Introduction to LASER
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Lecture - 19
Transverse Modes of a Spherical Mirror Resonator

Welcome to this MOOC on LASERS, we are now studying the optical resonators. So, far we have discussed about the resonance frequencies or the longitudinal modes, then from the ray picture we discussed about ray confinement and the stability of optical resonators.

And then of course, we have discussed various sources of losses and parameters which characterize losses. Today we will see the Transverse Modes of a Resonator.

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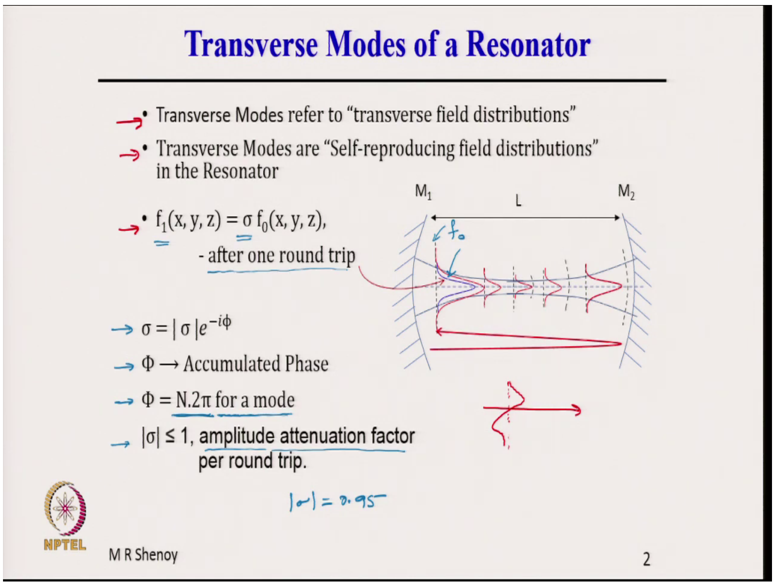

Transverse Modes of a Spherical Mirror Resonator, transverse modes here refer to transverse field distributions. Here transverse refer to if this is the direction of propagation then transfers to the direction of propagation perpendicular the field distribution.

For example, a field distribution something like this, so it refers to transverse field distributions and transverse modes are those field distributions which are self reproducing, they are self reproducing field distributions in the resonator.

What it means is if the field propagates to the other end and comes back then after one round trip it completely reproduces the distribution. So, if f 1 of x, y, z here f 1 of x, y, z let me change the color, f 1 of x, y, z is the field after one complete round trip, then we should be able to write f 1 of x, y, z is equal to sigma into f 0 of x, y, z f 0 of x, y, z is the starting field.

Let us say we start from here and let this be f 0. So, what is shown here? For example, the red colored field distribution is f 0 then it propagates in the resonator gets reflected, here comes back and gets reflected and comes back to the same plane z. Now, we look at the blue, so the blue field distribution is identical to the red field distribution f 0. The blue field distribution is f 1 which is identical to f 0 but for a multiplicative factor sigma, where sigma may have an amplitude attenuation coefficient.

For example, you see that the blue field distribution is smaller in size which means it has lost energy in propagating one round trip and it will have a round trip accumulated face. And therefore, if we are able to write the field after one round trip as sigma into the initial field, where sigma can be written as mod sigma into e to the power minus i phi, phi refers to the accumulated phase.

And this accumulated phase the round trip phase must be equal to an integral multiple of N 2 pi, here N is an integer and phi must be an integral multiple of 2 pi for a mode. For the field distribution to be a mode the round trip accumulated phase must be an integral multiple of 2 pi.

Mod sigma which represents the amplitude attenuation factor per round trip is less than or equal to 1, when it is equal to 1 it means there is no attenuation. But all practical resonators as we have discussed have a finite amount of loss and mod sigma which represents the amplitude attenuation factor per round trip is less than 1.

So, let us say if mod sigma is equal to 0.95; that means the amplitude will drop by 5 percent after one round trip. We can see here the blue one and the red one the blue one represents the field distribution after one round trip. But the field distribution after one round trip reproduces itself, the distribution is the same, but the amplitude may vary slightly and the accumulated factor will be equal to an integral multiple of 2 pi.

This is the condition for the transverse mode of a resonator, every transverse mode has to satisfy this condition if it has to be a mode.

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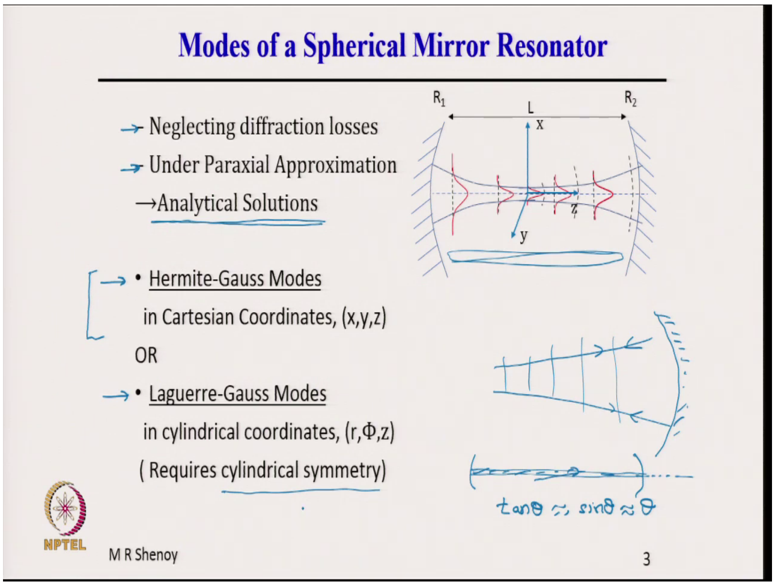

Let us discuss this mode. So, in a spherical mirror resonator if we neglect the diffraction losses, we have already discussed about this diffraction loss refers to the loss due to diffraction of the beam. We have already discussed this. Every finite beam diffracts as it propagates and if we can if we have a focusing mirror here such as a concave mirror which would focus it back, so that the diffraction effect is compensated then we can neglect diffraction losses.

So, neglecting the diffraction losses and under the paraxial approximation, we have also discussed under the paraxial approximation which means in a resonator we are normally looking at beams which are very narrow compared to the length of the resonator and therefore the rays comprising this beam will be essentially making very small angle.

So, the rays which are travelling are making very small angle with the axis and under this condition we know it is called paraxial approximation where the rays make small angle with the propagation axis, so that we could use the condition tan theta nearly equal to sin theta nearly equal to theta this we have discussed in detail in matrix optics.

So, with these two conditions it is possible to obtain analytical solutions for the propagating beams which form modes of a spherical mirror resonator. So, we will not go here into the derivation of these modes, but we will straight away we will start with the answer with the solutions. The solutions are 2 families of solutions are there Hermite-Gauss Modes and Laguerre-Gauss Modes.

In Cartesian coordinate we get Hermite-Gauss field distributions and in cylindrical coordinates if you solve the propagation problem that is the beam going back and forth, the propagation problem with the appropriate boundary conditions then we get Hermite-Gauss field distributions as the solutions for these modes.

Modes are allowed solutions to the propagation problem with appropriate boundary conditions. And if the resonator has a cylindrical symmetry then we can also get a second family of solutions which is Laguerre-Gauss Modes in cylindrical coordinates. So, in this course we will discuss only one family of modes and I will skip this Laguerre-Gauss Modes in this discussion alright.

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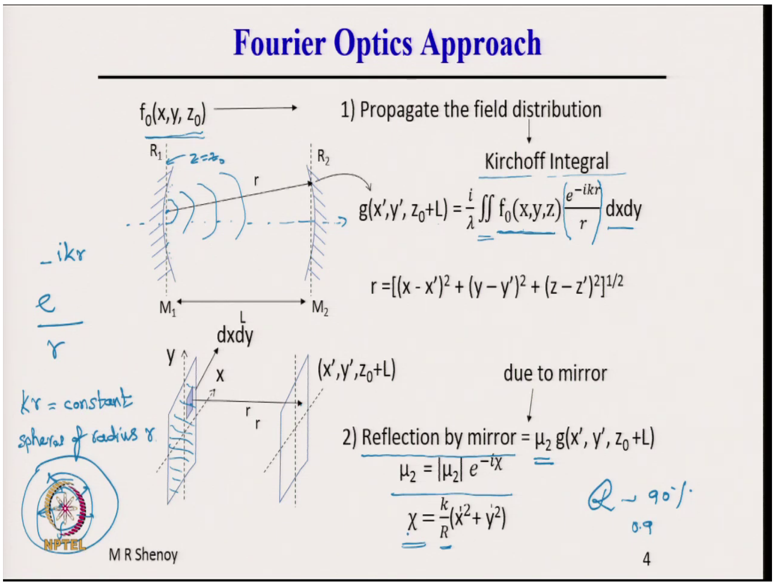

So, what is this approach? So, how to obtain the modes of a resonator? So, this is called the Fourier Optics Approach. In Fourier optics approach as I discussed that we start with the field distribution f of x, y, at z is equal to z 0 plane. So, let us say this is z equal to z 0 this plane is z equal to z 0 this direction is the z direction in all our discussions. So, if f 0 of x, y, z 0 is the initial field on the first mirror then the field is propagated to the other mirror.

So, propagation of the field distribution is done by what is called as Kirchhoff's Diffraction Integral. To obtain the field at the other mirror which is at a distance of L from the first mirror, therefore, at 0 plus L on the plane z equal to z 0 plus L. So, this integral essentially tells that the field at any given point x, y, z the initial field is propagated.

So, this if we consider the amplitude of the field at a certain point here then like a point source, so this will so this will propagate to the other end. So, the propagation of a point source is given by e to the power minus i k r by r. So, e to the power of so this represents the output of a point source, because e to the power of i k r.

So, k r is the phase, so k r equal to constant are spheres k r equal to constant are spheres of radius r spheres of radius r. In other words there are spherical waves starting from the points. So, if you have a point source then this gives out spherical waves as the waves start moving out in the form of a sphere spherical wave front. So, k r is equal to constant that is why we have e to the power of i k r.

And r in the denominator takes care of the amplitude drop, as we know that with r the amplitude drops as 1 by r, because the energy or the power or the intensity drops as 1 by r square as the light propagates from the point source. And therefore this term essentially represents the point source propagation term and this is the field distribution and it is integrated over the entire plane x, y, at z is equal to z 0.

So, it is propagated to the plane z is equal to z plus L and we get the field at z is equal to z 0 plus L by this diffraction integral. So, this integral essentially propagates point sources to this end to find out the effect at the point x dash y dash comma z 0 plus L. And integration gives contribution due to all point sources here. It is illustrated here at x, y, and z if we consider a small infinitesimal area element d x d y, then we find the effect due to this element here at the point x dash y dash z 0 plus L.

And then you integrate obtain the amplitude at any arbitrary point, you integrate the contribution of similar area elements over the entire plane here and that is what is done in the Kirchhoff integral to obtain the field distribution after propagating through a distance L. So, using that integral we can obtain the field distribution at the second mirror and then it is multiplied by a factor due to reflection by the mirror.

So, the field here is multiplied by a factor mu 2 which can be written as mod mu 2 into e to the power of minus i chi, where chi is the phase introduced by the spherical mirror of radius of curvature r and mod mu 2 gives us the amplitude attenuation factor at that mirror. That is we know that every mirror has a reflectivity let us say 90 percent, then the 10 percent is the loss and therefore the field will be multiplied by a factor actually the intensity will be multiplied by a factor of 0.9.

So, mod mu 2 will be 0.9 if the reflectivity is 90 percent, if the reflectivity of the second mirror is 80 percent then mod mu 2 will be 0.8 and chi is the phase due to curvature of the spherical mirror.

As you can see if we use a plane mirror then r tends to infinity and therefore chi is equal to 0 there is no phase change if you use a plane mirror. Of course, we assume that we are discussing about dielectric mirrors, if it is a metallic mirror then there will be a phase change of pi alright.

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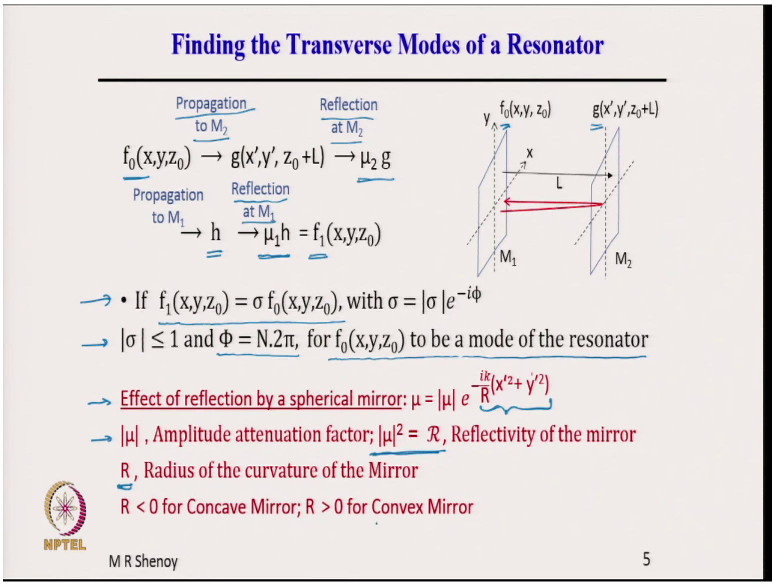

So, continue with that let me explain the procedure here. So, we start with the field f 0 of x, y, z 0 at mirror M 1 propagate to mirror M 2 which means the Kirchhoff's diffraction integral we get the field here at g, we started with f 0 then we get the field at g. At g there is reflection at the mirror M2, so M 2 is here which means as I explained multiplied by a factor mu 2. This is a complex factor it will have an amplitude attenuation factor which is mod mu 2 into the phase change provided by the mirror.

So, this mu 2 into g is propagated back to the mirror M1. So, as shown here it is propagated back to the mirror M 1 using the same diffraction integral or Kirchhoff integral. And if h is the field at mirror M 1 before reflection, after reflection it will be multiplied by a factor mu 1, as before mu 1 is a complex factor which has a mod mu 1 which corresponds to the reflectivity and a phase factor due to the mirror M1.

And after multiplication here that means one complete round trip the field we designate as f 1 of x, y, z 0 this is one complete round trip propagating the field through one complete round trip starting with f 0 of x, y, z 0 we came back to the same place x, y, z 0 same plane. But now we designate is as f 1, f 1 representing the field distribution after one complete round trip which included propagation through 2 lengths L and reflection at mirror M 2 and reflection at mirror M1.

If f 1 of x, y, z 0 can be written as sigma into f 0 of x, y, z 0 with sigma is equal to mod sigma into e power minus i phi and mod sigma less than or equal to 1 and phi happens to be an integral multiple of n pi then f 0 of x, y, z 0 will be a mode of the system. For f 0 of x, y, z 0 to be a mode of the resonator it must satisfy this condition f 1 of x, y, z 0 is equal to f 1 is after 1 complete round trip sigma into f 0 of x, y, z 0.

For a given resonator if you impose this condition and if you can find out a field distribution f 0 which will reproduce itself with a multiplication factor, then f 0 will be a mode of the system. For many resonators we can have analytical solutions but there are resonator. For example, plane mirror resonator where we cannot get analytical solution, but we can obtain numerical solutions for the modes by imposing the same condition.

You start with a certain field distribution propagate it and then you see after f 1 whether the terms can be written as sigma into f 0, if not then iterate f 0 and propagate again iterate f 0 propagate again, till you have you are able to write f 1 as sigma into f 0. This is the way numerically one can determine the field distribution for any given resonator.

The effect of reflection by the spherical mirror is again explained here, multiplication by a factor mu which is equal to mod mu into e to the power minus i k by R into x dash square plus y dash square. We can show that the phase introduced due to reflection by a spherical mirror of radius of curvature R is given by this factor.

Where mu mod mu as we discussed is the amplitude attenuation factor, mod mu square is the reflectivity capital R please see this R is flower R which is different from this R, this R is the radius of curvature this R represents the radius of curvature of the mirror and this R here is the reflectivity of the mirror. The radius of curvature we know the sign convention that R is less than 0 for concave mirror and R is greater than 0 for convex mirror.

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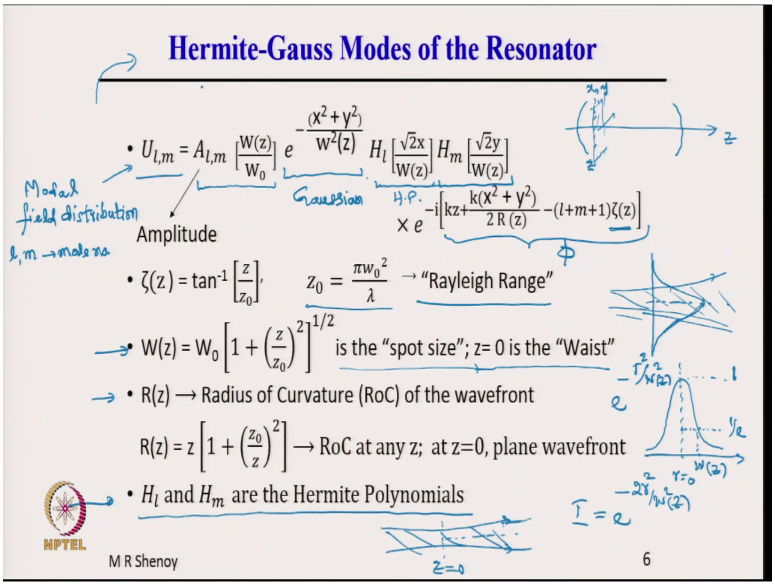

So, the Hermite-Gauss Modes of the resonator which are solutions of spherical mirror resonators are given by this expression, as I mentioned we are not going into the derivation to obtain these modes. I recommend for the interested people to look at the references given for this course where the derivations are available.

So, U l, m l and m are integers which are called the mode numbers, U l, m is the field distribution so this is the field distribution of the mode. So, this is the modal field distributions modal field distribution, the complete field distribution including amplitude and phase distribution and l m l comma m are the mode numbers mode numbers.

So, A l, m is amplitude which is multiplied by W of z by W 0 I will explain what is this W 0, W of z is the spot size it is a Hermite-Gauss field distribution.

This is the Gaussian distribution e to the power minus R square by W square of z, where W of z is called the spot size, the spot size at the waist is called W 0 we will see that again here this is the Gaussian. So, in Hermite-Gauss this is the Gaussian. So, the Gaussian field and these are the Hermite polynomials H l and H m are the Hermite polynomials.

So, here it is H l and H m are the Hermite polynomials we will discuss more about this and this is the phase factor phi e to the power minus i phi that is the round trip phase factor phi. So, this is the phase factor phi with the propagation where z is the propagation distance.

So, we will discuss each of these very carefully, first this is the amplitude the complete amplitude will have an amplitude constant multiplied by W of z by W of 0. We will see what is this? This is the Gaussian part and this is the Hermite polynomial let me write it as H.P the product of 2 Hermite polynomials of order l and m. So, this is the same l and m which is the mode number and we will discuss now.

In the phase factor here zeta of z there is a term called zeta observed l and m are mode numbers zeta of z is equal to tan inverse of z by z naught. What is that? z is the propagation distance at any z, so this is the z value. So, this is the z direction and therefore at any plane x, y, and Z. This is z tan inverse of z by z naught, z naught is a parameter called Rayleigh range, z naught is given by pi w 0 square by lambda it is called Rayleigh range W of z here is the spot size of the beam.

So, if you have a beam which prop which is shown whenever we show a beam like this. So, what are we showing? So, let us say it is a Gaussian beam which is propagating. So, the Gaussian is like this, so this is the field distribution of the Gaussian let me show that.

Now W of z, so when we show the beam like this we are showing actually the locus of the spot size that is spot size. So, if I now rotate this and show the Gaussian like this let me rotate the Gaussian and show it is in the form of a bell.

So, this is r is equal to 0 or x square plus y square is equal to 0 and W of z is. So, if we look at this Gaussian so e to the power, let me write here e to the power minus x square plus y square is r square that is the transverse coordinate into W square of z. So, this is the point r is equal to 0, of course r equal to 0 means I should not plot to this side I have to plot only to 1 side let us assume that I am along a diameter.

And therefore, at R is equal to W of z at R is equal to W of z is e power minus 1. In other words W of z the spot size is defined as the radial position where the amplitude drops to 1 by e of its maxima, at R is equal to 0 the maximum is 1 at this point it is 1 by e. So, those of you are not familiar with the Gaussian. So, please note that the spot size W of z is the radial position where the amplitude drops down to 1 by e or if we talk in terms of intensity then you take mod square of the field.

Then where the intensity drops to 1 by e square plus if we take intensity then it will be e to the power minus. So, mod square therefore 2 r square by W square of z here and therefore, 1 by e square at r is equal to W of z this will be 1 by e square of the maximum of intensity.

So, W of z is the spot size and at z is equal to 0 is the waist. What is this waist? We will discuss this in the next diagram waist is the smallest width of the beam. So, whenever we show the beam like this then this is the waist where the beam width is the smallest. So, what I am what am I showing?

I am showing the propagating beam like this a beam which is propagating and waist is where z is equal to 0, where the size of the beam is minimum and if W 0 is the waist, then it can be shown that the W of z that is the spot size at any value of z can be written as W 0 into 1 plus z by z naught square to the power half. R of z is the Radius of curvature ok.

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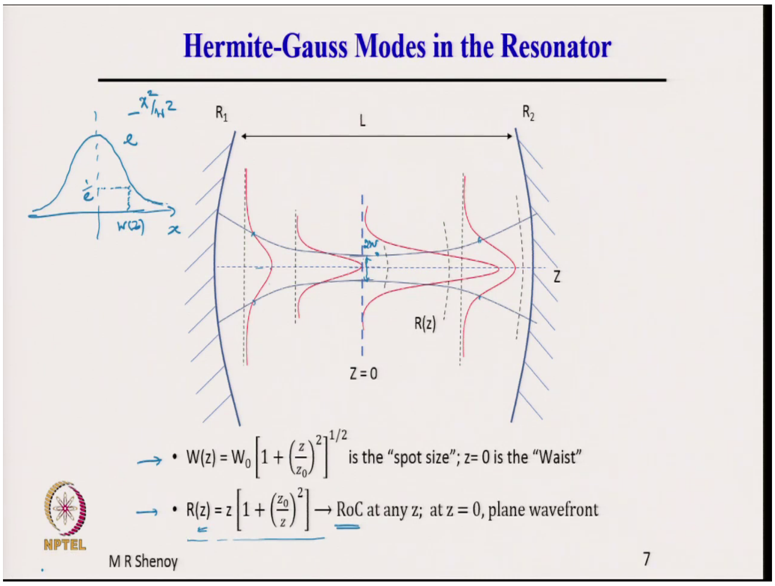

Let me show the diagram here more carefully. So, this is the resonator in which let us say a Gaussian beam is propagating assume that the Hermite polynomials are 1 and then it will be a pure Gaussian distribution. And as you can see z equal to 0 is where the waist is minimum, the line here the blue line here shown is the locus of the 1 by e point. So, this is the 1 by e point. So, 1 by e point 1 by e of the maximum. So, everywhere this is the 1 by e point.

So, what is shown is with this beam because if it is a Gaussian beam. So, those of you so the Gaussian field distribution becomes 0 asymptotically at so if I plot this as e power minus x square by w square then becomes 0. But to define a beam width normally what is done is you take the amplitude where it drops down to 1 by e of the maximum and this is called the spot size. So, this is the locus of the spot size as it propagates along the z and at z equal to 0 the width is smallest.

So, this width or this diameter here is 2 W 0. So, 2 W 0 is the diameter of the beam smallest diameter of the beam at the waist. So, W of z is given by this formula this relation and z equal to 0 is the waist, you can put z is equal to 0 in this you will see that W of z equal to W 0 at z is equal to 0. R of z is the radius of curvature at any z radius of curvature of what radius of curvature of the wave front. So, there is a wave front here wave front is surface of constant phase.

So, R of z is the radius of curvature of the wave front the radius of the curvature of the wave front also changes with z and it is given by a relation of this form. So, that is the radius of curvature. So, let me go back and see. So, R of z the radius of curvature of the wave front is we have discussed and R of z is given by this formula and H l and H m are the Hermite polynomials.

So, if we look at the entire field distribution now, now we understand all the parameters here. So, A l, m into W of z by W 0 represents the amplitude of the field distribution. This distribution is the Gaussian distribution which is multiplied by 2 Hermite polynomials, one in the x direction and one in the y; x and y are the transverse direction, please see this is z and x and y is the transversal.

So, if we show it as a plane then x and y are the transverse directions. So, 2 Hermite polynomials and the propagation phase accumulated or propagation phase term. So, that is the distribution that is why the name Hermite Gauss, Gauss multiplied by Hermite polynomial, hence the name Hermite-Gauss Modes of the Resonate; let us proceed.

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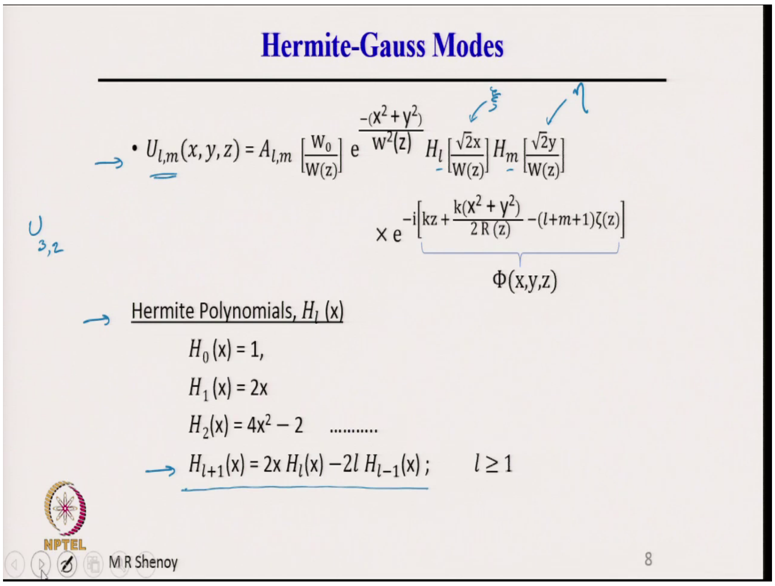

Let us see more carefully now the Hermite polynomials. So, it is the same field distribution I have written again here, the Hermite polynomials are given by H 0 of X. So, when you write H l of x l is an integer l or m. So, land m are integers 1, 2, 3, 4 etcetera, 0, 1, 2, 3 etcetera.

So, H 0 of x is 1, H 1 of x is 2x, H 2 of x is given by this and so on and if you know any 2 then the next one can be determined by the recurrence relation this is called recurrence relation for L greater than or equal to 1. So, these are the Hermite polynomials. So, whenever you want to determine a particular mode U l, m.

So, let us say U32 which means l is equal to 3 and m is equal to 2, then you have to use the polynomial here H 3 Hermite polynomial of the third H 3 of this into H 2 of this. A product with the Gaussian gives us the Hermite-Gauss modes.

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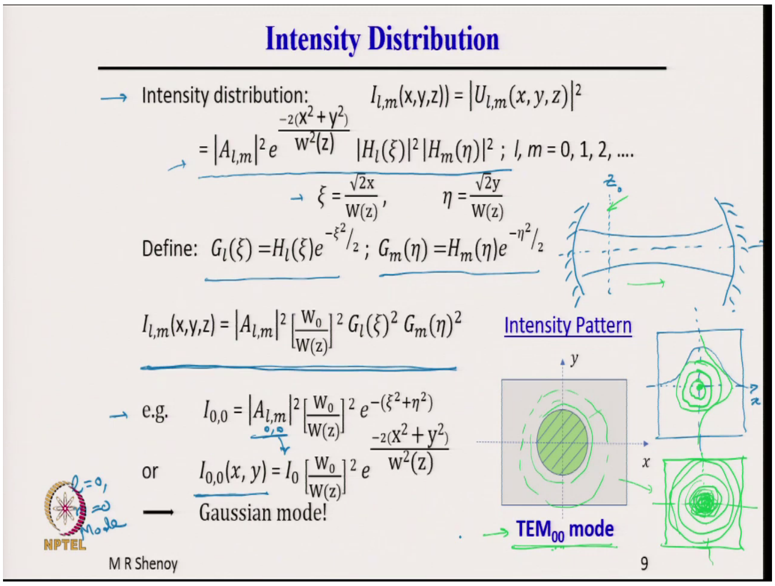

Let us see the intensity distribution of these modes, intensity distribution of the Hermite-Gauss mode. To obtain the intensity distribution you take mod square of the field U l, m of x, y, z that gives us the intensity, if you take mod square of the field. So, let us look at the field. If you take mod square first of all the phase term this part goes away and then we have square of this square of this e to the power minus 2 times this and mod square of H l and H m.

So, that is what is written here. So, we see so this is what is written the intensity distribution is written in this form. So, square of this there is a 2 here and of course the phase term disappears. Where psi just to simplify if we write this psi if you look back so this term if we call as psi, so this is psi and this as eta then we can simply write this as H l psi mod square into H m eta mod square for l m 0, 1, 2 etcetera.

Now, if you define because there is a x by W of z psi is defined by x by W of z into root 2; therefore, you can take e to the power so this is minus 2 x square by W square of z and into multiplied by e to the power minus 2 y square by W square of z. Therefore, taking the 2 x square by W square z with the psi and the second part with eta we can define G l of psi is equal to H l of psi into e to the power of minus psi square by 2 and G m here.

Then the Gaussian will be absorbed in this G and we can write the intensity given by this expression here. The amplitude so this is the amplitude multiplied by the Hermite-Gauss Fields the product of the Hermite-Gauss Fields. So, for example, the simplest one if you want to find out the intensity distribution of the 0 0 ordered mode I 0 0 land m if you put 0 0.

So the 0 is 1 H 0 of X, please see H 0 of x and H 0 of y or H 0 of psi is 1; H 0 of whatever be the argument is 1 and therefore for I 0 0 we have A l, m. So, this is a 0 0 mod square W 0 by W z whole square into e to the power of so these will be there because there is no l m, but both H l and H m are 1 because l is equal to 0 m equal to 0.

So, these are one and therefore we simply have a product e to the power minus z square plus eta square or it is e to the power minus. So, the same term here the first term e to the power minus 2 x square plus y square, essentially this is 1 this is 1 and therefore the intensity distribution. So, if I want to write I 0 0 here is A l, m mod square multiplied by this.

So, I 0 0 of x, y, is equal to we call this as I 0. So, this we call it as I 0 into this. So if you plot this what is shown is the intensity pattern? So, what is this? At a given z this is constant at a given z.

So, what is plotted here? So, we can have the resonator in which. So, let us say this is the spherical mirror resonator and here is the z propagation. You consider a plane some plane z z is equal to z naught. So, let us say this is z naught z is equal to z naught, in this plane you have W 0 divided by W of z naught.

So, this is now a constant as far as x, y, in the transverse plane at z naught if we plot the intensity distribution. So, what is shown is the intensity pattern actually it is like this, if you see the transverse plane at z is equal to z naught we have the x and y, here in the x direction we have e power minus 2 x square by w square. So, we have a Gaussian which I so the intensity now I am plotting as a field. So, this is plotting as a distribution e to the power minus 2 x square by W square of z.

Similarly, if I plot let me use a different color in this direction we have so this is with respect to y, in this direction it is y that is e power minus y square by W square of z. Note that at y is equal to 0 at x is equal to 0 we have maximum at y is equal to 0 also the function is maximum, because e to the power minus 0 is 1 and therefore we have maximum here for both.

If you take a product of this then what you get is you will get maximum intensity at the center, if you go to the sides the intensity decreases.

So, if I plot a contour map so maybe I will show it separately here a contour map, then with the density of contours maximum where intensity is, so then it will look like this I am trying to plot. So, the contours will become thinner and thinner as you go outside and the density will be maximum at the center. So, this is the contour map of the intensity pattern in a transverse plane at z naught. So, that is what is shown here.

So, it does not mean there is no field outside here, so this is shown as a spot, but actually note that there are the field is becoming weaker and weaker as you go outside. So, that is why I have shown an equivalent contour map of the TEM 0 0 mode the U l, m l m are 0 0. So, it is called the TEM 0 0 mode because the modes in such open resonators are TEM Transverse Electromagnetic Modes.

TEM refers to transverse electromagnetic modes this is an open resonator there is a beam propagating in the open resonator and the field distribution is a TEM 0 0 mode I 0 0 is equal to mod A l, m square here l and m should be 0. So, these are 0's so 0 comma 0 l and m are0. So, this is the complete expression is here.

So, when l and m are 0 this is a 0 zero mod square which I designate as I 0 is the it is a constant therefore it is I 0. I 0 0 x, y, represents the intensity distribution of the l equal to 0 m equal to 0 mode. So, l is equal to 0 m is equal to 0 mode and this is called the TEM 0 0 mode.

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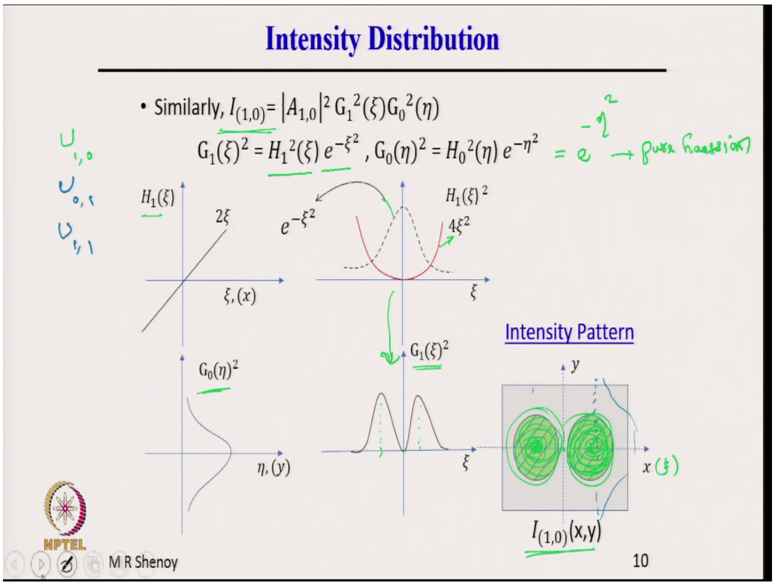

Now, if we look at another mode similarly let me take the next mode I 1 0 that is corresponding to U 1 0, l is equal to 1 m is equal to 0. I 1 0 1 and 0 are the indices l and m is given by mod A 1 0 mod square into G 1 now it is G 1 because l is 1. So, Hermite polynomial of the next order which is H 1 of psi is equal to 2 psi or H 1 of x is equal to 2 x is given here multiplied by the Gaussian e to the power of minus psi square.

So, this is the first one G 1 and this is G 0 is of course H 0 which is 1 and e power minus eta square. So, this is simply equal to e to the power minus eta square, which is the second function which is simply pure Gaussian so pure Gaussian. So, let us look at the field distribution the first field is H 1 square multiplied by the Gaussian. So, this is the Gaussian e power minus psi square. So, this is the Gaussian here H 1 square is 4 psi square H 1 is 2 x therefore H 1 square is 4 psi square.

So, that is why it is a parabola variation 4 psi square and G 1 is a product of H 1 square of psi into e to the power minus psi square that is a product of the Gaussian and the parabola here.

Note that the parabola goes down to 0 at psi is equal to 0 although the Gaussian is maximum and therefore when you take a product of these 2 functions what you get is G 1 square of psi which is like this, at psi is equal to 0 the product is 0 the function goes on Gaussian goes on dropping but the parabola goes on increasing.

And therefore at some place there will be a maximum, but later on again the field drops down because the Gaussian continuously drops down to 0. So, what is shown here is the product of the Hermite and Gauss functions which is the G 1 of psi square. G 0 f course, G 0 of eta is a pure Gaussian which is written here e power minus eta square, now a product of these 2 intensity distribution. So, the product is 0 at psi is equal to 0, so the product is 0 on the x axis here at psi is equal to 0.

And it is maximum where this is maximum where psi is maximum it is maximum where psi is maximum. So, we can see that the maximum so this axis is x or psi x is proportional to psi, therefore it is maximum. So, these points here correspond to maxima, again please remember that this is intensity distribution if I were to show a contour map the intensity will go on dropping down and the maximum at the center. So, this is maximum at the center and drops down in a contour map.

So, this is the intensity pattern in a transverse plane at some z value I 1 0 intensity distribution in the x, y, plane, so it is a product of G 1 into G 0. As far as y distribution is concerned in this direction. So, with y we would have had let me just change the pen. So, we would have had this distribution here maximum at y is equal to 0 and then it would so everywhere. If I plot on this with y if we plot this variation will be like this, everywhere but the x variation tells us that it is 0 at x is equal to 0.

And therefore, when you take the product you get a pattern with 2 lobes 2 intensity lobes with a minima in the center. Exactly like this we can find out the intensity distribution for the U 0, 1 or U 1, 1 and so on.

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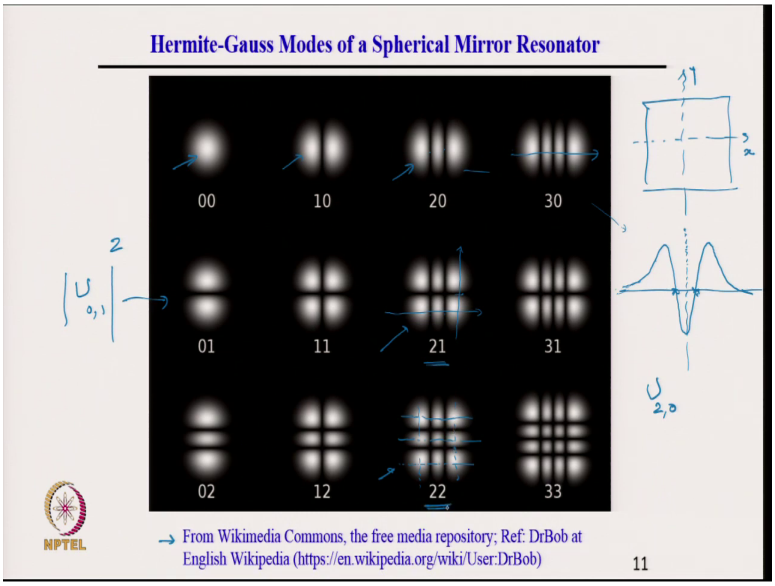

Now, what I am going to show is an actual computed Hermite-Gauss Modes of a Spherical Mirror Resonator, this is from Wikipedia commons the free media repository which is attributed to DrBob at English Wikipedia at this site. Note that the 0 0 is here so this is the 0 0 this is the 1 0 the 2 fields which I have described how to obtain the field distributions and if you similarly find out H 2 that is U 2 and 0 then you would get a field distribution like this and so on 30, 01.

So, this is 01 corresponding to U 0 comma 1 intensity distribution of this, so mod square of this is what is shown here. What is important is to note that the first index represents the number of 0's encountered. The number of intensity 0's encountered as you move along the x axis or the psi axis.

So, we have so this is the a plane a transverse plane and this is x or xi, so x and this is y here. So, if you are moving along the x direction then the number of 0's leaving the asymptotic 0 at the end if I want to plot the field corresponding to this the field in the x direction would be so we will have this here and then where this is the maximum.

So, note that there are 2 0's the field passes through 2 0's this is corresponding to the 2 0 mode. So, this is U 2 0 I have plotted only with respect to x, there is a 0 here there are 0 the field also goes to 0 asymptotically. So, this 0 is not counted only the 0's only the 0's which are of the field are counted as we move along the x direction, 2 0 when I give a field distribution 2 0 2 refers to the number of 0's along the x direction and 0 refers if we go in this direction y direction there are no 0's if we go in the y direction.

But here if you go along this direction again note that there are 3 0's which are encountered leaving the asymptotic 0 apart. Now if we take 2 1 what does that mean? That means, if we are travelling in the x direction if we move in the x direction there will be 2 0's which are encountered and if you are moving in the y direction there will be only 1 0 which is encountered here that is why the mode number is 2 1.

If you take 2 2 then note that if you are moving in the x direction whether here or here or here you will encounter 2 0's and if you are moving in the y direction then also you would encount 2 intensity 0's and that is why the mode number is 2 2.

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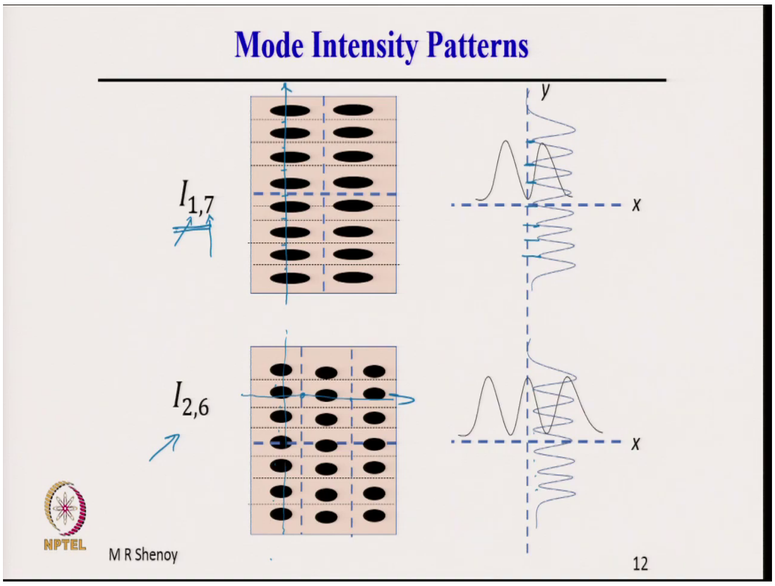

Therefore, if I have a general intensity distribution for I 1, 7; what does it mean? If I am moving in the x direction here I must encounter only 1 0 whereas, if I am moving in the y direction I will encounter 7 0's look at the field distribution. So, if you are moving in the y direction which is this you see one 0 2 0 3 0 4 5 6 7, leaving the last 0 last 0 that is why I have plotted here.

The corresponding field distribution note that the field 1st 0, 2nd 0, 3rd 0, 4th 0, 5th 0, 6th 0 and 7th 0 this 0 is the asymptotic 0 this end and this end they are not counted. And therefore I 1, 7 means in the y direction there are 7 0's of the field and in the x direction there is only 1 0.

Similarly, if you see I 2 6 when you are moving in the x direction you encounter 1 0 here and 1 0 here, whereas if you are moving in the y direction there are 6 0's encountered again the corresponding field variation actually the intensity variation corresponding intensity variation in the y direction and in the x direction are shown here.

So, the intensity passes through 0's 6 0's here leaving the asymptotic 0 at the ends. Therefore given any intensity pattern we would be able to identify the mode numbers alright.

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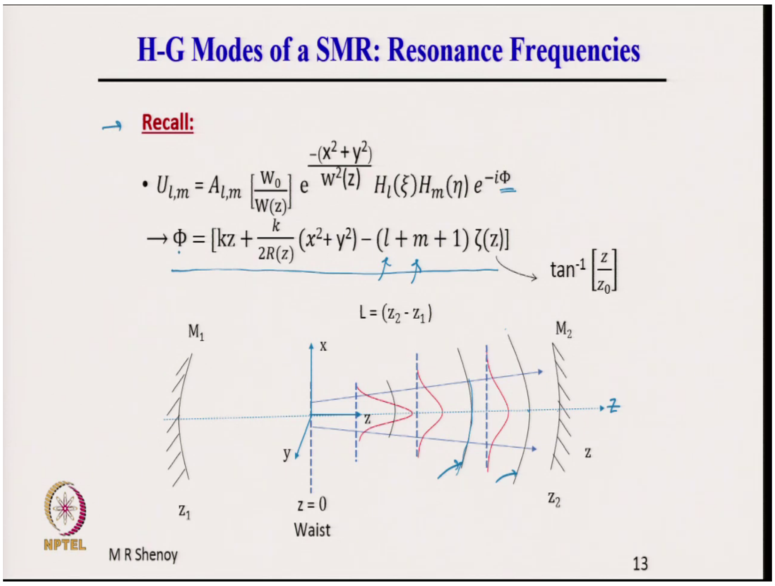

Let us proceed further finally we come to the resonance frequencies we have seen the intensity distributions. Now let us look at the resonance frequencies of these modes. Recall the complete field here where phi is the phase, so the phi is written as the full phi is written here is equal to k into z plus k divided by twice R of z x square plus y square minus l plus m plus 1 into zeta of z.

Where R of z is the radius of curvature here it is shown radius of curvature, so these are the radius of curvatures R of z at different z and this is the z equal to 0 is the waist. And with z propagation is shown here l and m in the phase are the mode numbers. Now the resonance frequencies are given by round trip phase must be integral multiple of 2 pi.

Now to determine the round trip phase accumulated it is sufficient for us to see the round trip phase on the axis, which means x equal to 0 y equal to 0 is the z axis. So, this is the z axis x equal to 0 and y equal to 0. Because the wave front every wave front is characterized by a z and therefore if we look at the, so let me show here and it becomes clear.

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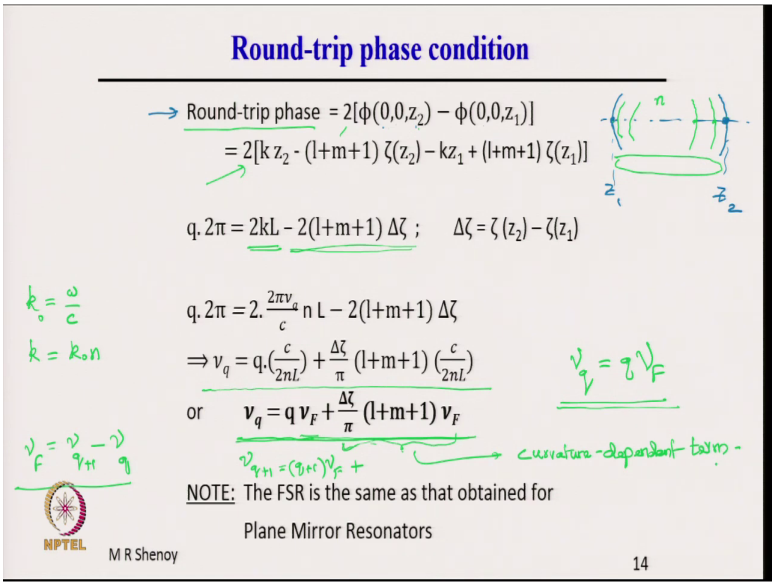

To calculate the round trip phase it is sufficient for me to find out the phase at the point 0, 0, z 2 minus 0, 0, z 1. What is this 0, 0, z 2 and 0, 0, z1. So, let me show here draw here and show. So, this is the plane z 1 and this is the plane z 2 0, 0, z 1 corresponds to the point here and 0, 0, z 2 that is x and y transverse coordinates are 0 z 2.

Now, because the wave fronts if I show with a different color if I show the wave fronts like this. So, the wave fronts propagate in this direction and go forward and come back. So, in calculating the round trip phase it is sufficient for us to find the point on the axis, because it is the same wave front it is not necessary to find out any other arbitrary point x, y, z instead the axial phase is sufficient to calculate the round trip phase. That is why the round trip phase is 2 times the propagation phase from z 1 to z 2 or z 2 minus z 1.

So, that is why 2 times phi 0, 0, z 2 that is the phase here at z 2 minus phi 0, 0, z 1 the phase which is here twice will give us the round trip phase and that is equal to 2 times k z 2 minus this; that x, y, term has vanished because of this concept that it is sufficient to consider the point on the axis, because the wave front is 1 and therefore we can consider any particular x, y, point. And therefore if you look back at the phase at the phase term here this term disappears because x equal to 0 y equal to 0.

Therefore, we have k times z 2 minus k times z 1 and L plus m plus 1 into zeta of z 2 minus zeta of z 1, so that is what is written here. So, 2 times k of z 2 minus this minus this which is equal to k times z 2 minus z 1 is k L z 2 minus z 1 is l. Therefore, 2 times k L minus 2 times l plus m plus 1 into delta zeta of z, where delta zeta of z is zeta of z 2 minus zeta of z 1 oh it is written here.

If we simplify this k is omega by c omega is 2 pi nu by c and because it is in the medium the medium may have a refractive index n. Then it is preferable to also keep the n and therefore it is k 0 into n, k 0 is the free space propagation constant which is equal to omega by c.

If it is in the medium then k is equal to k 0 into n, where n is the refractive index and that is why we have retained that n here minus 2 times l plus 1 into delta zeta this gives. So, you can simplify this 2 pi 2 pi cancels and the expression for nu q take this to the other side.

Then nu q is equal to q times c by 2 n L plus delta zeta by pi into this. Now c by 2 n L is nu f the free spectral range and therefore nu q is equal to q times nu f plus delta zeta by pi into L plus m plus 1 into nu f. If we recall the resonance frequencies that we had calculated earlier, then nu q was equal to q times nu f this we had considered plane mirror resonators and obtained this expression.

Now, we have considered spherical mirror resonators and we see that there is an additional term and this additional term is called curvature dependent term. What curvature radius of curvature of the mirrors? So, this is the curvature dependent curvature dependent term.

So, we have an expression for nu q and therefore nu q plus 1 if you write an expression for nu q plus 1 is equal to q plus 1 times nu f. So, we replace q by here plus this term does not have q. So, it is the same identical same term and therefore the free spectral range nu f is equal to nu q plus 1 minus nu q will still come out to be simply equal to mu f.

There is no change the free spectral range is the same as that obtained for plane mirror resonator. Recall for the plane mirror resonator we obtained the resonance frequencies as nu q is equal to q times mu f, but now for spherical mirror resonators we have got an additional term here and that term is called curvature dependent term.

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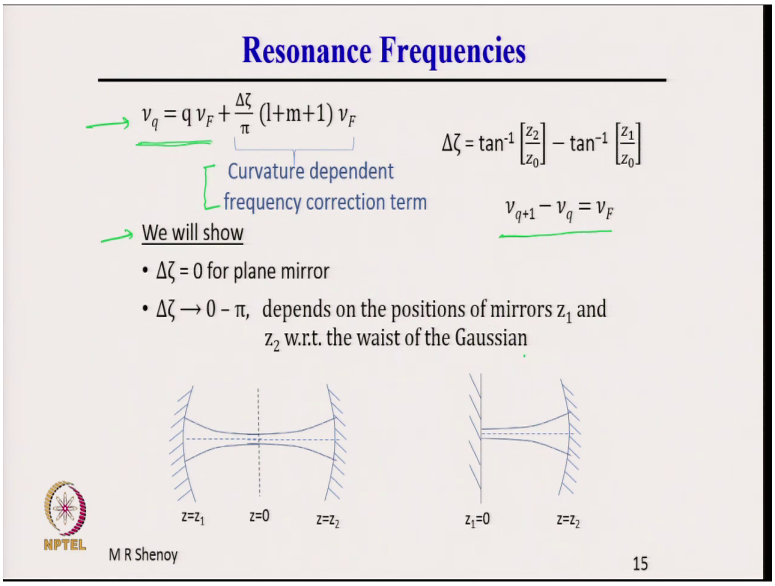

Let me discuss this a little bit more so nu q is equal to q times nu f plus delta zeta by pi into this which as I already discussed, curvature dependent frequency correction term where delta zeta is this and the free spectral range is here. Now, subsequently we will show I have written we will show that delta zeta is equal to 0 for plane mirror resonators.

If this is 0 then we get the original expression for resonance frequency for plane mirror resonators and delta zeta varies between 0 and pi, a small correction term depending on the positions of the mirrors z 1 and z 2. With respect to the waist depending on the positions z1 and z 2 of the mirrors we will get a value for delta zeta and therefore this will be a correction term which is curvature dependent.

So, depends on the position of the mirrors with respect to the waist of the Gaussian, we will show this we will discuss this curvature dependent term for practical resonators, alright.

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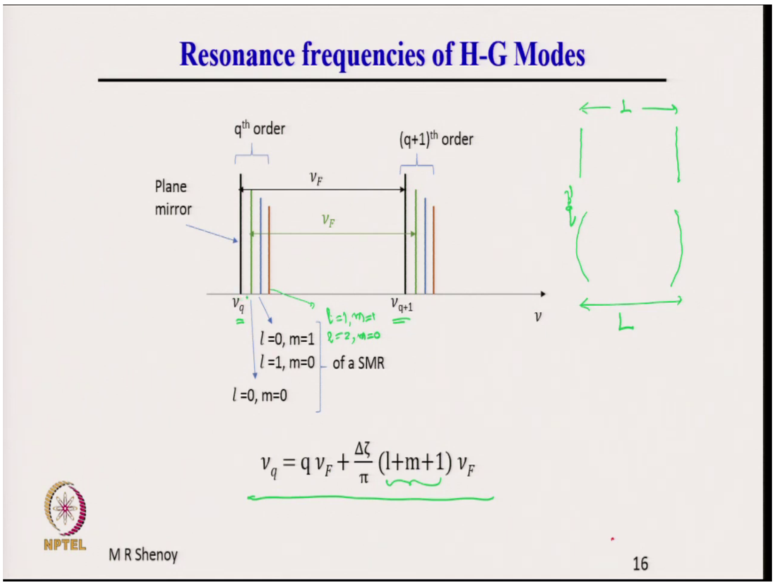

Therefore finally, the resonance frequencies of the Hermite-Gauss Modes are now illustrated here. So, note that for the qth order the resonance frequency of the plane mirror resonator. So, plane mirror resonator separated by a distance l here. So, the resonance frequencies if I call as nu q and nu q plus 1 shown in black color here, then for the spherical mirror resonator of the same separation l.

So, this is also l the frequencies the resonance frequency, so this is for plane mirror these ones the black ones for the spherical mirror resonator it is slightly shifted. If you put lis equal to 0 and m is equal to 0 then we get it as so look at the expression here the expression is here, if we put l is equal to 0 m is equal to 0 then we have q times nu f plus this delta zeta by pi into nu f.

If we put l is equal to 1 m equal to 0 or l is equal to 0 m equal to 1 because the sum is the same that is why we will have slightly increased frequency, because this will become now 2 times if l is equal to 0 m equal to 1 then we have 2 times nu f into delta zeta by pi. And therefore, this line here corresponds to l is equal to 0 m equal to 1 and the next one here would correspond to l is equal to 1 l is equal to 1 m is equal to 1 or l is equal to 2 m is equal to 0, so it is a degenerate position.

But the main point to see is that the FSR remains the same and the frequency of the spherical mirror resonator is slightly shifted, resonance frequencies are slightly shifted with respect to the plane mirror for a given order qth order. That is why we have shown that in a spherical mirror resonator different transverse modes will have slightly different resonance frequencies, different transverse modes of different order will have slightly different resonance frequencies.

We think about this and we will continue from here in the next.

Thank you.