Introduction to LASER
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Lecture - 25
Optimum Output Coupling

Welcome to this MOOC on LASERs. Today, we will see a an important concept which is very useful in the design of the laser cavity that is called Optimum Output Coupling. Let us see what is this.

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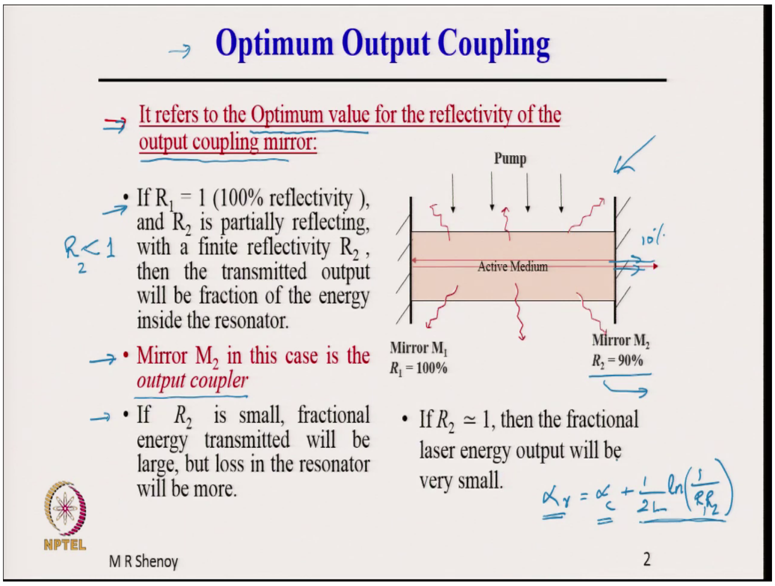

Optimum output coupling refers to, so here. It refers to the optimum value it refers to the optimum value for the reflectivity of the output coupling mirror. Hence it is called optimum output coupling. So, output coupling is here, optimum is here, the 3 words. So, let us see what does this mean?

So, a schematic of the laser system is shown here, the active medium, the two mirrors at the ends mirror M 1 and mirror M 2. Normally, we take output from one of the mirrors and therefore, we have assumed that R 1 is equal to 100 percent, but it need not be. It could be 95 percent and this could be some other R 1 and R 2 could be different values.

But for simplicity if you are assuming that the laser output is coming from this end here, then we can have a perfectly reflecting mirror at the other end. So, R 1 is equal to 100 percent and R 2 is equal to 90 percent, a typical values. So, if R 1 is 100 percent that is 100 percent reflectivity means R 1 is equal to 1, reflectivity is 1 means 100 percent reflecting. And R 2 is partially reflecting with a finite reflectivity R 2. That means R 2 is less than 1. So, R 2 is less than 1. So, if it is 90 percent; that means, R 2 is equal to 0.9, 0.9.

Then, the transmitted output will be fraction of the energy inside the resonator. So, the output would come from here a fraction of the energy because 90 percent reflecting means if we neglect any absorption losses 10 percent would be coming out. So, 10 percent of the incident being on the mirror would come out which means 10 percent of the power inside intensity inside or energy inside would come out of the resonator.

In this case, mirror M 2 therefore, in this case is the output coupler. So, when we discuss about optimum output coupling, we are referring to what should be the optimum value of reflectivity R 2, that is of mirror M 2. So, optimum value of the reflectivity refers to optimum value for the output coupling mirror which means in this case it is R 2.

If R 2 is small, see the logic, if R 2 is small, small means it could be 0.4, 0.5, I have taken 90 percent which means 0.9 R 2 is equal to 0.9 if R 2 is 0.5, 0.4, 0.3 and so on. If R 2 is small, the fractional energy transmitted will be large. For example, if R 2 is 30 percent; that means, 70 percent of the energy would go out.

So, the fraction of the energy will be large. But the losses in the resonator will be more. So, the reflectivity determines the loss of the resonator. So, smaller the reflectivity larger will be the losses, larger the reflectivity smaller will be the resonator loss, alpha R, we recall that alpha r was given by alpha c plus 1 over twice L into ln natural log 1 by R 1 R 2.

So, if R 2 is small which means less than 1, 0.7, 0.8 as it becomes smaller this number will become larger and therefore, alpha r will become larger. So, smaller reflectivity means loss coefficient of the resonator alpha r will be larger. If R 2 is nearly equal to 1, on the other hand if R 1 and R 2 if both are 1, then ln 1 is 0 and therefore, the second term will not contribute any loss, only the intrinsic loss alpha c.

So, alpha r is the resonator loss coefficient, alpha c is the intrinsic loss coefficient, and this one is the loss coefficient due to the finite reflectivity of the mirrors. And therefore, if R 2 is nearly equal to 1, then the fractional energy which come for example, if R 2 is 99 percent which means 0.99, then only 1 percent of energy which is inside the resonator would come out. So, then the fractional laser energy output will be very small. Therefore, there must be an optimum value of R 2.

So, if R 2 is large; that means, the transmission is very small the fractional energy coming out of the resonator is very small. Whereas, if R 2 is small the resonator loss is large and therefore, the energy built up inside the resonator is small, although a major fraction a higher fraction would come out the energy itself which is built inside the resonator is small. And therefore, there must be an optimum. Let me illustrate this in another way.

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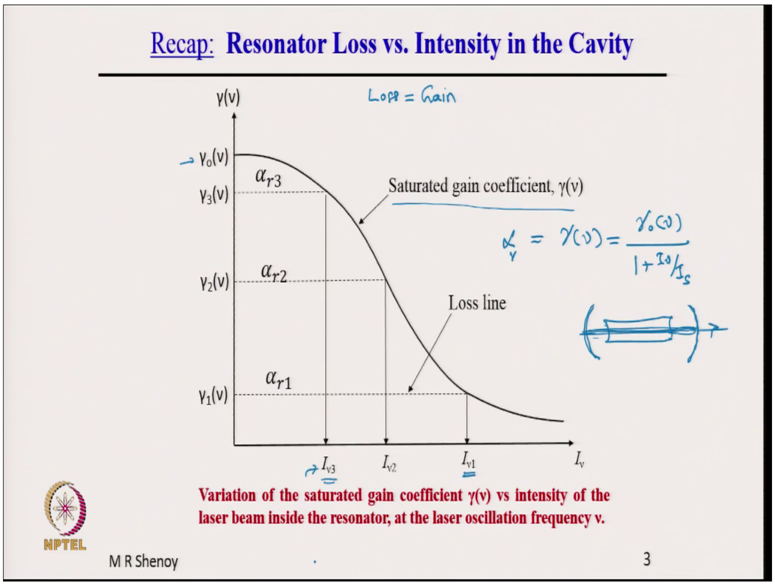

Let us recall. We have this graph resonator loss versus intensity in the cavity, either you could say resonator loss or gain coefficient. In fact, what I have plotted is the saturated gain coefficient alpha r. The title reads resonator loss because we know that loss is equal to gain when the laser is oscillating.

So, we could write here gain versus intensity in the cavity or resonator loss versus intensity in the cavity. So, gamma 0 of nu is the small signal gain coefficient that is when the intensity is very small in the amplifier. So, this is the gain coefficient of the medium.

We see that if alpha r is large which means when the gain coefficient, so when the laser is oscillating gamma is equal to alpha, when the loss coefficient is large the corresponding intensity here inside the resonator.

So, this is I nu is the intensity inside the resonator. Please recall that this gamma of nu is equal to gamma 0 of nu into 1 plus I nu divided by I s, where I s is the saturation intensity. And this is equal to alpha r when the laser is oscillating in steady state and therefore, if the intensity is small then the gain will be large.

And this indicates that if the loss is large the gain has to be large and if the gain has to be large which means the intensity inside the cavity must be small. Let me repeat that again. If alpha r 1 is the resonator loss coefficient, if the resonator loss coefficient is very small, then the gain inside the resonator can become small which means the intensity inside the resonator can become large.

This is for a given system. Intensity can become large because larger the intensity the gain will be pulled down and that is the saturation effect. So, the saturation will take place corresponding to an intensity level here. So, what is this intensity? This is the intensity inside the laser resonate. So, the laser resonator is here, and this is the laser medium, and the intensity of this beam that we are talking inside the resonator. Intensity is power per unit area.

So, if the intensity here is very high then the corresponding intensity, the fraction which would come would also be higher. And therefore, if loss is small then the intensity inside the resonator will be high, and if loss is large then the intensity inside the resonator will be small. And the loss is determined by the reflectivity of the mirrors and therefore, we must have an optimum.

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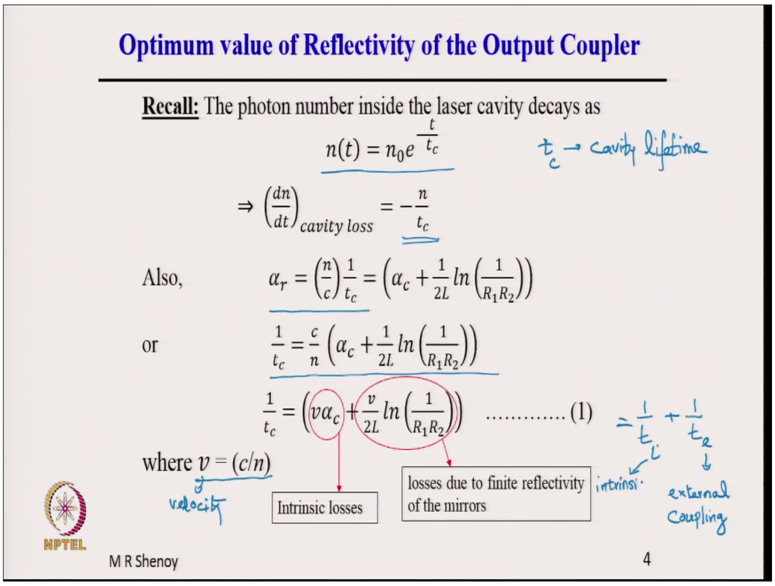

So, let us look at this more carefully. Let us look at that what is that optimum value. So, optimum value of reflectivity of the output coupler. Let us recall the photon number inside the laser cavity decays as n of t is equal to n 0 into e to the power minus t by t c, where t c is the cavity lifetime. So, t c is the cavity lifetime, so passive cavity lifetime.

And we have also seen in the last lecture that dn by dt that is rate of change of photon number in the cavity because of cavity loss is minus n by t c. We derived this in the last lecture. And alpha r is equal to n by c into 1 by t c. This derivation we had done earlier that alpha r is related to the cavity lifetime through the relation n by c into 1 by t c. And 1 by t c is given by the expression here. This also we have already derived, 1 by t c is equal to c by n into alpha c plus 1 by 2L into ln 1 by R 1 R 2.

So, this is alpha r, this is 1 by t c equal to c by n into alpha r and therefore, we can write this as if we multiply by c by n, c by n is nothing, but the velocity. So, v is equal to c by n; this is not nu, this is v which is the velocity. So, c by n is v into alpha c we have taken this inside, and this is v by 2L into ln 1 by R 1 R.

Now, this is therefore, an intrinsic loss coefficient at 1 by t c that is time inverse or a time coefficient it corresponding to the intrinsic loss and this part is due to finite reflectivity of the mirrors. Because we already know that this part is because of the loss, because of the loss due to mirrors.

So, we can write these which I am sure is written in the next slide 1 over t i, this is because of intrinsic loss plus 1 over t e. We are not using t r because of reflectivity, but we are using t e, e standing for external coupling, external coupling. So, loss, a time corresponding to the intrinsic loss coefficient and a time corresponding to the external loss, the coupling due to external coupling that is why the subscript and this I is for intrinsic.

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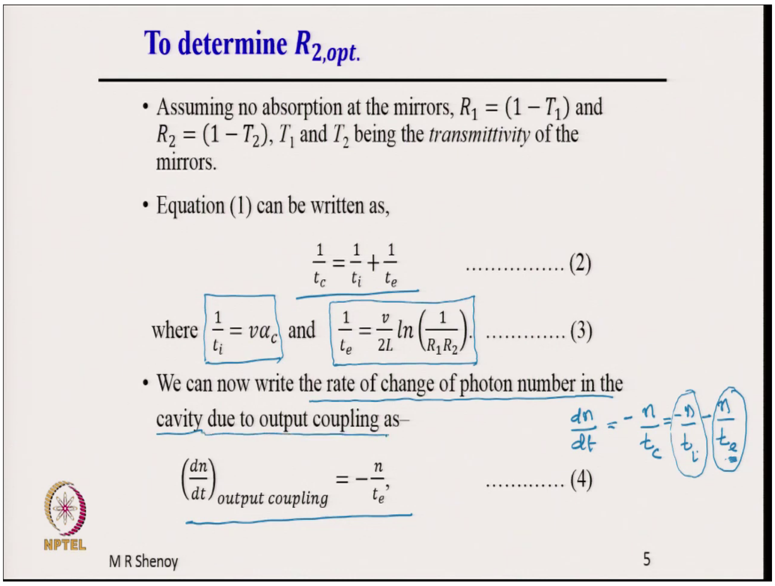

So, let us see the next slide. Assuming no absorption at the mirrors R 1 is equal to 1 minus T 1, R 1 is the reflectivity and T 1 is the transmitivity. And R 2 is equal to 1 minus T 2, T 1 and T 2 being the transmitivity of the mirrors. Equation 1 can be now written as this is what I had written here. So, 1 over t c is equal to 1 over t i plus 1 over t e, where 1 over t i is v times alpha c and 1 over t e is v by 2L into ln 1 by R 1 R 2.

We can now write the rate of change of photon number in the cavity due to output coupling. As I mentioned this is due to intrinsic losses in the cavity, this term, and this term is due to output coupling that is loss due to the mirrors. Loss due to the mirrors is if R 1 is 1, then R 2 is less than 1 and a fraction of light would go out and therefore, this term corresponds to external coupling.

And therefore, the rate of change of photon number in the cavity due to output coupling can be written as dn by dt output coupling is equal to minus n by t e. See that dn by dt in the previous slide, we had seen dn by dt is equal to minus n by t c. This is equal to minus n by t i minus n by t e. So, this term is rate of change due to intrinsic loss and the second term is due to rate of change of photon number inside the cavity because of external coupling that is what is written here.

That dn by dt output coupling is equal to minus n by t e. What is this? The rate of change of photon number in the cavity. It is very important rate of change of photon number in the cavity due to output coupling. Let us see next.

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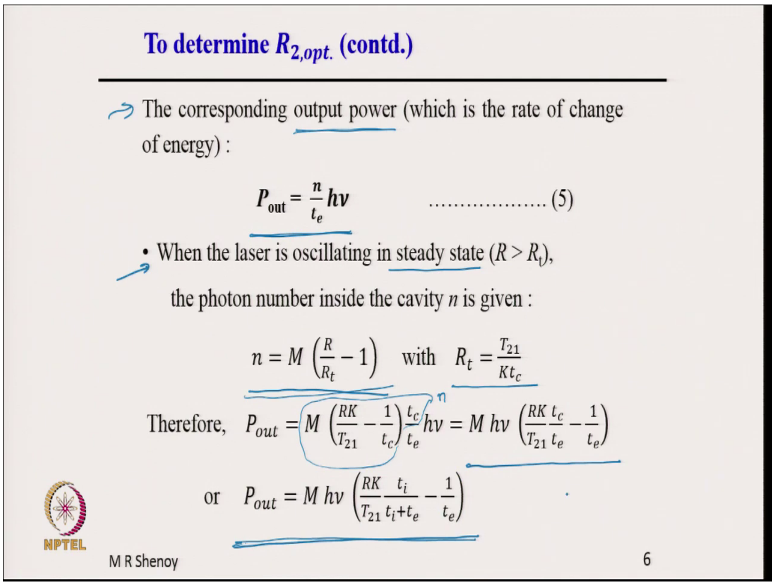

The corresponding output power therefore, which is the rate of change of energy, power is rate of change of energy will be equal to P out is equal to n by t e into h nu. The minus sign is dropped because power is always positive. What is n by t e? n by t e is the rate of change of photon number, n into h nu is the energy, therefore n h nu by t e is the rate of change of energy which is nothing, but the output power. It is a very important expression that P out is equal to n by t e into h nu.

When the laser is oscillating in steady state we know that the pumping rate R is greater than R t and therefore, the photon number inside the cavity n is given by this expression. We derived this in the last lecture, for R less than R t, R equal to R t and R greater than R t that is above threshold.

So, when a laser is oscillating in steady state which means above threshold, the pumping rate is greater than the threshold pumping rate. And therefore, n is given by this expression. We have got this analytical expression with R t, the threshold pumping rate as T 21 by K t c. Therefore, P out is equal to n h nu by t e.

So, M into this is n multiplied by; oh there we just jumped one step; M h nu by t e and this R by R t minus 1 is R by R t, R t is T 21 by K t c, so T 21 in the denominator, K is into t c, t c has been taken out and therefore, it is 1 minus t c here because there is a t c which has been taken out. And therefore, this whole term, please see that this whole term is now nothing but n.

So, this is n; n into h nu by t e, which is M h nu now we have kept h nu to the other side and t has been taken out this t c by t e has been taken out. Taken inside and therefore, we have the expression here. Or P out is equal to M h nu into R K by T 21 into t i by t i plus t e minus 1 by t e. So, everything is now written in terms of the intrinsic time constant and extrinsic time constant. Extrinsic here due to external coupling.

Our objective is to find out optimum reflectivity, optimum reflectivity. The reflectivity is related to the time coefficient, time constant t e and therefore, if you want to optimize the reflectivity to get maximum output power. Optimum reflectivity here refers to the reflectivity corresponding to which the laser gives maximum output power and therefore, we have to determine dP out by dt e and put that equal to 0 and find out the optimum value.

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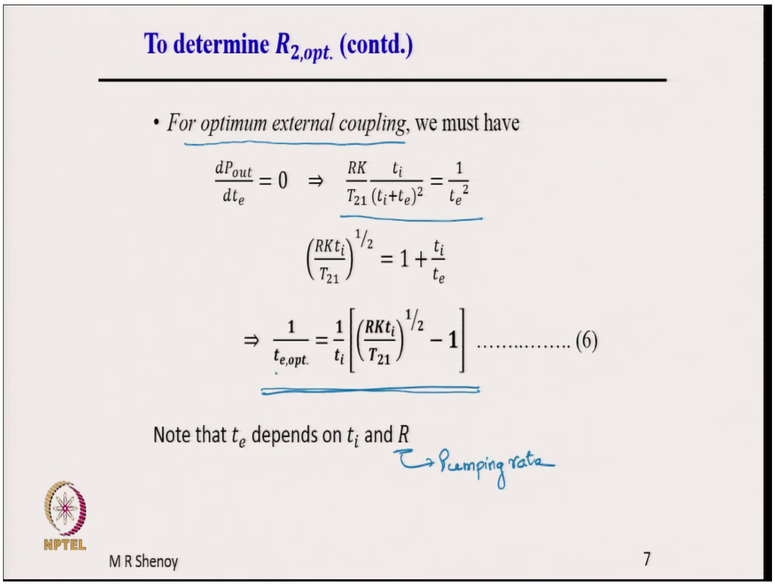

So, let us see what we have here. So, for optimum external coupling we must have dP out by dt e is equal to 0, which implies R K by T 21 into this must be equal to 1 by t e square. So, you can just have a look. So, if we do dP by dt e, then this is of course, constant and here t e is in the denominator, so we get t i plus t e the whole square. And again here also t e is in the denominator, so this also gives 1 by t e square.

And then we put this equal to 0 dP out by dt e equal to 0, which means the derivatives of this is equal to 0 and that 1 by t e square goes to the other side very elementary algebra and you get this expression here. Or this term can again be further simplified. So, you can take this to the other side and write 1 plus t i by t e is equal to R K into t i by T 21 to the power half or 1 by t e optimum.

So, now we have taken this minus 1 here and 1 by t i, t i comes to the denominator is 1 over t e optimum is equal to 1 by t i into this. So, if we know alpha r that is alpha c, R 1, R 2 and the pumping rate R at any given pumping rate R, K is of course, constant for the laser system, T 21 is the transition rate which is inverse of the lifetime of the upper level, then we can determine what is the optimum value of the t e which gives us the corresponding reflectivity.

So, note that t e depends on t i and R. This R is pumping rate. It is not reflectivity, pumping rate. For the reflectivity we have used R 1 and R 2 and this is pumping rate R. This R here is the pumping rate. Depends on the pumping rate, which immediately tells you that the optimum t e will change with the different pumping rate that is as you increase the pumping power the corresponding t e would also change.

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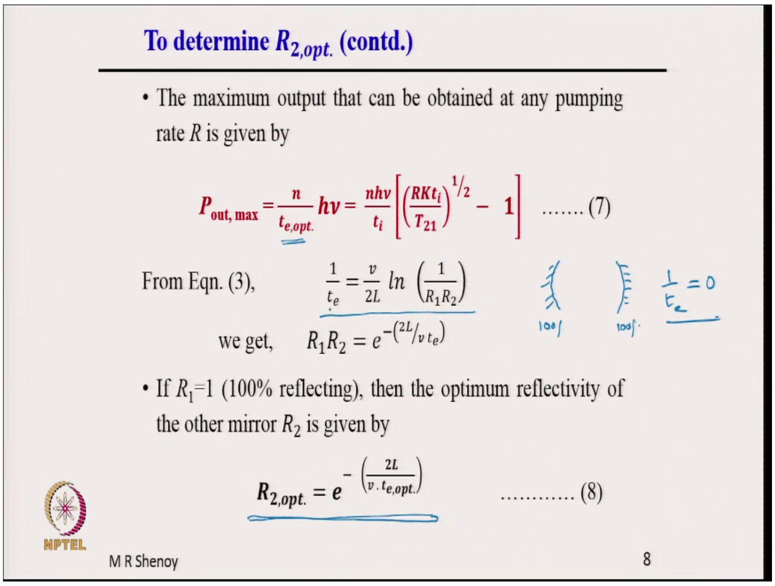

So, let us see further. The maximum output can be obtained at a given pumping rate. If we use P out max is equal to n divided by t e opt into h nu. Output power is n h nu by t e, the maximum power will correspond to the optimum value of t e opt and therefore, that is equal to 1 over h nu into we have substituted for 1 by t e and that is why we have this expression for maximum output power. The picture will become clear as I plot this P out as a function of 1 by t e.

From equation 3, we know the relation between 1 by t e and the reflectivity is given by this. And therefore, we can transpose this and determine R 1 R 2 is equal to e to the power minus 2L by v t e. Please see these are all velocity there is no frequency coming into the picture. And if R 1 is equal to 1, then we have the optimum R 2 is given by e to the power minus 2L divided by v into t e opt.

So, this is the expression for the optimum reflectivity of the output power, where 1 by t e is given by the expression for optimum 1 by t e is given by the expression here which depends on the pumping power, alright.

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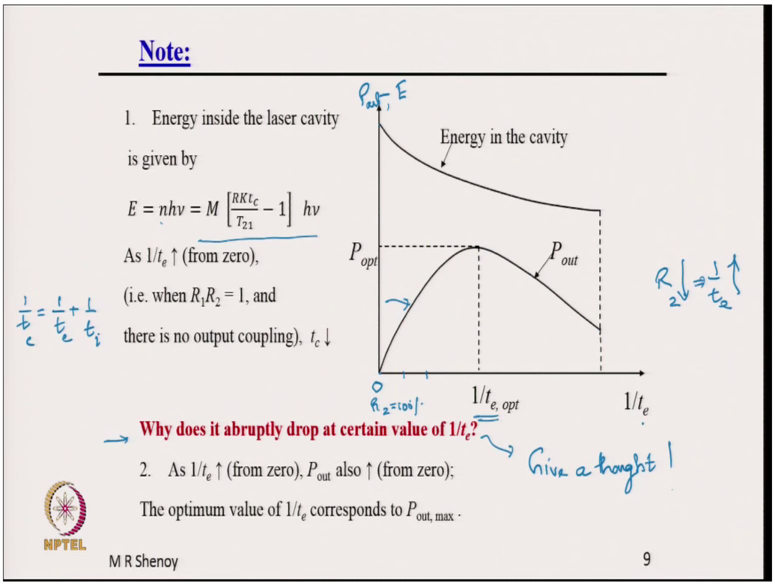

So, let us discuss further. Therefore, if I were to plot; so, note the energy inside the laser cavity is given by e is equal to n h nu, n is the photon number inside the cavity for which we got all the relevant expressions in the last lecture, which is equal to now M into R K t c by T 21 minus 1 into h nu.

As 1 by t e increases from 0, 1 by t e increases, so what we have plotted is P out here. So, this is P out. This curve is for P out. Also plotted is the energy, both are plotted. So, as 1 by t e increases from 0, if t e increases means t c decreases, 1 by t e increases please recall that 1 by t c is equal to 1 by t e plus 1 by t i. 1 by t i is the time corresponding to intrinsic loss coefficient is a constant whereas, 1 by t e will change with the pumping rate and the reflectivity of the output coupler.

And that is why we are plotting here the output power and the energy. So, this is for energy. So, here the both are plotted, energy and output. So, as 1 by t e increases from 0. So, 1 by t e is 0 means what? The reflectivity is 1. So, you could see in the previous expression for 1 by t e.

So, if the reflectivity is 1, R 1 anyhow we can assume 1, R 2 is the output coupler; if the output coupler is also reflectivity 1, 100 percent, then log of this is 0 and therefore, 1 by t e is 0. Means there is no external coupling. If both the mirrors are 100 percent; so, these are two mirrors if both the mirrors are 100 percent and 100 percent then 1 by t e is equal to 0.

So, 1 by t e would increase if R 2 will start decreasing from 1. If R 2 decreases below 1 then this term becomes more and more and therefore, 1 by t e would increase. So, that is the meaning of this statement. As 1 by t e increases starting from 0, 0 is when the reflectivity is 1. That is when R 1 R 2 is 1, it is 0. And there is no output coupling at that point. That is why the power output is 0. All the energy is inside the resonator. So, this is energy in the cavity.

The output power is 0 as 1 by t e starts increasing from zero. And as it increases, t c decreases and therefore, this starts increasing. So, this starts continuously increasing. But the energy continuously decreases. See this t c starts decreasing and therefore, the energy monotonically decreases, because it is R K t c by T 21 minus 1. And that is why the energy this is a qualitative plot, but it is almost correct. So, if you actually put numerical numbers, you will see a similar plot coming there.

Now, why does it drop abruptly at a certain value here? So, give a thought. Now, as 1 by t e increases P output also starts increasing. But as we said R 2 decreases when R 2 starts decreasing please remember R 2 decreasing implies 1 by t e increasing. So, whenever I am saying 1 by t e is increasing, it means the reflectivity is decreasing as you go from here.

So, reflectivity R 2 is 100 percent here. So, this is 100 percent. Then it is 0, 1 by t e is 0, so no output, but as R 2 decreases or 1 by t e increases more and more fractional power from the cavity. You see the energy is decreasing, alright. But the fractional power which is coming out is increasing, R 2 is 100 percent, R 2 is 90 percent, 90 percent means 10 percent of the power is coming out; 80 percent means 20 percent of the power is coming out. So, the output power continuously increases.

However, after some time when as R 2 further increases the loss in the resonator becomes so high that the energy built up continuously starts dropping down and after some time the fractional power will start coming down. Because fraction of a smaller quantity. That is why this starts coming down and we have an optimum value of reflectivity or 1 by t e.

Why does it abruptly drop at a certain value of 1 by t e? Please give a thought or think about it, alright. So, as 1 by t e increases, P out also increases. The optimum value of 1 by t e corresponds to P out maximum.

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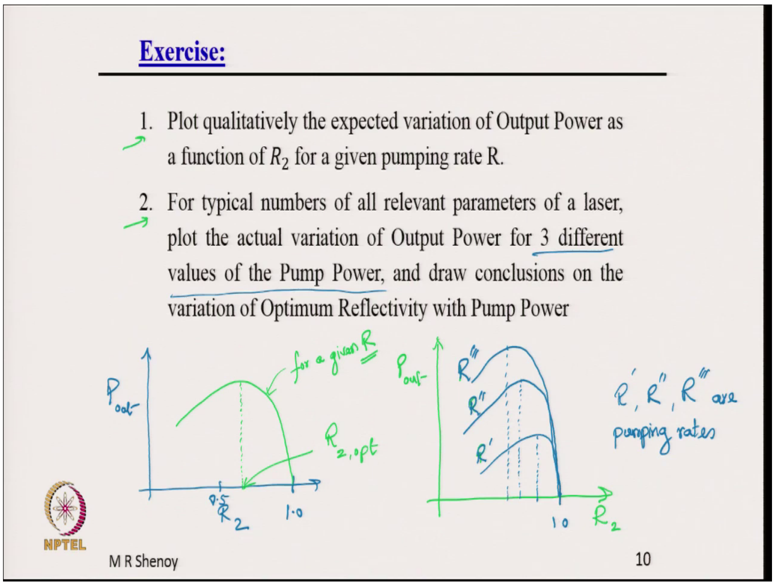

I have listed two exercises, one plot qualitatively the expected variation of output power as a function of R 2 for a given pumping rate R. So, what we have plotted here is the variation of P out as a function of 1 by t e. Now, what I am asking you here is to plot R 2 versus P out. Please see R 2 maximum value is 1.0 that is 100 percent reflecting, so this is 50 percent reflecting 0.5.

So, R 2 versus P out, if you plot what kind of variation do you expect? Let me give a hint that it should vary something like this. You please verify, put actual numbers. Here I have given a 2nd exercise for typical numbers of all relevant parameters of a laser plot the actual variation of output power this variation of output power for 3 different values of the pump power.

That is pumping rate R 3 different values of R capital R pumping rate and draw conclusions on the variation of optimum reflectivity with pump power. So, if the power variation comes out like this as a function of reflectivity then this value of reflectivity here is the optimum reflectivity, R 2 optimum. In the second exercise, what I would like you to do is plot this for 3 different same curve R, R 2 versus P out.

So, this is at one particular, for a given for a given pumping rate R. This is pumping rate, for a given pumping rate R. For 3 different values of pumping rate how would it vary? Let me qualitatively tell you how it would vary. So, if this is 1.0 here reflectivity the output power would vary, it would vary like this. For a particular pumping rate R if you increase the pumping rate then it would vary like this. And if you further increase the pumping rate it would further increase like this.

What do you see? You see that the optimum reflectivity was here, the optimum reflectivity for this pumping rate is here, and the optimum reflectivity is here, so R optimum. So, what are these curves? So, let me use the red color. So, this is for pumping rate R 1 pumping rate, this is for pumping rate R 2, this is for pumping rate R 3. 3 different values. Let me not write R 1, R 2, R 3, it may lead to confusion. Just 1 second let me erase that, alright. The whole thing got erased, ok.

Let us go back here to my curves have got erased, but alright here. So, let me use this as R, R dash, R double dash, and R triple dash. So, R dash, R double dash, or triple dash are pumping rates, pumping rates for 3 different values. That is an exercise, for 3 different values, for 3 different values of the pumping rate or pumping power draw conclusions on the variation. So, what you see is the optimum reflectivity shifts to lower values as you increase the pump power.

This very important in the laser design therefore, what it means is if the output reflectivity the output coupler reflectivity of the laser is designed to be optimum for a given pumping power. If you change the pumping power the reflectivity does not remain optimum.

Therefore, normally a manufacturer would specify the maximum output power and the laser is designed output reflectivity is chosen to have that maximum output at that pumping power.

So, if you change the pumping power of course, the output power will increase or decrease depending on the pumping power.

As you can see if you increase the pump power the output power increases, but the optimum value occurs at a different reflectivity. This is the concept of optimum reflectivity, output coupling, optimum output coupling or optimum reflectivity of the output coupler. Output coupler is the mirror at the output end. Hope this concept is clear.

So, with this we have covered most of the essential theory and physics aspects. In the next week or in the next lecture, we will take up the output characteristics of a laser.

Thank you.