Introduction to LASER

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Lecture - 23
Spectral Hole Burning

Welcome to this MOOC on Lasers. Today, we will see a specific aspect, namely Spectral Hole Burning.

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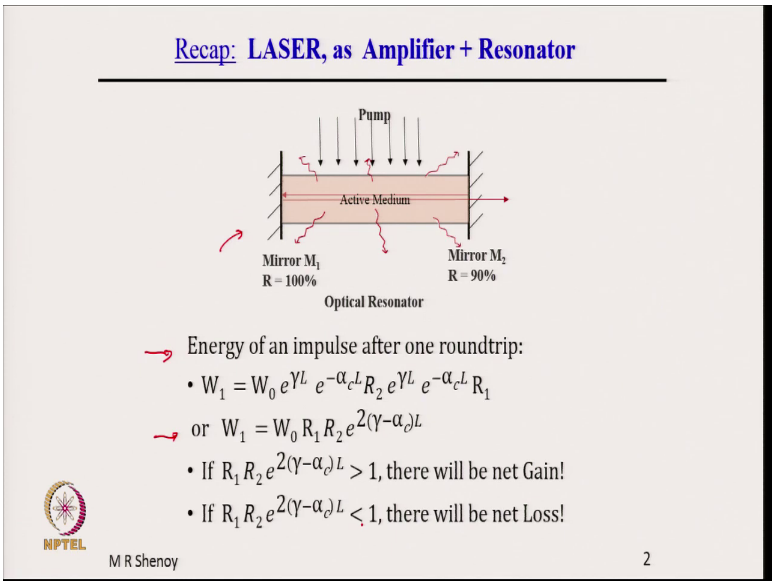

A very quick recap, laser as an amplifier plus resonator; in the last lecture, we saw how the laser, which is a oscillator is a combination of amplifier plus resonator. So, what is shown here is the laser medium, gain medium and two mirrors coated at the ends forming the resonator.

And we have seen that the energy of an impulse after one round trip is given by an expression here. And we have also seen that if R 1 R 2 into e to the power of twice gamma minus alpha c into L is greater than 1, then there will be net gain; otherwise if it is less than 1, then there will be net loss.

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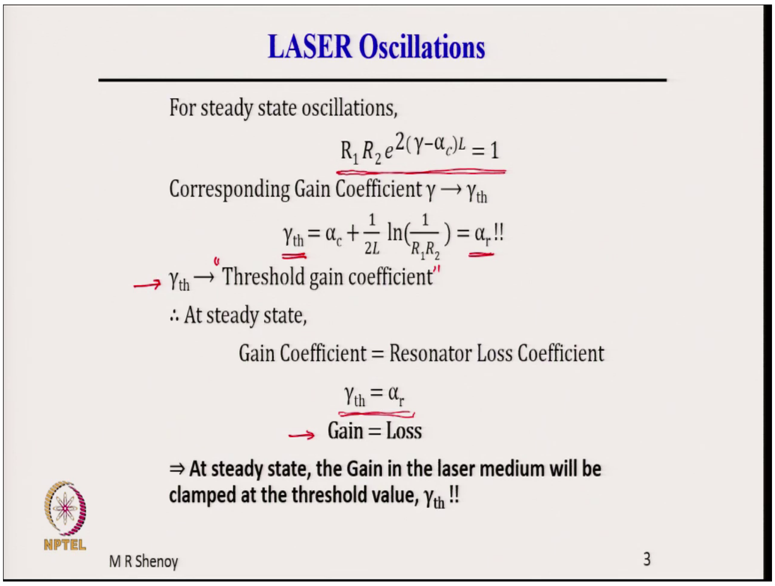

And for steady state oscillations, we want this factor to be exactly equal to 1. So, that there is neither buildup, nor attenuation. The corresponding gain coefficient gamma threshold is given by this expression, which we know that equal to the resonator loss coefficient. The gain coefficient which satisfies this condition, the threshold condition here is called the threshold gain coefficient.

And at steady state, the gain coefficient is equal to the resonator loss coefficient. And we have discussed that, no matter what the power is output power is, when a laser oscillates in steady states; the gain coefficient in the gain medium is clamped to the threshold value, which is equal to the resonator loss coefficient. And therefore, in simple terms, the laser physics or laser dynamics boils down to gain is equal to loss, alright.

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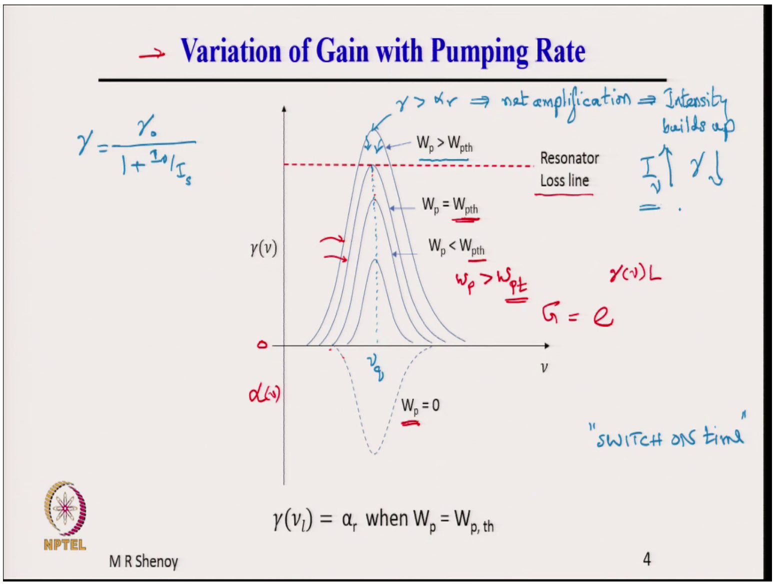

So, let us now recall the variation of gain with pumping rate. So, this diagram we discussed, when there is no pumping W p is equal to 0; it is absorption, so this is alpha of nu here and we have the absorption curve. So, alpha is negative. So, this is 0; therefore alpha is negative, which means it is attenuation.

So, please see that it is e to the power of gamma of nu into L, length of the medium is the gain factor. So, the gain G is equal to this; if gamma is negative, then we call it as alpha of nu and that corresponds to loss.

As the pumping rate increases, then the loss goes over to the gain; that is absorption changes over to amplification and a set of curves here shown, all these values correspond to W p greater than W pt, pt here is the threshold pumping rate, threshold pumping rate for amplification.

So, there are clear distinction, one pt, pt is the threshold pumping rate for amplification; pth , the W pth here refers to the pumping rate at threshold, that is when gain coefficient is equal to resonator loss coefficient. This W pt is when gain coefficient is 0; that is neither loss nor gain, that is W pt and W pth , th here standing for threshold.

So, at W p is equal to W pth, we have gain is equal to loss; we have discussed in the last lecture, why this is called loss line. So, what is shown is the resonator loss line and these are the gain curves, gain curves for different pumping rates. At W p is equal to W pth, the gain we have also assumed that there is only one oscillating mode; we just change the color, let there is only one oscillating mode at the line center. And therefore, for this mode, so this is nu q; for this mode, gain is equal to loss when W p is equal to W pth. If the pumping rate increases beyond W pth.

Then I have shown here the corresponding gain curve. But will it remain like that at steady state? Note that when W p is greater than W pth, the gain coefficient here gamma is greater than alpha r, which implies there is net amplification, ; which implies the intensity inside the resonator builds up, intensity builds up inside the resonator.

And when the intensity builds up; we know that as I nu increases, gamma decreases. This is because of the saturated gain coefficient; gamma is given by gamma 0, which is a constant, which is the small signal gain coefficient divided by 1 plus I nu by I s, where I s is the saturation intensity.

As I nu increases, gamma decreases; why I nu increases? I nu increases, because gamma is greater than alpha r and therefore, gamma decreases; which means the gain curve which is here will be pulled down. Gamma decreases because intensity increases, which means the gain curve is pulled down. So, note that at threshold gamma of nu is equal to alpha r; but if it goes beyond, pumping rate goes beyond the threshold pumping rate, gain temporarily increases, but then it is pulled back.

We have discussed in the last lecture, the corresponding intensity variations and how during the switch on time; we have discussed during the switch on time, switch on time, during the switch on time, there is a variation oscillation in I nu before it reaches it is steady state value.

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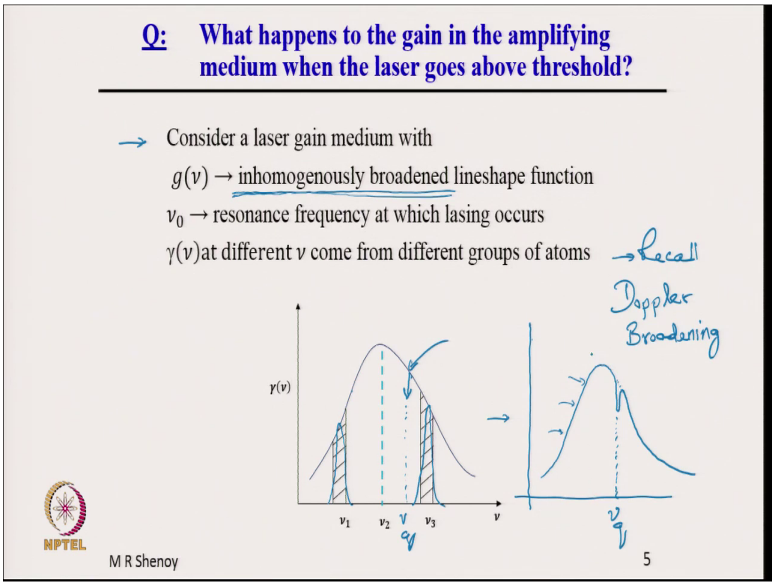

Let us discuss this issue further. So, what happens to the gain in the amplifying medium when the laser goes above threshold? So, first let us consider a laser gain medium, where g nu the line shape function is characterized by an inhomogeneous broadening. Or g nu is in homogeneously broadened line shape function.

Let nu 0 be the resonance frequency at which the laser is lasing. Gamma of nu at different frequencies nu comes from different groups of atoms in the case of in homogeneously broadened line shape. Recall Doppler broadening, which we have discussed in detail, recall Doppler broadening, where we have seen that different parts of the gain curve.

So, this is the gain curve is contributed by different groups of atoms. So, as shown here, different groups so, we have certain group of atoms contributing to gain here and another group of atoms contributing here and so on. So, nu 1, nu 2, nu 3 are different resonance frequencies, where amplification or gain is contributed by different groups.

And the importance of in homogeneously broadened line shape we have discussed that, if the gain is pulled down at any one place. Let us say the gain is pulled down here; then the gain at other frequencies, let us say this is nu q. If this is nu q and if the gain is pulled at nu q the lasing frequency; then the gain at other places do not get affected.

What does this mean? In such a case, I would have a gain curve, which would look like this. So, the same original gain curve here but, at the frequency nu q, the gain has been pulled down and then everywhere else it remains the same. So, this is the frequency nu q, where gain has been pulled down. If there was no intensity at nu q, then the gain curve would have continued as before.

But if the gain is pulled down because of intensity build up at a certain frequency nu q, then the gain profile will be pulled down at that region because of the gain saturation effect.

However, in the case of an in homogeneously broadened line shape function, which characterizes the gain profile; at all other places there is no effect on the gain profile. And we can see therefore, there is a dip which is occurring in the gain profile. Let us see this more carefully for the case where I was discussing about the line center with a single oscillating mode.

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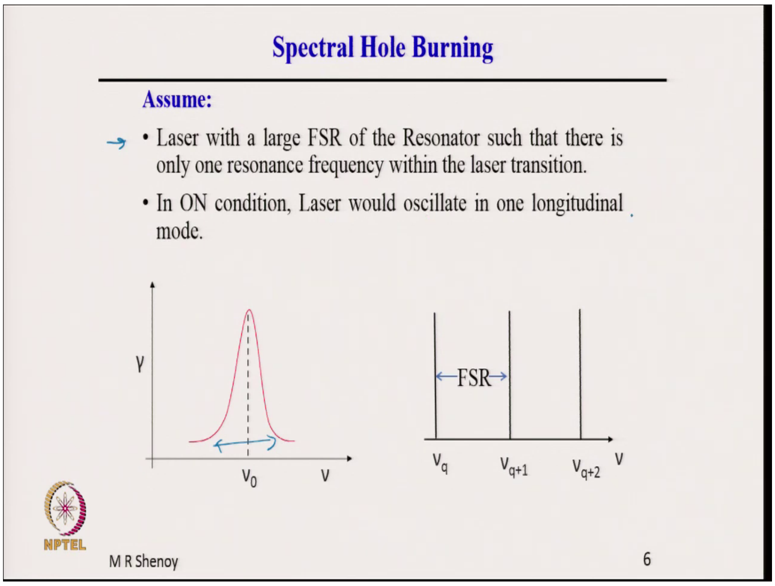

Now, assume a laser with a large FSR of the resonator; we have already discussed this that, if the FSR Free Spectral Range is sufficiently large within the laser transition or within the laser gain region, we could have only one oscillating longitudinal mode.

So, if FSR is sufficiently large, there is only one resonance frequency within the laser transition. In the on condition, therefore the laser would oscillate in one longitudinal mode. With this assumption, let us again discuss what happens to the gain when we cross the threshold.

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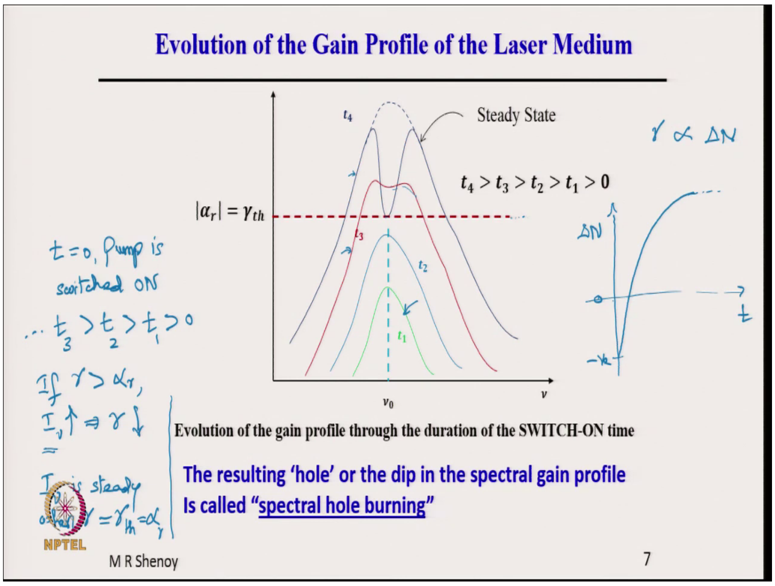

So, here it is. So, the pump is switched on at a t is equal to 0. So, at t is equal to 0 pump is switched ON is switched ON; t 1, t 2, t 3 are all. So, t 1 is greater than 0. So, t 1 is greater than 0, t 2 is greater than t 1, t 3 is greater than t 2 and so on. So, we see that when the pump is switched on at a certain time t 1, the gain profile has come up to this; because the pump is building up.

And therefore, it is building up the population inversion. We have already discussed this that the in the last lecture that, the population inversion is building up with the time. So, this axis here is time and this axis here is delta N. So, the population initially, so this is 0; initially it is negative when the pump is not there, but at t is equal to 0 when the pump is switched on, delta N increases and therefore, the gain increases.

That is what is shown here. As time increases, the gain profile continuously evolves and at a certain time t 3, it has crossed the loss line. When it has crossed the loss line as I discussed in the previous slide that, the gain is pulled down; because the intensity is building up, there is net amplification.

If gamma is greater than alpha r, a little bit of repetition; if gamma is greater than alpha r, then I nu builds up, increases, which implies gamma decreases, gamma will be pulled down. So, that is why we see that there is a small dip appearing; although the gain is getting pushed, but there is a dip appearing because of intensity built up. At a later time t 4, so this curve is for t 3. At a later time t 4, the gain at other places have been pushed up, pushed further; because the delta N has continuously increased.

And we know that gamma is directly proportional to delta N, the population inversion. But while the gain gets pulled up or increases at frequencies other than the oscillating frequency; at the oscillating frequency, the gain is pulled down. Because the intensity I nu is building up.

As I nu builds up the gain gets lowered and it gets lowered and finally, gets clamped to its threshold value; because at threshold value, there is no net amplification, gain is exactly equal to loss and I nu remains steady, I nu is steady. So, this details we discussed in the last lecture; I nu become steady when gamma gets clamped to its threshold value, gamma threshold which is equal to alpha. So, what has happened to the gain profile?

If you look at the gain profile, we see that in the profile there is a dip now and this is called a hole; a hole in the gain profile and the phenomena is called spectral hole burning.

The resulting hole or the dip in the spectral gain profile; it is spectral gain, profile because gain versus frequency. So, it is a spectral profile is called spectral hole burning. So, this is a very important phenomena, which I wanted to discuss here; spectral hole burning is because of gain saturation, alright.

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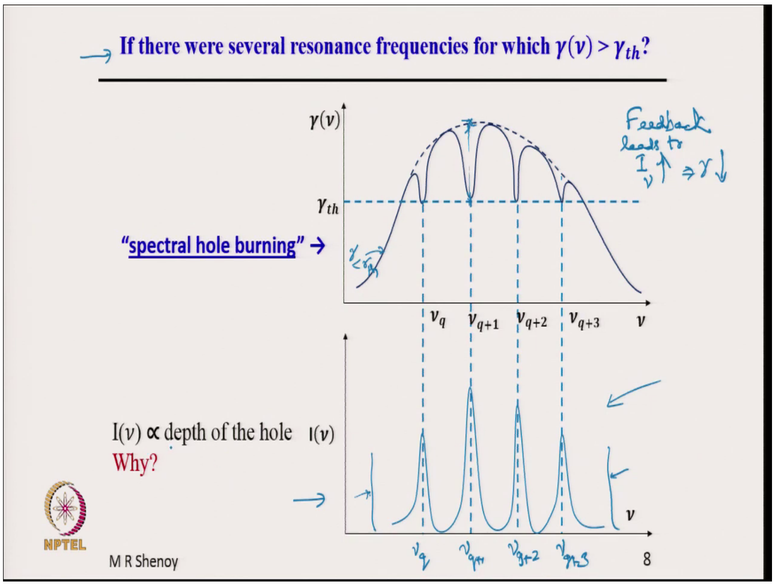

Let us go further. If there were resonance frequencies more than 1; that is there were several resonance frequencies for which gamma of nu is greater than gamma threshold initially, we see the gain profile. So, the gain profile would have, if the resonator were not there; then the gain profile would have been this dotted line here continuation.

So, this would have been the gain profile; but because of the resonator, there is a feedback and I nu building up. So, because of the feedback leads to I nu building up increasing and as I nu increases, this implies gamma decreases. And therefore at each mode, in the previous slide I discussed for one particular oscillating mode.

But now if the resonator supports more than one oscillating modes, for which the gain is greater than loss, there will be modes here also, here, here equally spaced. But the gain here gamma is less than gamma threshold; so gamma is less than gamma threshold, so there is no question of building up.

But for all resonance frequencies for which the gain is greater than the loss; we would get there are holes which are built up here, the spectral hole due to the phenomena of gain saturation. And if we see the output of the laser, I will discuss this aspect at a later stage and I will also show a real diagram corresponding to these longitudinal modes.

So, what are shown here are longitudinal modes nu q, nu q plus 1, nu q plus 2 and in this particular example, there are only four laser lines, four longitudinal modes for which the gain is greater than loss initially. For all the other modes which are on this side and this side, whether this one or this one; the gain is less than the loss and therefore, they die down.

And those longitudinal modes for which the gain is greater than the loss builds up. And that intensity of the mode is proportional to the difference in gain before the resonator builds up the I nu. So, the difference in gain this and this, that difference would correspond to the intensity of a particular longitudinal mode.

In other words, deeper the dip, higher will be the intensity here; we can see here the dip is small, therefore the intensity built up is small. Why is that? That is because when the dip is small; it means the gain has to be pulled down only by this much and therefore, I nu need not build up to a very large value and I nu will not build up to a large value, that is why the corresponding intensity is lower.

In this case for example, the gain has to be pulled down all the way from here up to the threshold line; that means the intensity should be sufficiently high to get it clamped at the threshold value. So, I will show a typical spectrum later in the course. So, the I nu is proportional to; this may not be directly proportional, but it depends on the depth of the hole as discussed.

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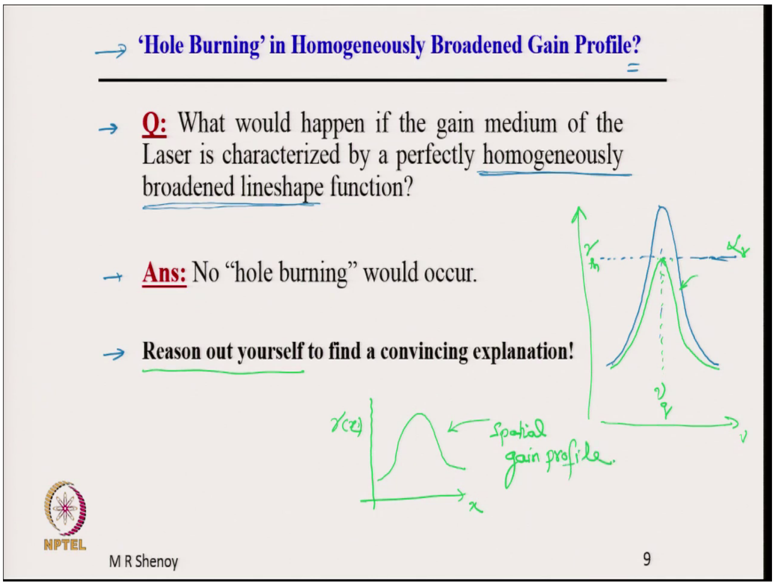

What about hole burning in the case of homogeneously broadened gain profile? There is a question mark. Will there be hole burning in homogeneously broadened gain profile? That is a medium which is characterized by homogeneously broadened line shape function. So, the question here is, what would happen if the gain medium of the laser is characterized by a perfectly homogeneously broadened line shape function?

The answer is given here that, no hole burning would occur in this case. And here what I have written is reason out yourself to find out a convincing explanation. If you go to the definition of a homogeneously broadened line shape function, you will find the answer why there is no hole burning.

There would not be any hole burning, but the entire gain profile. So, I will give the answer here, but reason out yourself; if the original gain profile, that is before the resonator was present. When the pump is switched on if it would have been this; then when the resonator is present, the gain would come down.

Let me use another color to the nu value, the whole gain profile would come down and get clamped to the threshold value. There is no hole burning, the entire gain profile will be pulled down, so that the gain gets camped down. If this is the oscillating mode nu q, then the gain gets clamped to it is threshold value, gamma threshold is equal to alpha r, resonator loss. So, reason out yourself.

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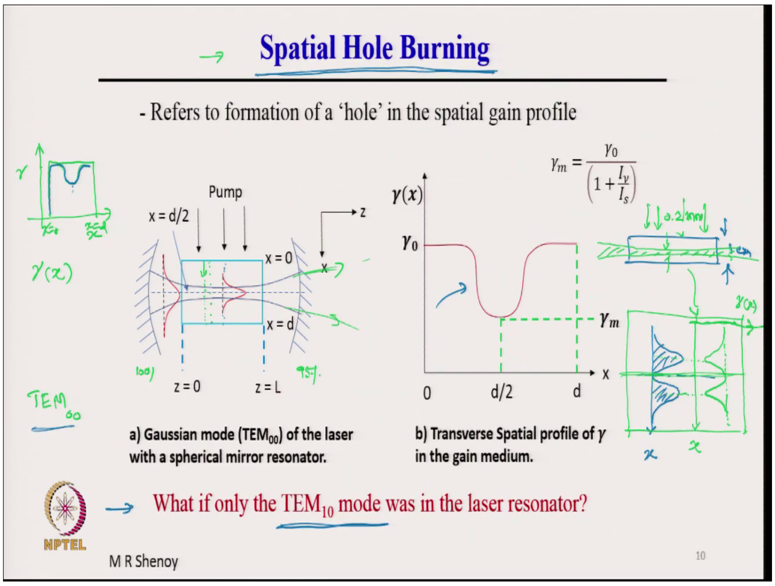

Alright, one important a parallel concept is spatial hole burning. As the name indicates earlier we discussed spectral hole burning; which means if I had a gain profile, that is gain versus frequency spectral. So, in the previous one, we see what is plotted here is frequency nu versus gain.

So, this is the spectral gain profile. But if I plot this gain as a function of spatial coordinate x for example, gamma of x for a particular frequency, at a particular frequency and then if the profile comes out to be like this; then this is called spatial profile, spatial gain profile; spatial gain profile. That is what I discuss in this slide.

So, look at the resonator here. So, this is a spherical mirror resonator supporting a Gaussian beam; the fundamental mode which is the TEM 00 mode, that is L is equal to 0, M is equal to 0. So, there is a TEM 00 mode is the Gaussian beam, which is propagating in the gain medium. Now, I have shown a gain medium here. So, the blue box is a gain medium, for example, N d YAG rod.

And the Gaussian beam is building up here and it reaches a steady state and at steady state if this is 100 percent reflecting and if this is partially reflecting, let us say 95 percent; then a fraction of light would come out here as the output. But if we look in the transverse coordinate; that is note that this is the z direction, always the propagation direction we will assume as z.

The transverse that is the perpendicular direction if it is x; then in the medium, in the medium if I were to travel from x is equal to 0 to x is equal to d, what would be the gain at different points? Gain at this value of x, this value of x at different values of x. And if I plot that gain gamma as a function of x; then what I would get is gamma of x.

If the laser beam was not there, if the pumping is uniform; I would have got a gain profile like this, gain would be constant with x. So, this would be the gain. So, this is a box type variation. So, x is equal to 0 and x is equal to d, x is equal to d and this total width is d, the width of the laser rod or laser medium.

I would have got this, gain versus x if I plot. But if the laser beam is present; then in certain portion of the rod, there is an intense laser beam, which depletes this gain. Just like spectral hole burning, it depletes the gain; because there is intense laser beam which brings down the gain and therefore, this would be, let me use a different color.

So, this would be the new gain profile in the presence of the laser beam and this dip would be like inverse of the Gaussian. So, this is Gaussian and this would be almost inverse; because higher the intensity, lower will be the gain. So, higher the intensity, lower will be the gain. And far away there is no intensity here; the Gaussian is present only in between, this is indeed true in practice.

For example, if we take an N d YAG rod. So, let me show here, if we take an N d YAG rod; typically this diameter is about 1 centimeter. And the laser beam, the laser beam which is building up here and coming out, the diameter here is typically about 0.2 millimeter.

We have taken an example earlier that is very very small, less than fraction of a millimeter; which means the beam is primarily confined to a small area here and rest of the medium is not exposed to the radiation, whereas pumping takes place everywhere. And therefore, there will be gain everywhere; but the gain will be depleted, where the intensity of the beam is present.

And that leads to this gain profile which we have here, shown here is a dip in spatial coordinate; hence the name spatial hole burning. So, spatial whole burning refers to depletion of gain with spatial coordinate x. One last point, what if only the TEM 10 mode were to propagate? What I have discussed here is the TEM 00 mode, which is the Gaussian mode; what about TEM 10 mode?

The TEM 10 mode will have a profile like this. So, the intensity profile I am plotting. So, the intensity profile will be like this. There will be 10 when I travel along the x direction; that is why 1, 1 here stands for the number of zeros and this is there is no 0 in the y direction.

So, if this is the intensity profile in the same medium. So, let me show the medium also here again with a different color. So, this is the box, the medium this is I have magnified the same medium here. So, this is the same medium. And this is the profile of the beam which is going back and forth; then note that there is no depletion of gain here in the midpoint, because the intensity has gone down to zero.

But the intensity is high on both the sides. So, what kind of profile can we expect? So, we can expect a profile. So, let me show first the profile that we would get when there is no beam, which means it is the uniform profile. And if the intensity is present; wherever the intensity is maximum, the gain would drop down and come back and again drop down and come back. So, this would be the gain profile. So, what I have plotted is the transverse direction x versus the gain gamma of x.

Note that, there are two dips corresponding to the intensity peaks here. Similarly, you can reason out for other gain profiles as well, modal profiles TEM 11, TEM 20, TEM 02 various profiles. What will be the profile of the spatial hole burning? So, spectral hole burning and spatial hole burning; spectral hole burning occurs in the case of inhomogeneously broadened line shape function.

And spatial burning has nothing to do with homogeneous or inhomogeneous; spatial burning takes place because of the intensity. In both cases, whether it is homogeneously broadened or inhomogeneously broadened, in both cases we will see a hole burning in the spatial gain profile. We will stop here and take up the other issue in the next lecture.

Thank you.