Introduction to LASER
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Lecture - 15
Resonator Loss and Cavity Lifetime

Welcome to this MOOC on LASERs. So, today, we will discuss about the Resonator Loss and Cavity Lifetime. The parameters which characterize loss in a resonator.

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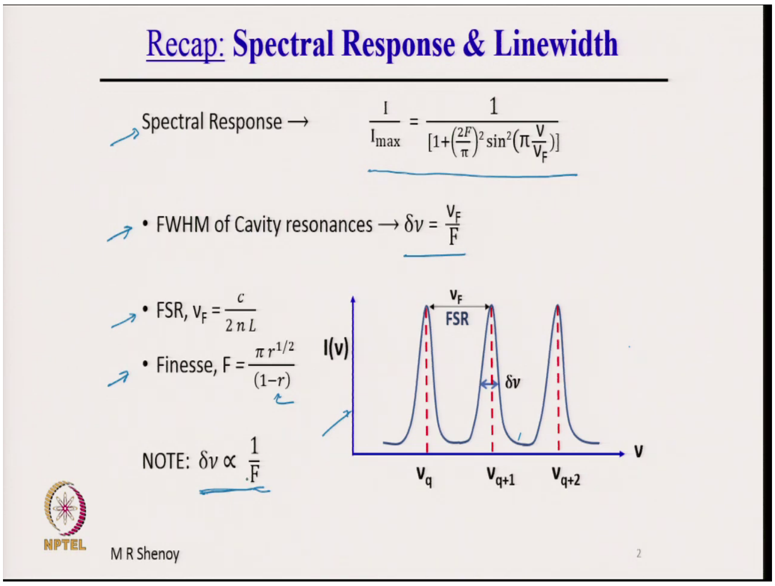

A very quick recap. In the last lecture, we discussed about the spectral response and the line width of a resonator. So, the spectral response, we had worked out an expression for spectral response and the spectral response is given by the expression here which depends on the finesse F.

And we had also got an expression for the full width at half maximum of the cavity resonances delta nu is equal to nu F by F; where nu F is the free spectral range, nu F is equal to c divided by 2 n L, n is the refractive index of the medium inside the resonator and L is the length of the resonator, F is the finesse.

Finesse characterizes the loss in the resonator, the expression is given here where r is an amplitude attenuation factor. We have discussed this in detail and therefore, we see that the finesse depends on attenuation in the resonator and finally, note that delta nu is inversely proportional to the finesse F.

So, here is the spectral response which I am just recalling again nu q are the resonance frequencies, nu q, nu q plus 1, nu q plus 2 are the resonance frequencies and delta nu is the line width of the cavity resonances and delta nu is inversely proportional to F and F depends on the loss.

And therefore, the spectral response of a resonator is determined by the losses in the resonator. The shape of the spectral response, the line width of the cavity resonances are determined by the losses in the resonator. And therefore, in this lecture, we will primarily discuss about parameters which characterize the loss in the resonator.

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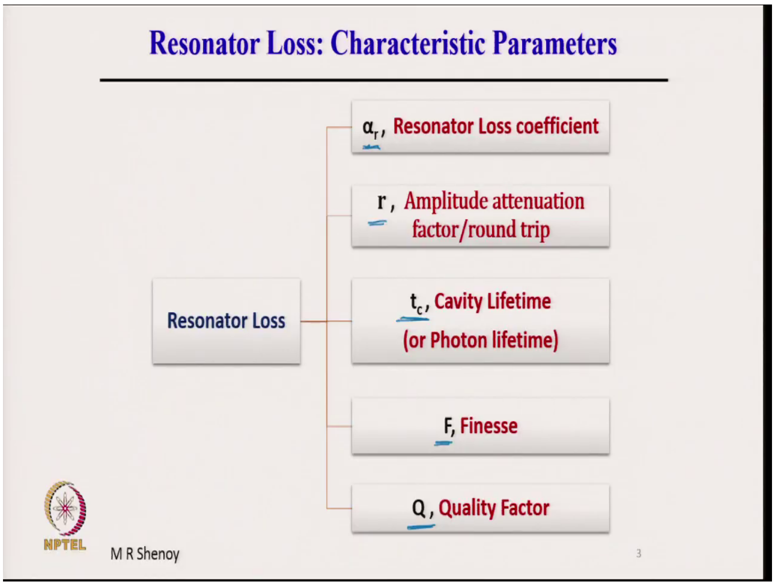

So, resonator loss, there are several parameters which can be used. So, I have listed here five different parameters alpha r, resonator loss coefficient, this is loss coefficient which means it represents resonator loss per unit length of the resonator, r is the amplitude attenuation factor per round trip.

We have discussed this in detail in the last lecture. We have taken some numerical examples also and the cavity lifetime t c, this is a measurable parameter cavity lifetime, or it is also called photon lifetime of the resonator and F, the finesse of the resonator, we have discussed this also in detail.

Today, we will also see sometimes the resonator is characterized by what is called the quality factor Q. So, typically, they say that it is the quality factor or the Q of the resonator is 1 million what does that mean? So, let us see these parameters which characterize the losses in a resonator.

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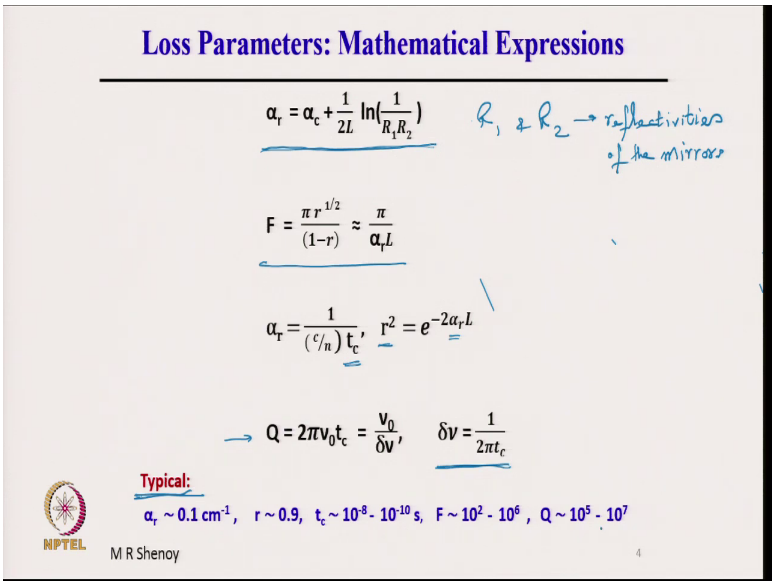

So, we will obtain the mathematical expressions. The resonator loss coefficient alpha r is given by an expression like this, we will show this where alpha r is equal to alpha c plus 1 by 2L into ln 1 by R 1 R 2. R 1, R 2 are the reflectivities of the mirror. So, R 1 and R 2, in this expression are reflectivities, energy reflectivities, reflectivities of the mirror; of the mirrors. If there are two mirrors, then of the mirrors.

F is the finesse; we have already defined this. We will show that for high finesse resonators, it can be written as pi divided by alpha r into L; which means alpha r and F are related and the resonator loss coefficient is also related to the cavity lifetime and the resonator loss coefficient is also related to the intensity attenuation factor per round trip.

So, we will obtain all these expressions and the quality factor Q which is related again to the cavity lifetime t c is given by an expression of this form and Q is equal to nu 0 divided by delta nu and delta nu, linewidth of the cavity resonances is given by a simple expression which directly relates it to t c the cavity lifetime.

I have listed some typical numbers here, if you have a numerical problem, then typical numbers you have to be familiar with the typical numbers which the resonators have alpha r could be of the order of 0.1 centimetre inverse, r the amplitude attenuation factor, we have already taken examples of the order of 0.9. t c the cavity lifetime depends on the losses in the resonator, F the finesse could be anywhere from a few hundreds to a million, similarly Q, the quality factor could also be of the order of million for high finesse resonators ok

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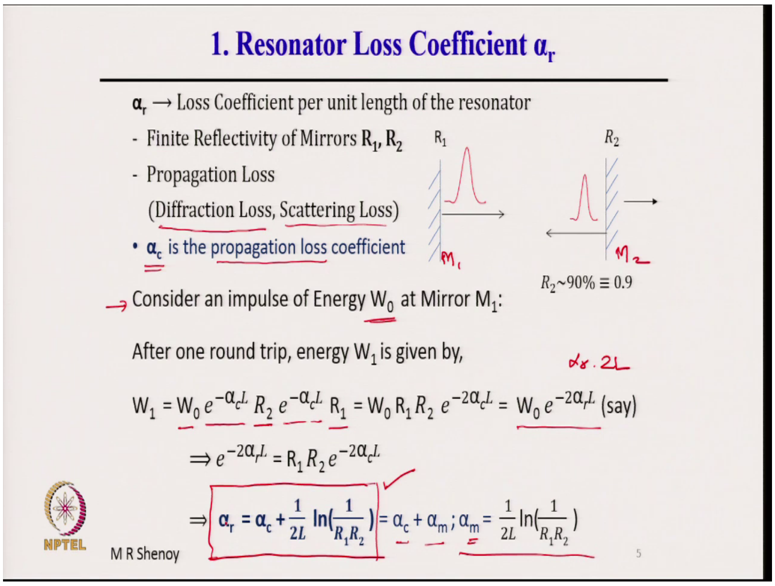

Let us start with the first parameter the resonator loss coefficient alpha r. Now, let us consider for example, a plain mirror resonator shown in the diagram here. This is mirror M 1 here and mirror M 2 here and what is shown is an impulse travelling back and forth inside the resonator, R 1 and R 2 are the reflectivities of the mirrors.

So, as the impulse propagates back and forth, it undergoes reflection at mirror M 2, comes back and undergoes reflection at mirror M 1, in between there could be propagation losses, inevitable loss of diffraction, diffraction loss and scattering loss, some of the losses which we combine together and call it as propagation loss. And let alpha c represent the propagation loss coefficient which means which represents the propagation loss per unit length.

Now, consider an impulse of energy W 0 at the mirror M 1. After 1 round trip, therefore, the energy W 1 is given by now, W 0 starts as it propagates through the distance L that is from one end to the other end, it is multiplied by a factor e to the power minus alpha c into L, alpha c is the propagation loss coefficient, at the other end, it gets multiplied by the reflectivity of mirror M 2, then the impulse starts back as shown by the arrow here.

As it propagates to the mirror M 1, it again gets attenuated by a factor e to the power minus alpha c into L and then, it gets multiplied by a factor R 1 which is the reflectivity of the mirror 1.So, in one round trip, we have two reflections and two propagation loss terms. And therefore, W 1 is equal to W 0 into R 1 into R 2 e to the power of minus 2 alpha c into L.

This if we call as W 0 into e to the power minus twice alpha r into L, it is actually alpha r into 2L, 2L is the total round trip propagation distance in that case, we can call this alpha r as the loss coefficient per unit length of the resonator and therefore, e to the power so, W 0, W 0 cancels and we have therefore, e to the power minus 2 alpha r into L is equal to R 1 R 2 into e power minus 2 alpha c into L.

If we transpose, then we get alpha r is equal to alpha c plus 1 by 2L into ln 1 by R 1 Rr 2. This is a important expression which gives the resonator loss coefficient per unit length. Alpha c is the propagation loss coefficient, L is the length of the resonator, R 1 and R 2 are the reflectivity of the mirrors.

Sometimes, this is also expressed as alpha c plus alpha m because the second term where alpha m is equal to 1 by 2L into ln 1 by R 1 R 2, the m notation or m subscript representing that it is primarily loss due to finite reflectivities of the mirrors that is why it is also represented as alpha c plus alpha m, the second term represents loss due to finite reflectivities.

If the mirrors were to reflect 100 percent, then R 1 is equal to R 2 is equal to 1 and then, we would have had ln 1 which is 0 and therefore, alpha m would be 0 if the mirrors were of 100 percent reflectivity which is not the case alright. So, we have got the expression for the resonator loss coefficient.

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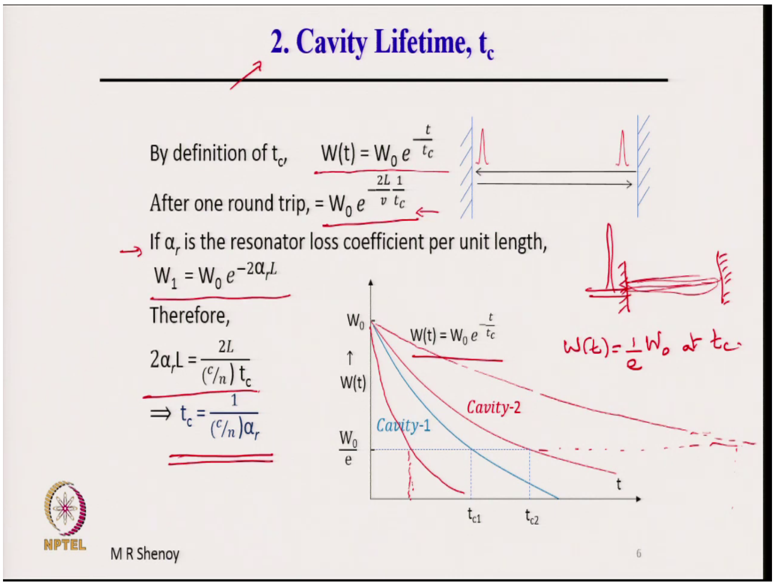

Let us go to the next that is cavity lifetime t c. The cavity lifetime is defined by this equation that is it gives energy in the cavity as a function of time which is described by W of t is equal to W 0 into e to the power minus t by t c. Now, what it means is if you consider any resonator and at t is equal to 0, if you input some energy into this let us say an impulse a burst which is input into the resonator.

Then as with time the energy goes back and forth, back and forth and because of the finite losses, both due to finite reflectivity of the mirrors and also due to the other losses, the energy will go on dropping that is what is shown here, with time, the energy drops down exponentially. So, W of t is equal to W 0 into e power minus t by t c is the defining equation for the cavity lifetime.

So, as can be seen, if the cavity lifetime is large that means, it takes more time for the energy to decay. For example, already there are two curves shown here, one for cavity 1 and another cavity 2. So, cavity 2 is less lossy because energy takes a longer time to decay. We are talking of cavity lifetime tau c, this also sometimes called passive cavity lifetime or passive photon lifetime in the cavity. Now, if a cavity is very lossy that means, the energy will get damped very rapidly like this and in this case, the cavity lifetime is here.

So, cavity lifetime is the time taken when the energy drops down to 1 by e of its value. So, in this expression, if we put t is equal to t c that is the time at which the energy drops down to 1 by e. So, if t is equal to t c e to the power of minus 1, therefore, W of t is equal to W 0 so, 1 by e into W 0 at t c and therefore, cavity lifetime is shorter means the resonator is more lossy.

Cavity lifetime is larger means the resonator is less lossy. If the resonator is very low loss, then it may probably go like this very slowly and the cavity lifetime may be somewhere here, far away which means a large cavity lifetime. Now, the definition W in this defining equation, if we substitute for t time taken for one round trip.

So, time taken for one round trip is here, e to the power this is e to the power minus 2L by v, v is the velocity, 2L is the round trip length and v is the velocity that gives us the time. And therefore, if we substitute this and then, if alpha r is the resonator loss coefficient per unit length which we just discussed in the previous slide, then W 1 is equal to W 0 into e power minus 2 alpha r L.

If we equate the two, this is also W 1 in terms of time and therefore, if we equate the two, the coefficients, the exponents must be the same. So, twice alpha r L is equal to this. We are equating this expression with this expression because this also represent energy after one round trip in terms of the resonator loss coefficient alpha r.

This represents the energy after one round trip in terms of the time, the cavity lifetime and therefore, if we equate this, we get an expression for cavity lifetime t c is equal to 1 divided by c by n the v velocity we have written as c by n, n is the refractive index of the medium and c by n into alpha r. The point is the first parameter resonator last coefficient and the second parameter cavity lifetime are related ok.

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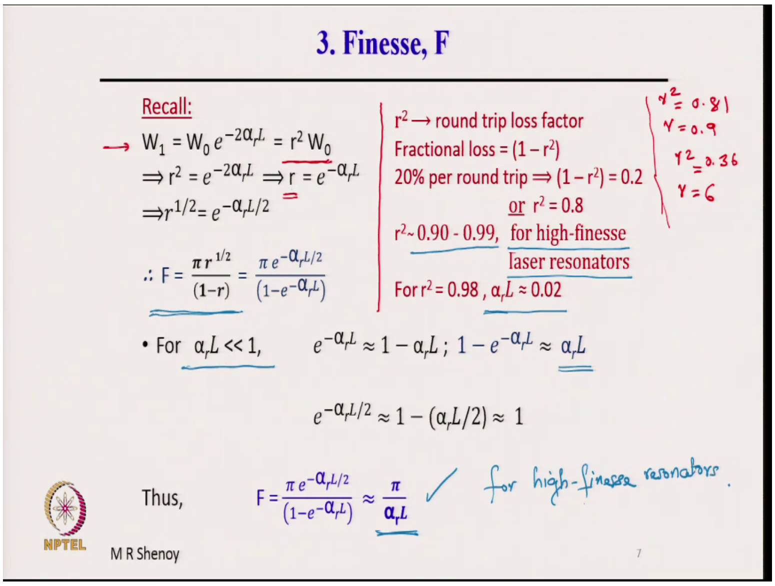

Let us now take the third parameter which is the finesse. We have discussed this in the last lecture finesse, when we obtain the spectral response of the resonator. Now, let us see finesse also depends on the loss in the resonator so, can we relate finesse to alpha r? So, here it is.

Now, recall that W 1 is equal to W 1 is energy after one round trip equal to W 0 into e power minus 2 alpha r into L. This is also equal to r square into W 0. In the last lecture, we discussed that the energy attenuation factor or intensity attenuation factor is r square, and r is the amplitude attenuation factor.

Therefore, if r square so, let me show it here, r square is the round-trip loss factor, then the fractional loss is 1 minus r square. Round trip loss factor if you remember that we had taken example such as r square, let r square be equal to 0.81 so that r could be 0.9. I had also taken an example with r square is equal to 0.36 which means this implies 64 percent loss so, r square is equal to this and r is equal to 6. These are the two examples which I had taken in the last lecture. So, you can see the last lecture where we discussed these.

Therefore, 20 percent loss per round trip means 1 minus r square equal to 0.2 or r square is equal to 0.8. In general, for high finesse laser resonators; for high finesse laser resonators, r square is generally of the order of 0.9 to 0.99 which means if I take for example, r square is equal to 0.98, then alpha r into L will be equal to 0.02 I am linking here so, r square is equal to e power minus twice alpha r into L therefore, r is equal to e power minus alpha r into L.

Therefore, if you know r, we have taken r square is equal to 0.98 so, we know what is r, if we substitute in this expression, we can get alpha r into L is equal to 0.02, this is the kind of numbers that we have because r square is very close to 1 and therefore, the exponent e to the power of 0 is 1 so, e to the power of a small number.

The point is we already have defined the finesse by this expression and r square is equal to e power minus 2 alpha r. Therefore, r is equal to e power minus alpha r into L and r to the power half is equal to e power minus alpha r L by 2. Substitute r power half here and r in the denominator that is what we have done here.

Now, the point is alpha r into L is much smaller than 1. Therefore, for alpha r into L so, this numbers I have considered only to say that this alpha r into L, the exponent is much smaller than 1, then we can write e to the power of minus alpha r L that is e power minus x as 1 minus x so, 1 minus this which is 1 minus or 1 minus e power minus alpha r L is simply alpha r L that is in the denominator. So, in the denominator term is simply alpha r into L.

Now, again e power minus alpha r L by 2 is 1 minus alpha r L by 2 and alpha r L is already very small and therefore, we can write it nearly equal to 1 so that the expression for finesse can be simplified as pi divided by alpha r into L. This is a very good approximation, but for high finesse resonators; for high finesse resonators. So, here, we have explicitly linked the resonator loss coefficient alpha r to the finesse of the resonator.

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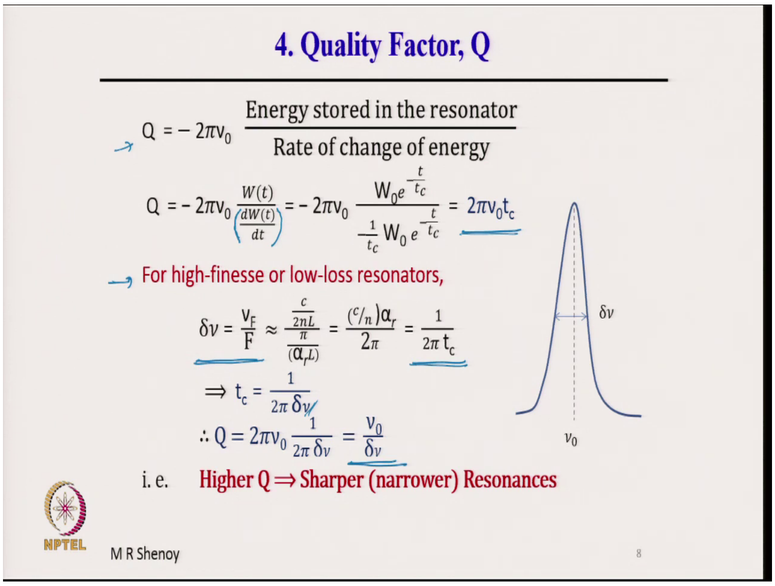

Now, let us take the next parameter which is the quality factor. So, quality factor the definition of Q is minus 2 pi nu 0 where nu 0 is the resonance frequency. So, we are looking at the resonances, cavity resonances and therefore, nu 0 is the cavity resonant frequency or the longitudinal modes, we have already got expression for the resonance frequencies nu equal to Q times nu F into energy stored in the resonator divided by rate of change of energy.

In writing this simplified expression, we have assumed that there is only one mode which is oscillating in the resonator, there is only one longitudinal mode which is oscillating in the resonator, then this expression is correct and therefore, Q is equal to minus 2 pi nu 0 into energy stored in the resonator at any instant t is W of t divided by d W of t by dt that is the rate of change of energy.

So, this whole thing is in the denominator, rate of change of energy. So, that is equal to minus 2 pi nu 0 W of t we substitute this expression, W 0 into e power minus t by t c, this divided by the rate of change that is the derivative of W of t is 1 by t c into W 0 into e power minus t by t c. So, it is actually minus 1 by t c.

Therefore, the minus minus sign goes and what we are left with is 2 pi nu 0 into t c. So, this is the expression for quality factor of the resonator in terms of cavity lifetime t c which is a measurable parameter. So, you can determine the quality factor by measuring the cavity lifetime.

For high finesse or low loss resonators, we already have obtained this expression. In fact, this expression is a very good expression even when F is just above 10, 15 or so, need not be F going to 1000 or a million just greater than 10 and we will see that the expression is correct to second decimals or so.

Now, delta nu is equal to nu F by F where we substitute for nu F c by 2nL and F, we just now derived the expression pi divided by alpha r into L, this is the expression which is valid for high finesse resonators. So, if you substitute, we get delta nu is equal to 1 by 2 pi t c, very important expression or t c is equal to 1 by 2 pi into delta nu. So, if you can measure the t c, then you can determine the line width or alternatively if you measure the line width of the cavity resonances, then you can determine what is the cavity lifetime.

Therefore, Q is equal to 2 pi nu 0 into t c here, t c is 1 by 2 pi into delta nu which is equal to nu 0 by delta nu and this is the very simple expression which we have in RC circuit resonance circuits, RLC circuits and so on. So, it is the same expression that we are getting, resonance frequency divided by width of the resonances. Therefore, higher Q means sharper or narrower resonances. So, narrower the resonance, larger will be the Q of the resonator.

So, we have now linked all the four parameters which characterized loss in a resonator and remember, loss determines the line width of the cavity resonances. This is very important because subsequently we will obtain output from the laser where the line width of the laser, it will be determined by the line width of the cavity resonances that is why we have given sufficient importance here and how the cavity resonance, the line width of the cavity resonance are linked to the last parameters

So, we will stop here and in the next class or next lecture, we will start with spherical mirror resonators. Now, we have all the basic parameters which characterize the resonator has been determined and we will now move on to spherical mirror resonators.

Thank you.