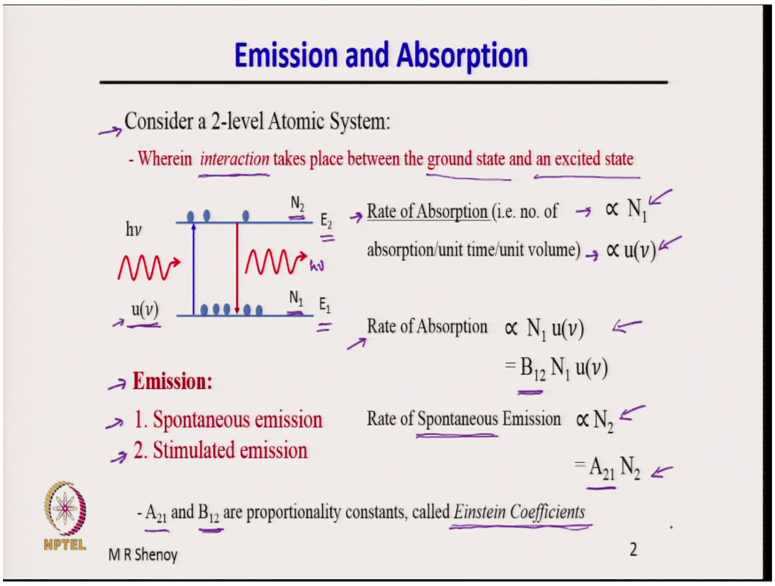

Emission and Absorption

Consider a 2-level Atomic System:

- Wherein *interaction* takes place between the ground state and an excited state

Rate of Absorption (i.e. no. of absorption/unit time/unit volume) $\propto N_1$
$\propto u(\nu)$

Rate of Absorption $\propto N_1 u(\nu)$
$= B_{12} N_1 u(\nu)$

Emission:

1. Spontaneous emission
2. Stimulated emission

Rate of Spontaneous Emission $\propto N_2$
$= A_{21} N_2$

- A_{21} and B_{12} are proportionality constants, called *Einstein Coefficients*

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Emission and absorption; consider a 2-level atomic system; a 2-level atomic system. A 2-level atomic system is one, wherein interaction takes place between the ground state and an excited state. Interaction, here interaction refers to emission and absorption takes place between the ground state and an excited state. So, such a system we call as a 2-level atomic system. So, it is shown here a 2-level atomic system.

So, E 1 is the ground state and E 2 is an excited state. N 1 is the number of atoms per unit volume in the ground state and N 2 is the number of atoms per unit volume in the excited state. u nu is the energy densities, spectral energy density of a radiation which is present in this medium.

This medium so, there are two energy levels therefore, emission and absorption is continuously taking place because of the presence of a radiation of energy density u nu, atoms are getting excited to the upper state and they also come down to the lower state giving out a photon of energy h nu. So, there are emissions and absorptions continuously taking place.

At steady state, the number of absorptions will be equal to the number of emissions. Therefore, we have an average number N 1 in the ground state and N 2 in the upper state at any given instant. Now, the rate of absorption here, the rate of absorption which is the number of absorption per unit time per unit volume is proportional to N 1. First, the rate of absorption is proportional to N 1 that is the number of atoms which are in the ground state.

It is also proportional to so, it is also proportional to u nu, the energy density because higher the density of radiation which is present here more number of atoms can make an upward transition. More number of atoms in the upper state implies more number of absorptions that can take place. Similarly, larger the energy density here, then larger will be the rate of absorption. Therefore, it is proportional to both of these and therefore, the rate of absorption is proportional to a product of these two that is N 1 into u of nu.

The proportionality constant is designated as B 12, 12 standing for from level 1 to level 2. So, B 12 into N 1 into u nu. Similarly, the emission so, this is about absorption. Now, emission here comprises of two components: 1 spontaneous emission and stimulated emission.

The fact that there are two types of emission; spontaneous and stimulated emission is pointed out by Albert Einstein in 1917. So, the rate of spontaneous emission as the name indicates it is spontaneous, spontaneous on its own. Spontaneous emission is proportional to the number of atoms N 2, the number of atoms in the excited state.

And therefore, we write that the rate of spontaneous emission is equal to A 21 into N 2 where A 21 is a proportionality constant for the spontaneous emission and these constants A 12 and A 21 so, A 12 here and A 21 are the proportionality constants called Einstein coefficients. Einstein introduced these and therefore, they are called Einstein coefficients.

Now, stimulated emission. We have seen emission comprises of two components: spontaneous emission and stimulated emission. So, spontaneous emission rate is given here A 21 into N 2. Now, let us see stimulated emission.

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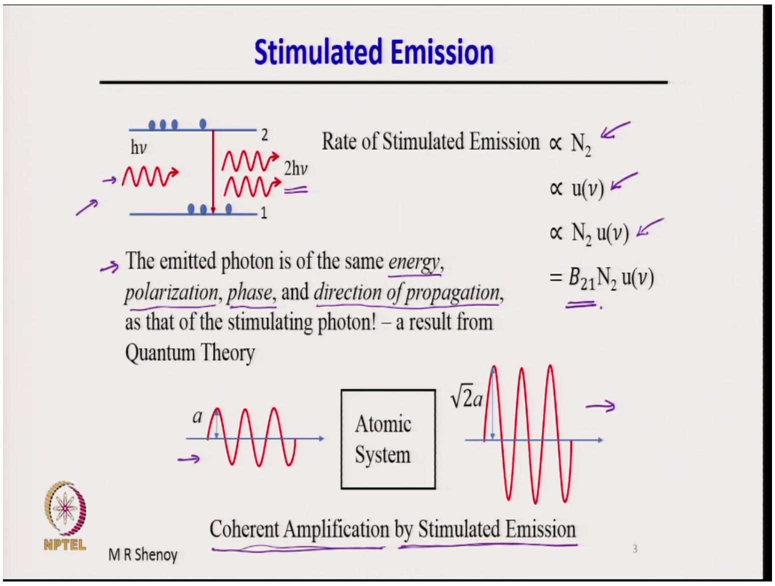

Stimulated Emission

Rate of Stimulated Emission $\propto N_2$

$\propto u(\nu)$

$\propto N_2\, u(\nu)$

$= B_{21} N_2\, u(\nu)$

- The emitted photon is of the same *energy*, *polarization*, *phase*, and *direction of propagation*, as that of the stimulating photon! – a result from Quantum Theory

Coherent Amplification by Stimulated Emission

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So, stimulated emission. Stimulated emission is illustrated in this way here that an incident photon which is called the stimulating photon, stimulates a downward transition of the atom in an excited state or de-excitation of an atom and such that the energy difference which is excited, which is emitted is of the same energy as that of the stimulating photons.

So, here it is the emitted photon is of the same energy polarization so, same energy, same polarization in phase and in the same direction of propagation as that of the stimulating photon a result which is from the quantum theory. So, we assumed that in stimulated emission, the emitted photon is in phase with the stimulating photon. Therefore, it leads to coherent amplification of this input here, coherent amplification.

And therefore, we can also illustrate it like this that in a wave picture, we can illustrate it as you have an incident wave of amplitude a, then by due to coherent amplification by stimulated emission, the output will have a higher amplitude which is root 2 times a because, the energy or intensity is proportional to mod square of the amplitude. Therefore, if you take mod square here, then you will get this as 2 mod a square that is 2 times the energy.

As in a photon picture, we have h nu as input, the output will have 2 photons of energy h nu and therefore, we call this as 2 h nu. It is actually 2 into h nu, 2 times h nu. Whereas, in a wave picture, we can schematically illustrate this as shown in the diagram and this gives an idea that stimulated, by stimulated emission; by stimulated emission, we can have coherent amplification of the signal.

Coherent amplification, when we say amplification, it is coherent amplification without change the input phase of signal the phase relationship of the input phase.

Now, the rate of stimulated emission here is proportional to the number of atoms in the excited state. It is also proportional because it is stimulated by another photon and therefore, it is also proportional to the energy density, spectral density of radiation at that frequency and therefore, it is proportional to the product N 2 into u nu and as before with B 21 has the proportionality constant we write rate of stimulated emission is equal to B 21 into N 2 into u nu.

So, we have seen rate of stimulated emission, rate of spontaneous emission and rate of absorption.

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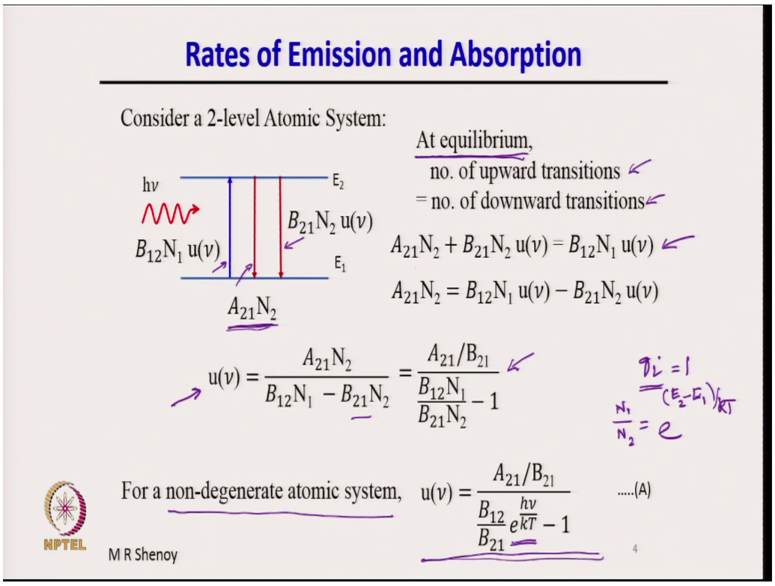

Rates of Emission and Absorption

Consider a 2-level Atomic System:

At equilibrium,
no. of upward transitions
= no. of downward transitions

$$A_{21}N_2 + B_{21}N_2\,u(\nu) = B_{12}N_1\,u(\nu)$$

$$A_{21}N_2 = B_{12}N_1\,u(\nu) - B_{21}N_2\,u(\nu)$$

$$u(\nu) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2} = \frac{A_{21}/B_{21}}{\frac{B_{12}N_1}{B_{21}N_2} - 1}$$

$g_i = 1$

$$\frac{N_1}{N_2} = e^{(E_2-E_1)/kT}$$

For a non-degenerate atomic system, $u(\nu) = \dfrac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1}$ .....(A)

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Now, at equilibrium. So, we are considering the same 2-level system at equilibrium. The number of upward transitions is equal to the number of downward transition because at steady state, you have to have number of absorptions must be equal to number of emissions per unit time.

So, what is shown here are the three components. The blue one here is the absorption which is B 12 times N 1 into u nu. The red one which is here is the spontaneous emission A 21 into N 2 and this component is the stimulated emission.

So, the downward component comprises of spontaneous emission and stimulated emission and therefore, we have the equation equating this the number of absorptions is equal to number of rate of spontaneous emission and plus rate of stimulated emission. So, we can simplify this. So, we take u nu terms together and then, u nu into this product and therefore, u nu can be written as u nu is equal to A 21 N 2 divided by this.

If we divide by this term in the denominator, then we have it given by this or for a remember that this is for a non-degenerate atomic system. We know that otherwise N 1 by N 2 will also contain the g degeneracy factor g i. We have discussed this in the last lecture that g i is a degeneracy factor which means it is it represents the number of levels present at a given energy.

So, we have considered for simplicity non-degenerate atomic system which means g i is equal to 1 for all levels and in that case, we have u nu is equal to this expression here. Because N 1 by N 2 is given by Boltzmann statistics that N 1 by N 2 is equal to e to the power of h nu by kT.

In the last class, we had seen this that N 1 by N 2 so, N 1 by N 2 is equal to e to the power E 2 minus E 1 so, if you remember that it is E 2 minus E 1 by kT and E 2 minus E 1 is here, the energy difference between the two levels which is h nu and therefore, we have substituted for N 1 by N 2 by e to the power of h nu by kT. So, let us designate this as equation A.

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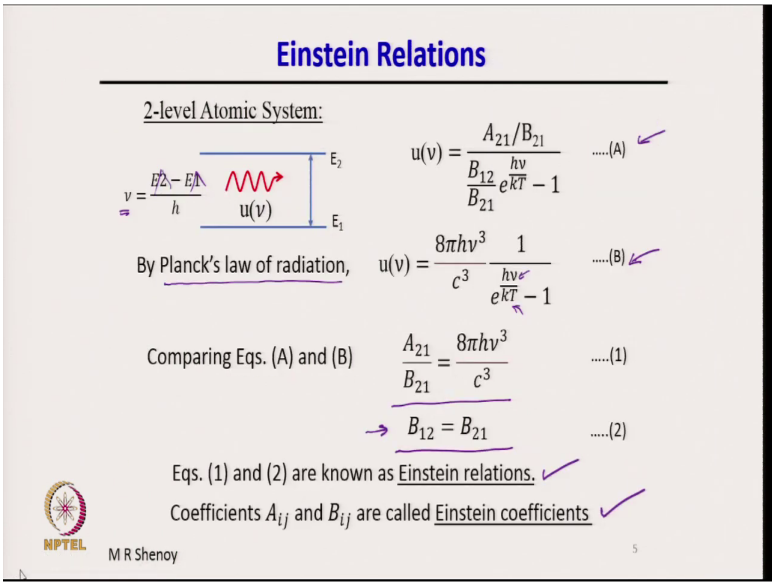

Einstein Relations

2-level Atomic System:

$\nu = \frac{E_2 - E_1}{h}$

$$u(\nu) = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1} \quad \text{.....(A)}$$

By Planck's law of radiation,

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \quad \text{.....(B)}$$

Comparing Eqs. (A) and (B)

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \text{.....(1)}$$

$$B_{12} = B_{21} \quad \text{.....(2)}$$

Eqs. (1) and (2) are known as Einstein relations.

Coefficients A_{ij} and B_{ij} are called Einstein coefficients

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Now, I have written here again. So, we are considering the same 2-level system nu the frequency is equal to E 2 minus E 1 by h or h nu is equal to E 2 minus E 1. So, these are subscripts here E 2 and E 1 are subscripts. We have also seen in the last class that the Planck's law of blackbody radiation says that the spectral energy density of radiation u nu is given by an expression of this type given by B.

Therefore, comparing 1 and 2, we have if the two equations have to remain valid for all frequency's nu and for all temperatures T, then we must have A 21 by B 21 that is the numerator here must be equal to this and B 12 is equal to B 21. These equations 1 and 2 are known as Einstein's relations and the coefficients A ij in this case A 21 and B ij, B 12 and B 21 are called Einstein coefficients.

So, note this logic for the given 2-level system for the frequency nu, this 2-level system is like a black body because this radiation can be absorbed or emitted of this particular frequency and therefore, u nu here is the same as that of the blackbody radiation because this system acts as a black body of course, only for one frequency nu.

And therefore, u nu comparing the two expressions, we have A 21 by B 21 is equal to 8 pi h nu cube by c cube and B 12 is equal to B 21. This is very important; it tells you that the rate constant for stimulated absorption is the same as the rate constant for stimulated emission. These are called the Einstein coefficient.

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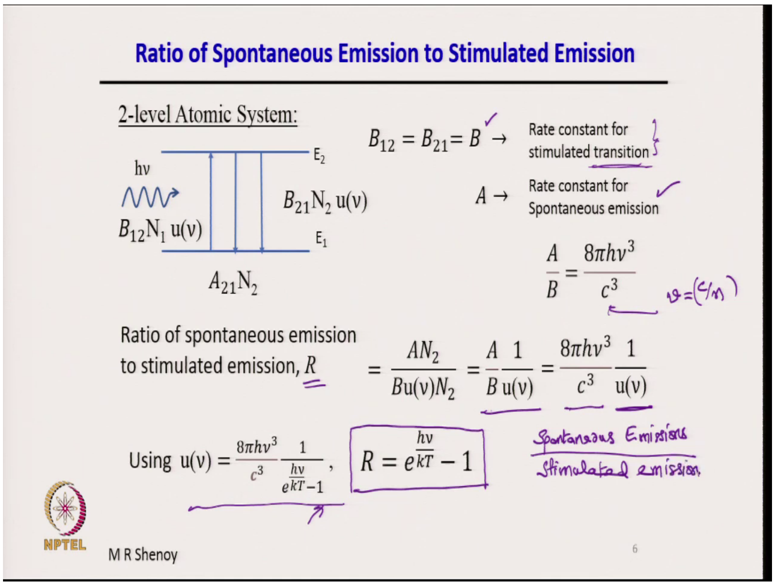

Ratio of Spontaneous Emission to Stimulated Emission

2-level Atomic System:

$B_{12} = B_{21} = B$ ✓ → Rate constant for stimulated transition

A → Rate constant for Spontaneous emission ✓

$$\frac{A}{B} = \frac{8\pi h\nu^3}{c^3} \qquad \nu = (c/n)$$

Ratio of spontaneous emission to stimulated emission, R

$$= \frac{AN_2}{Bu(\nu)N_2} = \frac{A}{B}\frac{1}{u(\nu)} = \frac{8\pi h\nu^3}{c^3}\frac{1}{u(\nu)}$$

Using $u(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{kT}}-1}$, $\boxed{R = e^{\frac{h\nu}{kT}} - 1}$

$$\frac{\text{Spontaneous Emissions}}{\text{Stimulated emission}}$$

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Now, let us see the ratio of let us try to get an idea about what is the rate of spontaneous emission as compared to stimulated emission. So, let us compare the rate of spontaneous emission to stimulated emission.

We will see that at room temperature that is when the system is in thermal equilibrium, the spontaneous emissions far exceed the stimulated emission whereas, by taking the atomic system to a condition of population immersion as briefly mentioned in the last class that we can get stimulated emissions far exceeding spontaneous emissions. So, let us see, put some numbers and see this.

In a 2-level atomic system so, we have B 12 is equal to B 21 is equal to B because there are only 2-levels. So, we need not keep this 12 and 21 and A 21 so, we have only two constants therefore, we call it as A and B. So, B is the rate constant for stimulated transitions. Transitions here means stimulated emission or stimulated absorption and the rate constant for spontaneous emission is here A.

So, A by B is equal to we know this is the Einstein relation 8 pi h nu cube by c cube. I must mention here that if this c is the velocity of light in the medium, then therefore, if the atomic system, if the medium has a refractive index n, then c must be replaced by v which is equal to c by n. So, subsequently we will replace this c by n whenever we consider a medium of certain refractive index otherwise, this is fine.

Now, the ratio of spontaneous emission to stimulated emission R, R is the ratio, then this is we can see in the previous page so, we have here the number of spontaneous transitions are A 21 into N 2 which is A time N 2 and this is B times N 2 into u nu.

So, we are trying to find out the ratio of stimulated emission to spontaneous emission and therefore, here. So, the ratio is equal to AN 2 divided by B into u nu into N 2. So, N 2, N 2 cancels and what we have is A by B into 1 by u nu so, A by B into 1 by u nu.

But u nu so, A by B is given here 8 pi h nu cube by c cube is A by B so, this first term and 1 by u nu, u nu A already have from Planck's law, 8 pi h nu cube by c cube into this one. So, substituting this in the expression here 1 by u nu, we get R, the ratio of spontaneous emission to stimulated emission is equal to e to the power of h nu by kT minus 1.

So, this is the ratio spontaneous emission; spontaneous emissions per unit time per unit volume that is what is meant by rate to stimulated emission. If we put some numbers, then we will see what I had mentioned. So, let us see, let us put some numbers in the next slide.

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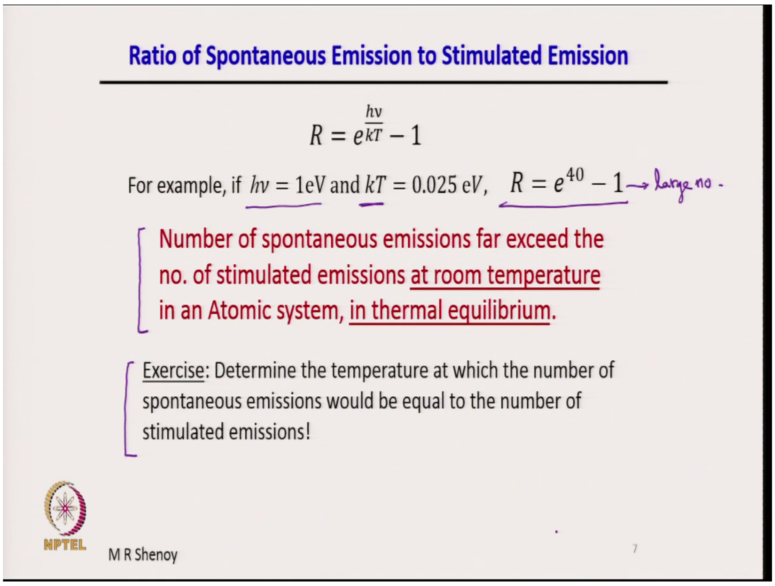

Ratio of Spontaneous Emission to Stimulated Emission

$$R = e^{\frac{h\nu}{kT}} - 1$$

For example, if $\underline{h\nu = 1\text{eV}}$ and $\underline{kT} = 0.025\ eV$, $\underline{R = e^{40} - 1} \rightarrow$ large no.

Number of spontaneous emissions far exceed the no. of stimulated emissions at room temperature in an Atomic system, in thermal equilibrium.

Exercise: Determine the temperature at which the number of spontaneous emissions would be equal to the number of stimulated emissions!

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So, R is equal to e to the power of h nu minus kT. If you take for example, h nu is equal to 1eV so, this is typically 1eV is 1eV electron volt is equal to 1 divided 1.24 divided by 1 that means, it is 1.24 micrometers wavelength in the near infrared, kT is the thermal energy and in the last class, I had mentioned that we can assume this to be 0.025 eV.

So, if we substitute here 1 by this is 40 so, R is equal to e to the power of 40 minus 1 which is a very very big number so, a large number. What does this mean? What is R? Ratio of spontaneous emissions to stimulated emissions and this expression is at thermal equilibrium and therefore, we have the number of spontaneous emission so, that is what is written here in red, number of spontaneous emissions far exceed the number of stimulated emissions at room temperature in an atomic system in thermal equilibrium.

So, we will see subsequently as I mentioned that this is in thermal equilibrium and there are no other external pumping. When you do an external pumping or excitation mechanism, it is possible to reverse this and we can have the number of stimulated emissions for exceeding the number of spontaneous emission which is true in a laser alright.

So, there is an exercise which is given here you may please work out. Exercise: to determine the temperature at which so, at room temperature, this is a situation. Determine the temperature at which the number of spontaneous emissions would be equal to the number of stimulated emissions. Is it possible? So, you see substitute the values and find out the numbers ok.

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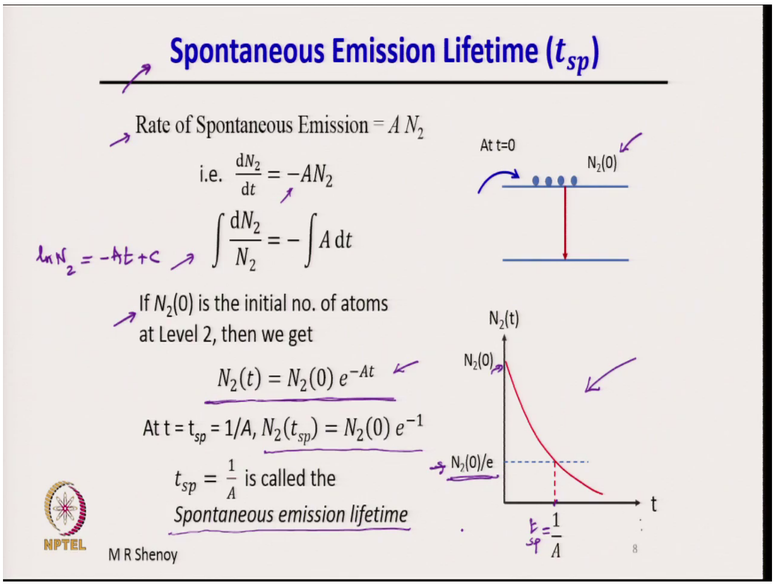

Spontaneous Emission Lifetime (t_{sp})

Rate of Spontaneous Emission = $A N_2$

i.e. $\frac{dN_2}{dt} = -AN_2$

$\ln N_2 = -At + c$ $\int \frac{dN_2}{N_2} = -\int A\,dt$

If $N_2(0)$ is the initial no. of atoms at Level 2, then we get

$$\underline{N_2(t) = N_2(0)\, e^{-At}}$$

At $t = t_{sp} = 1/A$, $\underline{N_2(t_{sp}) = N_2(0)\, e^{-1}}$

$t_{sp} = \frac{1}{A}$ is called the *Spontaneous emission lifetime*

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So, let us go further. We want to introduce an important parameter called spontaneous emission lifetime. Why we are interested in this spontaneous emission lifetime? Because this is a measurable parameter. Right now, we do not know we know A and B are rate constants for emissions and absorptions, but we do not know what are these.

So, to get a numerical value, we should have some this must be related to some measurable parameter and this t sp or spontaneous emission lifetime is an important parameter. So, let us discuss about this alright.

So, recall that the rate of spontaneous emission here is equal to A times N 2, rate of spontaneous emission that is dN 2 by dt number of spontaneous emissions per unit time per unit volume, unit volume is contained in N 2 because N 2 by definition is number of atoms in the excited state per unit volume of material. Therefore, dN 2 by dt is equal to minus AN 2.

The negative sign here shows that N 2 is decreasing because of spontaneous emission that is why we have introduced the negative sign.

Now, if you transpose bring N 2 here and dt to the other side, we have dN 2 by N 2 is equal to minus A times dt. So, we can integrate this. So, this will come out to be l n N 2 so, l n N 2 and on the other side, we have minus A, A is a constant into t plus of course, it is a indefinite integral so, you have to have a constant c. If N 2 of 0 is the number of initial number of atoms at level 2, then we get, we can find this constant c and we get the expression N 2 of t is equal to N 2 of 0 into e to the power minus At.

So, what is shown here is at t is equal to 0 by some mechanism, if we put a burst and N 2 of 0 is the number of atoms which are excited into the upper level at t is equal to 0, imagine that a burst, an energy burst which lifts 1 million atoms or 1 billion atoms to the excited state, then N 2 of 0 it is the number of atoms in the excited state and there are the burst, it is a burst means it is like an impulse in one go N 2 of 0 number of atoms are excited here and afterwards there is nothing.

Now, these atoms will start coming down and therefore, N 2 of 0 will go on reducing the time and therefore, so, what is shown here it is easier to understand from this graph here. So, at t is equal to 0, N 2 of 0 is the number of atoms which are raised or which are placed here in the excited level, then with time, the number of atoms will decay because of spontaneous transitions, spontaneous transitions coming here, coming down and what is shown in the graph is the decay of the number of atoms if which is given by this expression here, N 2 of 0 into e power minus At.

At t is equal to 1 by A, if we consider t is equal to 1 by A so, if we substitute for t 1 by A, A, A cancels and we will get N 2 of so. At t is equal to 1 by A, N 2 of t sp is equal to N 2 of 0 into e power minus 1 which means the number drops down to 1 by e of its value. So, the number of atoms in the excited state is continuously dropping down and at a time t so, t is equal to so, let me designate this as t sp is equal to 1 by A, the number of atoms dropped down to 1 by e of this.

In other words, t sp the spontaneous emission lifetime is defined as the time after which the number of atoms or the time taken for the number of atoms to drop down to a value 1 by e of N 2 of 0. This is defined, the spontaneous emission lifetime is defined as the time taken for the atoms to drop down to 1 by e of its original value at t is equal to 0. We will see using this definition, we can measure the spontaneous emission lifetime and then, if we measure t sp then, we will get the constant A.

If we get the constant A, then we can get the constant B because A by B is equal to 8 pi h nu cube by c cube, we know all the parameters at a given wavelength and therefore, we know A and B that is the importance of this spontaneous emission light. Let us see further.

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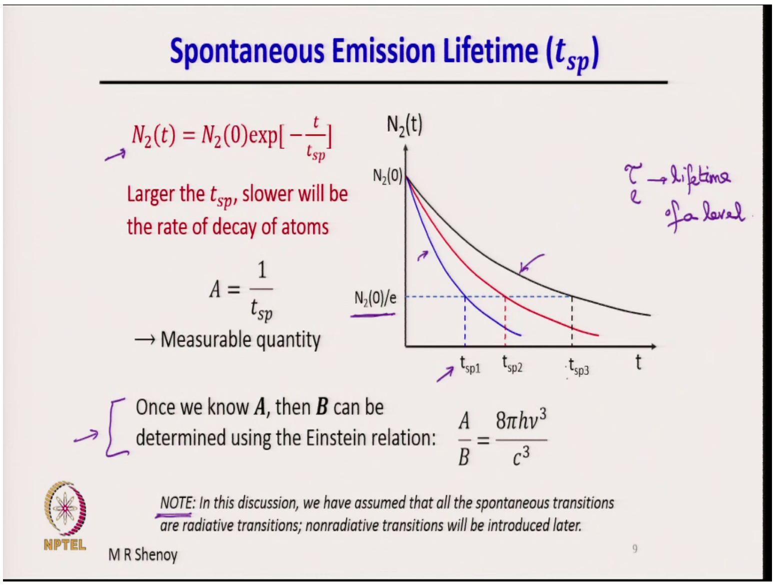

Spontaneous Emission Lifetime (t_{sp})

$N_2(t) = N_2(0)\exp[-\frac{t}{t_{sp}}]$

Larger the t_{sp}, slower will be the rate of decay of atoms

$A = \frac{1}{t_{sp}}$

→ Measurable quantity

$\tau_l \rightarrow$ lifetime of a level

Once we know **A**, then **B** can be determined using the Einstein relation: $\frac{A}{B} = \frac{8\pi h\nu^3}{c^3}$

NOTE: In this discussion, we have assumed that all the spontaneous transitions are radiative transitions; nonradiative transitions will be introduced later.

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So, what is shown here is N 2 of t therefore, is equal to N 2 of 0 into exponential minus t by t sp. It tells you that larger the value of t sp implies, it takes a longer time for the decay or vice versa. If the number of atoms dropped down rapidly as in the blue curve here, if they are dropping down rapidly, it means they will drop down to 1 by e of original value in a shorter time t sp.

So, there are three curves shown here corresponding to three different atomic systems. So, the atomic system in this case, the spontaneous emission lifetime is large, it means that the decay rate is much smaller. So, that spontaneous emission lifetime is related to the rate of spontaneous emissions.

So, larger the t sp implies slower is the spontaneous emission lifetime and I will show in the next slide that in a typical laboratory setup, we can measure this t sp and once we know so, once we know this is the point which I am emphasizing, once we know A, then B can be determined using the Einstein's relation A by B is equal to 8 pi nu by h nu cube; 8 pi h nu cube by c cube.

Note is an important note here that in this discussion, I have assumed that all spontaneous transitions are radiative that is give out a photon, non-radiative transitions we will introduce later and then, we will see that this spontaneous emission lifetime is different from there is another lifetime which we will introduce later called tau l that is the lifetime of a level so, lifetime of a level.

I will introduce this a little later, but right now, we have assumed that every atom making a downward transition leads to emission of a photon and therefore, with that definition, this is a procedure by which we can measure the spontaneous emission lifetime alright.

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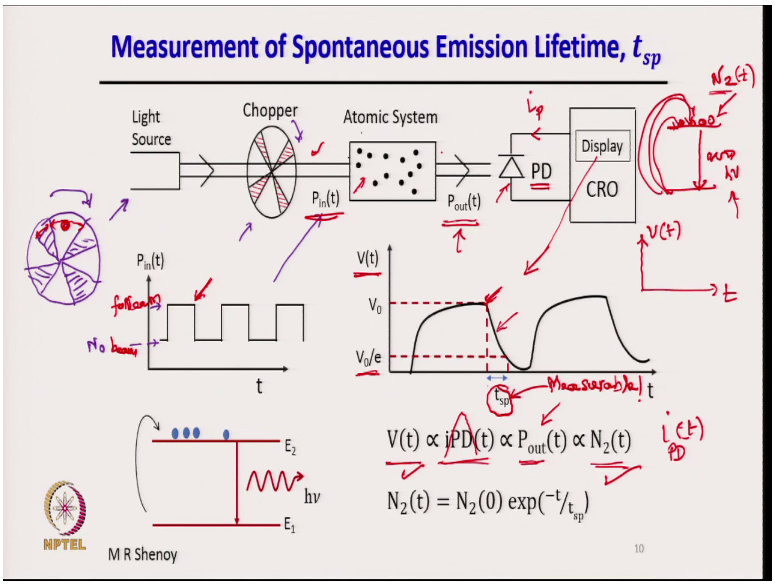

Measurement of Spontaneous Emission Lifetime, t_{sp}

$$V(t) \propto i_{PD}(t) \propto P_{out}(t) \propto N_2(t)$$

$$N_2(t) = N_2(0)\exp(-t/t_{sp})$$

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So, in this diagram, I have shown a simple laboratory setup where you have a light source here. So, this is a light source let us say laser light which is chopped by a mechanical chopper maybe you have seen a chopper, it is a rotating chopper. So, it has blades and no blade region so, here blades and blanks so, it is rotating, when it rotates, it gives a chopped output, the light beam at the output will be chopped so, what is shown here P in is here with the time this is light, no light.

So, this is when there is full light, no light, this level is no light here. No light because the blade has blocked the light beam, blade has blocked the laser beam. So, actually if you see from a perpendicular so, it is a circular blade like this with blades which are opaque so, these are the opaque blades, I am showing just four blades. So, these are the opaque blades here and it is rotating at a high speed let us say 2000 rpm that is revolutions per minute or more.

So, if the laser beam is vertically let us say the laser beam is here maybe I will use a red color so, let us say the laser beam is incident normally at this point, then as the blade moves for some time during this duration corresponding to this time here, the laser beam is on and for the duration corresponding to this part, the blade is blocking the light and therefore, the light output here; the light output here will comprise of full light so, full beam sorry full beam and no beam.

So, what we have is like a pulsed output, light coming out as a pulsed output here at P in it passes through the atomic system.

So, when the pulse is on, when the light pulse is on, it is exciting the atomic system. Let us say this is a 2-level system like this so, when the light pulse is on, atoms are continuously excited to the upper state and when light pulse is off that is in the off period, the atoms which are already here are making downward transitions, there is no upward transition because the pulse is off, the light is off, the light which is off so, there is no excitation, but de-excitation takes place or photons are emitted here.

In this setup, here we have pulsed light entering the atomic system and what we are measuring is at the output is the spontaneous emission which is coming. So, this is the spontaneous emission, which is coming at the output, which is detected by a photo detector so, PD here is a photo detector.

The incident light is incident on a photo detector which results in a reverse carrying photo generated current i p and that across a load resistance here leads to a voltage signal on a CRO connected to a cathode ray oscilloscope display and, in the display, what you will see is this.

So, what is shown here is variation of the voltage versus time. So, see in a cathode ray oscilloscope, you know that the x-axis is time and the y-axis here is voltage signal or current. So, this is what you see on a cathode ray oscilloscope. So, in the display, you will actually be able to see a waveform which is like this. I will explain what is the wave form? Why the wave form is like this?

Initially, when the pulse started arriving, it starts exciting the atomic system here, it starts exciting the atomic system and atoms are going to the upper level and they start emitting. After some time, when the pulse is completely, when the pulse reaches this edge here. Please see when the light pulse reaches this edge up to that there are atoms continuously being pumped up and emissions are taking place, the spontaneous emission photons which are coming here incident on the photo detector generate the photo current.

It is this V of t is proportional to photocurrent here, you can see V of t is proportional to the photo and this is PD is actually the subscript i PD. So, i photodiode, the current of photodiode, i of t PD like this which is proportional to the output power P output which is coming from here; so, this is the P out, this is optical power.

Please see what is entering is optical pulsed input to the atomic system. What is coming out is also optical spontaneously emitted optical signal which is coming here. Now, P out which is incident on the photodiode gives current generated proportional to the; proportional to the input optical power. So, current generated is proportional to the input optical power.

The input optical power, the optical power which is coming out is proportional to the number of atoms N 2 in the upper level because larger the number of atoms more atoms would be coming down and therefore, more photons would be emitted and therefore, more optical power will come that is why the P out which is coming from the atomic system is proportional to N 2.

The current in the photo detector is proportional to the optical power which is incident, and the voltage signal is proportional to the photocurrent generated by the photo detector and therefore, we get a voltage signal which is like this, this edge here corresponds to the edge I have marked here, this edge here is corresponds to the arrow which I have marked here and at that point, the light goes to 0 because the blade is now blocking the laser beam.

Light goes to 0 therefore, there is no excitation, and we have per, but we have the de-excitation continuously taking place. Therefore, there is signal which is incident on the photo detector, but the signal is dying down because the number of atoms N 2 is dropping down with the time t and therefore, the signal is also exponentially dropping down.

This dropping down portion corresponds to the decay of atoms at a rate N 2 of t equal to N 2 of 0 into e to the power minus t by t sp and therefore, if you determine the time taken for this current or voltage signal to drop to 1 by e of its value, then because voltage here is proportional to N 2 of t, this time would give the spontaneous emission lifetime.

Please see V of t is proportional to N 2 of t. N 2 of t is given by N 2 of 0 into e power t by t sp. So, when the time elapsed happens to be t sp, the number of atoms will drop down to N 2 by e and therefore, the voltage will drop down to V 0 by e and this t sp is measurable on a cathode ray oscilloscope CRO.

This is what I meant by measurable, this is the way it is being measured, this is the way that the lifetime of an atomic system or in a solid maybe those of you who are familiar with minority carrier lifetime measurement in a semiconductor, the procedure is the same that you excite with a pulse and then see the de-excitation or decay of the outcoming signal and where the signal drops down to 1 by e of its value, we take it as a characteristic time constant or time which characterizes the decay rate and that is the procedure followed in measuring t sp.

So, I have briefly described this procedure to measure t sp and it is very important because therefore, A becomes a measurable quantity and then, we can determine the Einstein coefficient B. So, we will continue from here in the next lecture.

Thank you.