Introduction to LASER
Prof. M. R. Shenoy
Department of Physics
Indian Institute of Technology, Delhi

Lecture - 24
Variation of Laser Power around Threshold

Welcome to this MOOC on lasers. Today we will take up Variation of Laser Power around Threshold. In the last couple of classes, we have seen that threshold is a very important region, where a number of changes takes place and the laser goes from below threshold to above threshold, how the gain profile changes; and the whole burning effect and how oscillations takes place in the output before the output intensity stabilizes. Today, we will see by actually the discussions earlier were qualitative.

So, today we will see actual derivation and see by putting numbers what kind of change in the photon number takes place as we go down from below threshold to above threshold. So, that is the topic variation of laser power around threshold.

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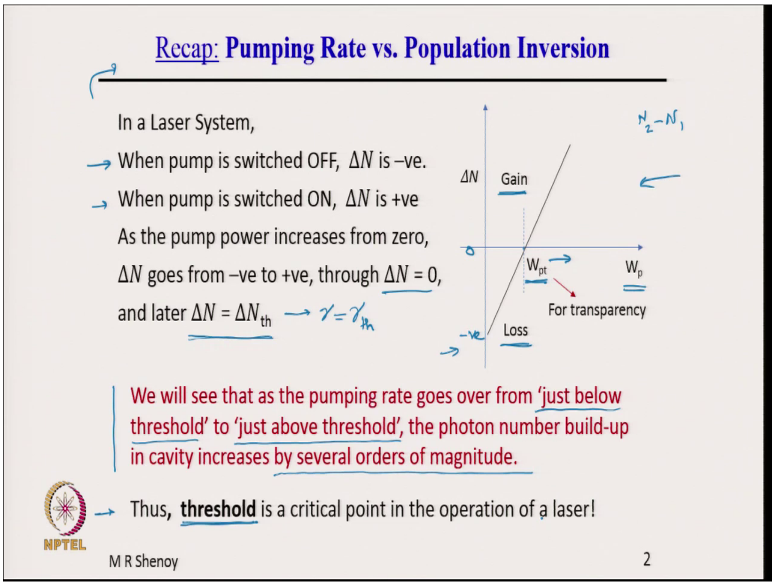

So, very quick recap, we had seen that with pumping rate how the population inversion changes the diagram which is here. So, delta N, delta N starts from negative, when there is no pumping on the x axis, what we have is the pumping rate W p, when the pumping rate is 0 that is when the pump is off delta N is negative, delta N is the inversion. So, when we say delta N it is higher level that is N 2 minus N 1 or N 3 minus N 2 depending on whether it is a 3 level system or a 4 level system.

But, delta N is negative when there is no pumping, as the pumping rate increases delta N monotonically increases it may not be linear increase, but it monotonically increases and delta N become 0, when the pumping rate is W p is equal to W pt t standing for transparency.

It means, delta N is equal to 0 means there is neither gain nor loss, when delta n is negative, there is loss which means the medium is absorbing and when delta N is positive then there is gain in the medium and when delta N is equal to 0.

We have neither gain nor loss or the medium acts as a transparent medium that is why the pumping rate is called W pt standing for transparency pumping rate. If we increase W p beyond that then, delta N becomes positive and the medium has gain.

Thus in a laser system, when pump is switched off, delta N is negative, we are here and when the pump is switched on, delta N is positive. As the pump power increases from 0 delta N goes from negative to positive, through delta N is equal to 0.

That is this point delta N is equal to 0, when W p is equal to W pt and later delta N is equal to delta N threshold, delta N threshold is the threshold population inversion in a laser corresponding to gain gamma is equal to gamma threshold, the threshold gain coefficient. And we will see, in this lecture that as the pumping rate goes over from 'just below threshold' to 'just about threshold', the photon number build-up in the cavity increases by several orders of magnitude.

We will actually, make this derivation and put numerical values and see that the photon number increases by orders of magnitude as we go from below threshold to above threshold. Thus, threshold just as it is literal meaning threshold is a critical point in the operation of a laser above which and below which the characteristics are very different is just like literal meaning of threshold that that is the point about which the characteristic changes drastically ok.

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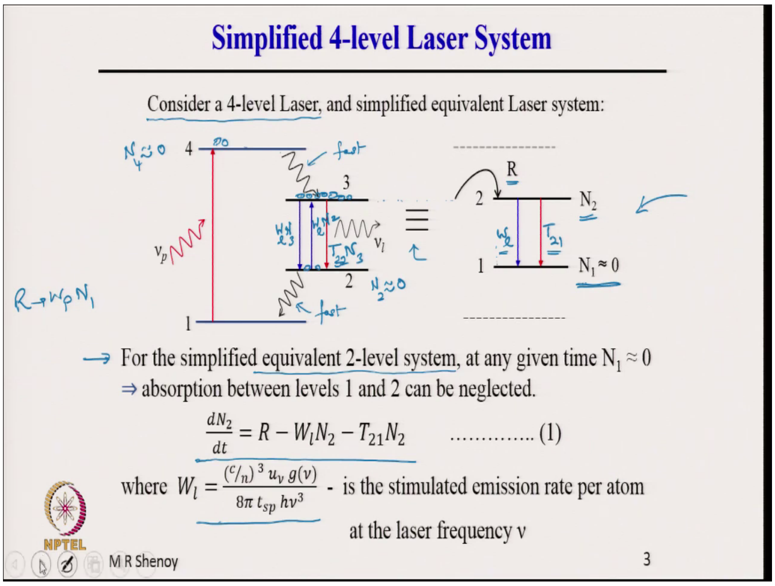

Let us consider a 4 level laser system, very quickly we recall that in a 4 level system from the ground state a pump excites atoms to the upper state here, from there the atoms rapidly come down and accumulate here provided 3 life time of the level 3 is large; and from level 2 here, they rapidly go down to the ground state. So, these two: this and are fast transitions, very fast transition fast transition, because the life time of those levels level 4 and level 2 are very small.

Therefore, if I have a pump essentially at any time N 4 the number of atoms in level 4 N 4 is nearly equal to 0 at any instant because, whatever atom which has been excited to level 4 rapidly comes down to level 3. Similarly, whatever atom which comes down the number of atoms which come down here they rapidly go down to the ground state and therefore, this N 2 is also approximately equal to 0. So, N 2 is approximately equal to 0.

So, essentially there is a population inversion between level 3 and level 2 in a 4 level system. Therefore, we write an equivalent simplified system we call this pumping rate which is going here so, W p into N 1 which is the pumping rate R.

So, essentially the atoms are pumped from here to level 3 from 1 to 3 through level 4 that is essentially the effect and therefore, what we have shown is a pumping rate R, which essentially lifts atoms or excites atom from the ground state to level 2 here.

Now, we are calling this as level 2 the other two level 1 and 4 are shown as virtual dotted line and the atoms are excited and brought to level 3, the level three is now shown as level 2. And similarly, whatever is the number of atom which comes here to the original N 2, here the level 2 they go down to the ground state very rapidly. In other words, in the level 2 there are hardly any atoms any time left and therefore, we write an equivalent.

Because, whatever laser action is taking place it has three participating transitions this one which is pumping and from there it goes down by stimulated emission radiation that is W l is the rate constant for stimulated emission; and T 21 now, we are calling this as level 1 and 2 and these two are virtual levels.

And note that N 1 is nearly equal to 0 because, atoms which come here rapidly go down to the ground state. So, what is essential is happening here, and that is why we are writing it as an equivalent two level system, where there is a pumping rate R and transition simulated emission transition and spontaneous emission transition which is taking place.

Of course, stimulated emission would have also had a upward transition, which is W l into N 1, but N 1 is 0 and therefore this upward transition which is shown here we see here this is W l into N 3 downward transition, upward transition is W l into N 2

And the 3rd transition if you recall the rate equations that we had studied T 3 2 into N 3 so, this is the transition rates of atoms in the normal 4 level system. So, what we have shown is because, N 2 is 0 nearly 0 this up going transition is neglected and we are left with only two transitions they are shown here W l and T 21; and N 1 is of course very nearly equal to 0.

Therefore, for the simplified equivalent 2 level system, we are calling this as a equivalent 2 level system at any given time N 1 is nearly equal to 0, which implies absorption between levels 1 and 2 can be neglected that is what I have been mentioning. There 1 to 2 this absorption is neglected and therefore, if we write rate equation for this system, then it is d N 2 by dt is equal to R minus W l n 2 minus T 21 N 2 R, because it is positive rate of change of the population of level 2 this N 2.

Now, onwards we are looking at only the 2nd system which is here, we are not looking at the normal 4 level system. So, d N 2 by dt where N 2 will get populated and positive, it will increase because of R, it will decrease because of a downward stimulated transition and it will decrease. Because, of a downward spontaneous emission transition, where W l is equal to so, this is the stimulated emission transition rate per atom at the laser frequency nu alright.

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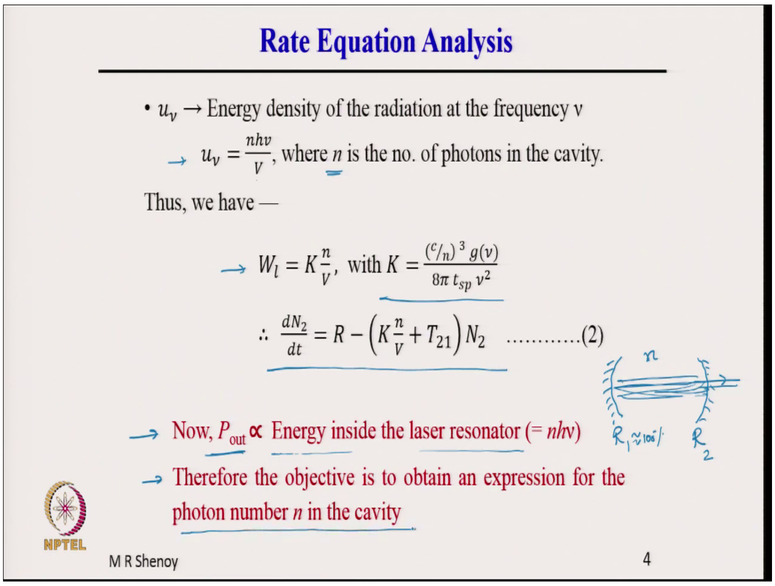

Where u nu in the previous expression here, u nu is the energy density energy density of the radiation at the frequency nu which means, it is if n is the photon number in the cavity then n h nu is the energy per unit volume V is the energy density, where n is the number of photons in the cavity.

Thus we have W l is equal to K into n by V so, here n by v the h nu one h nu cancels with one h nu here, the h and one nu cancels here. So, we are left with only nu square in the denominator and that K is equal to c by n cube into g nu divided by 8 pi t sp into nu square, one h nu has cancelled.

So, W l is equal to this and therefore, we rewrite the equation as delta N 2 by dt is equal to R minus K times n by V plus T 21 into N 2. Now, note that the power output of the laser, because the title of the torque is variation of laser output power with pumping rate, as we go from below threshold to above threshold. Note that the laser output power is proportional to the energy inside the laser resonator because, recall the laser resonator here and energy is building up here.

And every time a fraction of the energy is coming out depending on the reflectivity R 2 of the mirror. So, R 1 which is usually close to 100 percent and R 2 is the output reflectivity of the mirror.

So, what I have shown is the resonator here therefore, if n is the photon number steady state photon number in the cavity, then the amount of output is proportional to n therefore, the objective of this discussion or torque is to obtain an expression for the photon number n in the cavity. So, in this lecture we will obtain an expression for the photon number n in the cavity alright.

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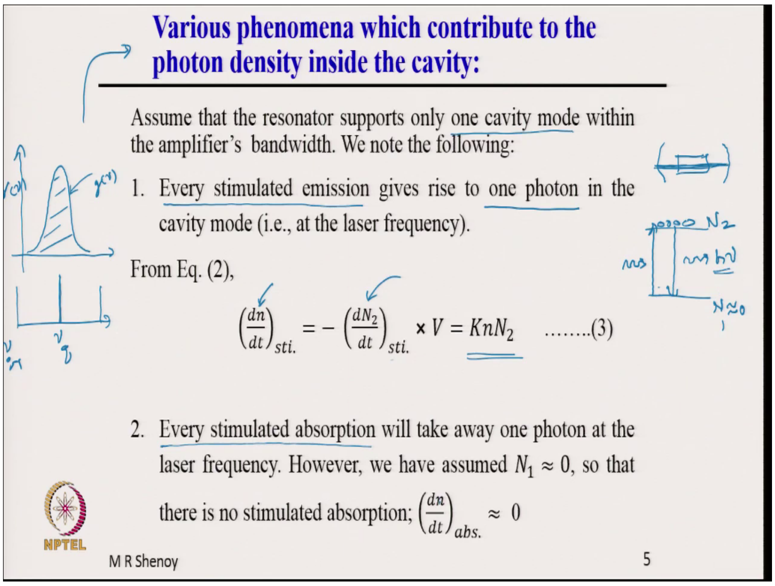

So, the various phenomena which contribute to the photon density inside the cavity let us see what are the factors which contribute to the photon density. So, assume that the resonator supports only one cavity mode within the amplifier's band width. This we have discussed in detail in the previous class; so, where we had said that we can always choose a resonator or a medium and a transition such that although this is the g nu spectrum or the gain spectrum so, this is g nu. We can have only one oscillating mode,

Longitudinal mode in the laser resonator nu q, for example, f s r could be large, so if the f s r is large free spectral range is large so, let us say this is nu q minus 1 then, within the laser transition where this is gamma of nu where the gain is available, there is only one longitudinal mode and that is what is mentioned here that we assume that there is only one cavity mode within the amplifiers bandwidth. And therefore, first component,

We are looking at various phenomena which contribute. Every stimulated emission gives rise to one photon in the cavity mode that is we have a stimulated emission taking place there are large number of atom. So, every stimulated emission which is coming down here gives rise to one photon h nu every atom coming down, gives rise to one photon h nu to the cavity mode to this longitudinal because, there is only one mode which is present please see that the laser is here and this is the amplifying medium.

So light is going back and forth and building up here and that is the cavity mode which is building up as it goes back and forth. So, every stimulated emission will contribute one h nu to the cavity mode that is the meaning of this statement that is at the laser frequency. Therefore, from equation 2 if we look at this equation 2 here look at the components so this is the stimulated emission component, this is the spontaneous transition component and this is the pump and therefore, this is the component that we are discussing.

And therefore, from equation 2 dn by dt due to stimulated emission dn by dt is rate of change of the photon number in the cavity. The photon number would change as we will see due to different components, but due to stimulated emission will be equal to d N 2 by dt due to stimulated emission that is the rate of change of atoms coming down from here to here that is d N 2 by dt due to stimulated emission multiplied by V is the volume, because this is per unit volume.

Whereas, n the photon number n is in the entire cavity, N 2 by definition is the number of atoms per unit volume n is the number of photons in the cavity irrespective of the volume and that is why we have multiplied it by volume. And this term the stimulated emission term is K in to n by V multiplied by V is K into n into N 2.

So, that is what is written here that equal to K into n into N 2 equation 3. Now, there can be stimulated absorption also as in general there is stimulated emission, but these photons can also be absorbed to make an upward transition.

The photon which is generated or in the cavity which is present can also lead to this however, we have assumed that N 1 is nearly equal to 0. Already, which is in practice true and therefore, is very very small compared to N 2 and therefore there is no stimulated absorption or the change in photon number per unit time due to absorption is nearly 0 alright.

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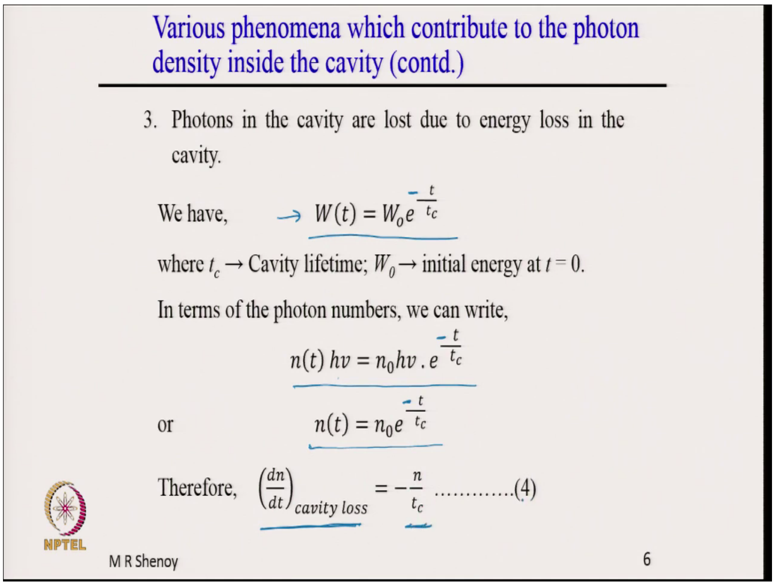

Let us see other components. Photons in the cavity are lost due to energy loss in the cavity every cavity has a finite loss and therefore, with time there is photons which are lost, because of the cavity loss.

So, that will result in depletion of the number of photons with time. So, that is given by the dn by dt due to cavity loss how to calculate this, dn by dt due to cavity loss the third term. So, recall that the energy W of t in a cavity is given by this expression W 0 is the initial energy, t c is the cavity life time and t is time e to the power of this is minus t by t c.

So, the minus sign has seems to have joined with this. So, it is minus t by t c oh here also it is the same minus t by t c, the negative minus sign is quite big and it gets combined minus t by t c alright. In terms of photon number, because energy is n into h nu so, energy at any time t is equal to the photon number at that time t multiplied by h nu. So, equivalent expression is given here, n of t into h nu is equal to n 0 into h nu into e to the power of minus t by t c, n 0 corresponds.

The photon number corresponding to the initial energy W 0 and therefore, h nu h nu cancels and we have n of t is equal to n 0 into e to the power minus t by t c. Now, we can differentiate this dn by dt there is the rate of change of n is simply equal to minus n by t c so, equation 4 so that is the third component contributing to dn by dt let us go further.

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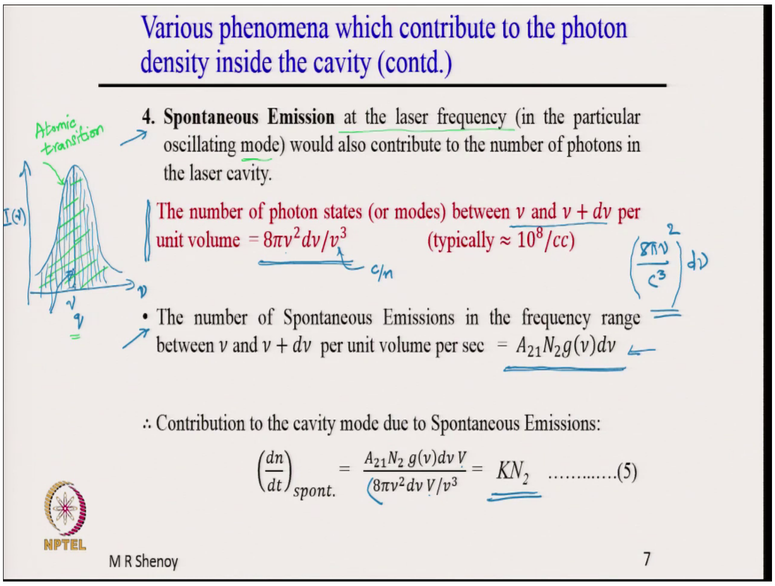

4th component, spontaneous emission spontaneous emission at the laser frequency would also contribute to the number of photons in the laser cavity. We will discuss this in detail, but here so if we have the atomic transition which is here. So, what I am plotting here is I of nu or I of lambda as a function of nu.

So, the intensity spontaneous emission intensity corresponding to the transition, laser will be one of the line so, the longitudinal mode if we consider this as nu q so the laser line will correspond to this frequency nu q.

But note that, the spontaneous emission comes over this entire band spontaneous emission is over the entire band corresponding to the laser transition. So, this is the transition atomic transition so atomic transition. There is a finite width, we have discussed all these in detail earlier and therefore, all those spontaneous emission comes over the entire frequency range, the spontaneous emission which comes exactly at nu q will also contribute to the number of photons in the cavity mode so that is the statement here.

Spontaneous emission at the laser frequency which means, in that particular oscillating mode here refers to this frequency nu q, mode here refers to the longitudinal mode would also contribute to the number of photons in the laser cavity. Now, the number of photon states so the number of photon states here or modes between nu and nu plus d nu per unit volume is given by this expression, 8 pi nu square by v cube this is v this is nu this is v c by n if you remember right at the beginning when we discussed Planck's law we had written that the number of states is 8 pi nu square by c cube.

So, c is the velocity so here it is v so, this v is actually, c by n. So, this is the number of modes between nu and d nu is this multiplied by d nu. So, that is what is written here 8 pi nu square into d nu by v cube.

What is this number of photon state? What is this, these are the intensities which are coming out? So, the photon states at different frequencies, how many such states are there between frequency nu and nu plus d nu and nu plus d nu and this is given by such a number. If you put typical numbers, you will see that it is approximately 10 to the power of 8 per cc so, number of photons states means the number between nu and nu plus d nu per unit volume that is why the name the density of states.

Therefore, the number of spontaneous emissions these many photon states are possible therefore, the number of spontaneous emissions in the frequency range between nu and nu plus d nu per unit volume per second is this much. This expression we have already written when we discussed about the Einstein coefficients in probably lecture number two.

Contribution to the cavity mode due to spontaneous emission please see, spontaneous emission comes over all these modes what is the contribution for this particular mode due to spontaneous emission and that is given by the rate of spontaneous emission here, this is per unit volume therefore, multiplied by V so, this is per unit volume multiplied by V tells us the rate of spontaneous emission which means, the number of spontaneous emissions per unit time, divided by the number of modes multiplied by

V because, it is a density of states is 8 pi nu square by v cube or c cube into d nu into V gives us total number of modes in the volume. And V of course, cancels and what we get is equal to K times N 2 N 2 comes out and the remaining all of it is nothing but K, this A 21 is one by t s p so, it will come down and therefore you will see this we call as K.

This point about the density of states, the number of photon states and the longitudinal mode oscillating mode I will just discuss again in this slide.

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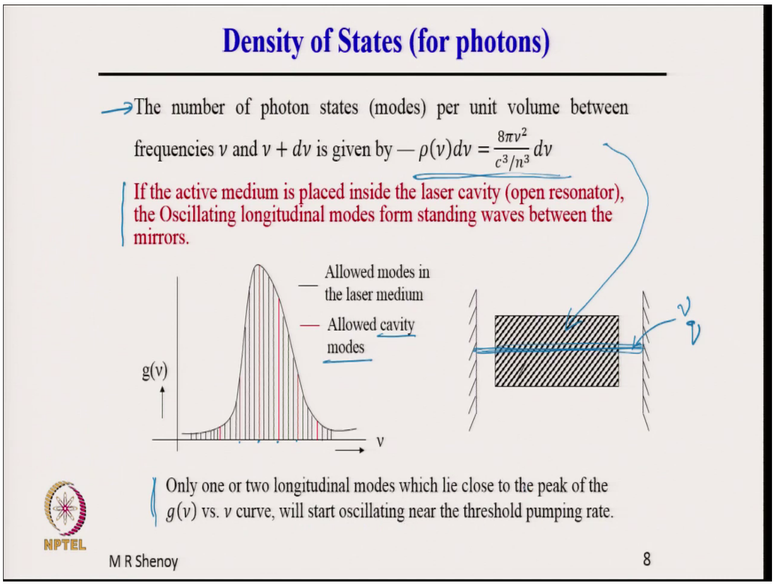

So, please see this, density of states for photons. The number of photon states per unit volume between frequency is nu and nu plus d nu is given by this expression, usually we use rho of nu as the density of state symbol for density of states.

If the active medium is placed inside the laser resonator, so this is the photon states in the active medium or in the amplifying medium. So, this density of states refers to this medium, when the medium is placed inside the resonator, there are longitudinal modes corresponding to the standing waves which are formed inside the resonator; and this frequency we have said is nu q one particular frequency, we have assumed that there is only one longitudinal mode if there is only one lot.

So, these modes are determined by the cavity the other mode is determined by the medium here. So, this is the density of states in the medium and the longitudinal modes or cavity modes are determined by the resonate and the resonator modes are shown here. So, they are shown by a different color.

So, you can see that the resonator modes are here and in between also you have plenty of photon modes. So, allowed modes in the laser medium allowed modes in the cavity allowed cavity modes.

Therefore, one of the allowed modes will also be corresponding to the cavity mode and that is why the contribution due to spontaneous emission is very small, we go back the total number of photons due to spontaneous emission per unit time divided by the number of modes spontaneous emission modes. That gives us the contribution due to spontaneous emission which of course, we will see later that is very small, because only a small fraction will go into the cavity mode alright.

So, as shown here only one or two longitudinal modes which lie close to the peak of the g nu curve, will start oscillating near the threshold pumping rate and we have assumed only one particular ok.

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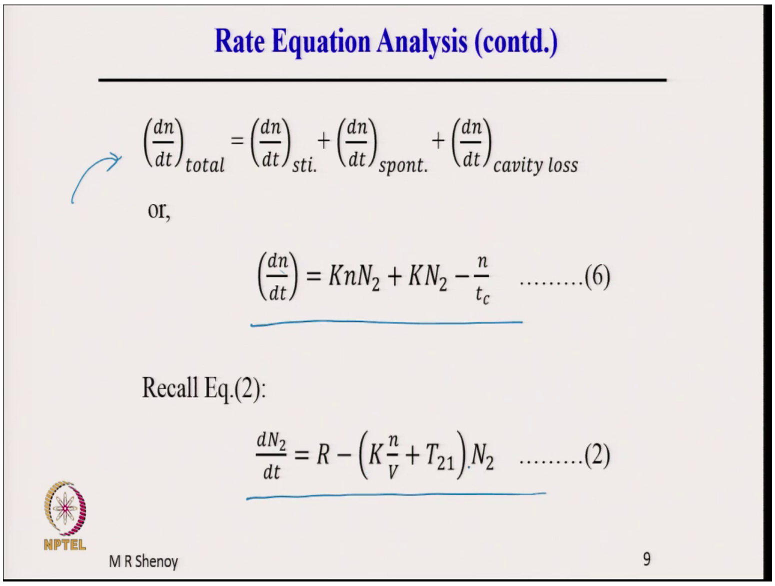

We come back therefore, the total contribution now, you see we have discussed four contributions one was zero due to absorption therefore, there are three terms remaining here, dn by dt total rate of change of the photon number in the cavity is given by the sum of these three and this is the so, we had seen all the three terms K times N 2 here minus n by t c here and K n N 2 here equation 3. So, equation 3 equation 4 and equation 5 added together here to form equation 6.

Now, recall this is the first equation that we wrote, this is for atomic population N 2 rate of change of the number of atoms and this is number of photons in the cavity. So, we have now two equations. So, please see that n is the photon number in both the equations and N 2 is the number of atoms. Now, we have two equations [FL] we can easily eliminate N 2 for example, because our interest is n we want to get n and therefore we can eliminate n two using these two equations and obtain an expression for n. So, that is what I will show in the remaining part.

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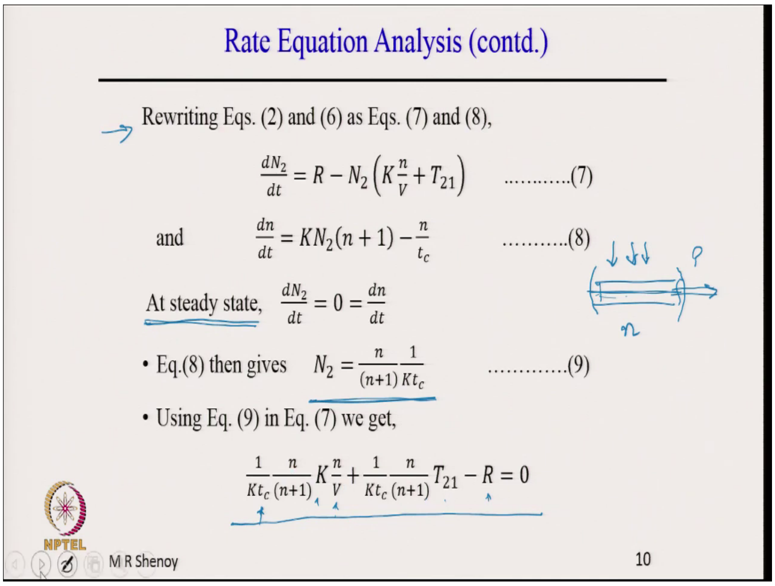

So, the rate equation analysis where we have now rewritten equation 2 and 6 as 7 and 8 is the same equation we have just combined the term you can see this. So, this is the same equation we have taken K n 2 together and put n plus one into K N 2 and this equation. So, we have written K N 2 into N plus 1.

Now, at steady state so, this discussion is for steady state or it is a steady state analysis d N 2 by dt is equal to 0 and similarly dn by dt is 0 at steady state means you have the laser and the laser is giving out a steady output power. So, there is a certain power which is coming out P out at steady state. So, there is a steady state, there is a pump and the laser is oscillating here and giving out steady state output power therefore, it has a certain n but, the n is constant and therefore, dn by dt is equal to 0 similarly, d N 2 by dt is also 0 and then equation 8 if we put this equal to zero we immediately get N 2 is equal to n by n plus 1 into 1 by KT c. Note that n is usually a very large number and therefore, this is approximately equal to 1 by KT c.

Using equation 9 in 7 so, we put this value of N 2 for N 2 we substitute from this then, we get an equation which is in n only small n only. So, substituting in this so, we have this expression. We are now eliminated n two and we have only n in this expression, note that all other parameters we know T 21 the transition rate which is inverse of the life time we know, this pumping rate we are determining the pumping rate.

So, we know this and K is course a constant, V is volume of the material and t c is a measurable parameter, which is the cavity lifetime. So, we know everything in this to determine n alright, let us find the solution of this.

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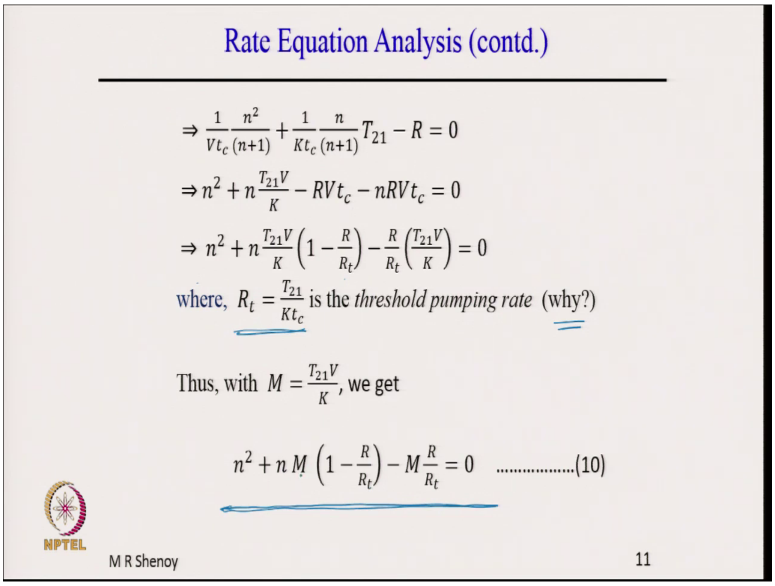

So, we further simplify this, very simple algebra all the steps are written here so, we are simplified this further you put this equal to 0 multiply the whole expression by this denominator then we have n square plus n into this term and so, here is and further you put them together in this form where, R t. So now, we have defined R t is equal to T 21 by K t c is the threshold pumping rate, why? We will discuss in the next slide.

But, now we are put this expression in this particular form and you still have a common T 21 V by K here T 21 V by K here. So, if we call that as M some capital M then we have the final quadratic equation for the photon number n in terms of the pumping rate R the threshold pumping rate and a number a dimensionless number M, we will see that it is dimensionless. So, we have got the final equation quadratic equation here.

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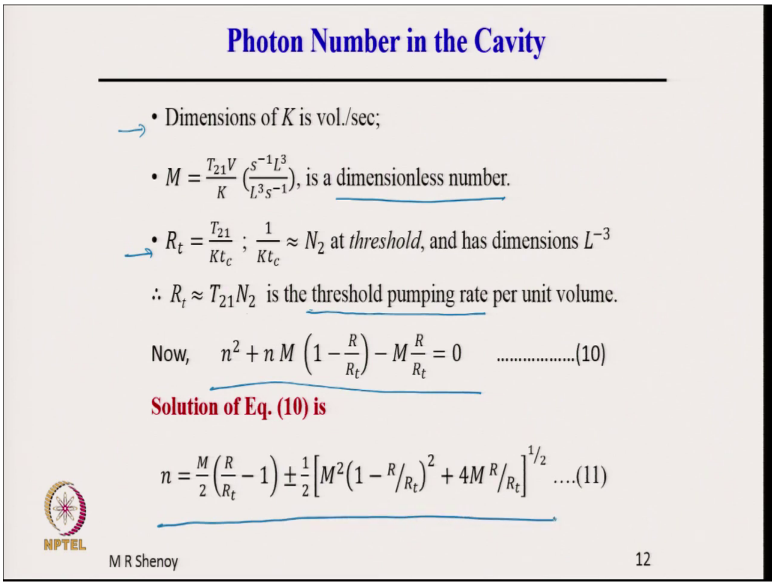

Note that the dimensions of K is volume per second because, if you remember if you look at this so, we can see here it is K into N 2 so, that is the rate which is just time inverse dn by dt n is dimensionless photon number dt so it is second inverse. Therefore, this K N 2 is second inverse, but N 2 is per cc because N 2 is the number of atoms per unit volume and therefore, K must have the unit of cc per second alright here. So, K is volume per second is the unit, M.

Therefore, is T 21 which is second inverse of lifetime we are just looking at the dimensions and V is volume so L cube K is volume per second and is a dimensionless number. And therefore, R t which is equal to T 21 by K t c and 1 by K t c is very nearly equal to N 2 at threshold, and thus has dimensions of L inverse, because N 2 is per unit volume. So, this has dimensions of L minus 3 and therefore.

R t is equal to T 21 into N 2 is the threshold pumping rate because, 1 by Kt c is nearly equal to N2 at threshold per unit volume therefore, we come back to the equation n square plus n into M. So, this is our equation that we have got and the solution is it is a simple quadratic equation so, you can write n is equal to so here minus b plus minus square root of b square minus 4 a c by 2 a. So, we apply the same thing and we have this expression here.

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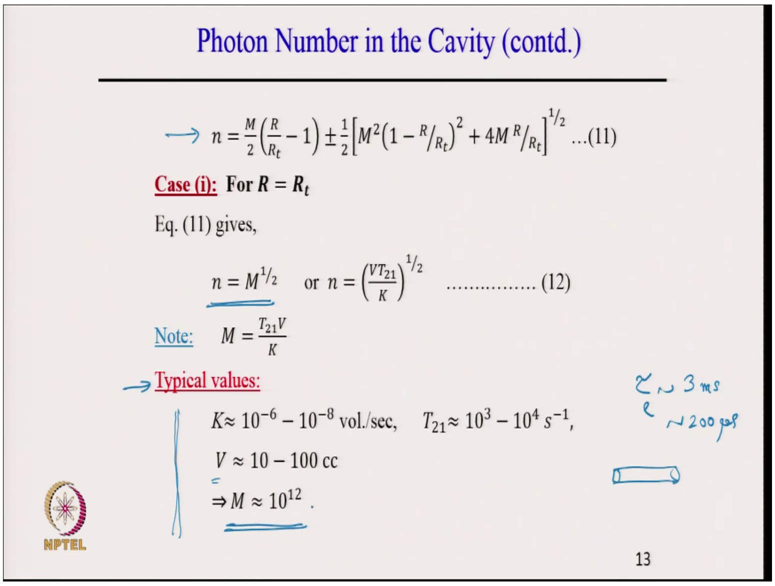

Now, let us look at the solutions. So, here is the same expression for n now, first case for R is equal to R t. We are taking this case wise because, there is R by R t minus 1 terms and therefore, if we take R is equal to R t the first term is zero, the second term here is zero so, what we are left with is only this with the half outside. So, we have equation 11 gives, n is equal to M to the power of half there is a half outside when it goes inside it becomes 4 in the denominator 4 cancels R is equal to R t that is one and therefore.

We simply have M to the power half. The photon number is given by n is equal to M to the power of half or here n is equal to where m is V T 21 by K. So, the typical values let us put some typical values, K is in this region typically if we put values for K we have an expression for K, if you substitute for typical numbers we will get K as 10 power minus 6 to 10 power minus 8 volume per cc, I have used volume in terms of cc everywhere cubic centimeter and T 21 is one over life time.

Please recall that life time, if you take ruby laser it is about 3 milli second, if you take ND YAG this is about 200 microsecond and therefore, one over this is approximately 10 power 3 in this range and V is the volume of the laser medium, if you take for example, the ND YAG rod, so, it is about one centimeter dia and 10 centimeter here or maybe 15 centimeter length and therefore, you can find out the volume so, the volume is typically in this range.

So, if you use numbers corresponding to the mid value here, M will come out to be approximately 10 to the power of 12. So, M is a big number so, why we have put these typical values, because once we know that M is a big number we can make some simplifications that is why we are using practical numbers alright.

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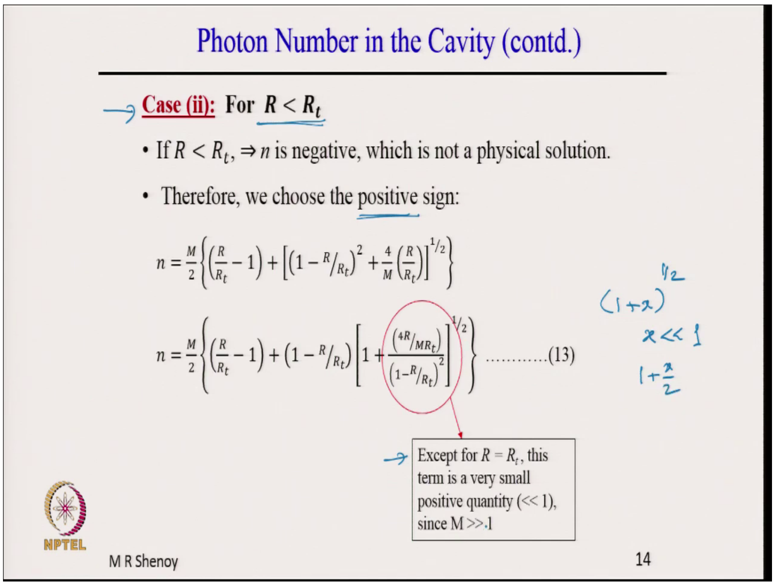

Let us go to case 2 for R less than R t, we have seen R is equal to R t, R is what pumping rate R t is threshold pumping rate, when the pumping rate is less than the threshold pumping rate then if we use so, there are two solutions if you see here plus or minus. So, if we use the plus sign or minus sign so, we will see that one is for R less than R t and the other is for R greater than R t. So, this plus and minus what I am referring to here, when I say that n is negative, if you take the negative sign which is not a physical solution,

And therefore, we choose the positive sign here refers to the plus minus there in the solution which I showed you just now so here. The solution plus minus so, you can take plus sign or minus sign. So, plus sign corresponds to the case when R is less than R t and minus sign corresponds to the case when R is greater than R t, how do we know this because, for R less than R t if you take the negative sign then, The photon number will come out to be negative. A number cannot come out to be negative photon can be zero or more than zero, you cannot have negative photon numbers, that is why we take the positive sign here. And if you take the positive sign so, now, we have chosen the positive sign then this is the expression we can take this outside this term if we take this outside then we will have this in the denominator here no approximations made yet. Now, we know that.

So this M is here actually in the denominator therefore, M is a very large number 10 to the power of 12 that we got and R by R t is may be 0.8, 0.9 it is less than 1. Except when R is equal to R t where we have a separate solution, this number is extremely small because M is in the denominator.

So, except for R is equal to R t this last term is very very small, this term which is here and therefore, we use 1 plus x to the power half when x is much much less than 1 we simply have this as if you put any n this is one plus n x or 1 plus x by 2.

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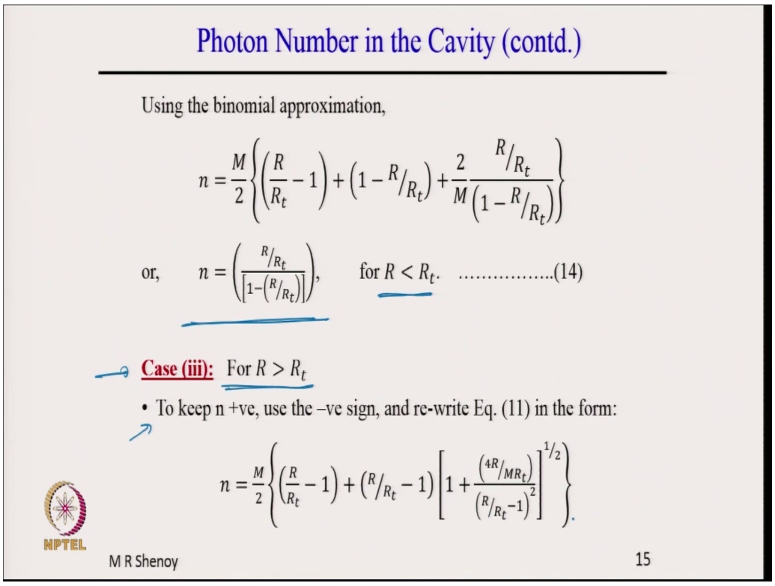

So, if we apply that then note these two numbers cancel and therefore, we have using the binomial expansion we get this plus this. So, these two numbers cancel and what we are left is the last term, which is and 1 2 outside here one M outside here so, 2 by M M by 2 cancel each other and we are left with only this term; and that is what we have written here or n is given by the expression R by R t divided by 1 minus R by R t for R less than R t.

So, we have now got solutions for R equal to R t and R less than R t. Now, we will see case 3 for R greater than R t so as before to keep n positive, we must use now negative sign in that solution and rewrite equation 11, please see the equation again just so, equation 11 here alright here. So, we have to take now, negative sign and therefore, we rewrite the equation 11 here in this form. So, because we have now flipped the R by R t earlier it was 1 minus R by R t.

So, we have interchange this and the negative sign is been made positive. So, no approximations are made yet, but we have simply put it in this form.

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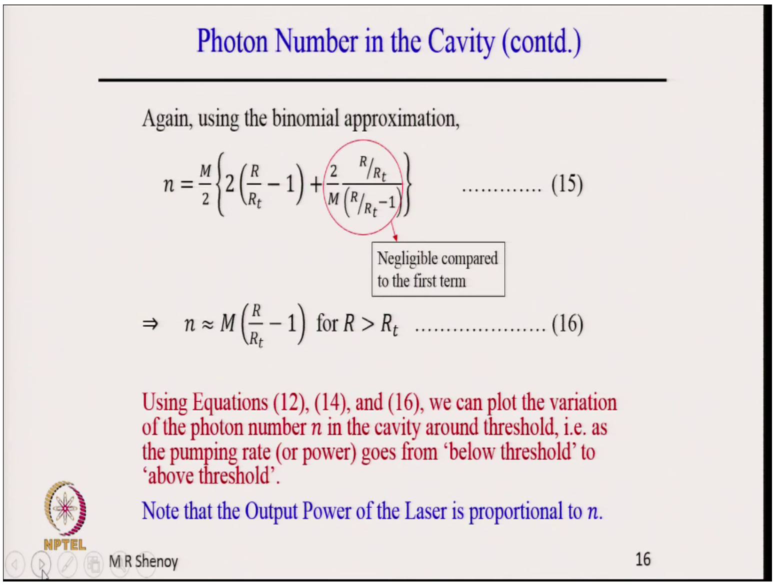

So, let us solve it. So, again using the binomial please see that M is in the denominator so as before so, here M is in the denominator and again this number is very small and therefore we can apply the binomial expansion and therefore, this will be one plus half into this term and that is what is written here. So, we get M by 2 into 2 into R by R t minus 1 you see there are two R by R ts here.

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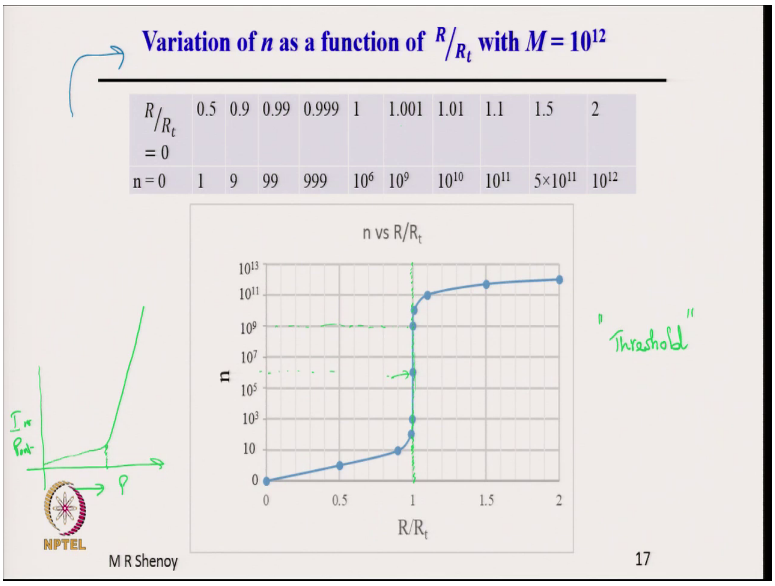

So, R by R t minus 1 plus R by R t minus 1 into 1 so, that is why we got two times of this. So, we have 2 times R by R t minus 1 plus this term. Now, note that this term is really negligible, because M is in the denominator and M is of the order of 10 to the power of 12. So, it is really negligible, but this is the complete expression, but we can even neglect that in that case we have only the first term so, 2 by 2 cancels and we have n nearly equal to m into R by R t minus 1 for R greater than R t.

So, we have got the expression for n, analytical expression for n when the pumping rate is below threshold equal to threshold and greater than threshold pumping rate. Therefore, using equations 12, 14, 16 which give answers for n, so, 16 is here others are in the previous slides. We can plot the variation of the photon number n in the cavity around the threshold, that is as the pumping rate or power goes from 'below threshold' to 'above threshold', when it goes from below threshold to above threshold.

And we have already discussed that the output power of the laser is proportional to n and that is why variation of n will also tell us variation of output power, because they are proportional alright here we are. So, finally, variation of n as a function of R by R t with we have assumed M is equal to 10 power 12. A different laser may have m is equal to 10 power 14 or m is equal to 10 power 10, but M is a large number.

Now, what is shown here is R by R t values 0.5, 0.99 like this and the corresponding n values here. So, at 0.5 R by R t is equal to 0.5 please see the expression 14. So, R by R t is equal to 0.5 if you substitute in this so, it is 0.5 1 minus 0.5 and 0.5 in the numerator and therefore this is just one. So, at 0.5 it is 1, if you put 0.9 you will get the photon number as 9, if you put 0.99 we are coming close.

What is this is the graph R by R t versus photon number this was our objective. Variation of photon number as we go from below threshold to above threshold and this is the threshold. So, this is R by R t is equal to one means at threshold.

So, you note that in these three expressions when we substitute, how the photon number changes at R by R t is equal to 1 that is at threshold pumping rate this is the point, 10 to the power of 6 you will see here 10 to the power of 6 and next point one point little bit above threshold it is already 10 to the power of 9.

So, this is the point. So, this is actually plotted so 10 to the power of 9 10 to the power of. So, what we see is when we go from below threshold to above threshold the photon number changes from orders of magnitude that is no lazing to lazing, it changes by orders of magnitude. Very very important this illustrates, the importance or real literal meaning of threshold and threshold is a most important property of the laser.

In fact, in many devices when you make a laser, whether the device is lazing or not is determined by the existence of a threshold.

So, you vary the pumping rate and then see the output and if the output suddenly changes for example, the slope changes for example, in a particular material you are continuously increasing the current or pumping power then, initially so, what is shown here is intensity I the output or P out or P out output power from the device, then initially it may be going like this almost like this and then suddenly it changes like this.

So, which means that this is the threshold point? So, this clearly indicates the onset of laser oscillations and we have shown here that how the photon number changes drastically, as we go from below threshold to above threshold. This illustrates really what this threshold is which is a very important property of a laser, we will discuss one last aspect in the next class and then go to characteristics of the laser.

Thank you.