

**Introduction to LASER**  
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**Lecture - 22**  
**Laser Oscillations & The Threshold Condition**

Welcome to this MOOC on LASERS. Today, we will start part 4, the laser. The first topic here is Laser Oscillations and the Threshold Condition. Let us see.

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**PART IV: The Laser**

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**Recall**

**PART I: Interaction of Radiation with Matter**


- Einstein Coefficients
- Condition for Amplification
- Atomic lineshape function  $g(\nu)$

**PART II: Scheme of Amplification**

- Rate Equations → 3-level system
- 2-level system
- + Laser Amplifier

→ **PART III: Optical Resonator**

- Plane Mirror- and Spherical Mirror Resonators
- Resonance Frequencies & the Field Distributions
- Gaussian mode of the Spherical Mirror Resonator

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So, very quick recap in part I, we discussed about interaction of radiation with matter. We saw the Einstein coefficients and obtained the condition for amplification. We have shown that population inversion is the necessary condition for amplification. Then we went on to see or discuss the atomic line shape functions that is  $g_{\nu}$ , so, the atomic line shape function  $g_{\nu}$ .

And, in part II we discussed about the scheme of amplification. So, how to obtain population inversion, so, that is what we mean by scheme of amplification and we have looked at 2-level, 3-level and 4-level lasers. We have written the rate equations and then discussed these various systems and then in particular the laser amplifier including fiber amplifier we have discussed in part II.

Part III was dedicated to optical resonators plane mirror and spherical mirror resonators; the resonance frequencies and the field distributions, these are the longitudinal modes and the transverse modes and in particular, we focused on the Gaussian mode of a spherical mirror resonator, because of the importance of the Gaussian mode.


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### The Laser (Oscillator)

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- The Laser → Laser Amplifier + Optical Resonator
- Gain Coefficient :  $\gamma(\nu) = \frac{(c/n)^2}{8\pi\nu^2 t_{sp}} g(\nu) [N_2 - N_1]$   
 $= \sigma(\nu) \Delta N$
- $\sigma(\nu)$  → Cross section;  $\Delta N$  positive → Population inversion
- Note:  $\sigma(\nu) \propto g(\nu)$   
 Normalized lineshape function
- Two lineshape functions discussed: 1. Lorentzian, 2. Gaussian

**NOTE:** For many practical lasers, the lineshape function of the laser transition is described as combination of several Lorentzians and/or Gaussians!



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Today, we will take up laser. As we had discussed in the beginning laser is amplifier plus oscillator. So, the laser is laser amplifier plus optical resonator ok. Now, we will see the laser.

Laser I have written here in the bracket oscillator, because when we say the laser we are referring to the source laser and in that sense it is an oscillator.

Whenever we will discuss about the amplifier, we will refer to it as laser amplifier and if we simply say laser it refers to the source or the oscillator and as we discussed the oscillator comprises of the laser amplifier plus the optical resonator. We have obtained the expression for the gain coefficient  $\gamma$  which is equal to here the expression is here, which we write as  $\sigma \Delta N$ , where  $\Delta N$  is  $N_2$  minus  $N_1$ , the population inversion term and the rest of the term is  $\sigma$  of  $\nu$ .

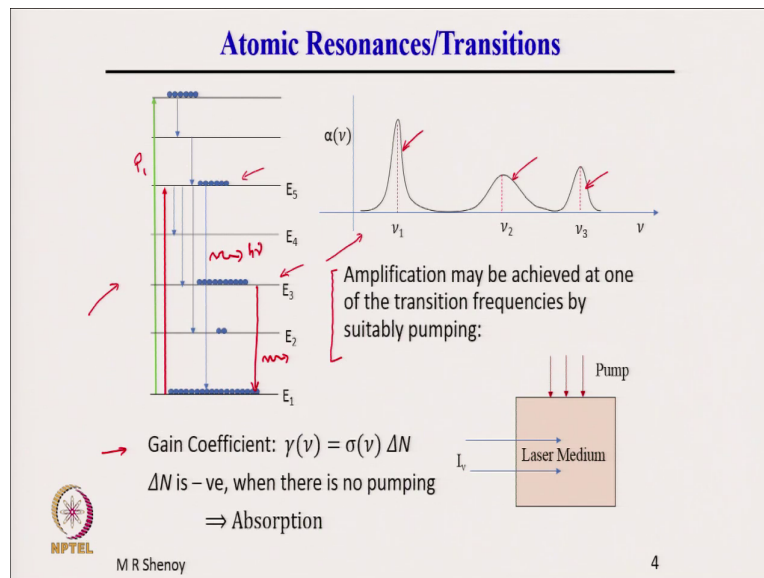
$\sigma$  is known as the cross section, cross section for the interaction and  $\Delta N$  is positive implies there is population inversion.  $\Delta N$  is positive it means  $N_2$  is greater than  $N_1$  and that corresponds to population inversion which is necessary to obtain the gain.

And, note that  $\sigma$  of  $\nu$  is proportional to  $g$  of  $\nu$  here,  $\sigma$  of  $\nu$  is this term here all this term is  $\sigma$  of  $\nu$ , it is proportional to  $g$  of  $\nu$  and  $g$  of  $\nu$  is the normalized line shape function. In particular, we have discussed two line shape functions in detail – the Lorentzian in homogeneous broadening and the Gaussian due to inhomogeneous broadening, we had taken the example of Doppler broadening in particular.

However, a note here, says that for many practical lasers, the line shape function of the laser transition may be described as a combination of several Lorentzians and or Gaussian. The point is one Lorentzian or one Gaussian may not be able to represent the line shape function of the medium the laser medium; in particular, this is true for solids.

And in such cases, when some analytical solutions have to be found out or analysis has to be carried out usually, the line shape function is represented as a superposition or a combination of several Gaussians or Lorentzians, alright.

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The atomic resonances and transitions what we are referring here, is an atomic system that is characterized by several atomic energy levels and the red lines here, indicate pump that is excitation mechanism.

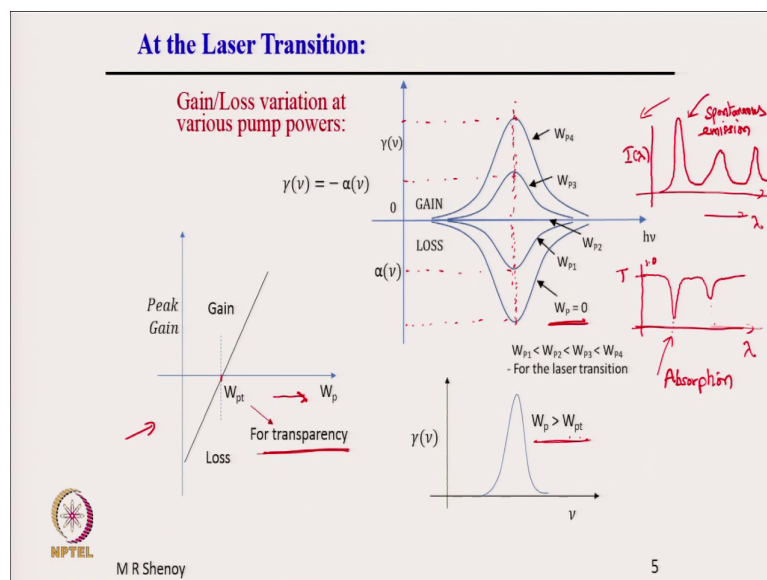
So, let us say there is a pump  $P_1$  which excites from  $E_1$  to a higher state here, then the atoms which are excited to the higher state would make transitions downward transitions with different probabilities, accordingly different levels are characterized by different lifetimes. And, depending on the lifetime there may be building up for example, in this diagram  $E_3$  and  $E_5$ , there is accumulation of atoms taking place. This depends on the lifetime of the levels.

And, there are several transitions which take place. For example, they may represent each transition would represent a resonance centered around a frequency  $\nu_1$  or  $\nu_2$  and  $\nu_3$  and so on. So, the amplification may be achieved at one of the transition frequencies by suitably

pumping. And, again the condition for population inversion has to be met to achieve amplification.

The gain coefficient is  $\gamma(\nu)$  is equal to  $\sigma(\nu) \Delta N$ .  $\Delta N$  is negative when there is no pumping because we know the Boltzmann statistics tells us that the dependence is exponential minus  $E$  by  $K T$  which means, the lower levels which have lower  $E$  will have higher population. In fact, we have put some numbers and seen that number of atoms in the lower levels are much much higher than that in the upper levels.

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At the laser transition, what is shown here in this diagram is one of the transitions I showed that in the previous there can be transitions entered around different resonance frequencies. Each of this is called as a resonance. So, this is a resonance, this is a resonance.

Now, let us look about a resonance, a particular resonance centered around a certain frequency, where let us assume that laser transition is possible or lasing is possible. So, what is shown here is the gain or loss, as we know that when there is no pump  $W_p$  is 0 when the pumping is 0, then there is loss.

So, this is the resonance, the resonance is inverted. If you recall the absorption curves; so, if we just recall the absorption dips of an atom then we have transmission and wherever resonance comes there is a dip. So, different dips corresponding to different absorption resonances.

Now, in the previous diagram, what I had shown is the absorption here  $\alpha$  of  $\nu$  is the absorption at the various resonances. What is shown here, is the transmission. So, this is transmission intensity maximum is 1. So, this is 1.0 and dips corresponding to different resonances.

Now, if we look at one of the laser resonances; so, this is absorption  $\alpha$  of  $\nu$  and transmission, if we had seen the spontaneous emission spectrum then we would have got we would have got resonances like this inverted. So, these are the resonances of the spontaneous emission spectrum, which you see in a spontaneous emission spectrum, this is again  $\lambda$ .

So, what is plotted is wavelength versus intensity  $I$  of  $\lambda$  or  $\nu$  versus  $I$  of  $\nu$ , then we see such resonances. This is spontaneous emission spectrum, this is absorption spectrum; dips corresponding to absorption. And, this corresponds to spontaneous emission; an atomic system which is excited gives out spontaneous emission and we see resonances at different wavelengths corresponding to specific transitions from different levels.

So, here we excite atoms from the ground state to an upper state, from there they make downward transitions and when they make downward transitions there are photons which are emitted. So, photons are emitted when transitions take place; for example, from here to here a transition comes like this, then photons are emitted around a certain frequency. These

resonances in the case of a spontaneous emission spectrum are the resonances corresponding to these specific transitions ok.

So, in this diagram, we have picked up one resonance, where when there is no pump there is absorption and the absorption coefficient is maximum around a certain wavelength. So, this is  $h\nu$  or wavelength  $\lambda$ . As you pump the absorption reduces and further pumping absorption becomes almost 0, which means it is almost transparent and further pumping the system starts giving out radiation, because there is a certain gain coefficient.

So, this aspects we have discussed in detail earlier and this is shown here. Now, for a certain pumping rate which is above  $W_{pt}$ ;  $W_{pt}$  corresponds to the pumping. So, what is shown here is peak gain versus  $W_p$ . So, this is for a laser transition and therefore, as the pumping power increases from 0 the system goes from a lossy system with  $\alpha$  of  $\nu$  here, towards 0 and then this is increasing.

So, what is plotted in this diagram is peak gain, peak gain corresponding to this line that I have shown. So, this is the peak position of the peak or resonance peak, and the absorption is maximum at the same place where gain is also maximum. When pumping is 0 that is no pumping, the transition corresponds to absorption, but as pumping increased then it goes over to the gain and now, you have peak gain.

So, what is plotted in this diagram here, is the peak gain that is here, it is peak loss, loss and then here it is gain. So, as you pump as you increase the  $W_p$  pumping powers. So,  $W_{p1}$  is less than  $W_{p2}$  or  $W_{p2}$  is greater than  $W_{p1}$ ;  $W_{p3}$  is greater than  $W_{p2}$  and so on.

So, as  $W_p$  increases we go from an absorbing system to an amplifying system. The pumping power the rate  $W_{pt}$  refers; when there is no gain no loss and that is called the situation is called transparency. So,  $W_{pt}$  is the rate that is pumping rate for transparency and if the pumping rate goes beyond  $W_{pt}$  is increased beyond  $W_{pt}$ , then there will be gain in the system.

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**Recap: The Optical Resonator**

$\rightarrow \alpha_r = \alpha_c + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right);$

$\rightarrow \nu_q = q \cdot \nu_F, \nu_F = \left[\frac{c}{2nL}\right]$  FSR

$\rightarrow \nu_F = \left[\frac{c}{2nL}\right] \sim 10^9 \text{ Hz} \rightarrow \Delta\lambda \sim 0.01 \text{ nm}; \nu_q \rightarrow 10^{14} \text{ Hz} \rightarrow \lambda \sim 1000 \text{ nm}$

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So, this is at the laser transition as a function of the pumping rate. Now, let us see further. Recap the optical resonator. So, the previous discussion was for the medium laser medium and now, the optical resonator, a very quick recap. This is the loss coefficient  $\alpha_r$  which is equal to resonator loss coefficient equal to  $\alpha_c$  the intrinsic loss coefficient plus  $\frac{1}{2L} \ln \frac{1}{R_1 R_2}$ . Here  $R_1$  and  $R_2$  are the reflectivity of the mirrors.

$\nu_q$ , the resonance frequencies, the resonators are characterized by resonance frequencies which is equal to  $q$  into  $\nu_F$ . So,  $\nu_F$  here is the free spectral range. So, this is the FSR free spectral range, which is equal to  $\frac{c}{2nL}$ ;  $c$  is the velocity of light,  $n$  is the refractive index of the medium and  $L$  is the length of the resonator,  $q$  is an integer.

Now, let us look at some typical numbers. If you take some typical numbers for the resonator say  $L$  is equal to 15 centimeter and refractive index equal to 1, then you will see that  $\nu_F$



comes out to be  $10^9$  hertz. The corresponding  $\Delta\lambda$  is of the order of  $10^{-2}$  nanometer, that is 0.01 nanometer and remember, the frequency  $\nu_q$ , this is really not required here.

So,  $\nu_F$  is of the order of  $10^9$  hertz that is the separation between adjacent resonances. Please see the resonances are shown  $\nu_q$ ,  $\nu_q + 1$ ,  $\nu_q + 2$  and so on. The resonances, what resonances are these? These are the cavity resonances. This is not the atomic resonance resonances.

So, cavity resonances these are the atomic resonances. So, this is the emission spectra, spontaneous emission spectra. These resonances are cavity resonances which means in the cavity a very quick recap in the cavity those frequencies which build up as they go back and forth and we have discussed this in detail that the round trip phase must be integral multiple of  $2\pi$ .

The frequency is light frequency itself  $\nu_q$ , the light frequency itself is of the order of  $10^{14}$  hertz and wavelengths of the order of 1000 nanometer. Now, if we see the spontaneous emission spectrum from an atomic system, then for example, this spontaneous emissions spectrum approximately corresponds to the neodymium Nd in YAG.

We see that there are three resonances here, or Yb in YAG typical of Nd in YAG and Yb in YAG; Nd is neodymium in YAG host yttrium aluminum garnet and Yb is terbium in YAG host. So, typically these are the resonances which are centered around wavelength 800 nanometer, 975 nanometer and 1060 nanometer. These are the resonances atomic resonances. What is shown here below is the cavity resonances.

When you place this Nd:YAG rod in a cavity, let us say the mirrors are right here, coated on the rod itself right here; then the resonances which are shown. So, this is Nd in YAG. The resonances shown here are due to the YAG atomic resonance. The resonance is shown here what is this is frequency axis  $\nu$  and these are the frequencies  $\nu_q$ ,  $\nu_q + 1$  and so on the various longitudinal modes.

These are the resonances because of the cavity. So, this is the cavity that is the difference. Please see both are called resonances. So, this is cavity resonance and this is atomic resonance. Now, the atomic resonances are well separated typically the separation for example, in wavelength is around 175 nanometers.

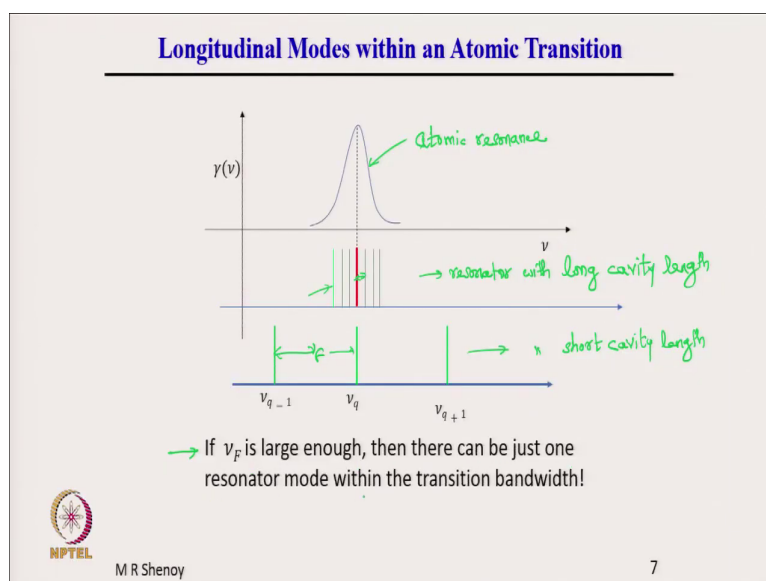
If you see the separation corresponding to  $\nu F$ ;  $\nu F$  is this, this frequency is  $\nu F$ . If you see the corresponding  $\Delta \lambda$  this comes out to be 0.01 nanometer whereas, the atomic resonances are separated by large  $\Delta \lambda$  175 nanometer, 85 nanometer and so on.

What does this mean? This means that within an atomic resonance within one atomic resonance, there will be large number of longitudinal modes of the resonator or cavity resonances of the resonator within one transition. Why am I interested in this, because laser will be at one transition lasing will take place at one transition, let us say 800 or 1060 in the case of Nd:YAG, then in one atomic transition there will be large number of resonances of the cavity.

If we place this inside the cavity the original spectrum, the emission spectra which is shown here is because, of spontaneous emission and when the laser starts lasing, we will see that the cavity resonances will appear and we will discuss this in more detail. So, in this discussion, the main point is that atomic resonances correspond to transitions between different energy levels and cavity resonances refer to the resonance frequencies of the resonator – 1.

2nd – atomic resonances are well separated in wavelength whereas, cavity resonances the separation between them is very small in wavelength, which means if we look at a particular laser transition within that transition there will be large number of in general there are techniques by which we can have only one, we will discuss that later, but in general there can be large number of cavity resonances corresponding to one laser transition.

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So, if I now look at only one atomic resonance, this is the atomic resonance. So, one atomic resonance or the laser transition, then there are many cavity resonances here, these lines are cavity resonances and if we can somehow increase the separation between the cavity resonances that is if  $\nu F$ , the separation between cavity resonances is  $\nu F$  free spectral range if  $\nu F$  is large enough, then there can be just one resonance.

For example: you see here  $\nu F$  is so much, large. This is a different resonator, this is a different, this is a resonator with long cavity length long cavity length when  $L$  is large, then  $\nu F$  is small cavity length just as an example. If I take this is a resonator with short cavity length when  $L$  is small short cavity length this is really true in the case of semiconductor lasers where the cavity length is typically 100 to 500 micrometer short cavity length.

So, if the cavity length is short the separation can be large and within one atomic resonance we can have just one longitudinal mode. Why? The discussion is because, in the next discussions I will consider that there is only one longitudinal mode in the atomic resonance or in the laser transition, that makes the discussion easier, subsequently we will bring all the multi longitudinal transitions.

If  $\nu F$  is large enough, then there can be just one resonator mode within the transition. bandwidth alright.

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**LASER: Amplifier + Feedback**

Consider a 'Laser Resonator' comprising of a pumped active medium. Let  $W_0$  be the energy of an impulse at the mirror  $M_1$ .

Energy after one roundtrip:

- $W_1 = W_0 e^{\gamma L} e^{-\alpha_c L} R_2 e^{\gamma L} e^{-\alpha_c L} R_1$
- or  $W_1 = W_0 R_1 R_2 e^{2(\gamma - \alpha_c)L}$

→ If  $R_1 R_2 e^{2(\gamma - \alpha_c)L} > 1$ , there will be net Gain!  $N_1 > N_0$ ;  $N_1 < N_2 < N_3 \dots$

→ If  $R_1 R_2 e^{2(\gamma - \alpha_c)L} < 1$ , there will be net Loss!  $N_1 < N_0$ ;  $N_2 < N_1 \dots$

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With this basics and revision, let us go to laser. So, laser is amplifier plus feedback. Consider a laser resonator comprising of a pumped active medium. So, what is shown here is the active

medium and we have shown the mirrors as if they are coated right at the end of the active medium.

In general, the mirrors could be separate then the length of the active medium and mirror can be different which is true in most of the lasers, but for simplicity we have now assumed that the mirrors are coated right at the ends of the active medium. This is also a practical situation; this is not a hypothetical situation. This is also a practical situation

If  $W_0$  is the energy of an impulse at the mirror  $M_1$  here, to begin with at  $t$  is equal to 0 let  $W_0$  be the energy of an impulse at one of the mirrors, it is now propagating to the other end. Then as the impulse propagates to the other end, so, the impulse starts from here and it is propagating to the other end, when it propagates it gets amplified. This is a pumped laser medium which means it is amplifying, it is ready to amplify.

Therefore, when the energy starts propagating to the other end it gets amplified by a factor  $e^{\gamma L}$  to the power  $\gamma L$ ,  $L$  is the length of the pump. So, this is  $L$  and therefore, the total gain in propagating from this end to this end is  $e^{\gamma L}$ , but it also gets multiplied by  $e^{-\alpha c L}$  as we have already seen an intrinsic attenuation factor  $\alpha c$ .

Therefore,  $e^{\gamma L - \alpha c L}$  power, then at the other end there is a mirror of reflectivity  $R_2$ . So, this is mirrors  $M_1$  and  $M_2$ . So, let us say this is  $R_1$  and this is  $R_2$  the impulse gets reflected and propagates backward. As it propagates backward it again gets amplified by a factor  $e^{\gamma L}$  to the power of  $\gamma L$ , attenuated by a factor  $e^{-\alpha c L}$  and on reflection it gets multiplied by a factor  $R_1$ , where  $R_1$  is the reflectivity of the mirror.

So, here  $R_1$  and  $R_2$  are the reflectivity of the mirrors. Therefore,  $W_1$  is equal to  $W_0$  into  $R_1 R_2 e^{2\gamma L - 2\alpha c L}$ . If this factor here  $R_1 R_2 e^{2\gamma L - 2\alpha c L}$  is greater than 1, then there is net gain  $W_1$  will be greater than  $W_0$  that is  $W_1$  is greater than  $W_0$ . And, if  $R_1 R_2 e^{2\gamma L - 2\alpha c L}$  is less than 1,  $W_1$  is less than  $W_0$ .

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### The Threshold Condition

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→ For steady state oscillations,

$$R_1 R_2 e^{2(\gamma - \alpha_c)L} = 1$$

→ Corresponding Gain Coefficient  $\gamma \rightarrow \gamma_{th}$

$$R_1 R_2 e^{2(\gamma_{th} - \alpha_c)L} = 1, \quad \gamma_{th} = \alpha_c + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = \alpha_r !!$$

*Resonator Loss Coefficient*

→  $\gamma_{th} \rightarrow$  Threshold gain coefficient


∴ At threshold,

Gain Coefficient = Resonator Loss Coefficient

$$\gamma_{th} = \alpha_r$$

**Gain = Loss**

*Threshold condition*



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If it is equal for steady-state oscillations we will show this that this must be equal  $R_1 R_2$  into  $e$  to the power of why? Because if this is the condition which is all the while satisfied that means, the light intensity is continuously building up with the time because  $W_1$  is greater than  $W_0$ , in the second round trip  $W_2$  will be greater than  $W_0$ ,  $W_3$  will be greater.

So, if this is the case then  $W_1$  less than  $W_2$  next round  $W_3$  will be greater and so on. So, the light intensity will keep on building up there is no steady state. If we look at this situation, then  $W_1$  will be less than  $W_0$ ;  $W_2$  will be less than  $W_1$ ;  $W_3$  will be less than  $W_2$  and so on.

And, in this case if I were to plot the corresponding energy or intensity at that then intensity will drop down continuously like this; in the other case the intensity will continuously

increase. In other words, with the time so, this is with the time and what I am plotting is either intensity or W energy W of t.

So, there is no steady state. This is continuously attenuating, this is continuously amplifying. Of course, it cannot continuously amplify, there are saturation effects which take this we will discuss that, but if this happens to be equal at some point, let us say initially it is building up, but after sometime at some point it becomes equal, then the intensity or the energy would remain constant.

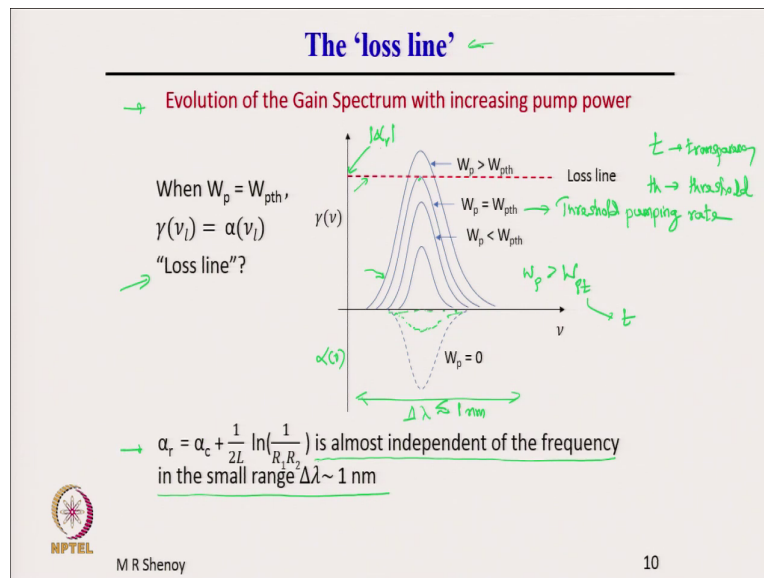
And, this is corresponding to the saturation, where  $R_1 R_2$  into  $e$  to the power twice  $\gamma$  minus  $\alpha c$  into  $L$  is equal to 1, where neither gain nor loss. So, that is the situation. This is with time that that may come after several round trips. So, that is what is indicated here that for steady-state oscillations this must be equal to 1.

And, the corresponding gain coefficient  $\gamma$  is called the threshold gain coefficient the  $\gamma$  which satisfies this equation  $\gamma$  which satisfies this equation is called  $\gamma$  threshold. And, then what we see is that  $\gamma$  threshold if I write this as  $R_1 R_2$  into  $e$  to the power twice  $\gamma$  threshold minus  $\alpha c$  into  $L$  equal to 1. So, then I get expression for  $\gamma$  threshold as this.

And, what is this? This is nothing, but  $\alpha R$ , the resonator loss coefficient is equal to  $\alpha R$  which is the resonator loss coefficient. Thus  $\gamma$  threshold which gives this stable condition is called the threshold gain coefficient and at threshold, the gain coefficient is equal to resonator loss coefficient.

So, the threshold condition is this that the gain coefficient  $\gamma$  threshold is given by this expression which is equal to resonator loss coefficient or the threshold condition is here. Gain coefficient is equal to loss coefficient or  $\gamma$  threshold is equal to  $\alpha r$ . So, this is the threshold condition gain is equal to loss threshold condition. We will see its importance and how it leads to steady-state oscillations.

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Let us proceed further now. What I am now, discussing is what is shown in the diagram is evolution of the gain spectrum with increasing pump power. So, we have already discussed this when the pump is off the medium is absorbing. So, what you have here is alpha of nu. So, this is alpha of nu because every medium will absorb at the resonance at the atomic resonance if there is no pumping.

As you pump then this curve we already discussed this goes like this and finally, becomes flat and then it is amplifying. So, all the  $W_p$ 's which are shown are  $W_p$  are greater than  $W_{pth}$ . what is this  $t$ ? This is for transparency. We will use  $t$  for transparency the subscript transparency and  $th$  for threshold.

So, all the pump powers which are here are greater than because, then only we have gain, but  $W_p$  is less than  $W_{pth}$ . What is the threshold pumping rate? Threshold pumping rate



is such that the gain coefficient here will be equal to the loss coefficient. What is this? The value here is  $\text{mod } \alpha R$  because,  $\alpha$  is actually negative that is why I have written  $\text{mod } \alpha R$ . This is called the loss line.

When the pumping rate at which the gain coefficient becomes equal to the loss coefficient that is called the threshold pumping rate. So, this is the threshold pumping rate threshold. So, threshold for lasing threshold pumping rate and if you pump more that is if the gain were to cross the loss line, we will see there is a new dynamics which takes place and this discussion we will do in the next class.

But, let us first discuss what is this couple of points which I want to discuss here is why loss line? What is this? This is the gain variation; gain with frequency or wavelength, loss is shown as a line because over the laser transition this wavelength region is very small. This whole this is of the order of  $\Delta \lambda$  is of the order or less than 1 nanometer or 0.1 nanometer.

We have already seen corresponding to the line width here it is of the order of 0.1 or 0.01 nanometer. So, this region of wavelength, the frequency range corresponding to a  $\Delta \lambda$  of about 1 nanometer over this wavelength range there is no change in  $\alpha r$ . So, see here,  $\alpha R$  is almost independent of the frequency in the small range of approximately 1 nanometer. That is why we have shown the loss as loss line.

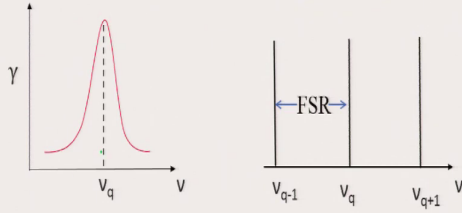
So, this is the loss line. We will be using this loss line several times in the next couple of next few classes and that is why I thought here, we should explain why loss is shown as a line. It simply says that loss is independent of wavelength. This is not true in general, but over the range of frequencies corresponding to an atomic transition, the loss is independent of the wavelength.

That is why we draw loss as a horizontal line which helps us in the subsequent discussions alright.


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### Laser Dynamics at 'Switch ON'

- • Assume that the gain medium is characterized by a homogeneously broadened transition.
- • Laser with a large FSR of the Resonator such that there is only one resonance frequency within the laser transition.
- In ON condition, Laser would oscillate in one longitudinal mode, say  $\nu_q$ .



The diagram consists of two plots. The left plot shows a gain curve  $\gamma$  versus frequency  $\nu$ . The curve is a smooth, bell-shaped peak centered at  $\nu_q$ . A dashed vertical line marks the peak at  $\nu_q$ . The right plot shows the laser's resonance frequencies  $\nu_{q-1}$ ,  $\nu_q$ , and  $\nu_{q+1}$  on a frequency axis  $\nu$ . The spacing between these frequencies is labeled as FSR (Free Spectral Range).

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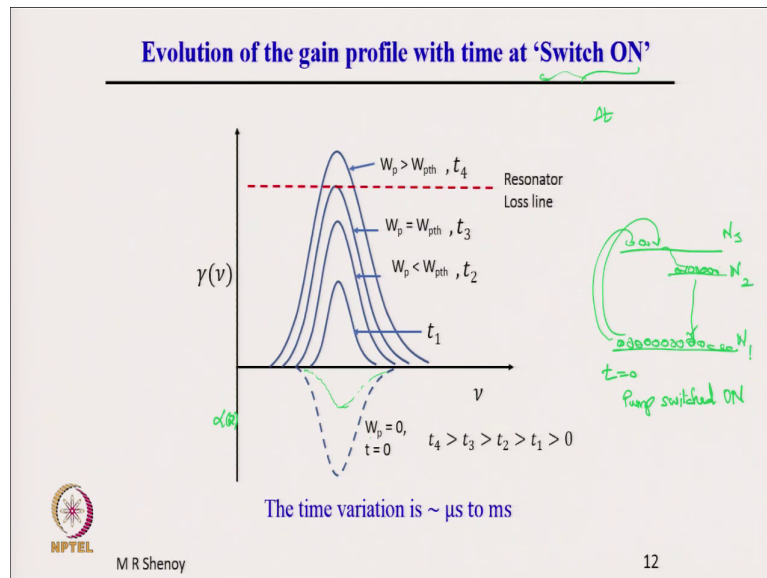
So, laser dynamics at switch ON. So, let us now, discuss the laser dynamics at switch ON. I must say that we will discuss this qualitatively, here without mathematics, but conceptually and qualitatively.

So, let us assume that the gain medium is characterized by a homogeneously broadened transition – first. The laser has a free spectral range. The cavity free spectral range is large enough, so that there is only one resonance frequency within the laser transitions. We have already explained this in detail in the earlier slide.

And, in the ON condition, the laser would therefore oscillate in one longitudinal mode only say  $\nu_q$ , corresponding to let us say corresponding to the peak or somewhere near the peak.

So, this is an assumption which is possible as I have already told you that it is possible to make the laser oscillate in only one longitudinal mode ok.

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The evaluation of the gain profile with the time is what I am now, discussing with the time at “switch ON.” At switch ON, the switch ON refers to a small period of time, switch ON corresponds to a small delta t just after switching the electrical power on or the pump on.

Just immediately after the pump is switched on, till the laser gives a steady-state output that time is called switch on time. Typically, this could vary from a few tens of micro second to few milliseconds even up to a second sometimes, alright. So, that is the switch ON time. We are discussing the dynamics around this switch ON time alright.

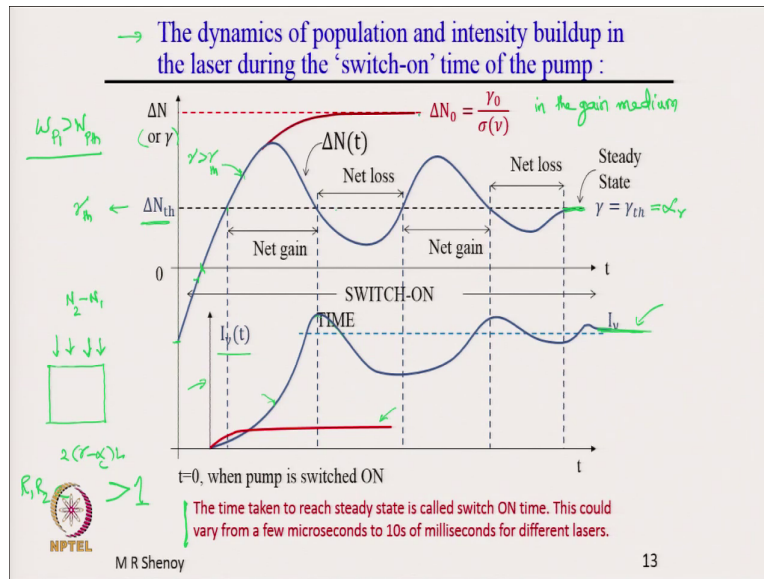
So, at  $t$  is equal to 0, the pump is not there so,  $W_p$  is 0. So, the medium is characterized by loss  $\alpha$  of  $\alpha$  of  $\nu$ . So, this is the gain profile. As time increases, so, at  $t$  is equal to some intermediate value, because  $t_1$  I have already shown positive. So, after some time these times are micro seconds or a fraction of a millisecond. So,  $t_1$  we are in the gain region.

Time increases the electrical the pump has switched ON, so, as you know when the pump is switched ON atoms start. So, to begin with all the atoms are on the ground almost all and there may be very few one or two atoms here at  $t$  is equal to 0. When the pump is switched ON, atoms start making transitions to the upper level.

And, then there may be a third level where they come down rapidly and start accumulating here. As they start accumulating here and if the number becomes greater than this number, then we have gain and that is why the gain profile is continuously increasing. So, the gain profile is increasing as the time goes above zero after we switch ON the pump. So, pump switched ON pump switched ON.

Now, at some point it reaches the threshold value and then of course, instantaneously, it would cross the threshold value, the gain would increase beyond the threshold value, but we will see that subsequently this will be pulled down. This discussion we will take in the next class.

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But, now what I want to show is when this happens how does the laser intensity build up inside the cavity. So, that is the laser dynamics which I will discuss in the next slide ok. So, here is what is shown the dynamics of population dynamics here refers to with the time of population and intensity buildup in the laser during the switch ON time of the pump.

So, at  $t$  is equal to 0, the pump is switched ON. At  $t$  is equal to 0, on the y-axis we have delta N population inversion. Population inversion is negative at  $t$  is equal to 0, because population inversion is  $N_2$  minus  $N_1$  or  $N_3$  minus  $N_2$  delta N. So, it is  $N_1$  is greater than  $N_2$ , when the pump is off.

When the pump is switched ON as I discussed in the previous slide when the pump is switched ON, atoms start going upward and then they start accumulating in the  $N_2$  level. So,

this is  $N_3$ ,  $N_2$  and  $N_1$ , alright. So, the population starts increasing it crosses the 0 that is  $\Delta N$  becomes positive; that means there is gain in the medium.

But, gain is not sufficient after some time it crosses  $\Delta N$  and threshold that is the threshold population inversion corresponding to  $\gamma$  threshold;  $\Delta N$  threshold corresponds to  $\gamma$  threshold. This axis is actually it can be  $\Delta n$  or  $\gamma$  easier to understand in terms of  $\Delta n$  that is why I have written  $\Delta N$ , but it is also nothing, but the gain coefficient.

Gain coefficient starting from a negative value reaches 0, it reaches the threshold value, but because, the pump is on the  $\Delta n$  keeps on increasing. Suppose, there was no resonator first point suppose, there was no resonator which means I just have the medium here and I have a pump on there is no resonator.

So, the population inversion will increase and it will finally, reach a steady-state value when the number of atoms going upward is equal to the number of atoms coming downward. And, this is the steady-state value  $\Delta N_0$  which is equal to  $\gamma_0$  by  $\sigma \nu$ . This is the population inversion in the gain medium in the gain medium without any resonator.

If the resonator were not there, then we would have had when the pump is switched ON, the medium is ready to amplify. It has a population inversion  $\Delta N_0$ . So, let me see the next slide.

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**Laser Dynamics at 'Switch ON' (contd.)**


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Pump is Switched ON at  $t = 0$ :

→ a) **If the resonator was not present** (red color curves):  
 $\Delta N \uparrow$  from -ve value, and approaches towards the steady state value,  $\Delta N_0$

→ b) **If the resonator was present** (blue colored curves)  
→  $\gamma = \frac{\gamma_0}{(1 + \frac{I_v}{I_s})}$  &  $\Delta N = \frac{\Delta N_0}{(1 + \frac{I_v}{I_s})}$   
as  $\Delta N > \Delta N_{th} \rightarrow \gamma > \gamma_{th}$ ,  
⇒ the resonance frequency  $\nu_q$  builds up exponentially

When  $\Delta N < \Delta N_{th}$ , there will be net loss  $I_v \downarrow$  with passing time  
 $\Delta N$  again  $\uparrow$  and  $\Delta N > \Delta N_{th} \Rightarrow I_v \uparrow$ , and so on.....



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So, if the resonator was pump is switched ON at  $t$  is equal to 0, this explanation is for the same previous graph. If the resonator was not present, then  $\Delta N$  increases from a negative value and approaches towards the steady-state value of  $\Delta N_0$ . Let us go back.

So, the red curve which starts from here and it reaches the steady-state value if the resonator were not there, if the resonator was present that is the blue curve; so, the next curve. So, and this is the discussion, if the resonator were present let us first read this. What it says we know that there is a saturated gain coefficient, alright, let us go back.

If the resonator were present as soon as we cross  $\Delta N$  threshold, the gain coefficient  $\gamma$  here is greater than  $\gamma_{th}$ . So, at this point  $\gamma$  is greater than  $\gamma_{th}$  threshold, because this is the  $\gamma_{th}$  threshold value. When  $\gamma$  is greater than  $\gamma_{th}$

threshold then we remember that  $W_{r1} - r_2$  into  $e$  to the power twice  $\gamma$  minus  $\alpha$   $c$  into  $L$  is greater than 1.

When  $\gamma$  is greater than  $\gamma$  threshold we have this condition and therefore, the intensity starts building up. Now, we see here at the inset what is plotted is intensity  $I_{nu}$  of  $t$  as a function of time. Initially, there was no intensity; when the resonator was not there the population reached a steady-state value and accordingly, the laser gives some steady-state output here due to spontaneous emissions, because transitions are continuously taking place, atoms are going up, atoms are coming down, but there is a steady state.

And, therefore, there is a certain amount of light which is generated and this is due to spontaneous emission. If the resonator were present, the moment  $\gamma$  exceeds  $\gamma$  threshold intensity inside the resonator starts building up exponentially. We recall this discussion and this diagram here.

So, the diagram here; the intensity in one case will build up exponentially when it is greater than and at steady-state it will remain otherwise, it will either get exponentially built up in the case when this is satisfied that is  $R_1 - R_2$  into this is greater than 1 or it will exponentially decrease when we have this situation. So, this is applied in that discussion now. Please see. Yes.

So, here it is exponentially building up. Why it is building up? Because  $\gamma$  is greater than this is, the same common time axis. The time axis is the same here as well as here is the same axis. So, the intensity starts building up, it will build up till the gain comes down, when the intensity starts building up the gain starts dropping down here the gain starts dropping down. Why does the gain drop down? Because, the gain depends on the intensity  $I_{nu}$  here,

So, the gain depends on as the intensity builds up the denominator becomes larger and larger and the gain coefficient starts dropping down. If the gain coefficient starts dropping down so, the gain coefficient now, starts dropping down, but still it is greater than  $\gamma$  threshold,



intensity is building up. The gain coefficient drops down continuously, and drops down below the threshold value, this is the threshold line.

So, the gain coefficient drops down below its threshold value. When the gain coefficient drops down below its threshold value, the intensity starts decreasing exponentially. Recall the graph, now we have the condition  $R_1 R_2 > e^{2\gamma L}$  into  $e^{2\gamma L} < 1$ .

So, it exponentially intensity starts dropping down. When the intensity starts dropping down gain coefficient again starts increasing and as soon as it crosses the threshold value, again intensity starts building up exponentially, till the gain coefficient again drops down below the threshold value when again it will drop.

So, this would happen over a few cycles and finally, it will reach a steady value which is the steady-state intensity output. So, this is the steady-state output intensity and at this point, what happens here? The gain coefficient becomes equal to the loss coefficient equal to the threshold gain. So, this is equal to  $\alpha_r$ .

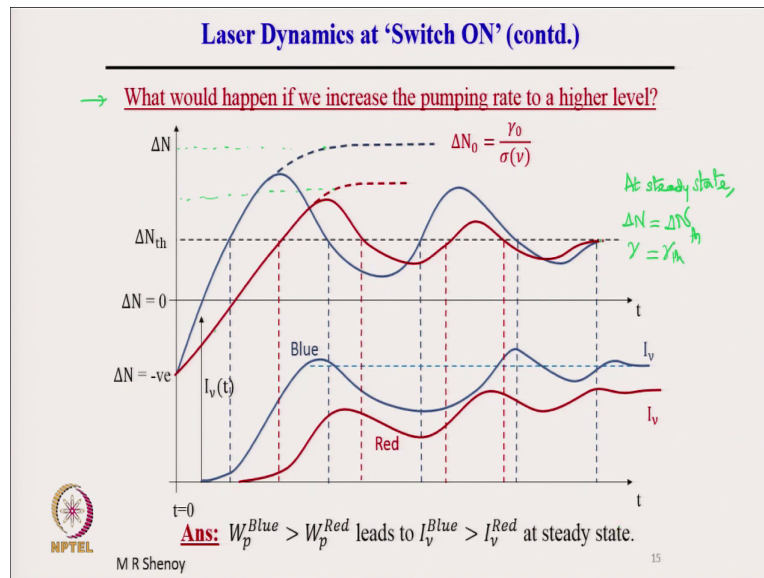
So, what does this mean? This means that at steady-state at steady-state when a laser is oscillating whatever be the intensity of the output the gain coefficient in the medium will be equal to the loss coefficient. This has to be. Because if it is more than the loss coefficient it will exponentially build up if it is less than loss coefficient it will drop down.

Therefore, at steady-state gain is equal to loss and we have a certain amount of intensity output from the laser because this is the intensity build up inside the resonator and a fraction of it comes outside. The time taken to reach this steady-state it is written here. The time taken to reach the steady-state is called the switch ON time; this could vary from a few microseconds to tens of milliseconds for different lasers, alright.

So, this is the explanation for the curves a and b; a for the red curve I have given this explanation a for the red curve and b for the blue curve. So, when  $\Delta N$  falls below  $\Delta N_{\text{threshold}}$ , there will be net loss and intensity decreases with passing time;  $\Delta N$  again raises

and  $\Delta N$  becomes greater than this intensity increases and so on till it reaches steady state. This is the explanation which I have already given.

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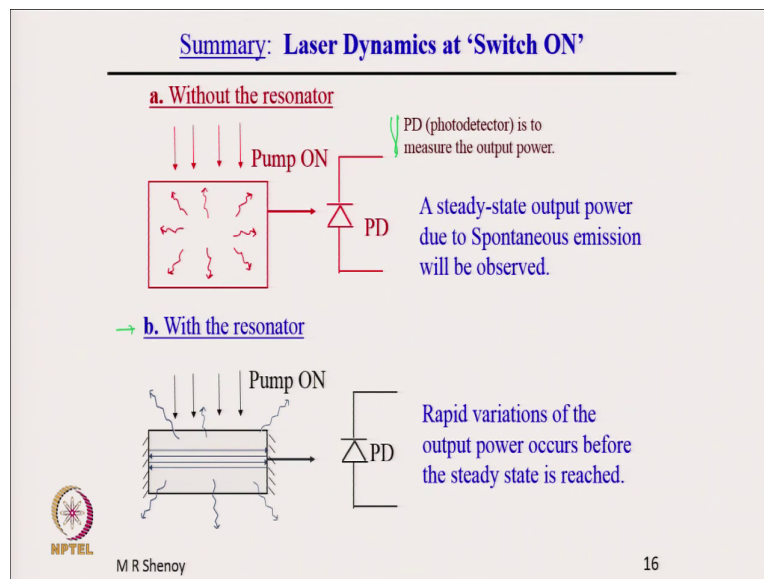
Now, the question is what would happen if we increase the pumping rate? The previous discussion was at a given pumping rate, this is for a particular value of  $W_p$ . Of course,  $W_p$  is greater than  $W_p$  threshold, but one particular value for which  $\Delta N$  would have come here.

Now, if I increase  $W_p$  1 to a new value  $W_p$  2, then what would happen? That is the question what would happen if we increase the pumping rate to a higher level? Now, the steady-state level would be higher if the pumping rate is higher. See this, this is the steady-state that is without the resonator; in the medium  $\Delta N_0$  would come to a would have come to a higher level.

But, in the presence of the resonator the same dynamics is explained here. See the difference while  $\Delta N$  will get clamped to the threshold value  $\Delta N_{\text{threshold}}$  or  $\gamma$  will become equal to  $\gamma_{\text{threshold}}$  at steady state. So, at steady-state the intensity will saturate at a higher value for higher pumping power.

In other words, the  $W_p \text{ blue}$  is greater than  $W_p \text{ red}$  leads to  $I_{\text{nu blue}}$  greater than  $I_{\text{nu red}}$  at steady state, but in both cases at steady-state the gain coefficient will get clamped to its threshold. It is a very interesting and important laser dynamics which tells that no matter what the laser output power is, the gain coefficient will always be clamped to the threshold value when a laser is oscillating in steady state ok.

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So, the summary is here of laser dynamics at switch ON without the resonator if you measure the output power. So, because of pumping there is spontaneous emission which is taking

place. So, light is emitted in all directions and on the photo detector you will detect a certain steady output power; the photo detector is to measure the output power. A steady-state output power due to spontaneous emission will be observed.

So, this is the steady-state that I had shown in this diagram, earlier diagram, let me show here. So, this is the steady-state. This is due to spontaneous emission a steady-state corresponding to a steady-state population inversion  $\Delta N_0$  will be observed, that is a. With the resonator the blue curve, which showed rapid variations in the output power before the steady-state is reached and that is the laser dynamics at switch ON.

So, we will stop here and in the next lecture, we will discuss what happens to the gain curve as it crosses the loss line, the gain profile and we will see that it leads to a very important effect called spectral hole burning in the gain profile.

Thank you.