Introduction to LASER

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Lecture - 21
Longitudinal Modes of a Spherical Mirror Resonator

Welcome to this MOOC on LASERS. We have been discussing about the Longitudinal and Transverse Modes of a Spherical Mirror Resonator. In the last lecture, we saw about the transverse modes and then in particular, the Gaussian mode of the resonator. One last thing remaining is the longitudinal modes of the resonator.

Of course, we had seen the longitudinal modes, but the expression was incomplete without a knowledge of delta zeta. Now, today we will discuss how to determine the delta zeta and hence the longitudinal modes of the spherical mirror resonator.

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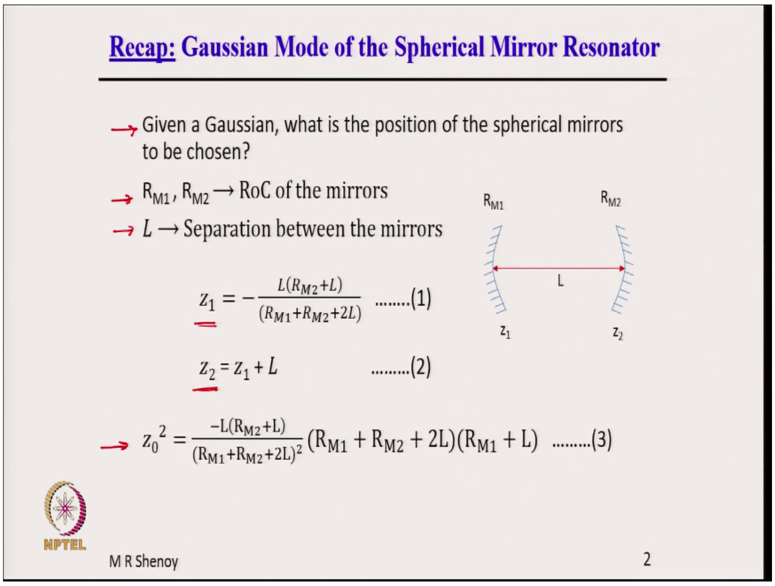

A very quick recap: Gaussian mode of the spherical mirror resonator. So, the problem that we had addressed is given a Gaussian what is the position of the spherical mirrors to be chosen? If R M1 and R M2 are the radius of curvature of the mirrors and L is the separation, then we have already worked out in the last lecture that the position of the mirror M1 is z 1 is given by an expression that is equation 1 and the position of mirror M2 z 2 is given by z 2 is equal to z 1 plus L.

More importantly, we have got an analytical expression for z naught square where z naught is the Rayleigh range. This is important therefore, for a given resonator which means R M1, R M2 and L are given we can determine what is z naught square.

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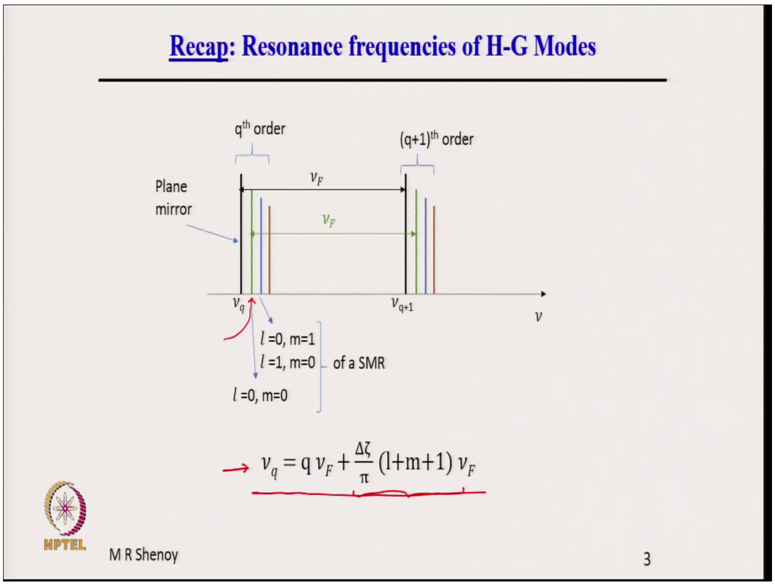

Therefore, we had seen that the expression for resonance frequency can be written in this form nu q is equal to q times nu F into delta zeta by pi into l plus m plus 1 into nu F, l and m are the mode numbers, nu F is the free spectral range.

So, we had seen therefore, that if l and m are 0, then for the spherical mirror resonator this will be the resonance frequency. Nu q is the resonance frequency of the plane mirror resonator which is only the first term nu q is equal to q times nu F, but whenever there is a spherical mirror then there is an additional curvature dependent term which is here which leads to a slight shift in the resonance frequency.

And, today we will determine this the magnitude of the curvature dependent term for some resonators.

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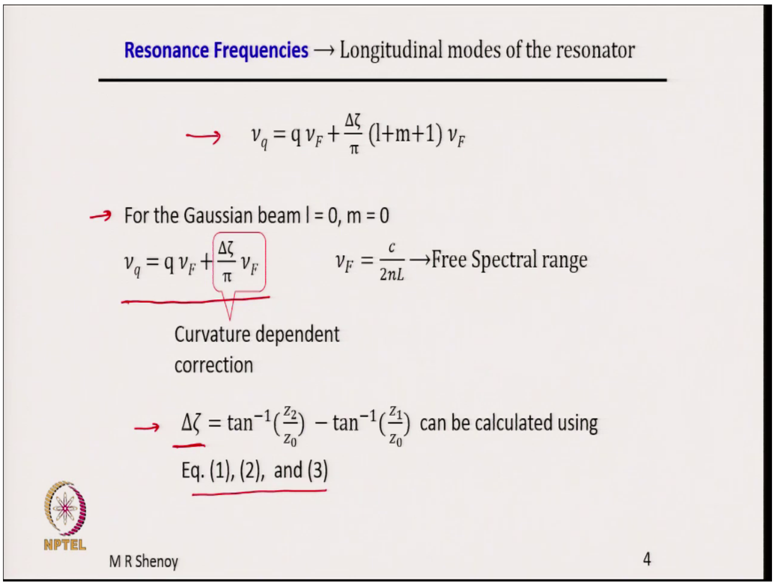

So, as I said we continue to have a recap for the Gaussian beam l is equal to 0, m is equal to 0 and therefore, the resonance frequency is given by this expression, where the second term on the right hand side is the curvature dependent correction, where delta zeta is equal to tan inverse z 2 by z naught minus tan inverse z 1 by z naught.

Now, using equations 1, 2, 3 shown in the earlier slide we can determine the positions z 1, z 2 and z naught and exactly determine the magnitude of delta zeta. That is what we will do in this lecture.

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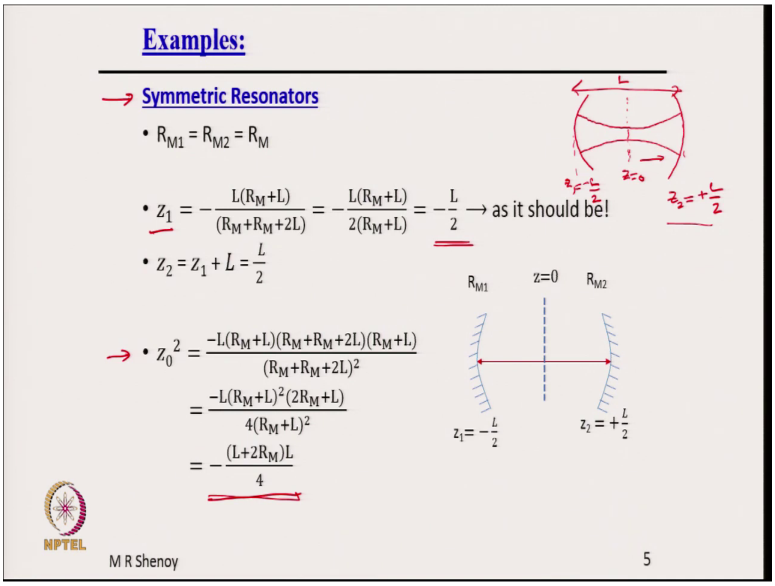

So, let me straight away take some examples and illustrate and calculate this delta zeta. So, first if we take symmetric resonators; that means, R M1 is equal to R M2 is equal to R M then z one is we simply substitute R M1 equal to R M2 is equal to R M, then we get z 1 is equal to minus L by 2, it says as it should be.

So, whenever you have a symmetric resonator which means you have a resonator with spherical mirrors identical spherical mirrors. This will support a Gaussian with its waist exactly midway because it is a symmetric resonator therefore, exactly midway we will have z is equal to 0; which means z 1, if L is the separation here if L is the separation between the mirrors, then z 1 will be equal to minus L by 2 because z equal to 0, this direction is positive and the other direction is negative.

And therefore, z 2 will be equal to plus L by 2 this is easily comprehendible and we can see that the mathematical expression also gives us that z 1 is equal to minus L by 2 and z 2 is equal to z 1 plus L which is equal to L by 2. So, z 1 is equal to minus L by 2 and z 2 is equal to plus L by 2. The z naught square in the case of symmetric resonators we substitute R M1 is equal to R M2 and gives us to a simple expression which is minus L plus 2 R M into L by 4.

So, we will use this simple formulae to determine the delta zeta for some symmetric resonators.

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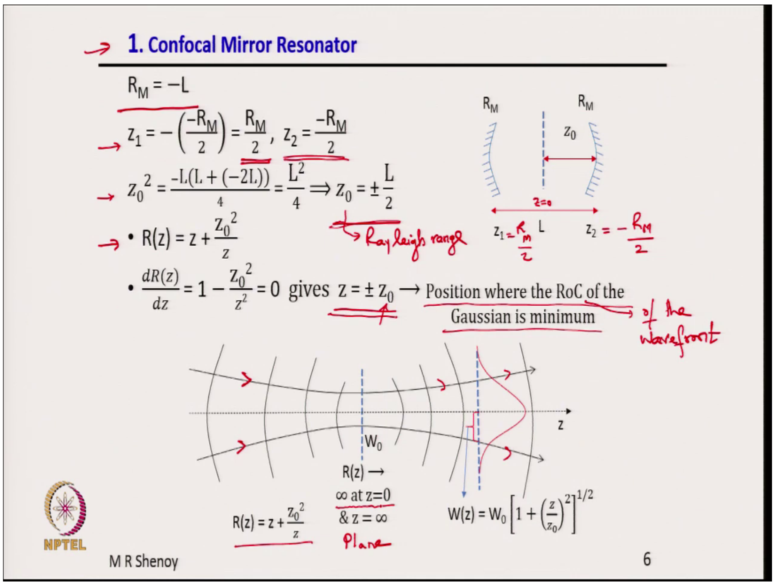

So, let us first pick up the confocal mirror resonator. So, the confocal mirror resonator is shown here where L is equal to the radius of curvature of the mirror. So, L is equal to R M.

So, it is written here. R M is equal to minus L because these are concave mirrors and therefore, L is equal to minus R M or R M is equal to minus L.

So, if we substitute for z 1 we get z 1 is equal to R M by 2 and z 2 is equal to minus R M by 2, the positions for the confocal mirror resonator. So, here are the positions. So, z 1 is equal to R M by 2. So, this is equal to R M by 2 and z 2 is equal to minus R M by 2. Note that R M is negative and therefore, z 1 will be negative because this is z is equal to 0.

And, therefore, z naught is equal to if we substitute in the formula we say that z naught is equal to plus minus L by 2 in other words for a confocal mirror resonator z naught this is called the Rayleigh range; the Rayleigh range. What is its importance we will see in a minute.

The Rayleigh range is at the position of the mirrors itself. So, this is one of the only resonators where the Rayleigh range z naught is equal to z 1 is equal to minus z 2. So, it is illustrated here z naught is the separation.

Now, what is the importance of z naught? We know that the radius of curvature of the wave front of the Gaussian beam is given by R of z is equal to z plus z naught square by z. If we differentiate therefore, with respect to z to find out where r of z becomes minimum or maximum then we get 1 minus z naught square by z square.

And, if we put that equal to 0 gives us z is equal to plus minus z naught. So, the importance of z naught is z naught is the position of the beam where it is radius of curvature is minimum radius of curvature of the Gaussian is minimum. We are referring to radius of curvature of the wave front of the wave front wave front of the Gaussian.

This is not radius of curvature of the mirror; this is the radius of curvature of the wave front. So, here it is illustrated in the figure below. So, here is a Gaussian which is propagating from minus infinity to plus infinity. As we see the converging beam has a wave front which is concave, then it becomes plane at the waist and then again it becomes convex and goes ahead.

Now, we know that the wave front is plane. So, R of z is infinity at z is equal to 0, infinity means it is plane; it is plane wave front and this expression tells us that R of z is infinity at z equal to 0 and also at z is equal to infinity which means the wave front is plane on the waist and also at infinity.

And, this we discussed in the previous lecture that therefore, it must pass through a minimum and that minimum corresponds to z naught is equal to 0.

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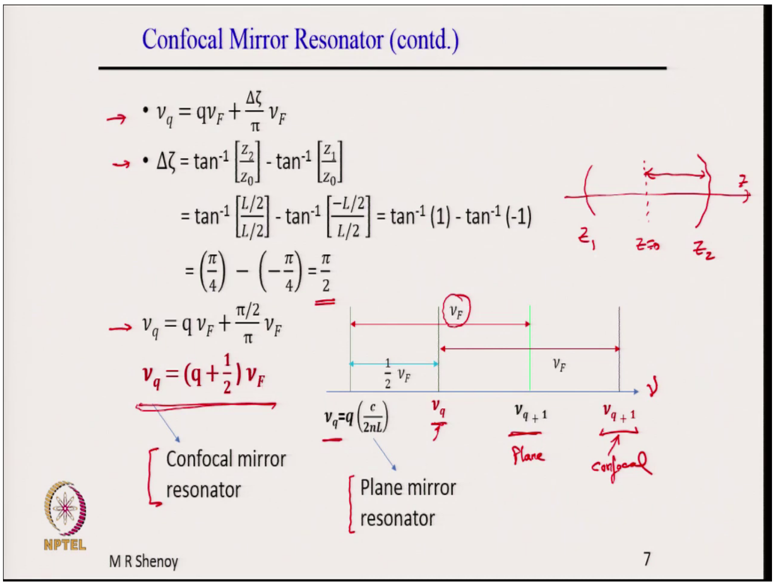

So, for the confocal mirror resonator the frequency nu q is equal to q times nu F into delta zeta by pi into nu F. Now, delta zeta is equal to tan inverse z 2 by z naught minus tan inverse z 1 by z naught. So, we substitute for z 2 and z naught; z 2 is also L by 2, z naught is also L by 2 minus tan inverse of minus L by 2 z 1 is minus l by 2, but z naught is a value, z 1 is the coordinate. It is a position here, z 1 and the z naught is z equal to 0 is here and z 2 is here and this is the z axis, this is z.

And therefore, z 1 is the coordinate of the position of mirror m 1 whereas, z naught is simply the distance the distance where the beam has a smallest radius of curvature. So, if we substitute we get tan inverse of 1 minus tan inverse of minus 1 which is pi by 4 minus minus pi by 4 is equal to pi by 2. So, this is delta zeta

And, therefore, if we substitute in the expression for resonance frequency q times nu F plus delta zeta which is pi by 2 by pi into nu F or nu q is equal to q plus half into nu F for all values of q. Nu q for the confocal mirror resonator is q plus half times nu F. Nu q was equal to q times nu F for the plane mirror resonator. Now, for the confocal mirror resonator it is q plus half into nu F which is illustrated now here in this picture, this is the nu axis.

On the nu axis if we say nu q nu q plus 1 the black colored points positions correspond to the qth order resonance and q plus 1th order resonance for the plane mirror resonate. qth order resonance which means q into nu F c by 2 n L and q plus 1th order that is the next one separated by a separation which is nu f. So, nu F is the free spectral range.

Now, for the confocal mirror resonator resonance frequency of the qth order is shifted by half times nu F which means it is exactly midway. So, it is shifted by half times nu F and therefore, nu q is here for the confocal mirror resonator and nu q plus 1 is here for the confocal mirror resonator. So, these are for the confocal and this is for the plane.

But, note that the separation is the free spectral range is still nu f. So, these are the resonance frequencies of a confocal mirror resonate. We can now determine the resonance frequencies of a concentric mirror resonator for example.

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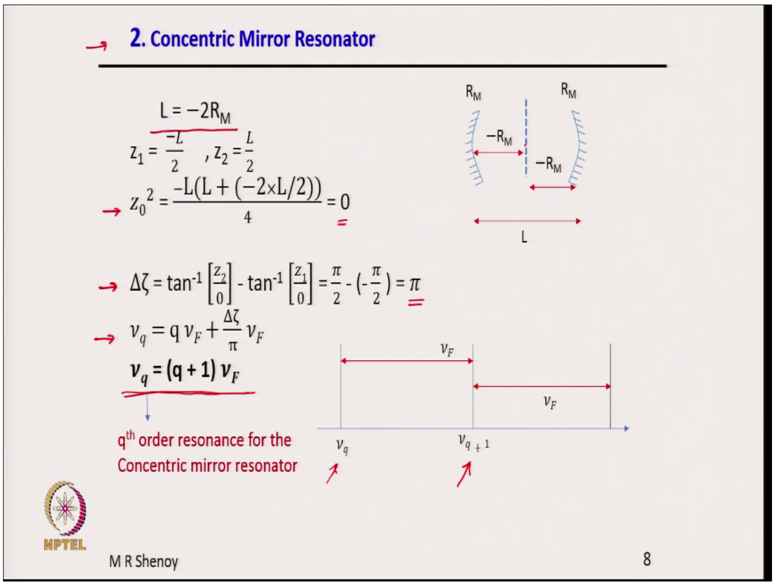

So, let us see. Second example concentric mirror resonator concentric which means the centers center point is common which means the distance from the center point to the mirror 1 is the radius of curvature of the mirror. So, therefore, L the separation between the mirrors is twice the radius of curvature of the mirror, the negative sign again stands for the concave mirror.

Therefore, z 1 is equal to minus L by 2, z 2 is equal to L by 2 which we already know which means it is z 1 is equal to minus R M and z 2 is equal to plus R M. So, therefore, z naught square is equal to comes out to be 0 and then delta zeta will be equal to tan inverse of z 2 by 0 minus tan inverse of z 1 by 0.

So, this is infinity tan inverse of infinity and tan inverse of minus infinity; minus infinity because this is minus z 1 is negative and therefore, it is pi by 2 minus minus pi by 2 which is equal to pi. So, delta zeta turns out to be pi and therefore, if we substitute in the expression for the resonance frequency of the Gaussian beam then, we get nu q is equal to q plus 1 times nu f.

So, the qth order resonance frequency for the concentric mirror resonator is shifted it is q plus 1 coincides with the q plus 1th order resonance frequency of a plane mirror. So, nu q here and nu q plus 1, the separation is still nu f. So, note that the order is shifted by 1 as compared to the resonance frequencies of the plane mirror.

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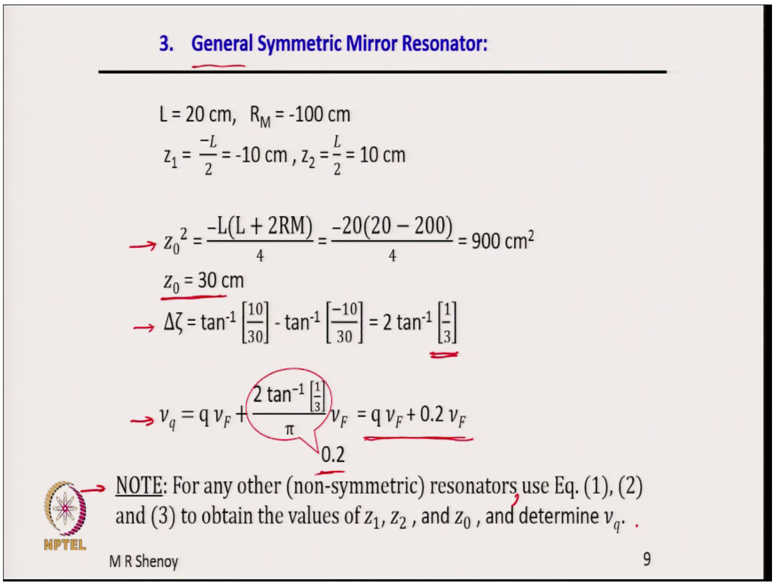

Let us take a general symmetric mirror resonator; general here stands for we normally discuss confocal and concentric mirror resonators, but general means you could have any other value and any other L, provided it is a stable resonate.

So, what we have chosen is L is equal to 20 centimeter and radius of curvature of the mirror is minus 100 centimeter. So, if we substitute we get z 1 is equal to minus 10 centimeter and therefore, z 2 is equal to 10 centimeter. More importantly z naught square now comes out to be if we substitute here it is 900 centimeter square or z naught is equal to 30 centimeter.

And, we follow the same procedure. Therefore, delta zeta is equal to tan inverse of z 2 by z naught minus tan inverse of z 1 by z naught. With z 1 there is a negative sign here always in a symmetric resonator because the waist is at the midpoint where z is equal to 0 z 1 is negative and this comes out to be twice tan inverse of 1 by 3.

So, if we substitute in the expression for nu q so, nu q is equal to q times nu F plus delta zeta which is twice tan inverse of 1 by 3 divided by pi into nu F. This term comes out to be 0.2 and therefore, the resonance frequency is q times nu F plus 0.2 times nu F. We have seen 0.5 times nu F for the confocal case and 1 times nu F the additional curvature dependent term for the concentric case and here it turns out to be 0.2 times nu F.

If we want to take any other spherical mirror resonator that is not necessary symmetric resonator any other spherical mirror resonator then we must use equations 1, 2, 3 to determine the positions z 1, z 2 and z naught using equation 1, 2 and 3. That is what is stated here for any other non-symmetric resonators use equation 1, 2 and 3 to obtain the values of z 1, z 2 and z naught and hence determine the resonance frequency nu q.

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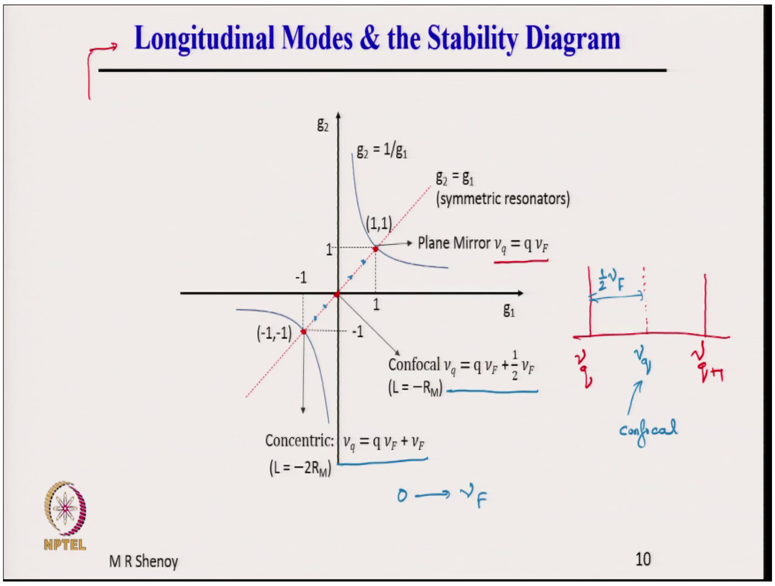

Now, finally, I want to illustrate the positions on this stability diagram. That is why the title is given longitudinal modes and the stability diagram. We have already seen the stability diagram in detail. We know that this position 1, 1 is plane mirror, the position 0, 0 is confocal mirror and the position minus 1, minus 1 is the concentric mirror.

So, these are the g 1, g 2 values. So, we have already seen these in detail, but now I am interested in seeing the longitudinal modes means the resonance frequencies. So, the resonance frequency for the plane mirror is nu q is equal to q times nu F, in other words the curvature dependent term is 0.

When we come to the confocal mirror, we get nu q is equal to q times nu F plus half nu F that is midway we recall that this is for the plane mirror. So, nu q and nu q plus 1. So, for the qth order of the confocal resonator is here. So, this is let me use a different color. So, this is nu q of the confocal mirror confocal mirror which is shifted by half times nu F half nu F nu f.

So, confocal mirror has nu q is equal to q plus half nu F and we just calculated for the concentric mirror resonator as nu q is equal to q times nu F plus 1 nu F. If we take another symmetric mirror resonator, remember all symmetric mirror resonators g 1 is equal to g 2 which has to lie on this line.

So, for the symmetric mirror resonator that I had chosen it was 0.2 times please see this is the additional term is 0, additional term is half times nu F and here the additional term is 1 times nu F that is shifted by 1 nu F. The one which I calculated for 0.2 is somewhere here.

So, it is about 0.2 times nu F if you take another symmetric mirror resonator it will be here, it will be here and so on. Because the line y is equal to x or g 2 is equal to g 1 represents all symmetric mirror resonators and the line from here concentric to plane mirror resonator represents stable resonators.

And, therefore, we see that the curvature dependent term will vary from 0 to 1 nu F. For all symmetric resonators stable resonators the curvature dependent term that is the additional term the second term varies from 0 to nu F. For any other resonator which is non symmetric resonator we need to calculate using equations 1, 2 and 3.

So, I would recommend that you work out some examples, some examples will be there in the weekly assignment as well and find out what is the frequency shift that we get for different resonators with respect to or with reference to the resonance frequencies of the plane mirror resonate.

So, that brings us to the end of this part of the resonator and in the next part, we will see the laser where the gain medium, the amplifying medium is placed inside a resonator and we have amplification and feedback by the resonator which leads to was a LASER.

So, we will see the dynamics of the LASER and how the LASER reaches steady state oscillation, the importance of pump, the importance of the resonator in determining the output characteristics of the LASER. So, we will stop at this point.

Thank you.