

Introduction to LASER
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Lecture - 27
Laser Beam Properties

Welcome to this MOOC on LASERS. Today we will discuss about the Properties of the Laser Beam so Laser Beam Properties.

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Recap: Characteristics of the Laser Output

→ **1. Monochromaticity** → linewidth: $\delta\nu$ (or $\delta\lambda$) of the output spectrum

c. g. $\delta\nu \sim 10^{10}$ Hz for normal laboratory He-Ne lasers
or $\delta\lambda \sim 10^{-1} \text{ \AA} = 10^{-2} \text{ nm}$


For single frequency lasers, $\delta\nu \sim 10^6$ Hz, gives $\delta\lambda \sim 10^{-5} \text{ \AA} = 10^{-6} \text{ nm!}$

→ **2. Coherence** : Coherence time, $t_c \sim \frac{1}{\delta\nu}$; Coherence length $L_c = ct_c$

$\delta\nu \sim 10^{10}$ Hz for MM lasers → $t_c \sim 10^{-10}$ s
 $\sim 10^6$ Hz for SM lasers → $t_c \sim 10^{-6}$ s

⇒ $L_c \sim 3 \text{ cm}$ for MM Laser, and $\sim 300 \text{ m}$ for SM Laser

• t_c → Average time duration over which there is a constant phase relationship among the component waves



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A very quick recap of what we discussed in the last class that is Characteristics of the Laser Output. So, the first one that we had discussed was Monochromaticity which is characterized in terms of the line width $\delta\nu$ or $\delta\lambda$ of the output spectrum. And we have seen that for a normal laboratory He-Ne laser $\delta\nu$ is of the order of 10^{10} hertz

and that would give a corresponding $\Delta\lambda$ of the order of 10^{-1} angstrom or 10^{-2} nanometers.

For single frequency laser $\Delta\nu$ is of the order of 10^6 hertz can be smaller as well gives $\Delta\lambda$ of the order of 10^{-5} angstrom or 10^{-6} nanometers. The second thing which we had discussed was Coherence, coherence is characterized by the coherence time t_c or τ_c which is $1/\Delta\nu$, where $\Delta\nu$ is the line width and the coherence length is given by L_c is equal to $c \times t_c$, where c is the speed of light.

For $\Delta\nu$ is equal to 10^{10} hertz which is typical of multi mode lasers, the t_c is of the order of 10^{-10} seconds. Whereas, for $\Delta\nu$ of the order of 10^6 hertz which is for single mode lasers single frequency lasers, t_c is of the order of 10^{-6} seconds.

The corresponding L_c the coherence length is 3 centimeter for multi mode lasers and about 300 meter for the single mode laser that simply $c \times t_c$. Recall that t_c is the average time duration over which there is a constant phase relationship among the component waves and we say that the source is coherent.

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Laser Beam Properties

1. Beam Divergence Angle:

Diverging Gaussian Beam (from the Waist)

$$w(z) = w_0 \left[1 + \left(\frac{z}{z_0} \right)^2 \right]^{\frac{1}{2}}; \quad z_0 = \frac{\pi w_0^2}{\lambda}$$

For large z , $w(z) = w_0 \frac{z}{z_0}$; $w(z) = \frac{\lambda}{\pi w_0} z$

$$\Rightarrow w(z_2) - w(z_1) = \frac{\lambda}{\pi w_0} (z_2 - z_1)$$

$$\tan \theta = \frac{\lambda}{\pi w_0}, \quad \text{or } \theta \approx \frac{\lambda}{\pi w_0}$$

Typical: $\theta \approx \frac{1.5 \mu\text{m}}{3 \times 0.5 \text{mm}} = 1 \text{ mrad}$

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Now, let us look today at the Laser Beam Properties. The first one is the Beam Divergence Angle we briefly discussed this in the last lecture as well. So, what is shown here is a diverging Gaussian Beam, so a beam diverging from the waist so this is the waist where the diameter is 2 times w_0 , w_0 is called the spot size.

And as it propagates so there are some z value is shown z_0 this is the Rayleigh range beyond which the laser diverges into the far field and let z_1 and z_2 be 2 positions along the axis. Where the cross section diameter can be measured or the spot size for example what we shown here from here to here is w_1 of z at z equal to z_1 and this is w_2 of z at z is equal to z_2 .

Therefore the w of z is given by this formula; however, for large z because z is in the numerator z_0 is the Rayleigh range given by πw_0^2 by λ we have already

seen this. For large z the 1 is very small when z is much larger than z_0 . This number is very big and we can write w of z is equal to w_0 into this term and that is z divided by πw_0^2 square divided by λ , λ goes to the numerator.

And we have w of z is equal to λ by πw_0^2 into z , that is the spot size varies directly proportional to z . So, that is what is plotted here for large values of z w of z is directly proportional to z . So, as z so this is the z axis the propagation of the beam is in the z axis, as z increases then the spot size increases that is this expression which is given spot size related to z .

Now, in this regime we can determine what is the angle θ that is the divergence angle which is shown here by the dotted line, the divergence angle θ here which is also shown here can be determined by using the formula w_2 minus w_1 is this λ of πw_0^2 into z_2 minus z_1 and w_2 minus w_1 divided by z_2 minus z_1 is $\tan \theta$. So, $\tan \theta$ is equal to λ by πw_0^2 , $\tan \theta$ is w_2 minus w_1 divided by z_2 minus z_1 .

For small θ because as we see numerator is λ which is of the order of micron 1 micron and w_0 here is 100's of micron generally this is of the order of 100's of micron 1 millimeter and therefore $\tan \theta$ is much less than 1; is very small and therefore θ $\tan \theta$ nearly equal to θ and θ can be approximated as λ by πW_0 . So, this is the divergence angle θ .

So, the divergence angle θ of a Gaussian is given by this formula λ divided by πw_0 , where w_0 is the spot size at the waist typically. You can substitute some numbers λ at 1.5 micrometer this is π approximated by 3 and w_0 let us say 0.5 mm then that gives you 1 milli radian. This is a typical number for commercial, so typical for commercial lasers.

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Laser Beam Properties (contd.)

2. Beam Diameter: $2w_0$ at waist; $w_0 = \sqrt{\frac{z_0 \lambda}{\pi}}$

- For a confocal resonator, $z_0 = L/2$
- For He-Ne laser, $L \approx 40$ cm, $w_0 = \sqrt{\frac{0.6328 \mu\text{m} \times 20 \text{ cm}}{2\pi}} = 0.14$ mm

At the output $w(z) = \sqrt{2}w_0 \approx 0.2$ mm

3. Diameter of the focused spot: $d \approx f \frac{\lambda}{D}$ ($f \cdot \theta \sim f \cdot 1.22 \frac{\lambda}{D}$)

$\theta \rightarrow$ Divergence angle; θ is the angular radius of the first Airy disc
 $f \rightarrow$ Focal length of the lens; $D \rightarrow$ Aperture diameter of the lens

$\theta \approx \sin^{-1}\left(\frac{\lambda}{D}\right) \approx \frac{\lambda}{D}$ (for first minimum of single-slit diffraction)

$d \approx \frac{\lambda}{2(N.A.)} \Rightarrow d \sim \lambda \sim 1 \mu\text{m} \Rightarrow \text{Area} \sim 10^{-12} \text{m}^2$
 $\approx \frac{\lambda}{2 \times 0.5} = 0.6$ of the focused spot

$\theta \rightarrow$ Angular Div. $= 1.22 \frac{\lambda}{D}$

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The next parameter of the beam is the Beam Diameter, let us discuss about the beam diameter which is $2w_0$ at the waist, w_0 is given by the formula here z_0 into λ by π . For a confocal resonator we have already seen that this z_0 is $L/2$, for a helium neon laser with L is equal to 40 centimeter.

Let us say we are in the confocal resonator configuration, then w_0 just to get an idea about the numbers w_0 is equal 0.6328 micrometer which is the wavelength of helium neon laser the orange red colored and this is z_0 20 centimeter because L 40 centimeter z_0 is $L/2$ divided by 2π this is already 20 centimeter there.

So, this 2 does not come, let me make this 20 centimeter and then w_0 because typically helium neon lasers laboratory helium neon lasers have a tube length of 20 to 30 centimeters. So, if we because it is $L/2$ and into π in the denominator there is only π , so which is 0.14

millimeter or 140 micrometer. So, that is the typical numbers that we have for the spot size at the waist of a helium neon laser.

The diameter of the focused spot let us see so if there is a laser beam which is incident and it is focused by a lens. So, let us consider a lens of diameter so the diameter of this aperture is D , let us say D is the diameter of the aperture and there is a laser beam which is incident for simplicity I am filling the aperture. So, the laser beam which is incident and this gets focused to a spot.

So, it really is a Gaussian beam and therefore, it comes down to a spot a very small spot here and this is the diameter D of the aperture and if this is the focal length f here. Then the diameter of the focus spot d here the diameter d , diameter of the focus spot d is given by f into λ by D .

So, this λ by D is nothing but the divergence angle. So, at the focal point it forms an airy disc if you have plane waves which are incident, then at the focal point you have an airy disc which is formed that is bright and dark rings.

And the diameter of the first ring here which is the main spot where most of the energy is focused is given by θ the angular diameter. So, θ is the angular diameter, is equal to 1.22λ by D . So, θ is of the order of λ by D and that is why we have substituted here d the diameter of the spot please see this the spot is formed here.

So, there is a spot which is formed here let me show it as a big spot and the angle θ is this, θ corresponding to the first ring here outside. So, this is the θ and this is the focal length from the lens, so this is the focal length f and the small angle which is formed here is θ , so this is θ and therefore the f into θ gives us the diameter of the spot because θ the divergence angle is 1.22λ by D .

And therefore d is of the order of f into λ by D , θ is the divergence angle, θ is actually the angular radius of the first Airy disc for those of you are familiar that when plane

waves diffract they form airy pattern at the focus of a lens. So, this is the airy pattern so airy disc.

So, airy disc is the innermost bright disc is called the airy disc, f is the focal length of the lens and D is the aperture diameter of this lens. So that means the diameter of the lens provided the entire beam fills the lens. So, that is the aperture diameter of the lens. We also know in single slit diffraction those of you are familiar with single slit diffraction you get a pattern like this for a single slit diffraction pattern and this minima here corresponds to an angle of λ by D .

Where d is the so if you have a single slit of diameter d . So, this is a single slit those of you are not familiar this is of diameter d , then if plane waves are incident here on the single slit then it forms on a screen here it forms single slit diffraction pattern with maxima and minima here and the first minima occur at an angle θ corresponding to λ by D . So, that is what is shown as λ by D .

So, this λ by d is this is considering airy disc actually it is 1.22λ by D , but which is of the order of that is why this of the order size is given λ by D . Here also you see that θ is equal to \sin^{-1} of λ by D , where D is the diameter of the aperture here. So, this is capital D diameter of the aperture then this will be capital D because they have used capital D here we have used and therefore since λ is much smaller compared to D this is approximately λ by D .

So, either way an order of magnitude estimate can be made for the diameter of the focus spot as f focal length of the lens into λ divided by diameter of the aperture of the lens. So, we go on repeating that its diameter of the aperture, because the lens is like this usually the lens has an aperture a block or a stop at the back.

So, when light beams is light beam is incident here it is up to the aperture only light up to the aperture. So, this is the aperture which is present which is a stop. So this only is incident on

the lens and then gets focused and therefore the effective diameter of the lens is the diameter of the aperture.

So, this is for those of you are not familiar, so this is the D that we are talking of diameter of the aperture. So, aperture diameter D here refers to this which is always it is smaller than the actual diameter of the lens alright.

So, the spot size is equal to λ by D and it can also be written in the form of numerical aperture of the lens and therefore we get D is which is of the order of λ , because numerical aperture is typically anywhere from so NA for a lens is typically 0.4 to 0.6. Let us say for a twenty x objective or a 20 x lens it is typically 0.4 to 0.6 that is why we have written this as λ by 1 or d is of the order of wavelength which is of the order of 1 micrometer.

And therefore the area of the spot area of the focused spot area of the focused spot just to give some idea about the numbers focused spot is of the order of 10^{-12} meter square. This is an of the order calculation just to give an idea that what is the size of the spot at the focus of a lens, if you focus a laser beam alright.

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Laser Beam Properties (contd.)

4. Intensity: $Intensity = \frac{Power}{Area}$

- Power is independent of z (considering total cross section of the beam)
- Intensity $I = I(r, z)$, since spot size changes with z .
- $Power = \int_{cross\ section} I(r, z) dA$

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Let us go further the next important property is the Intensity, intensity at the focused spot or intensity in the beam itself. So, let us say a laser beam is let me use a different color. So, there is a laser beam which is coming like this of certain power let us say this is 10 milli Watt laser beam. Then if A is the area here then the intensity of the beam is I is equal to power P out P divided by area, this area A is area of cross section. So, this cross sectional area so cross sectional area.

The power is independent of z power in the beam whether you measure the power here whether you measure the power here or wherever you measure the power. So, long as the total beam is incident on the detector, let us say we take a photo detector we take a photo detector of this diameter. So, this diameter let us say 1 centimeter typical silicon photo detectors large area detectors have a diameter of about 1 centimeter.

If the laser spot if the laser beam is incident here or if it is incident up to this or if it is incident up to this, why am I showing bigger and bigger? Because as we can see when the beam propagates. So, this is the z direction as the beam propagates the diameter becomes larger and larger.

So, long as the entire beam is inside the detector, so this is the photo detector. So, long as the entire beam is inside the active area of the photo detector which is typically about 1 centimeter in diameter it will measure the total power. And therefore the power is independent of z; which means if you measure the power here or here or here it will show the same power.

However the intensity is z dependent the intensity at a point or in a small area of cross section is what we are looking for. So, if you are measuring the intensity here in this plane then this is the z and r at a certain position r from the axis. So, that is why it is intensity is a function of r of z. So, if I show the beam cross section again let me show the beam cross section here, then we know that the intensity varies if I were to plot graphically the intensity actually varies like a Gaussian.

The power distribution is Gaussian and therefore at the center here we have maximum intensity. So, what is plotted is I of r actually it is along a diameter. So, I can write I of x or I of y I have shown this graph let me again use blue. So, this variation here is the intensity profile I of x is what I have plotted here and this is x is equal to 0 or r is equal to 0 or if you were to plot as a function of r then we should plot like this.

So, this is r from 0, so the Gaussian is dropping down like this so this is r equal to 0. The point is this is I of r at a particular z. It is a function of z because the spot size increases the spot size increases as z increases, that is why the intensity is a function of r and z, it is a function of r because the intensity here is different from the intensity here and the intensity is maximum at r is equal to 0 that is this point the r is equal to 0.

The intensity is maximum on the axis; we can see the same thing which is shown here what is already shown is the Gaussian profile. The intensity is very high here, because the beam diameter is small. As the beam propagates to a larger value of z the beam spreads the diameter of the beam is larger here, the power is the same the energy content is the same and therefore the profile changes like this it spreads.

The height has reduced the height has reduced with z , here the height is maximum height of the intensity profile is maximum at the waist actually. So, we will make a calculation and see what kind of numbers that we are talking of. So, power total power is intensity integrated over the entire cross section. So, $\int I \, dA$ gives us the total power P and intensity is a function of r and z , with this picture let us make a calculation.

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Laser Beam Properties (contd.)

For the TEM₀₀ mode, $I_{00}(r, z) = I_0 \left(\frac{w_0}{w(z)}\right)^2 e^{-\frac{2r^2}{w(z)^2}}$; $dA = r \, d\theta \, dr$

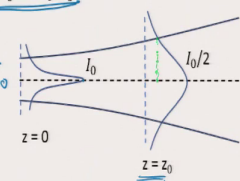
Total Power $P_0 = \int_0^\infty \int_0^{2\pi} I_0 \left(\frac{w_0}{w(z)}\right)^2 e^{-\frac{2r^2}{w(z)^2}} r \, d\theta \, dr$


Integrate and show: $I_0 = \frac{2P_0}{\pi w_0^2}$ (Intensity at the waist)

$\therefore I_{00}(0, z) = \frac{2P_0}{\pi w_0^2} \left(\frac{w_0}{w(z)}\right)^2 = \frac{I_0}{\left[1 + \left(\frac{z}{z_0}\right)^2\right]}$ (Intensity on the axis)

Thus, at $z = z_0$, $I_{00}(0, z_0) = \frac{I_0}{2}$

$\int_0^\infty \rightarrow$ Amplitude at $r=0, \theta=0$





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For the TEM 00 mode which is the Gaussian mode, we know the intensity distribution is given by this I_{00} this is for the 00 mode number here of r of z is equal to I_0 into w_0 by w of z , w_0 is the spot size at the waist and w of z is the spot size at any z into e to the power of minus $2r^2$ by w^2 of z .

And dA the area of cross section is $r dr d\theta$. If you have a circle then a circular cross section then the small area element if you consider an area element at r then the area element here is $r dr d\theta$ gives you the area element. So, this is dA so this is this written here dA is $r dr d\theta$.

So, intensity into dA integrated over the entire area here entire cross section will give us so entire cross section means what? r goes from 0 to infinity and the angle $d\theta$ goes from 0 to 2π . So, $d\theta$ is 0 to 2π you integrate so this with respect to some reference angle if this is θ and r , then r goes from 0 to infinity please note that the field or the intensity distribution dies down to 0 at infinity, it asymptotically comes down to 0 and that is why we have to integrate from 0 to infinity.

Infinity means of course sufficiently large distance up to a large distance from the axis where r is equal to 0, so 0 to 2π I_0 . So, please integrate this simple integral here and show that you will get an expression for P_0 ; P_0 is equal to I_0 by $2\pi w_0^2$ or I_0 . What is I_0 ? I_0 is the intensity the maximum intensity on the axis at z is equal to 0. So, I_0 is the amplitude here the maximum intensity the amplitude at amplitude of the intensity distribution at r is equal to 0 and z is equal to 0 that is the intensity.

For any z if you take an arbitrary z the intensity distribution is given by $2P_0$ by πw_0^2 this is I_0 we have substituted I_0 multiplied by w_0 by w of z the whole square. And this is equal to I_0 divided by because w of z is w_0 into $1 + z$ by z naught the whole square and therefore we get I_{00} this is the mode number this is r is equal to 0 and this is as a function of z is given by an expression like this.

Note that if we put z is equal to z_0 then I_0 of z_0 is I_0 by 2 again I_0 is the amplitude or maximum intensity on the axis of the beam at the waist. At a certain distance z is equal to z_0 the intensity on the axis drops down to half of its value. So, z_0 we know is called the Rayleigh range. So, at the Rayleigh range the intensity peak drops down to half of its value, for any other z it is given by the expression which is here alright.

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Laser Beam Properties (contd.)

Example: Find the power contained within a ring of radius r_0

$$P_{r_0} = 2\pi I_0 \left(\frac{w_0}{w(z)}\right)^2 \int_0^{r_0} e^{-\frac{2r^2}{w(z)^2}} r dr$$

$$= \frac{\pi}{2} I_0 w_0^2 \left[1 - e^{-\frac{2r_0^2}{w(z)^2}}\right]$$

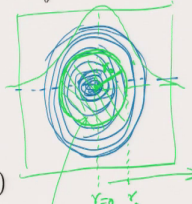
If $r_0 = w(z)$ (i. e., within the spotsize of the beam)

$$\eta = \frac{P_{r_0}}{P_0} = [1 - e^{-2}] = 0.855 \Rightarrow 85.5\%$$

- If $r_0 = 1.5w(z)$, $\eta = 0.99!$

Exercise: Calculate typical intensity at the focussed spot of a practical laser and the corresponding Electric field. E

Handwritten notes:
 $P = 10 \text{ mW}$
 $I = ?$
 $I_0 \propto |E|^2$



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So, let us take a simple example find the power contained within a ring of radius r_0 , that is if you have the Gaussian beam. Let me show now in terms of the this is the cross section in terms of the contour diagram. Because the contours will spread and become farther and farther as you go away like this and it will dense at the center.

So, if you plot the corresponding line diagram then it will show the line diagram let me use a different color, the line diagram it will show like this it will be maximum across the diameter

if you measure the intensity and plot, so this the peak will occur here. In terms of the contour diagram this is in the cross section the beam cross section.

Now, the question is find the power contained within a ring of radius r_0 , that is at the center r is equal to 0 up to a certain circular ring let us say consider a circular ring of radius r_0 , so this is the radius r_0 . The question is what fraction of power is contained in this disk or in this ring.

Within this ring what is the fractional power which is present? That is designated as P_r . So P_r is the fractional power, the total power of course is P_0 which we have calculated already. So, here after integrating this is the total power in the beam in a cross section is P_0 . So, we integrated over the entire cross section.

Now, we have to integrate of course 0 to 2π will be there because it is a disc or a circle, but this will go from 0 to r_0 that is the only difference. So, that is what we have shown here. So, $2\pi I_0$ into w_0 by w_z the whole square the other term outside this this 2π has come from integrating 0 to 2π $d\theta$ and this is 0 to r_0 instead of infinity.

So, the total power means I am repeating this actually from here this is r is equal to 0 to infinity, total power integrating over the entire cross section. But now we want from r is equal to 0 up to some radius r_0 . So, what is the power content in this? So, if we integrate we see that we get an expression like this it is a simple integral if r_0 were equal to w of z , w of z is the spot size where the amplitude drops to 1 by e of the amplitude at the center or where the intensity drops to 1 by e^2 where the if this is the intensity plot.

So, here the intensity has dropped down to 1 by e^2 . So this is w of z . So, if r_0 were equal to the spot size at that value of z within the spot size of the beam the fractional power contained is 0.855 which implies 85.5 percent of the power is contained within the spot size within an area represented by the spot size within an area of radius equal to the spot size.

And more importantly if r_0 so this is if r_0 were w of z if r_0 happens to be 1.5 times so this is if let us say this was w of z then 1.5 times, if we go a little bit further 50 percent more than eta

the fractional power is 99 percent is very interesting and that is why the spot size is used as a parameter to represent the dimension the spot the dimension of the spot of a Gaussian beam.

Because 85.5 percent of the total power is contained within the spot size and if 1.5 times the spot size then the power content is 99 percent within that disc or within that ring circular ring. There is an exercise which is given here, calculate typical intensity at the focused spot of a practical laser and the corresponding electric field.

So, first calculate what is the intensity on the axis of a focused spot start with a practical laser, let us say P is equal to 10 milli Watt simple laser, then you can increase this to 100 milli Watt or 100 Watt and then see what is the intensity that changes corresponding to this calculate the intensity.


So, I_0 is on the axis the maximum intensity possible at the waist on the axis is I_0 . So, calculate what is I_0 ? I_0 is related to electric field we know that the intensity is I_0 is proportional to $\text{mod } E^2$, there is a factor which is outside here but it is proportional to E^2 . So, the E is the electric field. So, this is the electric field electric field E .

So, the question is calculate first I_0 and determine what is the field E corresponding to 10 milli Watt for example, then you make it 1 Watt or 100 Watt or 100 milli Watt and every time see what is the corresponding electric field. And how this electric field compares with the electric practical electric fields when you apply an electric field to a material or the electric field that exists between the nucleus and the electrons in an atom ok.

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Recap: Selection of Single Longitudinal Mode


- - Multi-frequency oscillation leads to large source-linewidth
- - many applications require single frequency oscillation
- - use of intra-cavity etalon for frequency selection



Transmission peaks of etalon:

$$\nu_p = p \frac{c}{2nt \cos \theta}; p = 1, 2, 3, \dots$$

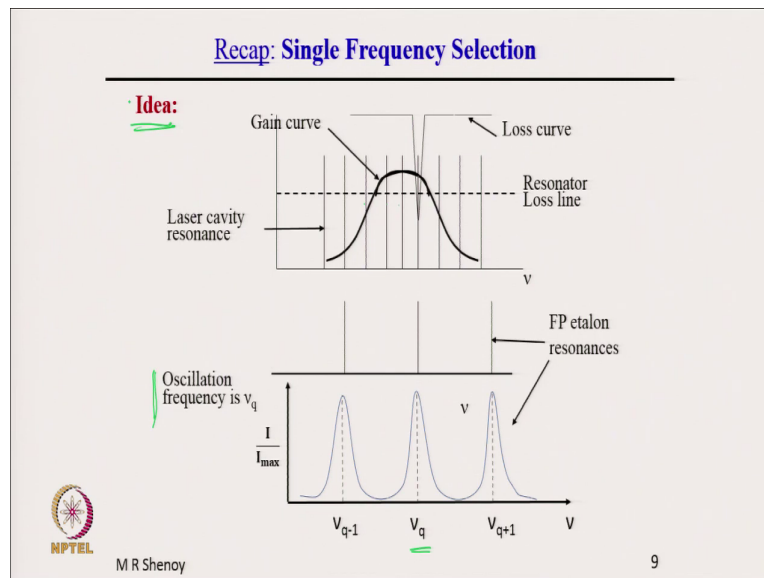
t – etalon spacing; n – refractive index ;
 θ – tilt angle w. r. t. vertical

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Let us now go to another property of the laser beam. So, recall that in the last lecture we had seen how to select a single longitudinal mode. Why did we want a single longitudinal mode? Because multi frequency oscillation leads to large source line width and many applications require single frequency oscillation.

The idea which is used is used of intra cavity etalon for frequency selection. This we have discussed in detail in the last lecture and using a tilted intra cavity etalon we can select a single longitudinal mode that is a single resonance frequency.

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The idea is illustrated here, so the idea used to select a single frequency is this somehow if we make the loss high for all other resonance frequencies except one then we will be able to choose only one this is what we had discussed. The laser will oscillate in just 1 frequency ν_q for which the loss the gain here.

So, this is the loss so the loss is less than the gain or gain is more than the loss. For all other resonance frequencies here the loss is on top it is much more than the gain and therefore we can select one single frequency this is the idea which is used. A similar idea can be used to select transverse modes.

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Selection of a Single Transverse Mode

- Multimode oscillations reduce the spatial coherence
- The intensity distribution across the beam varies randomly
- Many applications require single transverse mode oscillation

Idea: Use of intra-cavity apertures to select a particular mode

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So Selection of a Single Transverse Mode, now why do you want a single transverse mode? Multi mode oscillations now we are referring to multi mode transverse modes; transverse modes as we have already seen refer to field distributions, longitudinal modes refer to resonance frequencies. So, these are field distributions field distributions.

So, multi mode oscillations that is several field distributions oscillating in the laser reduces the spatial coherence, there are 2 types of coherence temporal coherence and spatial coherence. Temporal coherence refers to coherency in time and spatial coherence in simple terms, spatial coherence referred to the spatial extent over which you have coherence.

That is if I have a screen which is like this where the laser beam is incident and there is let us say some interference is taking place, then over what diameter or what is the range over

which we have coherence. Coherence whenever there is coherence then only we will see their interference fringes. So, beyond that we will not be able to see interference fringes.

So, spatial coherence in simple terms refers to the area spatial position up to which we have coherence alright. So, the intensity distribution across the beam varies randomly, this is because if you have several different distributions for example you have the Gaussian you have the next field distribution you may have let me use another color. So, you may have a distribution the third distribution which may be like this so field variations.

So, due to the superposition of these different field variations the intensity distribution across the beam, because all these field distributions are coming into the same beam the same beam comprises of let us say the fundamental mode here, the next mode which is here. If all the beams all the modes so this is these are I am showing in 1 dimension here.

Then if we look on a screen here the super position of these will vary randomly. So, the intensity distribution across the beam varies randomly and in many applications we do not want this and we need require a single transverse mode of oscillation. Just like a single frequency was required there are applications where you would like to have a single transverse mode. So, that the intensity distribution does not vary with the time or from position to position alright.

So, how to use how to select a single transverse mode? So, the idea in this case so deliberately I had written here the idea used was to create low loss for only one frequency which you need, the same idea is used here so use intra cavity apertures to select a particular mode we will discuss this in detail in the next slide.

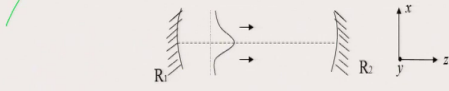
But let us first see what do I mean by this here is the laser. So, this is the laser the mirrors m_1 and m_2 and the gain medium and we have a pump and the laser beam is going back and forth. So, let us say the beam is going back and forth, this is the laser beam. Now, if we use intra cavity, intra cavity means inside the cavity inside the resonator apertures of appropriate

shape and size it is possible to select a single transverse mode, a required transverse mode can be selected by using appropriate intra cavity apertures.

So, intra cavity aperture this is what is shown here. Now, let us see what type of aperture we should use to select a certain mode.

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Transverse Modes of a Spherical Mirror Resonator




➤ The complete set of Hermite-Gaussian function given by

$$U_{l,m}(x, y, z) = A_{l,m} \left[\frac{w_0}{w(z)} \right] H_l \left[\frac{\sqrt{2}x}{w(z)} \right] H_m \left[\frac{\sqrt{2}y}{w(z)} \right] \times e^{-\frac{(x^2+y^2)}{w^2(z)}} e^{-i\phi}$$

H_l, H_m - Hermite Polynomials; $w(z)$ - Gaussian spot-size parameter

➤ The fundamental mode (TEM_{00}) has a Gaussian Intensity Distribution

$$\left| U_{0,0}(x, y, z) \right|^2 = I_{0,0} \left[\frac{w_0}{w(z)} \right]^2 e^{-\frac{2(x^2+y^2)}{w^2(z)}}$$


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Let us very quickly recall this transverse modes of a spherical mirror resonator are given by this Hermite Gauss functions and the fundamental mode has a Gaussian Distribution we know this.

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Choice of Intra-cavity Apertures ...

Recap:

Intensity Pattern

TEM₀₀ mode
High Loss to the modes

Intensity Pattern

I_(1,0)(x,y)
Very low loss

Intensity Pattern

I₀₁
Very low loss

- choice of **intra-cavity apertures** to select a particular mode

- use of metallic wires/grids, Teflon aperture for TEM₀₀ mode, etc.

Grid

Antenna Mode

I_n mode
Very low loss

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Let us go further we have already discussed that the intensity pattern of the TEM 00 mode looks like this in a cross section, that is if you plot a contoured graph then it will be maximum at the center and slowly slowly the intensity drops down; because it is a bell shaped curve.

Similarly, the intensity pattern of the next mode that is I 1, 0 is given by this there is 1, 0 at the center as you go along the x axis and if we very quickly recall that if it was I 0 1 we would have had a variation like this. So, we would have had maximum here at the center and 0 so something like this, so this is the y axis.

So, when you go along the y axis here we encounter a 0 here, here when we go along the x axis there is a 0 here that is why this is I 1, 0 and this is I 0, 1 intensity distribution intensity

pattern in the cross section. Now, just let us look at these 2 this has maximum in the center this has 0 in the center.

Therefore, if this cross section here represented the cross section of the laser then the same cross section I am drawing x and y, if I use a wire use of metallic wires or grids. So, let me change the color here suppose I insert an metallic wire. Why metallic wire because metal is conducting and the electric field associated with the mode will see this metal boundary and the boundary condition is that the tangential component of the electric field must go to 0 on the metal.

And therefore the boundary condition requires that the field should go to 0 here and therefore there is a huge loss which is taking place for this mode. Because the mode has maximum as we plot the line diagram, the mode has maximum and the center and we have placed a metallic wire. So, this is a metallic wire so metallic wire. You could also place some other obstacle, but metallic wire is very effective because the electric field has to go to 0.

So, right now the field is very high maximum at the center, but the metal requires that the field has to go down to 0 there which means this mode will suffer huge loss. So, this leads to huge loss, loss for the TEM 00 mode TEM 00 mode. Now, let us take the same metallic wire and place here in this cross section.

So, the same metallic wire we have placed here now, if we place here then this mode suffers huge loss. If we now place here for the $TE_{1,0}$ mode this makes hardly any difference because the field varies like this it goes to maximum it comes down to 0 here and I am plotting the field now not the intensity. So, the field varies like this.

In other words the field is already 0 field associated the electric field is 0 here that is why there is an intensity 0 and therefore by placing a metallic wire here it hardly affects the field distribution of the $TE_{1,0}$ mode. Please recall those of you who are familiar let us consider the modes of oscillation of a string which form standing waves alright. So, let us say it is forming standing waves here like this they have to be identical 3 lobes.

Where the displacement this is displacement from the [noise so this is the standing wave means the string was like this and if you have forced some oscillations, then the string would oscillate like this. There are famous experiments to get such standing views on a string, the point is these are nodes and these are anti nodes.

So, this is anti node and this is node at the node the displacement of the wire is 0. So, this is the displacement basically what is shown is this is the displacement maximum displacement this is an anti node this is an anti node and this is a node. At the node the displacement of the string is 0 and this is steadily oscillating standing waves have formed.

At this point if you touch let us say if you bring a small rod and try to place here like this a small pin or rod or something, if you place here it does not affect these oscillations, because there is no displacement of the string at this point. But if you keep the same thing you try to bring the pin or the pointer here, then it will immediately damp because the wire comes in contact with this and therefore immediately it will start damping.

The same thing is true here, here this is actually wire or the string here it is the electric field variation. So, if you keep the wire this metallic wire along the y axis, then it makes very little effect for the field distribution of the mode and hence the intensity and therefore the loss is very low loss. So in this case very low loss, whereas in this case the wire induces high loss to the mode to the.

Similarly for $I_{0,1}$ here if i place the wire like this, so the same wire but now if I place the wire like this this will also cause very low loss so very low loss. But if I place the wire vertically so let me use another color here let me use the black color. So, if I place the wire like this then of course it will cause huge loss here because, the maxima are in contact with the wire. So, in this case it is high loss.

Suppose you have the next mode that is $I_{1,1}$ mode. So, if we take the intensity distribution of the $I_{1,1}$ mode then we know that it will have a variation like this. So, maximum here and

then contour maximum here so 1 lobe here and 1 lobe here, this is intensity distribution of the $I_{1,1}$ mode.

In this case suppose you want only this mode and none of these modes should come, but I want only $I_{1,1}$ mode, if I want only $I_{1,1}$ mode what kind of aperture would we take. So, if we take an aperture let me now show again in blue color. If I take an aperture which is like this like a grid which comprises of 2 perpendicular wires like this this is the metallic wire mesh or grid.

If I use such a grid then this grid will cause very little loss for this mode because the grid is in positions where the field distribution has gone to 0 and therefore, this will cause very low loss very low loss. If I take an aperture like this a wire grid like this the grid I am using is this, so this is my grid. So, I have placed 2 wires perpendicularly and let us say I have tied it there. So, that I have a grid like this so this is the grid

So, if I have used this grid which is tied here at the center and placed here it will cause very very low loss for $I_{1,1}$ mode. But it will cause high loss for this mode this mode this mode all the 3 modes TEM 00 mode 1 0 mode and 0 1 mode this grid would cause high loss.

And therefore you can select only one transverse mode whether it is 0 0 mode 1 0 mode or 01 mode or 1 1 mode you can select only one transverse mode by using appropriate metallic wires and grids. I have also written here that practically people use certain Teflon apertures to select only the TEM 00 mode let us discuss this in the next slide.

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Choice of Intra-cavity Apertures ...

- The 'transverse extent' of *higher order modes* is more, as compared to that of the TEM_{00} mode.

Choose TEM_{00} mode!

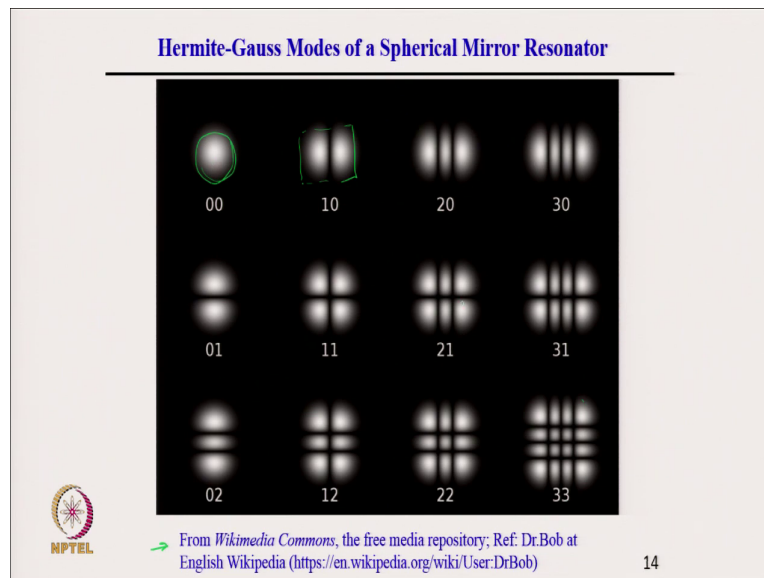
TEM_{11}

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So, here an important point to note is the transverse extent of higher order modes is more as compared to that of the TEM_{00} mode. What does this mean? Transverse extent in the sense if you look at the cross section like this, let us say this is the cross section then the 00 mode may be here and then of course it is very low intensity and like this this is a 00 mode goes down to 0.

But if you take the 11 mode let me use a different color then it has 4 lobes. So, it has maximum here, maximum here and maximum this is the 11 mode. So, this is TEM_{11} the central one is 00 . Now, you note that the transverse extent that the spatial extent of this mode is higher than this mode.

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For example, let me show this beautiful figure which I had shown earlier attribute is here this is the reference. Note that, this occupies over a small region here in space, compared to a higher order mode which occupies much larger space it is the same cross section all of them are for the same cross section same dimension. But the spatial size for example you see now this size is bigger compared to this size this is even bigger and so on.

So, that is the statement that I have made here the transverse extent this refers to spatial extent of higher order modes is more as compared to that of the TEM 00 mode. And therefore if you need let us say the laser is oscillating with many many multi modes many many modes, then if we use an aperture a circular aperture a circular hole with a certain small dimension here. Then it will cause minimum loss only for the TEM 00 mode all other modes will suffer much higher loss.

If we have the gain profile here let me draw this, if we have the gain profile varying like this, then for the TEM 00 mode the loss may be just here for the TEM 00 TEM 1 1 it may be here TEM 1 0 it may be here this is the loss line for different modes. So, this is the gain curve this is the loss line for TEM 00 and this is for TEM 1 0 this is for TEM 1 1.

The point I am making is the laws for different modes are higher and least for the TEM 00 mode, if we choose an aperture like this a small aperture a Teflon aperture a small aperture like this this is opaque and there is only 1 hole that is what I have shown here in the transverse cross section and this is the method used to choose only the TEM 00 mode TEM 00 mode. This is the technique which is used to select a required transverse mode. So, we will stop here and then in the next lecture we will continue.

Thank you.