Semiconductor Optoelectronics
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Lecture-09
Quasi Fermi Levels

In this lecture we will discuss a little bit more about Quasi Fermi levels as I had mentioned that Quasi Fermi levels are very important in opto-electronic devices, normally in p-n junction diodes we right what is Quasi Fermi levels but the levels we do not discuss much but in opto-electronic devices as you will see it is the difference between the Quasi Fermi levels which will determine the band width of the semi conductor laser amplifier.

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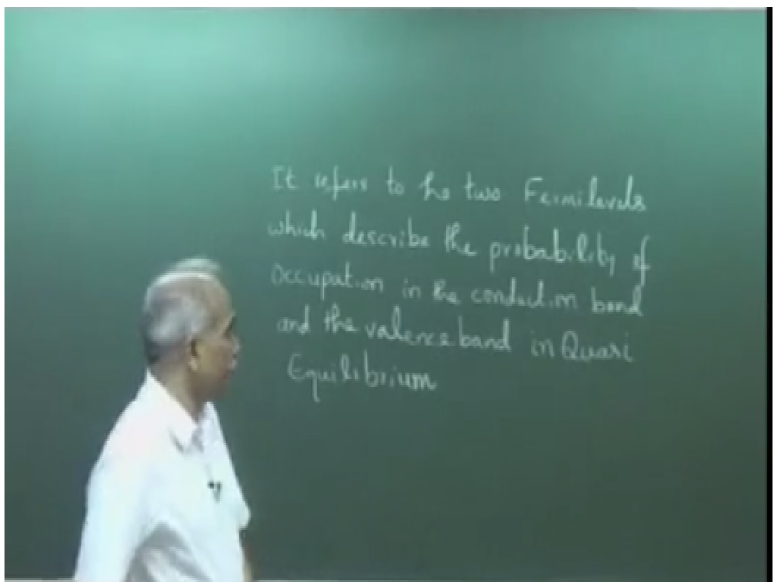

So, what are Quasi Fermi level it refers to the two Fermi levels which describe the probability of occupation in the conduction band and the valance band now there are two that is why for Quasi Fermi levels valance bands in Quasi equilibrium hence the name Quasi Fermi levels.

So, it refers to the 2 Fermi levels that describe the occupation probability of electrons in the conduction band and in the valance band. One Fermi level to describe the occupation probability in the conduction band and another Fermi level to describe the occupation probability in the valance band when the semiconductor is in Quasi equilibrium. So, we will see what is this Quasi equilibrium.

That before we go to Quasi equilibrium let us first discuss the thermal equilibrium. Thermal equilibrium refers to a steady state an equilibrium a steady state at constant temperature and when there are no other sources of excitation. So, the energy is supplied the energy under consideration is mainly thermal energy. So, let us first see at thermal equilibrium what are the electronic transition processes in a semiconductor.

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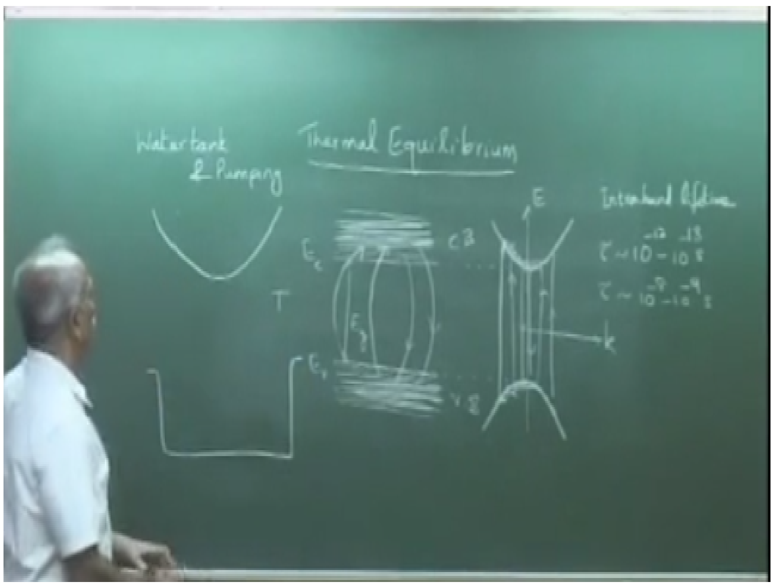

So, at thermal equilibrium it is an equilibrium it is not static it is dynamic but here is an equilibrium, so if you recall the energy band diagram, so this is Ev, Ec we have large number of allowed states in this band and the density of states go on increasing as you go to higher values. Similarly there are density of states here and the density of states become more and more which means they are becoming denser and denser as you go further down.

So, Ev, Ec and separated by Eg this is valance band and this is the conduction band along with this we can also plot the EK diagram the EK diagram where this is basically a large number of allowed states here almost 4continue and large number of states in the lower band as well, so both are equivalent, equivalent picture this is EK. At thermal equilibrium when t is equal to 0 in semiconductor a t is equal to 0k the lower band is completely full and upper band is completely empty.

And at a finite t there are electrons which are making continuous upward transition please remember it is not that there are some fixed number of electrons here and fixed number of holes here it is a dynamic equilibrium there are electrons which are making upward transition and downward transition. In the presence of at a finite temperature t electrons are continuously getting excited to the conduction band.

And also they are recombining with the hole but there is a study state at study state number of electrons making upward transition is equal to number of electrons making downward transition this is the steady state it does not mean the electrons are there. They are continuously in a state of uhhh transition upward and downward but there is a equilibrium at equilibrium there is an average number of carriers and an average number of carriers which are present here.

In this so there are electrons which are making transition upward transition it could make upward transition here also and also there are electrons which are making downward transition usually if an electron which makes a transition to a higher energy level here it will rapidly come down within the band by process called thermalisation this is due to phonon transitions an electron excited to in the upper level in the conduction band rapidly comes down to lower vacant states by the process of thermalisation.

Thermalisation means it gives energy to phonons and phonons are quanta of lattice vibration, so and it rapidly comes down here and keep accumulates at the bottom of this plot although it maybe making a transition upward it comes down and accumulates here whether electron has left to this place there is a hole here the hole is at a higher energy. So, electron which is sitting here comes down to the position of the hole and the hole makes an upward transition here.

So, the hole also similarly makes transition upward to the top of the band hole has lowest energy when it comes to the top of the band. So, the holes accumulate near the top, so at steady state when the number of electrons going up equal to number of electrons coming down there is a certain number of holes here and there is a certain number of electrons in the conduction band that is the carrier concentration which we talk of at any given temperature t.

The phonons the phonon transitions are very very rapid typical time for these transitions is tow is the order of 10 to the power of -12 to 10 to the power of -13 seconds. This is the transition time recombination time that is the time over which the electron comes down by thermalisation or this is called the intra band recombination time or intra band carrier time carrier lifetime within the band intra band.

So, the intra band like that lifetime here refers to the time over which it is it at an excited state it is not lifetime of the electron within the band intra band like that or it is also the same as recombination time or it is also the same as the photon transition time. So, that means this is extremely rapid it comes to the bottom. The transition time here that is from the conduction band to the valance band typical time tow that is inter band transition time id of the order of 10 to the power of -8 to 10 to the power of -9 seconds.

The inter band transition which means the intra band transition is much faster therefore at anytime although there are continuously transitions taking place at anytime you see a steady state you can imagine this by a an example here you take a container please see this example an example of water tank and pump so, I have a container here alright let me draw it at the same level as Ec just a second.

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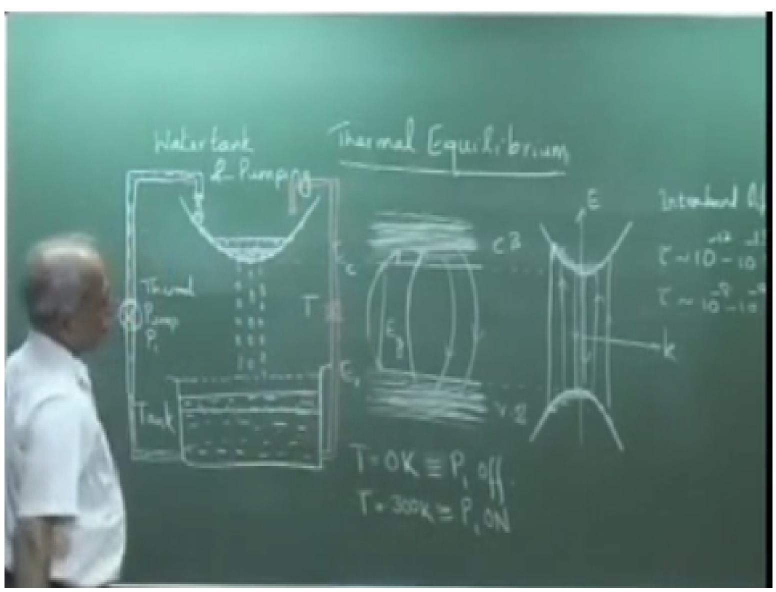

This is not band diagram this is a container, so there are 2 container 2 tanks, so this is the tank where there is water I am describing the process of thermal equilibrium with the analogy of water in a tank at t is equal to 0 the lower tank is completely full upper tank is completely empty this upper tank has holes there are holes here through which if I pump water it can twirled down.

These are holes in the upper tank, the lower tank does not have any hole I now take a pump, so this is a pump, pump P1 when the pump is off this tank is full this tank is empty I switch on the pump, so the pump is pumping water her e, so water is falling down and water is flowing down here to the bottom of the tank now and there are holes therefore water is also trikling down here, so water is also falling down.

Because I am pumping up into this tank water is getting accumulated here but it is also flowing down through this holes if the holes are sufficiently small if they are small then there will be some amount of water which is accumulated in this. The amount of water here the level of water here depends on the pumping rate if I pump very slowly very small amount of water is entering this tank and it is also going down.

So, the new steady state when the pump is on the new steady state is this that the water level has come up to this there is no vacancy here and there is water up to this you compare with this water to electrons and the vacancy is to holes. So, electrons are pumped up because of temperature because of thermal energy the thermal pump. So, this pump is for thermal energy, so I want to call this as thermal pump or temperature pump, a pump because of thermal energy.

So, there is a steady state here you can imagine bucket of water also you take a bucket of water and there is a hole from the side leave the tap into it, it depending on the rate at which water is falling in the bucket the hole is very small, so there will be some steady level you increase the rate of flow the level will go up and of course here it will go out at a faster way because whatever come the steady state is the amount of water which comes must be equal to the amount of water .

It is exactly like this at thermal at finite t the pump is off is equivalent to t equal to 0. So, t equal to 0k implies is equivalent is not implies it is equivalent to pump P1 off, this tank is completely full this tank is completely empty this band is completely full this band is completely empty you switch on the pump, so now we are at a finite temperature some vale t equal to let us say 300 k which is equal to on P1on at some rate and therefore you have some level which is steady level.

If I increase the pumping rate that is if i increase the temperature from 300 to 400, if I increase the pumping rate it means it will now go to a new level, so more water is coming, so it will fill here this level will go down and you will have a new level which is here and this is here this is carrier concentration temperature dependence of carrier concentration. Now why did I bring this form set at a given temperature I may have a second pump nor the temperature pump temperature is fixed 300 k.

But I have another pump so I use a second pump from here and which is also pouring water into this, this is pump 2.

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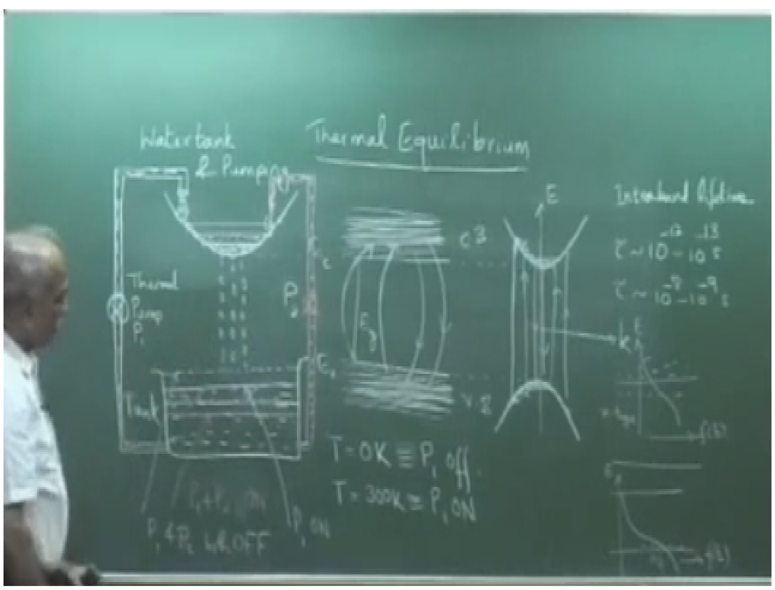

Pump P2 which is also lifting water and pouring into this, this is not the temperature pump, so it is not now because water is coming from here also the level will now change to a new level because there is more water which is being pumped. So, this comes to a new level and similarly here this goes down to a new level. So, this is the new level this is with P1+p2 both ON P1+P2 ON.

This level is with P1 ON same thing is to here also, so P1 ON and this level is when P1 and P2 both OFF what is the point the point is when you have a second pump there is a new equilibrium the levels are different you know there is a new equilibrium there has no change in temperature, so it is not temperature pump the normal Fermi function describes the occupation probability of electrons at thermal equilibrium.

It describes occupation probability for both conduction band and valance band but when you have a new equilibrium in the presence of an additional pumping source what is this additional pumping source we will see when you have pumping an additional pumping source 1 Fermi function cannot describe both of them because you have simultaneously very large number of electrons and very large number of holes a large vacancy.

In this tank water has gone down because half the water is here, so you have but this rate at which the water is falling is much slower compare to the rate at which water is settling down here, why does this have a steady state here, you imagine the bucket if the bottom of the bucket is open no matter what the rate at which you pour there will be no water in the bucket there is no steady level at all.

But if the hole is very in the bucket or if the rate at which water is going down from here is much smaller than the rate at which it is pouring down and settling quickly here then you have steady state distribution here and a steady state distribution here each one of them can be described by the Fermi function but you need 2 Fermi functions one to describe this and the another one to describe this why to Fermi function.

Because the number here the carrier density here is so large that you have to bring the Fermi level here up you know in a semiconductor is the number of electrons here is very large like in n doped semiconductor the Fermi level is shifted up because I want to describe the probability like this if it is P type then the Fermi level is shifted down. So, if it is P level P type where you have large number of holes when the Fermi function is shifted down.

So, this is 0.5 probability, so 0.5 what I have plotted is ffe versus E always the vertical axis E sometimes I might not have marked what is the vertical axis but always everywhere vertical axis is E. So, this is ffe so this is a n type and this is a P type now we have a situation where we have large number of electrons and large number of holes simultaneously and this can be described and all at steady state they all at steady state.

Because the time this time here where the rate at which transition which taking place is much smaller compare to this time which is causing stabilisation rapid stabilisation or rapid thermalisation therefore at anytime if you see if you take a photograph you have there is a steady level sitting here there is steady level sitting here it steady state therefore if you sue 2 Fermi functions one to describe the population of the valance band**.**

Simultaneously 1 will be here and 1 will be here 2 Fermi functions these are called the Quasi Fermi levels in the same semiconductor here it was n types another semiconductor here it is P type separate semiconductor. But now you have a situation of quasi equilibrium where you have an additional source of pumping you have 2 Fermi levels to describe the population of the 2 bands and these are called the quasi Fermi levels.

Is this picture clear that this is a 0k no point P1, P2 both OFF the lower band is full upper band is this will I plotted it equivalently just by the side. Here the level goes down which means there is vacancy here that means there are holes here the vacancy here R gap is like holes and water drop is like electron this is a very good analogy at every state you will later on there are complicated uhhh h band diagrams where you can understand them every time you think that electron is like a water drop.

Because water drop always flows from a higher level to a lower level whereas air bubble always goes from lower to higher level. So, we will see that this picture will definitely will be very helpful, so let me rub this off and. So, under the condition that the intra band transition time or thermalisation is much smaller compare to the inter band transition you will always have a steady state distribution and that steady state distribution in the 2 bands can be described by 2 different Fermi function called the Quasi Fermi level.

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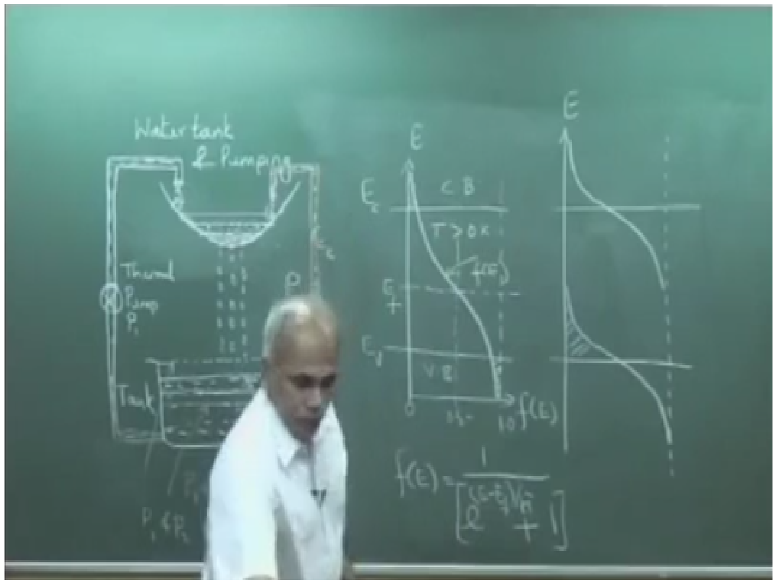

So, we have at thermal equilibrium again let me plot Ev, Ec and E versus f (E) the Fermi function and you have a distribution, so this is 1.0, so it is 0.5 f (E) equal to 0, 0.5 you draw the vertical line where it intersects that gives you Ef. So, this is at a particular temperature t, t greater than 0 1 f(E) describing the probability of occupation of the conduction band and the valance band and f(E) is given by 1/E to the power E-f/kT+1.

In Quasi equilibrium have you have an additional source which brings in more electrons to the upper band and therefore you have **oh** I am sorry this is where you do not have anything.

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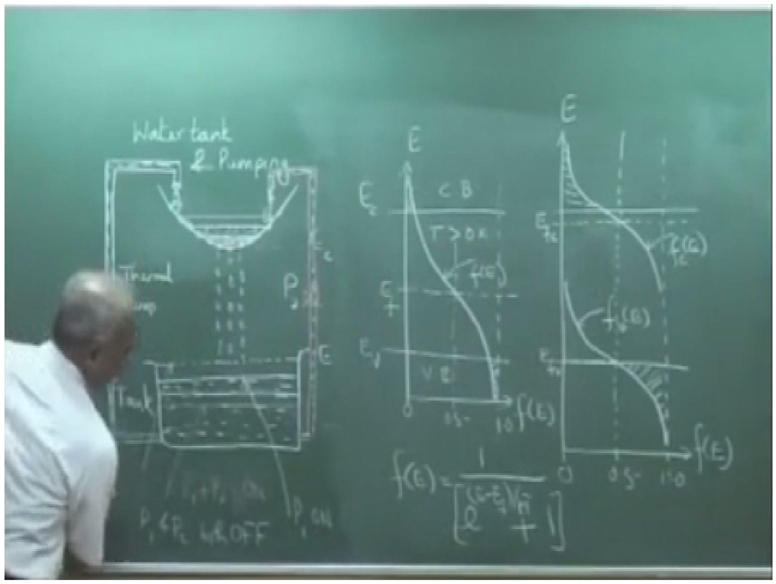

I could have kept a drawn figure already for display but I thought if I plot it in front of you you will know what is the difference. So, this is Efc Fermi level or Fermi function describing the conduction band occupation probability of the conduction band this part has no role only this upper part has a role to describe the population of the conduction band occupation probability of electrons in the conduction band.

Here this is at 0.5 it so happens that it comes out at the same value as by chance it is at Ev, so Efv is here this function describes the population of occupation probability of electrons in the valance band, so this is f(E) and this is 1-f(E) which describes the occupation probability of holes in the valance bands, 2 functions are separate, so this is called fv(E) and this is fc(E) one Fermi function describing the occupation probability of the conduction band.

And a second Fermi function describing occupation probability of the valance band fc(E) okay let me write on this side this will go out of view let me erase this picture, where fc(E) .

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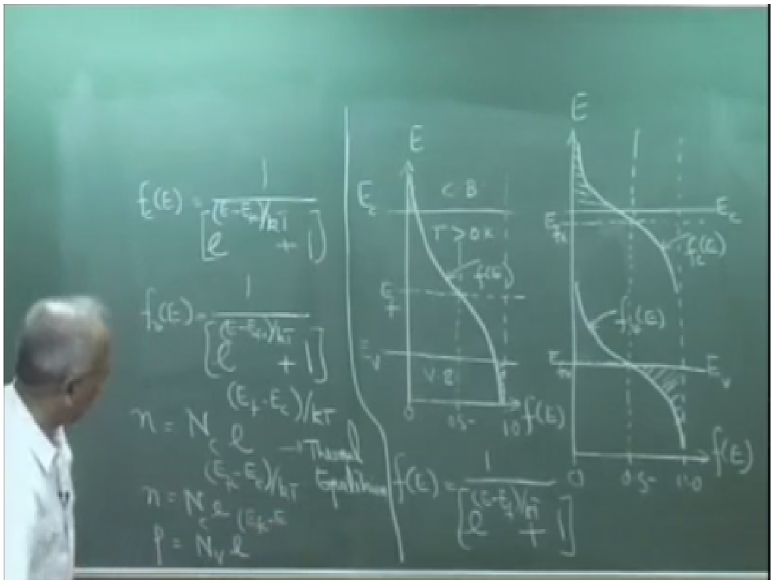

So, fc(E) is equal to 1/E to the power E-Efc/kT+1. So, instead of Ef we have Efc and here we have Efv, so fv(E) is equal to 1/E to the power E-Efv.kT+1 is this okay is there any doubt, no no I went on saying this is just by chance in my diagram it came out to be coinciding it need not it does not or need not in generally it does not.

But by chance the way I have drawn it the 0.5 thing came out to be at the same value as Ev but there is no relation at all, so this is Ec and Ev by chance it is there but the important point that you get is 1 Fermi function characterised by 1 Fermi energy Efv will describe the occupation probability of the valance band and therefore 1-fv (E) will give you the occupation probability of holes and a second Fermi function.

Simultaneously in the same material this is 1 time 1 is n type 1 is P type it is the same material but it is now in Quasi equilibrium when a material is in thermal equilibrium you will describe it by 1 single Fermi function when a material is in Quasi equilibrium which means there is an additional source of pumping additional source of excitation this additional source of excitation could be radiation.

A semiconductor which is illuminated by light or it could be the p-n junction where carrier injection is taking place and at the junction in the junction region you are pouring in electrons and holes. So, if you look at that junction region there are simultaneously large number of electrons and large number of holes this is carrier injection and you have 2 Fermi levels in that in that region.

Before I proceed to the excitation mechanism more about the excitation mechanism under Bozeman approximation we had an expression for carrier concentration n is equal to Nc into E to the power Ef-Ec/kT at thermal equilibrium. In Quasi equilibrium everything remains the same if we still need the Bozeman approximation this is a clause because Efc if it is still somewhere below here then I can still use Efc and I can still use Bozeman approximation.

And then this will become, so this is at thermal equilibrium in Quasi equilibrium the same expression is valid with Ef replaced by Efc –Ec/kT simply the Ef is replaced by Efc because what was Ef doing it was describing the Fermi function. Now the Fermi function is described by Fc(E) which means Efc of and p the carrier concentration of holes is Nv into E to the power Efv-Ev Ev-Efv/kT.

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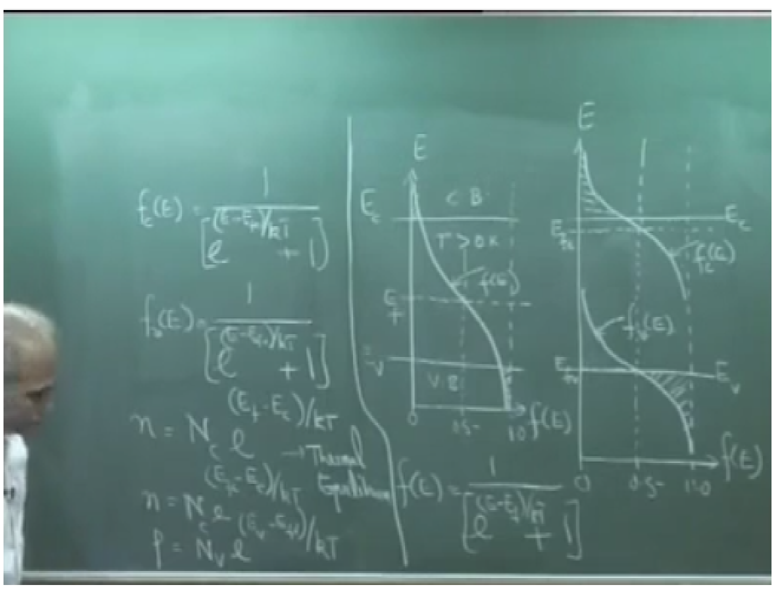

Let me write the 2 expressions again because it is congested there.

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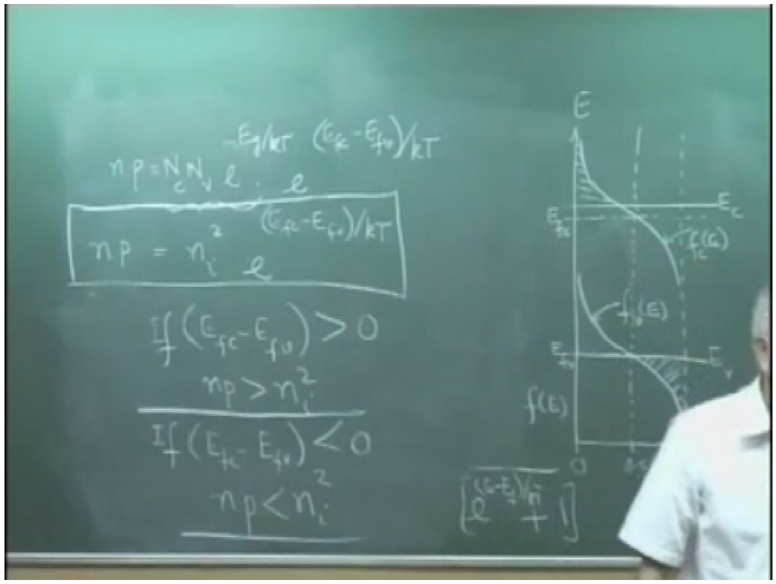

And therefore N into p is equal to Nc into Nv into the product of these is equal to E to the power –Eg/kT N into p is Nc into Nv E to the power Ev E to the power –Ec therefore it is –Ec-Ev therefore –Eg/kT into what do we have there is a second term E to the power let me go further into E to the power Efc-Efv/kT what is this term it is Ni square this term is Ni square therefore this is equal to Ni square into E to the power Efc-Efv/kT, so N into P is equal to this/kT.

In themal equilibrium Efc is equal to Efv equal to Ef there is only one Ef which means this is 0 the numerator here because Efc equal to Efv there is nothing like 2 Fermi functions so only one and you are np equal to Ni square in thermal equilibrium. In Quasi equilibrium we have Fermi functions and I mention that it is the difference between this therefore if Efc Efc-Efv is greater than 0.

If the difference is greater than 0 then Np is greater than n the number of carriers that we have the product of carriers is greater than Ni square and if this is less than 0 when we this be better than 0 when will this be less than 0 I will show you the p-n junction now. But if this is less than 0, so if Efc –Efv is less than means Efc is below Efv, Efv is above less than 0. Then we have np less than ni square under Bozeman approximation which means we are considering relatively lightly doped p-n junction.

And relatively lightly doped p-n material and our Quasi equilibrium is quite mild. So, in quasi I erased it in quasi equilibrium the large mass action is not np equal to ni square. But n into p equal to ni square into E to the power of Efc-Efv/kT.

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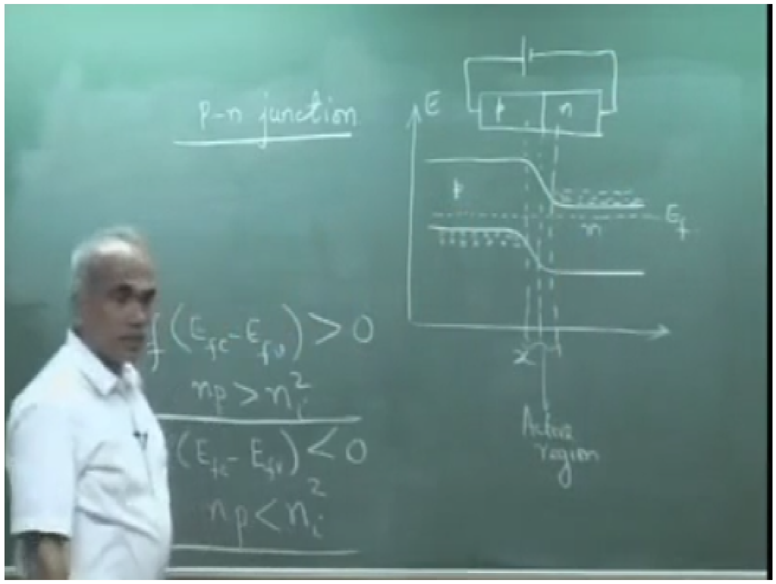

Let us quickly take the band diagram of a p-n junction and see what is this we will discuss p-n junction a little bit more in a later class but let us quickly take up the band diagram and see what will be the situation when this is greater than 0 and then will be this is less than 0. All of you must be familiar with the p-n band diagram of a p-n junction I hope we will see p-n junction. So, I am drawing the energy band diagram of a typical p-n junction.

So, this is P type material p and this is n and we have at equilibrium we have 1 Fermi level Ef, so this is this axis is energy E and this axis is the distance x, so x you can take x equal to 0 at the junction the metallurgic junction between p type material and n type material and there is a energy band variation. So, you have a plenty of electrons here, so plenty of electrons water so whenever electron comes just think of water very easy to follow.

And plenty of holes air bubbles and near the junction there are neither holes nor air bubbles or very little electrons holes at the junction which we usually call as the depletion region. In electronic we usually call this region as the depletion region but in opto-electronics we do not call it depletion region because for opto-electronic devices this is the most important region and we call it the active region it is called the active region.

Depletion region is a little bit of a negative is shown that oh it is depleted but it is the most important region which is the active region. So, you have 1 Fermi function this is the p-n now you forward biase this which means to the p end so let me show, so this is the junction p-n now you apply a forward biase is entire p-n material I am not showing just the junction. So, p-n p applied a forward biase.

So, when we apply a forward biase what happens the right side goes upwards with respect to the left side because electrons to get more potential energy it is connected to negative terminal. So, electrons now have a higher potential energy and this energy axis, so this end starts moving up relative to this end. So, we have a forward biased p-n junction.

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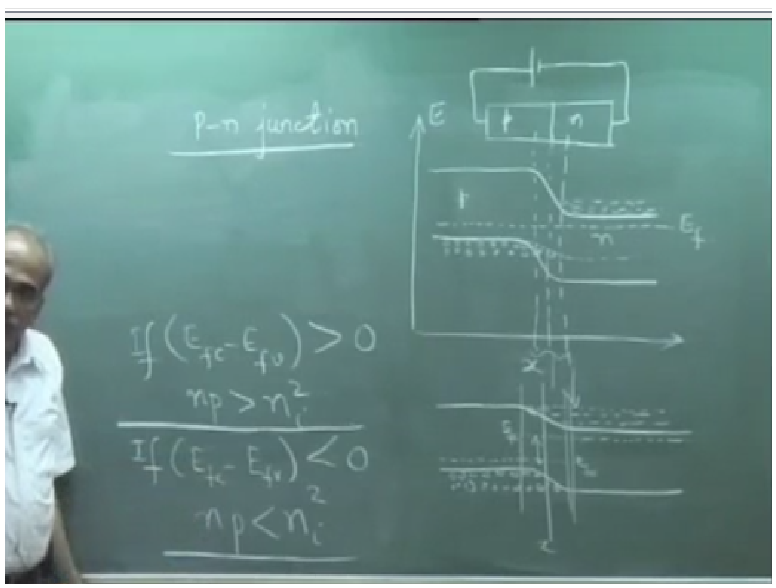

And the band diagram now so originally it was like this, so now it has been lifted up from the original position relative to that, so this was it is original position, so let me not show the original position, so this is after forward biasing so forward biased junction what happens to the Fermi function, so Fermi level in this side remains where it is far away from the junction there is no potential felt, so Fermi level is here.

And far away from the junction the Fermi level is here what about the junction region through this. So, in the junction region you see there is a there are 2 Fermi functions in the junction region now this is the moment you forward biase it which means in addition to temperature you are injecting you are giving it supply from outside which means you are introducing carrier injection then in the junction region you have what does happen this is gone up.

So, the when this has gone up water has started moving here you see the level has been raised, so water has come into the there was earlier a gap depletion region. Now when you have raise this water is moving this side because the barrier is lower and what happens to the air bubble, so this has imagine here itself, so if this goes like this the air bubble can come here, so exactly like that you have air bubbles coming into this side which means you look at the same original junction.

And the new forward biase junction you have lot of electrons and lot of holes at the same value of x is at the same physical position you have simultaneously large number of holes and large number of electrons, this is Quasi equilibrium this has come up because of the external power source because of forward biasing and accordingly you see the there are 2 Fermi functions here.

And this is now Efv and this is Efc for the junction region and therefore as far as if you focus only on the look at the junction region only it is a region where you have simultaneously large number of holes and large of electrons and it is described by 2 Fermi functions. The separation here Efc-Efv is positive Efc-Efv is positive Efc-Efv is greater than 0 which means in this junction now n into p is greater than ni square.

You have simultaneously large number of carriers when will we have this kind of the second tye of a situation if we reverse biase. So, if we reverse biase the junction then this will further go down which means this will remain there but the other portion goes down. So, let me erase the forward biase.

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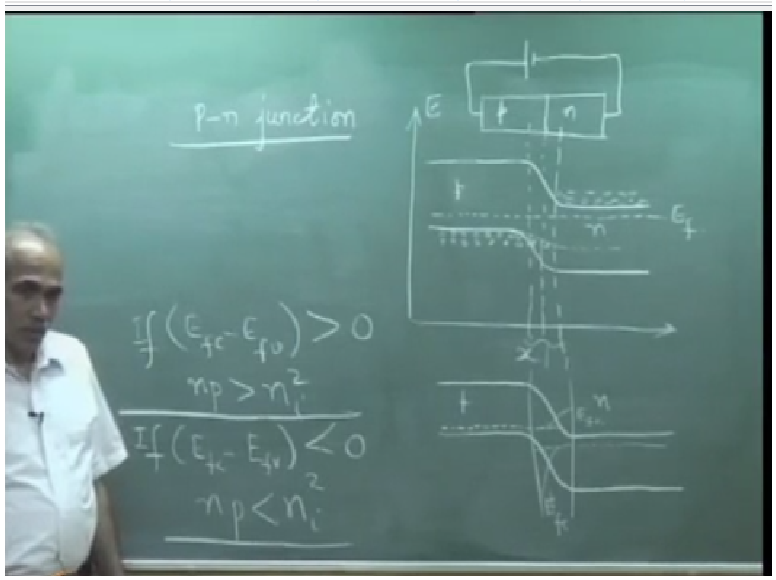

And the band diagram now becomes so it is goes down further the n still remains here Ef but in the junction region this is Efc and Ef is here far away for the p type and n type but in the junction region Efv is now here. So, if you look at the junction region this is Efv and this Efc the difference now is negative obviously when you revere biase the depletion region further widens the end carriers are pull back.

And you have much less carriers in the depleted, depletion region. In fact this the forward biasing is the principle of realising optical sources and reverse biasing is the principle of realising optical detectors as we will see later that this is the case corresponding to optical sources and this is the case corresponding to optical detectors.