Introduction to LASER
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Lecture - 20
Gaussian Mode of the Spherical Mirror Resonator

Welcome to this MOOC on LASERs. In the last lecture, we saw Hermite-Gauss modes of the resonator which is a family of modes of which form the transverse modes of a spherical mirror resonator. And we have also seen that the fundamental mode of this family that is when the mode numbers l and m are 0 is a Gaussian beam or a mode which has a Gaussian transverse distribution.

Gaussian modes are very important in resonator physics and therefore, today we will take up Gaussian Mode of the Spherical Mirror Resonator.

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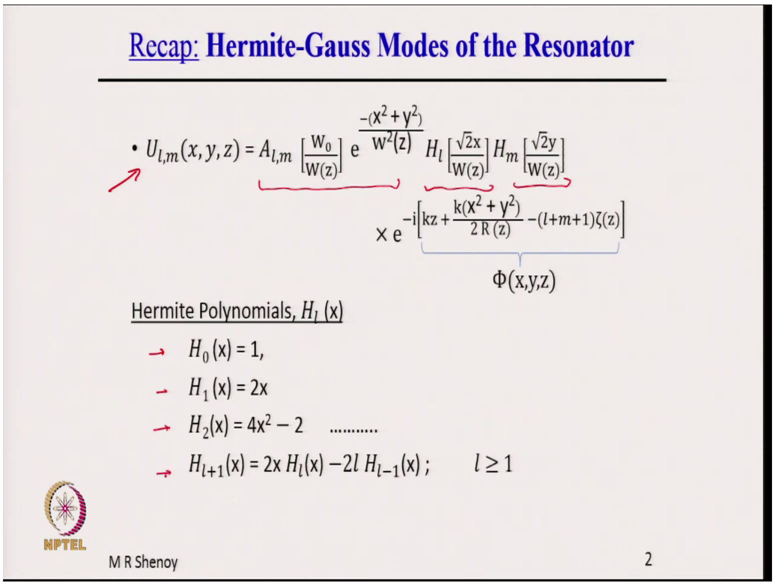

A very quick recap. So, the Hermite-Gauss modes are shown here and as I mentioned for l is equal to 0 and m is equal to 0 we have the Gaussian mode. So, this is the amplitude distribution and then the Hermite polynomials and then the phase term phi of x, y, z. The Hermite polynomials are given by this a very quick recap we have discuss this in detail.

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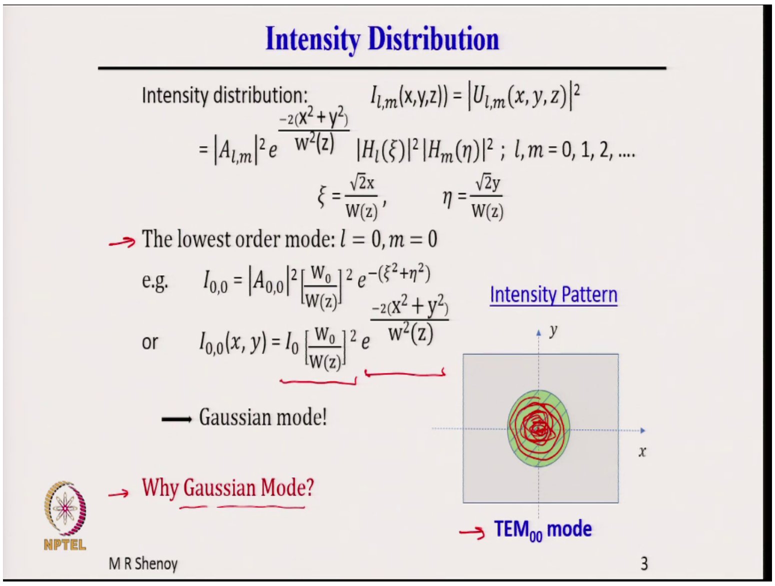

The intensity distribution is given by mod square of U l, m x, y, z. So, that will come out to be like this in this expression, where l comma m is equal to 0, 1, 2 etcetera. The lowest order mode that is for l is equal to 0 and m is equal to 0 the intensity distribution is given by this expression here. And, as we have already seen in the last lecture that this is an amplitude distribution multiplied by a Gaussian e to the power minus 2 r square by w square of z it is a radially symmetric field distribution and that is the Gaussian mode.

Why Gaussian mode? As you can see, we have seen the other distributions also the Gaussian mode has a maximum at the center and then the intensity monotonically drops as you radially go outward. It has a maximum intensity peak here and it is radially symmetric. There is no azimuthal dependence that is there is no phi dependence in this and it is a pure spot.

And, in most application one prefers to have the Gaussian mode or its also called the TEM 00 mode transverse electromagnetic; 00 is l is equal to 0, m is equal to 0 that is the fundamental mode. The Gaussian mode can also be focus to the smallest size and being a Gaussian field distribution the far field of the mode when the mode diffracts it still maintains the Gaussian amplitude distribution and in most of the applications one uses the Gaussian mode.

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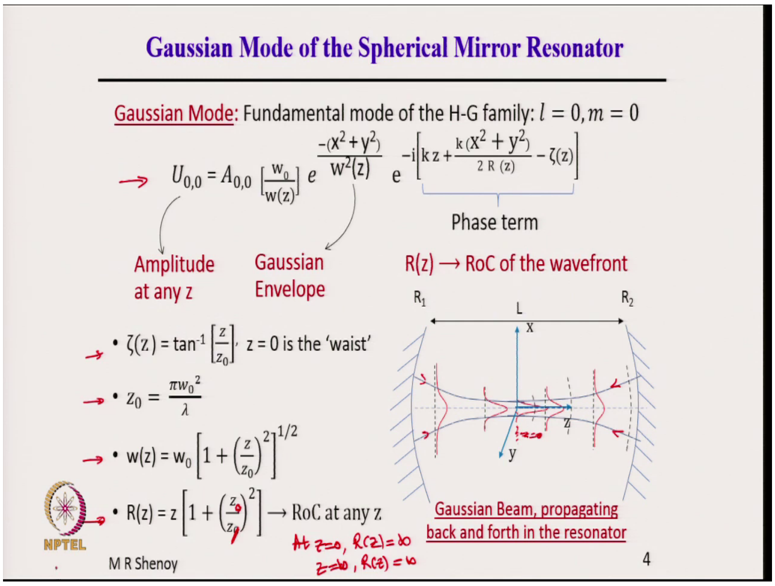

So, the Gaussian mode of the spherical mirror resonator it is the fundamental mode of the Hermite-Gauss family. So, we have already repeated this. The amplitude at any z, the Gaussian envelope term here and where zeta of z is equal to tan inverse of z by z naught, z is equal to 0 is the waist. So, we can see here in the resonator, the Gaussian beam is going back and forth is going back and forth.

The field distribution is evolving it is spreading because of the diffraction, but then again focused back by the spherical mirror resonator and this forms a standing wave and a field distribution which repeats itself. z naught is called the Rayleigh range is given by this expression pi w 0 square by lambda; w 0 is the spot size at the waist and spot size w of z at any z.

So, z is equal to 0 is here at the waist; waist is where the width of the beam is the smallest. So, this is z is equal to 0 point. And, at any other z the spot size is given by this formula here and R of z is the radius of curvature of the wave front RoC of the wave front at any z. We can see that the wave front also evolves as the beam propagates. We will discuss more about this a little later.

Therefore, as we have seen the Gaussian mode propagates, it diffracts and then it is focused back by the spherical mirror and then it propagates in the reverse direction focused back by the second spherical mirror here and therefore, it goes back and forth; a Gaussian beam propagating back and forth in the resonator.

We want to see what is the effect of the spherical mirror? What should be the condition on the spherical mirror or on the Gaussian such that the mirror reflects the beam exactly in the same direction backwards? Or the beam is essentially reverted when it is reflected propagates along the same direction in which it has come. So, we will see this.

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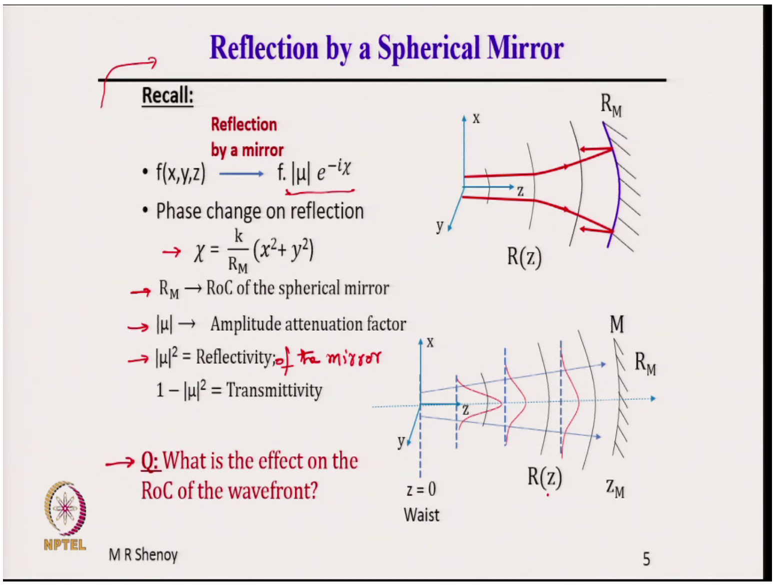

And, therefore, let us look at the reflection by a spherical mirror a very quick recall if there is a function if there is a field distribution f of x, y, z at x, y, z then the effect of reflection by a mirror is to multiply by a factor mu which can be written as mod mu into e to the power of minus i chi; chi is the phase change on reflection and mod mu is the amplitude attenuation factor.

We have already discussed this in the last lecture and mod mu square is the reflectivity of the mirror. So, reflectivity of the mirror and therefore, 1 minus mod mu square is the transmitivity of the mirror. Of course, assuming that, there is no absorption in the mirror. So, mod mu square is the reflectivity and 1 minus mod mu square is the transmitivity.

The phase change on reflection chi is given by a term like this k divided by k is the free space propagation constant and R M is the radius of curvature of the mirror. R M is the RoC of the spherical mirror multiplied by x square plus y square. So, the question is what is the effect of so far we have seen we have considered the phi and we wanted that the round trip phase must be integral multiple of 2 pi and mod mu is the amplitude attenuation factor.

Now, the question that we are asking is what is the effect on the radius of curvature of the wave front? So, here is a Gaussian which is propagating or here it is propagating. As you can see the wave front also evolves as it propagates it changes the curvature of the wavefront changes with the propagation. That is why R of z we have just seen that R of z it depends on the z the radius of curvature of the wave front and then R M is the radius of curvature of the spherical mirror.

So, it reaches here and then gets reflected back. So, we want to see what is the effect on the radius of curvature that is R of z of the beam upon reflection.

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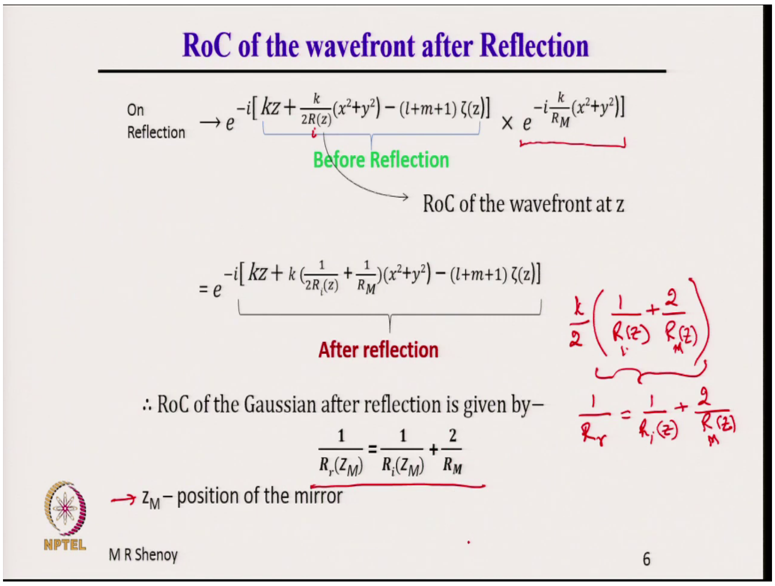

So, let us look at. Now, on reflection the incident beam, so, this is the phase term now we are looking only at the phase term because the radius of curvature of the wave front appears only in the phase term and that is why we are now looking at the phase term. So, e to the power of i kz plus k divided by twice R of z into x square plus y square minus l plus m plus 1 into zeta of z.

Because of the phase change at the mirror it will be multiplied by this term e to the power of i k by R M; R M is the radius of curvature of the mirror into x square plus y square. So, if we take it together, then we can write it in this fashion. So, after reflection the total phase phi will be given by this reflection.

Now, if we compare with the original term, then we see that. So, this term can be the second term can be written as k into k by 2; let me take 2 outside and then we have 1 by R i of z plus 2 by R M of z.

Comparing it with the original term here the first term before reflection, this says that the radius of curvature after the reflection must be given by this term that is 1 by R of R that is R of R here stands for the radius of curvature after reflection of the beam is equal to 1 by R i of z R i of z plus 2 by R M of z.

R M is the radius of curvature of the mirror. So, we do not have to write z because we know the position of the mirror. But, that is what is written RoC of the Gaussian after reflection is given by this expression where Z M is the position of the mirror. So, we see that an incident radius of curvature which is R of z. Now, we have called it as R i of z the incident beam is R i of z.

So, let me write here incident R i of z, then the reflected radius of curvature of the reflected beam at the mirror just after reflection is given by 1 by R r of Z M is equal to 1 by radius of curvature of the incident beam plus 2 by radius of curvature of the mirror this is important.

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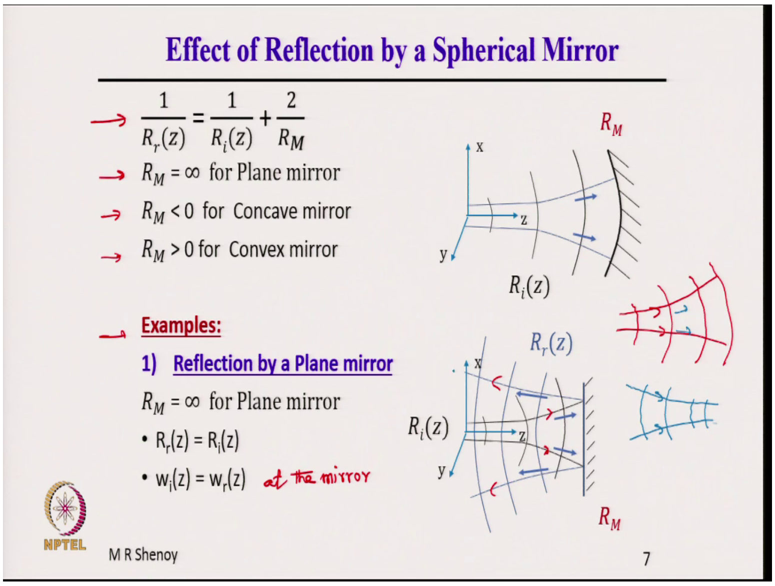

So, the effect of reflection by a spherical mirror is here now, the same formula. 1 by R r of z is equal to 1 by R i of z plus 2 by R M of z. Note that R M is equal to infinity for a plane mirror and therefore, in the case of a plane mirror we can immediately see that if R M is infinity then R r of z will be equal to R i of z that is the first example which is shown here reflection by a plane mirror.

If R M is equal to infinity for a plane mirror then R r of z is equal to R i of z just at the reflection and the incident spot size will be equal to the reflected spot size at the mirror. This is only on the surface of the mirror as it propagate the spot size would change at the mirror. The radius of curvature R M is less than 0 for a concave mirror R M has to be taken as negative and R M is positive for a convex mirror.

So, this is what is illustrated here that there is a wave which is propagating these black wave fronts are incident wave front on a plane mirror and upon reflection from a plane mirror the beam continues to diverge; if you have a diverging beam like this then after reflection it continues to diverge and a diverging beam means it has a convex wave front. So, we see that the wave front of the diverging beam is convex.

So, whenever you have a diverging beam like this a beam which is diverging then the wave front will be convex like this and if the beam is converging let me take a different color and if the beam is converging like this, so, you have a focus it a focused beam which is converging this will always have a concave wave front like this. So, concave wave front means its a converging beam and a convex wave front means its a diverging beam.

So, we see here there is a diverging beam which is incident on the plane mirror which continues to diverge because the radius of curvature of the incident beam is the same as radius of curvature of the reflected beam on the mirror. Incident beam is convex, radius of curvature of the wave front, then the reflected beam also has a convex radius of curvature. Convex radius of curvature means it is diverging and that is consistent with what is shown in the diagram.

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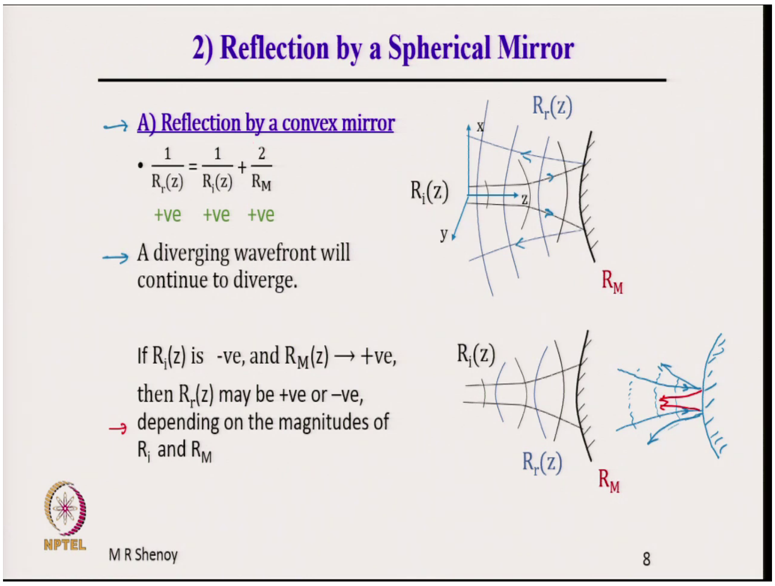

Let us take a 2nd example, reflection by spherical mirror. Here there are reflection by convex mirror and concave mirror. So, first consider reflection by a convex mirror.

So, convex mirror; that means, R M the radius of curvature of the mirror is positive, the radius of curvature of the reflected them will be positive because incident beam is positive, reflected beam is positive then the mirror radius of curvature is positive then we have reflected beam also having a diverging wave front or a positive radius of curvature gives a diverging wave front and we will continue to diverge that is what is illustrated here.

An incident diverging beam with a convex wave front incident on a convex mirror of positive radius of curvature results in a positive R r of z which means it is a. So, this is the incident beam and this is the diverging beam. If R i of z is negative which means if it is a converging beam and if R M of z is positive because we have considered convex mirror, then R r of z may be positive or negative.

If R i of z is negative. So, what is shown here is a diverging beam? So, if I show here a converging beam like this, this is the mirror convex mirror this is converging wavefront. Therefore, the wavefront is convex concave. So, it is a beam which is coming like this. After reflection this may diverge like this it may diverge further like this or it may also converge it may also converge like this depending on the magnitudes that is the statement.

If R i of z is negative that is a converging wave front and if R M of z is positive that is a convex mirror, then R r of z may be positive or negative depending on the magnitudes of R i and R M because the formula is here. So, if 1 by R i which is negative term, 1 by R M is positive term is such that if the sum is positive then R r of z would be positive and if the sum is negative R r of z would be negative, that is the meaning of this.

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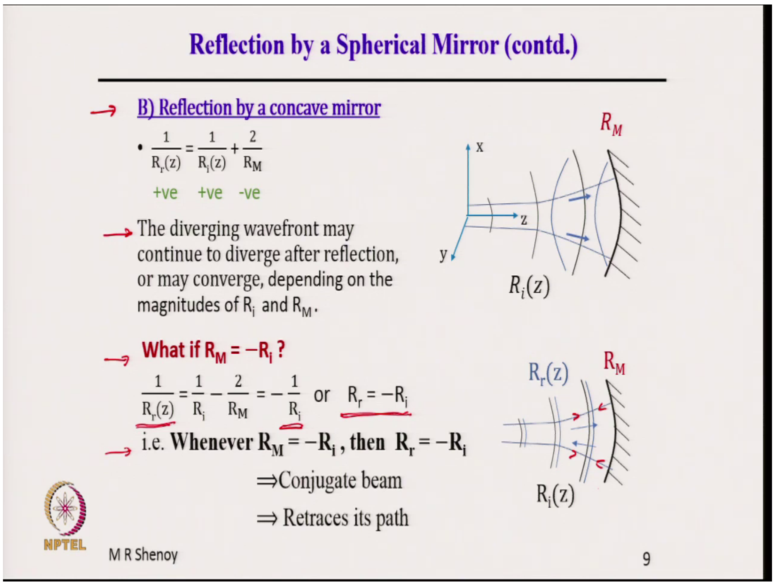

Now, let us see what happens on reflection by a concave mirror. Now, we are looking at reflection by a concave mirror this time R M is negative R M is less than 0 for a concave mirror. So, again as before we have a diverging beam which means R i is positive these black wavefronts here are positive or convex wave fronts.

After reflection it may be a convex wave front or maybe a concave wave front. So, the diverging wave front may continue to diverge after reflection or may converge depending on the magnitudes of R i and R M. Just as before R M is now negative, R i is positive depending on their magnitudes if the sum is positive then R r of z will be positive; positive means convex wave front a diverging beam.

Now, the question is what if R M is equal to minus R i; look at it only from a mathematical point of view R M is equal to minus R i. Then 1 over R r of z is equal to 1 by R i minus 2 by

R M. Now, R M is negative that is why 2 by R M and R M is R i therefore, minus 2 by R i this is minus 2 by R which is equal to minus 1 by R i. 1 by R of z that is radius of curvature of the reflected beam is negative of that of the incident beam or R r is equal to minus R i.

What does this mean? This means that if you have a convex wave front then the reflected beam will have the same magnitude, but a concave wavefront. Which means if this is diverging this will be converging. Therefore, whenever R M is equal to minus R i then R r is equal to minus R i which implies that it is a conjugate beam which retraces its path. Its illustrated here.

See this, the black wave front here is the incident wave front which is a diverging beam with a convex wavefront incident on the mirror, then if the magnitude of the convex wavefront is the same as the magnitude of the concave mirror then the wavefront will simply be reversed and it will be a converging beam with a concave wavefront.

So, the R r will be equal to minus R i which means it is simply flipped and therefore, the beam retraces its original path. That is what happens in a resonator where we saw that the Gaussian beam is going back and forth. Now, this has important implications.

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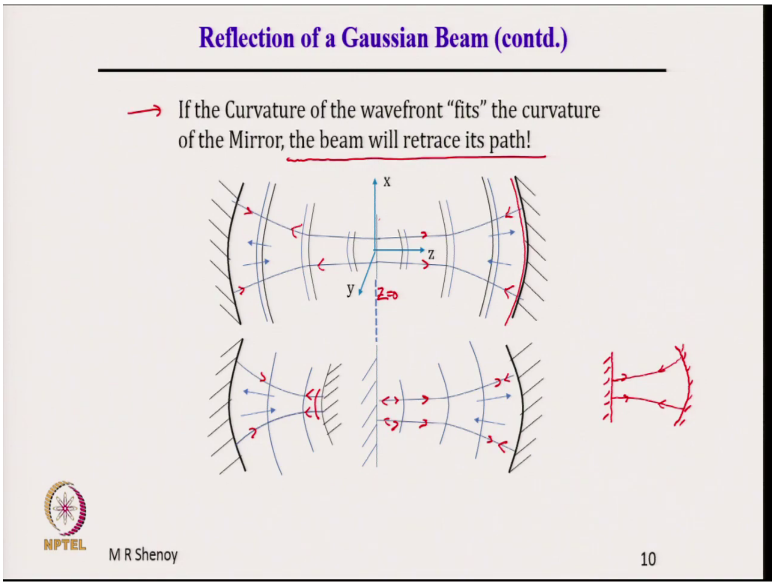

So, if the curvature, so, see the conclusion; if the curvature of the wavefront fits the curvature means magnitude is equal means what? It fits the curvature; incident beam has the same curvature as that of the mirror. So, it fits the radius of curvature, it fits the mirror here then the beam will retrace its path.

So, a diverging beam will converge back and therefore, it can go back and forth; back and forth. This is a Gaussian beam which has the smallest width at the waist that is z equal to 0. So, this is z equal to 0. Therefore, from here onwards as the beam propagates it diverges. So, the beam propagates it diverges because of diffraction. If it fits this, then the wave front will reverse and it will start converging back to the waist that is a conjugate beam.

The face is inverted and therefore, its called a face conjugated beam which retraces its original path. When it comes here it continues to propagate to the other end. So, shown here it continues to propagate and then it again gets reflected provided the radius of curvature of the beam fits the radius of curvature here of the second concave mirror.

Therefore, if we look at the figure it is not necessary, for example, not necessary to have two concave mirrors. If I have at the waist the wave front is plane; at z is equal to 0 the wavefront is plane, R of z is infinity. If we just see the expression for R of z see R of z at z is equal to 0 z is equal to 0, the radius of curvature there is an error here. This is z naught square by z this is z naught square and this is z.

So, there is a mistake here. So, it is z naught by z the whole square in w of z it is z by z naught the whole square, but in R of z it is z naught by z the whole square and therefore, if z is equal to 0 at z is equal to 0 R of z is infinity. So, at z is equal to 0 R of z is infinity. What does it mean? R of z infinity means its a plane wave its a plane wavefront.

Similarly, we see that if we put to z is equal to infinity then also R of z will be infinity because this term will become 0 and outside there is an infinity and therefore, we will get R of z is equal to infinity. So, at z is equal to 0, R of z is infinity and at z is equal to infinity then also R of z is equal to infinity. Means a Gaussian beam will have a plane wave front at the waist and at infinity. Let us come back.

We will discuss this issue again. So, we were here at z is equal to 0 the wavefront of the beam is plane. Therefore, if I place a plane mirror at the waist and the concave mirror here then the beam would of course, propagate like this and get reflected back exactly along the same path provided R M is equal to minus R i at that point and then the beam would come back.

As the beam approaches the mirror it is wavefront becomes plane and therefore, the plane wavefront will fit the plane mirror here and will get reflected back again along the same line. In other words, we can have a resonator with the one plane mirror and one spherical mirror. In fact, we have already seen this in ray optics that we can have such a resonator in which rays remain confined. But, what we have seen is beam confinement, a Gaussian beam confinement.

The condition is that at the waist the plane wave front exactly fits the plane mirror and then it gets reflected back. So, this forms a stable resonator. Similarly, if we see that we could have here a convex and a concave mirror.

The convex mirror look at the beam which is coming from here, when the beam comes from here at this point it has a concave wave front like this and if it fits exactly the convex mirror curvature of the convex mirror, then R M will be equal to minus R i and therefore, the beam will get reflected back along the same line.

In other words, we can also form a stable resonator with one concave mirror and one convex mirror which we had seen during the discussion on the stability condition. We have seen that it is possible to have stable resonators with one concave mirror and one convex mirror.

Now, we can see that even with this condition that R i is equal to minus R M we clearly see that it is possible to form stable resonator for Gaussian beams with one concave mirror and one convex mirror and one plane mirror and one concave mirror, alright.

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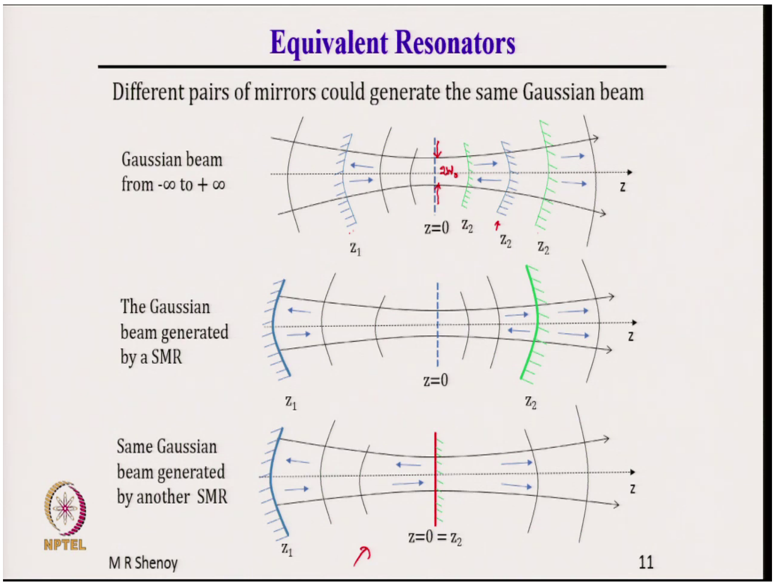

Let us continue further. Therefore, if you have a Gaussian beam one Gaussian where the wavefront is evolving like this plane then the wave front is evolving, you can place the mirrors at different positions and obtain a stable resonator. Depending on the position, the radius of curvature of the mirror should be different. In other words, if you have mirrors of different radius of curvatures and if you need a particular Gaussian, you can place the mirrors at exact positions.

What are those positions? Positions at which the radius of curvature of the wavefront fits the radius of curvature of the mirror or the curvature of the mirror and therefore, accordingly, we can have a resonator by having one mirror here. So, this is the Gaussian the black coloured one is the Gaussian. So, we want to get this Gaussian. So, how can we get such a Gaussian? We place one mirror here and one mirror here that is this is z 1, this is z 2.

I can have z 1 here if the radius of curvature of the mirror is different, then I place it at a different position such that the curvatures fit z 1 here and z 2 here or z 1 here and z 2 here; z 1 here, z 2 here.

All these configurations will give the same Gaussian, what do I mean by the same Gaussian? The Gaussian which has to this 2 waist as 2 w 0. So, this is 2 w 0, the w 0 defines the Gaussian and therefore, this Gaussian can be achieved by using different combinations of the mirrors.

For example, it is shown here in this the lowest graph that we can have a plane mirror and a concave mirror to get the same Gaussian. In all the three figures the Gaussian is the same, but the mirrors used are different. Here there are different mirrors, here there is a different pair of mirror and here there is a different pair of mirrors. But, the Gaussian generated will be the same.

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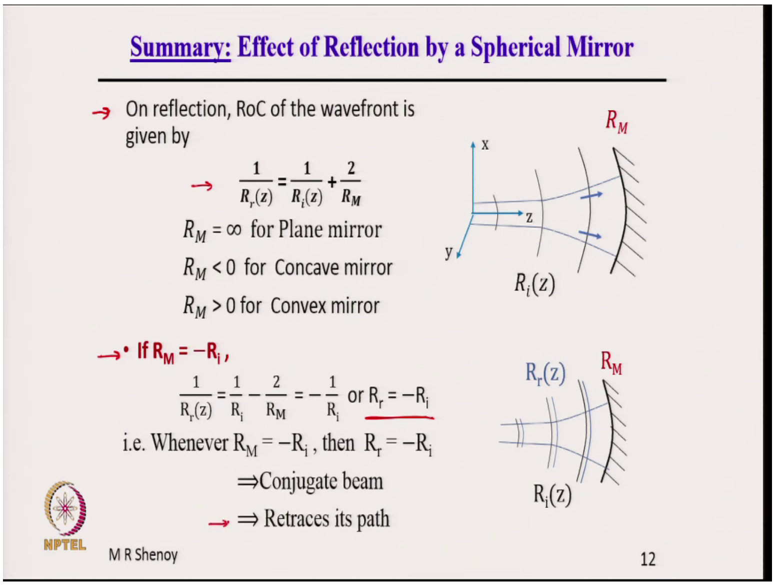

This leads us to the design problem, but before that let summarize now. The effect of reflection by a spherical mirror is on reflection. The radius of curvature of the wavefront is given by this formula, where R M is equal to infinity for plane mirror and less than 0 for concave mirror and positive for convex mirror.

And, as we discussed if R M is equal to minus R i then R r will be equal to minus R i. That is, whenever the radius of curvature of the mirror is equal to the radius of curvature of the wave front in magnitude, but in different sign then R r will be equal to minus R i which leads to a conjugate beam which retraces its path. This is a summary of discussion.

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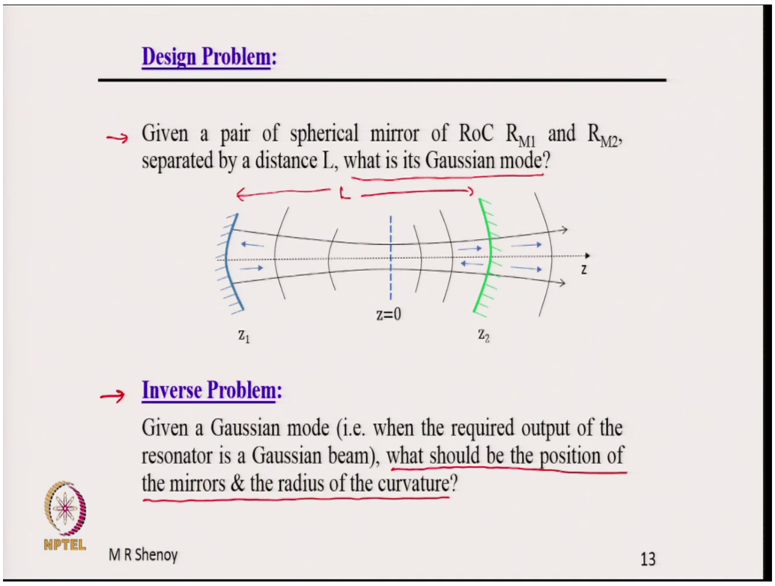

Now, let us go to the design issue. So, what is the design problem? The design problem we see given a pair of spherical mirrors of RoC radius of curvature R M 1 and R M 2 separated by a distance L, what is its Gaussian mode? How to determine the Gaussian mode? So, we have to determine the Gaussian mode for a given resonator; given resonator means a mirror of radius of curvature R M 1 is given, R M 2 is given and the separation L is given, that is a given resonator.

For a given resonator, what is the Gaussian mode? The inverse problem is given a Gaussian mode; that means, we require a certain Gaussian mode what should be the position of the mirrors and the radius of curvature to get a required Gaussian beam. In a particular application, you need a certain type of Gaussian beam then what should be the positions of the mirrors and the radii of curvature of these mirrors that you should choose.

So, these are the design problems. One given a resonator what is the Gaussian mode that it will support or if you need a certain Gaussian how to design the resonator or what should be the radius of curvatures of the mirrors that you have to choose and what should be their separation.

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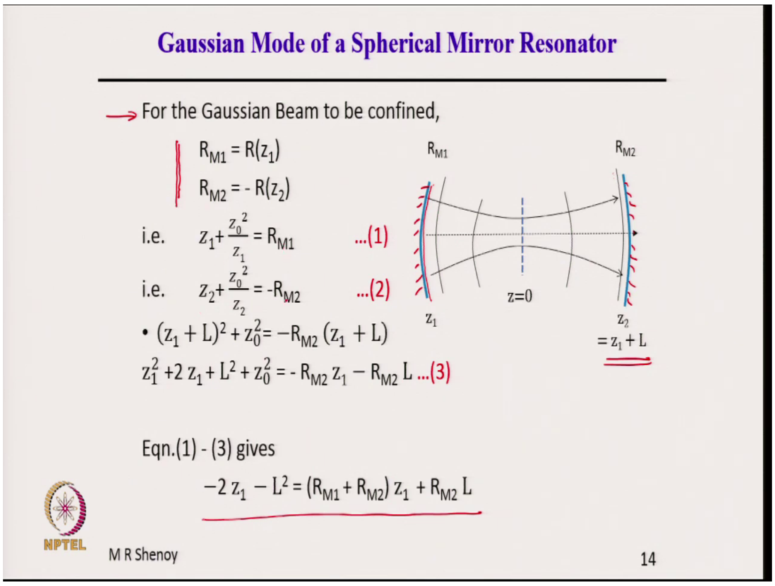

Let us see how to get this mathematically now. For the Gaussian beam from the discussion about what we have got is R M 1 must be equal to R of z 1; z 1 is the position where mirror M 1 is kept, z 2 is the position where mirror M 2 is kept. See the resonator diagram here. So, this is mirror M 1 placed at z 1 and this is mirror M 2 placed at z 2; R M 1 and R M 2 are the radius of curvature.

Therefore, as per the earlier discussion the radius of curvature of the wavefront here must fit the curvature of the mirror which means so, if the mirror is concave the radius of curvature must also be concave. Therefore, R M 1 must be equal to R of z 1, R M 2 here at the other end must be equal to minus R of z 2 because here it is a diverging beam with a convex wavefront. We must have a concave mirror which fits it and that is why we have these two conditions.

Now, we substitute for R of z 1. So, R of z 1 is given by z 1 plus naught square by z 1 and R of z 2 is z 2 plus z naught square by z 2 is equal to minus R M 2. The same conditions are now written as equation 1 and 2. We can simplify this. In the 2nd equations z 2 is z 1 plus L because z 1 is here, z 2 which is equal to z 1 plus L because L is the separation between the two mirrors.

Therefore, if we substitute this in the second equation we get this equation here and then you can rearrange this to open this bracket and all the steps are here. So, we got equation 3. Equation 1 minus 3; so, 1 here minus 3. So, minus 2 z 1 minus L square is equal to R M 1 plus R M 2 because there is a negative sign. So, minus minus is plus, minus minus is plus. So, we have this equation 1 minus 3 gives us this.

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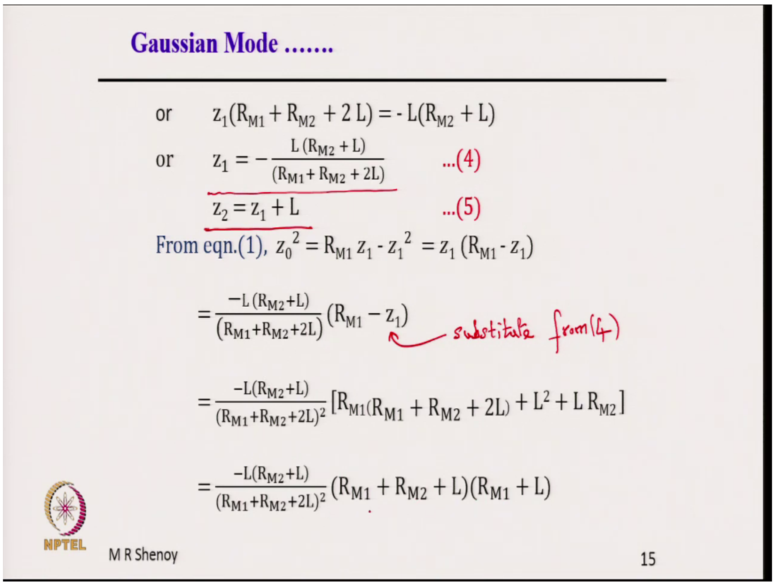

Now, let us simple algebra here that is z 1 into we have taken z 1 terms together is equal to L into this or z 1 is equal to a formula which is obtained here by simple starting point is simply this. This is the starting point which we have conceptually discussed and then you simply substitute for this and simplify this simple algebra to get z 1 is equal to that and z 2 is given by this.

Now, from equation 1 the 2st equation here equation 1 we can get expression for z naught in terms of z 1, z 2 and R M 1. So, if we substitute, then we get z naught square is equal to R M 1 into z 1 minus z 1 square which is equal to z 1 into this. We already have expression for z 1. So, if you substitute for z 1 then we get z naught square is equal to. So, we get this expression here and simplify it further.

So, for z 1 now we substitute from equation 4; for this substitute from 4. It is very simple algebra from 4 equation 4 we get this expression. Simplify it further and you can put it in this form.

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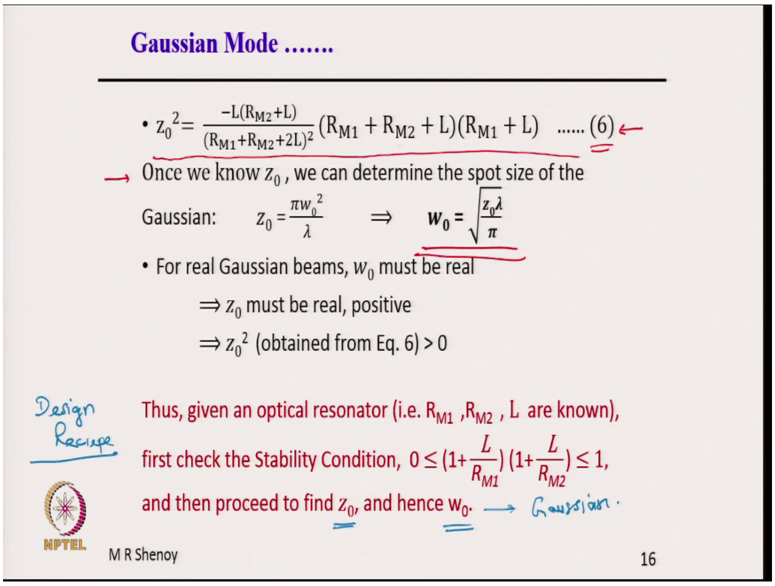

So, we have therefore, z naught square is equal to this. So, we have got an expression for z 1, z 2 is simply z 1 plus L and we have got an expression for z naught. So, what do we have? We have a given resonator which means we know the radius of curvature R M 1 and R M 2 and L.

Now, we have obtained expressions for z 1 and z naught in terms of R M 1, R M 2 and L. Therefore, once we know z naught we can determine the spot size of the Gaussian; if you know the spot size of the Gaussian then you know the Gaussian. So, the Gaussian is equal to z naught is equal to pi w 0 square by lambda or w 0 is given by this expression. So, given a resonator, equation 6 tells you what is z naught square and if you know z naught, then you can calculate the spot size of the Gaussian.

For real Gaussian beams w naught must be real which means z naught must be real and positive here because square root of z naught is the term. Therefore, z naught square obtained from equation 6 above here must be greater than 0. So, in this expression when you substitute the radius of curvature of the mirror 1, mirror 2 and L you must get this as positive z naught square as positive. Thus, so, what is the recipe? So, this is the design reciepe now.

Let me. So, this is the design recipe. Thus given an optical resonator that is R M 1, R M 2, L are known. First check the stability condition because we have to know whether it is a stable resonator or not that is this one g 1, g 2 should be between 0 and 1 and then proceed to find out z naught equation 6 and once you know z naught you know w naught. If you know w naught you know the Gaussian. So, the Gaussian is obtained.

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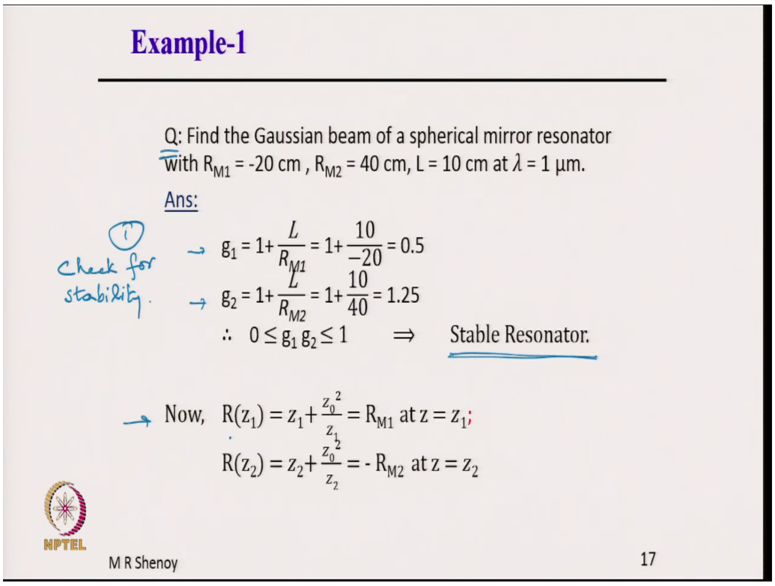

Let us take some examples and it will become very clear now. Question: find the Gaussian beam of a spherical mirror resonator with the R M 1 is equal to minus 20 centimeter which means this is concave mirror; R M 2 is 40 centimeter which means this is convex mirror and L is equal to 10 centimeter at lambda equal to 1 micrometer.

First check g 1, g 2 and stability condition. g 1 is given by 1 plus L by R M 1 and g 2 is 1 plus L by R M 2. So, you can substitute and we see that g 1 is 0.5, g 2 is 1.25. Therefore, the product is less than 1 which means it is a stable resonator. So, this is the first point. So, first check for stability first step check for stability, do not waste time if it is not stable check for stability.

Then, now we go for the condition R of z 1 must be equal to R M 1, R of z 2 must be equal to R M 2.

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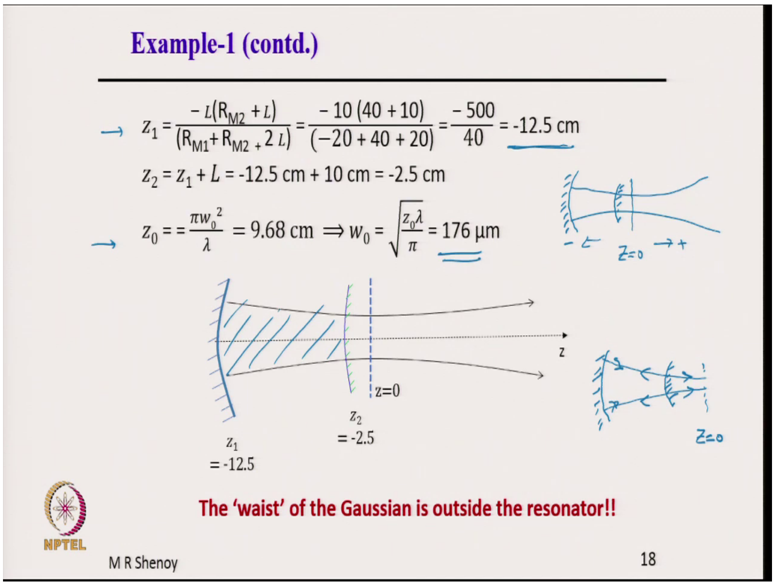

So, let us therefore, find out z 1. z 1 is given by this formula, we have derived just now. Substitute the values. Please remember to keep the signs appropriate signs and then we see that this is minus 12.5 centimeter. For a given Gaussian recall a Gaussian the position of waist is always z equal to 0; if it is minus; that means, you are on this side, if it is plus; that means, you are on the other side.

So, this is plus side, this is negative side. That is how we have got z 1 is equal to minus 12.5 means your first mirror is on this side on the negatives on the left side minus 12.5 and z 2 is equal to z 1 plus L; L is given as 10 centimeter which means it is minus 2.5 centimeter which means the second mirror is also on this side. So, the second mirror is also here.

So, this is the first mirror which is a concave mirror and this is a convex mirror positive radius of curvature and that is what is shown here.

That the resonator now has to be placed in this way to get the Gaussian beam. Note that in this particular example the waist is outside the resonator. So, this is the resonator where the beam is present, inside and the beams coming out and the waist is outside which means if I want to show that the focused the position of the waist is outside, the resonator is here.

Inside the resonator the beam is going back and forth so, this is beam is going back and forth like this, but outside the resonator it is coming out here. And, the focus is here this is z equal to 0. So, inside it is going back and forth here, but the beam which is coming out is a converging beam which has the smallest size spot size or the waist outside the resonator.

There are many applications where such resonators are useful because you need the smallest size of the Gaussian and that can be obtained outside if you choose one concave mirror and one convex mirror. And the size w 0 is now given by so, you know z 1, z 2 and z naught and you can immediately determine w 0 as 176 microns.

Typical helium neon lasers or practical lasers that you take they near the waist the size is typically 200 micrometre or 250 micrometer that is about 0.2 mm, alright.

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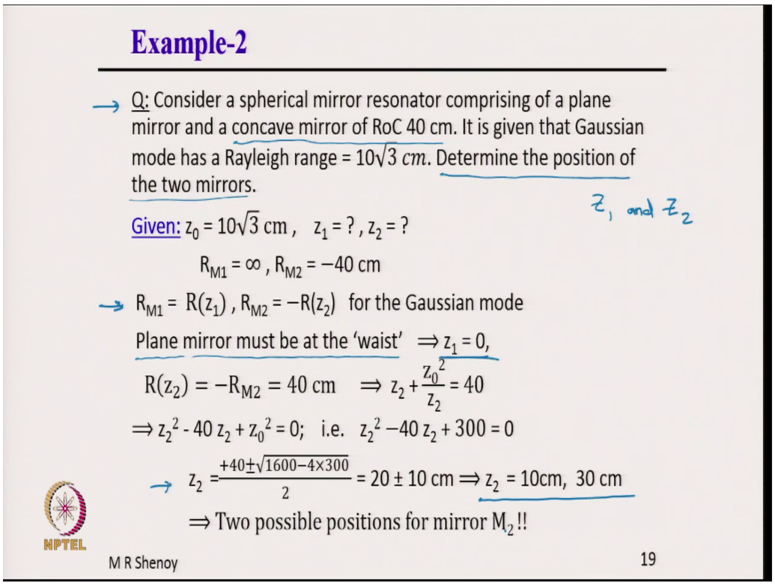

Let me take one more example, example 2. So, consider a spherical mirror resonator comprising of a plane mirror and a concave mirror; earlier I had considered one concave and one convex. So, now, I am looking at a plane mirror and a concave mirror of RoC radius of curvature 40 centimeter, plane mirror radius of curvature is infinity it is given that the Gaussian mode has a Rayleigh range of 10 square root of 3 centimeter.

Determine the position of the two mirrors. You are asked to determine the position of the two mirrors; that means, we have to find out z 1 and z 2. Mirrors are given; one is a plane mirror, another is a concave mirror. Concave mirror of radius of curvature 40 centimeter. Find out the position z 1 and z 2, z naught is given. So, what is z 1 and what is z 2 this is the question.

So, R M 1 first mirror is plane mirror therefore, radius of curvature is infinity; second mirror is a concave mirror therefore, R M 2 is minus 40 centimeters. This is our condition. The requirement is R M 1 must be equal to R of z 1 and R M 2 must be equal to minus R of z are 2 for the Gaussian mode. The plane mirror must be at the waist we have already seen the picture that the plane mirror must be at the waist because at the waist the wave front is plane.

So, we discussed in this problem here that if we are using a plane mirror then it must be at the waist. So, we can see here. So, one of the mirrors positions z 1 is already fixed. So, let me come back to the problem here. So, the plane of curvature must be at the waist which implies z 1 is equal to 0. Therefore, the second condition R of z 2 equal to minus R M 2 is equal to 40 centimeters. Its minus sign is already taken here 40 centimeters which means z 2 plus are z naught by 2 is equal to 40.

So, if we substitute in this, then we get expression here and its a simple quadratic equation and therefore, we have the result z 2 is equal to minus b plus minus square root of b square minus 4 ac by 2a. So, which comes out to be 20 plus minus 10 centimeter what does that mean? That means, z 2 can have two values – one is 10 centimeter, another is 30 centimeters; two possible solutions, which one is correct?

So, we should check the stability condition and see which one of them satisfies the stability condition. It is possible that both may satisfy the stability condition. Let us check.

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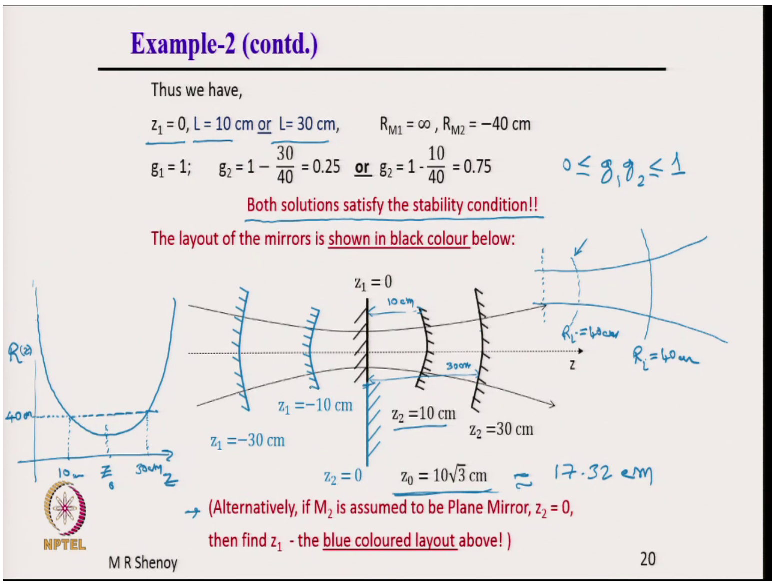

So, thus we have z 1 is equal to 0, L is equal to 10 centimeter or L is equal to 30 centimeter that is if z 2 is 10 centimeter, then L length is 10 centimeter; if the z 2 is at a 30 centimeter then length is 30 centimeter and R M 1 is infinity, R M 2 is minus 40. g 1 calculate g 1. g 1 is 1 because of plane mirror and g 2 is 1 minus 30 by 40 that is 1 plus L by R r is minus 40 therefore, 1 minus 30 by 40, 0.25 or g 2 is 1 minus 10 by 40 if you take L as 10 centimeter, then it is 0.75.

Both solution satisfy the stability condition because 1 into 0.25 is also less than 1, 1 into 0.75 is also less than 1. So, both of them satisfy the condition 0 less than or equal to g 1 g 2 less than or equal to 1. Therefore, both resonators must be stable which means we may have a resonator first let us look at the black colour layout.

The plane mirror is M 1. So, I have the plane mirror at z equal to 0 and this is at z 2 equal to 10 centimeter which means this separation is 10 centimeter or I may have z 2 is equal to 30 centimeter. According to the mathematics, I may have z 2 is equal to 30 centimeters is also permitted which means the resonator may be of length 30 centimeters.

So, this is 30 centimeter. How is it possible? Because we have another condition which says the radius of curvature of wave front must fit the curvature of the mirror here or here which means the radius of curvature of the wave front here must be the same as the radius of curvature of the mirror here. Or the beam which is propagating like this a Gaussian beam which is propagating like this starting from the plane wave front diverges like this, at 10 centimeter it has some radius of curvature which is minus 40 centimeter is the radius of curvature because R M 2 is minus 40 centimeter.

It says at 30 centimeter also it has the same radius of curvature 40 centimeters. So, this is R i of the beam is equal to 40 centimeters; R i of the beam is equal to 40 centimeters. How is it possible? It is possible if the radius of curvature passes through a minimum. The radius of curvature is infinity for a Gaussian beam the radius of curvature is if I want to plot the radius of curvature, then the radius of curvature is infinity.

We have already seen it is a plane radius of curvature and again at z tending to infinity it is again going to infinity which means there is a minima. And therefore, if the radius of curvature minus 40 centimeter happens to be here; so, this is 40 centimeter, then it can have the same value at two distances 10 centimeter and 30 centimeter provided the minima lies between them.

So, what I have plotted is R of z versus z for the Gaussian beam indeed the radius of curvature, this point you can show from the expression that the minimum occurs at z is equal to z naught. So, if z naught lies between these two values somewhere in between it is not exactly mid way, then it is possible.

And, note that in the given problem z naught is equal to 10 root 3; root 3 is 1.732 therefore, this is approximately equal to 17.32 centimeters and z naught indeed lies between 10 and 30. And, therefore, both at z 2 is equal to 10 centimeter and z 2 is equal to 30 centimeter the Gaussian beam has the same radius of curvature which is 40 centimeter is the RoC of the Gaussian beam. So, that is what I have shown here in this diagram. It is a very interesting problem.

Alternatively, here if I had chosen M 2 as the plane mirror then the plane of curvature has to be placed like this and then we will get z 1 is equal to minus 10 or minus 30 as the two solutions. But, note that there are many problems where out of the two solutions only one of the solutions becomes stable, the second solution may not be stable.

But, here is a special case where both the solutions are stable and therefore, the plane mirror position is fixed at z equal to 0, the concave mirror maybe z 2 equal to 10 centimeter or it may be at z 2 is equal to 30 centimeter because the minimum is in between these two positions. The minimum radius of curvature occurs at z is equal to z naught.

Please verify this. So, differentiate the formula for R of z with z and find out put that equal to 0 we will find that R of z is minimum when z is equal to z naught and z naught this special value is called the Rayleigh range, where the radius of curvature of the Gaussian is a minima. We stop here.

Thank you.