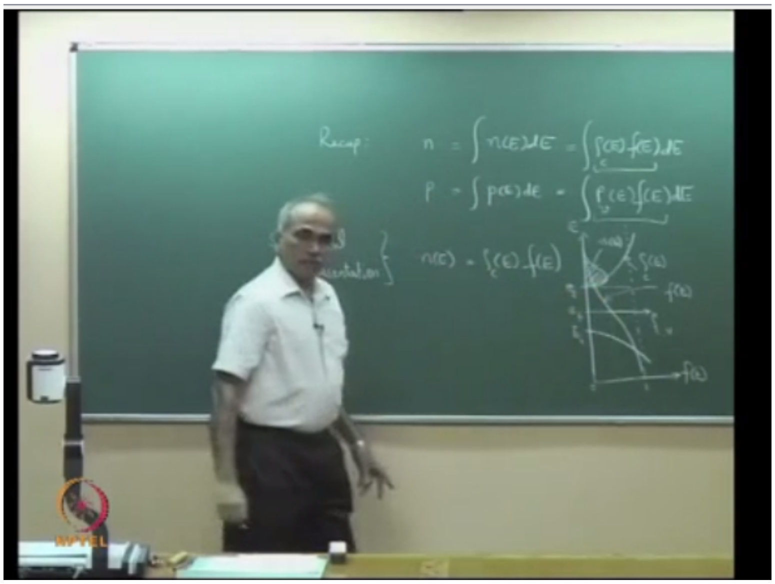
Semiconductor Optoelectronics
Prof. M.R. Shenoy
Department of Physics
Indian Institute of Technology-Delhi

Lecture-08
Carrier Concentration & Fermi Level

Today we will discuss carrier concentration and Fermi level given a carrier concentration or a current through the device you calculate the carrier concentration. And knowing the carrier concentration it is possible to calculate the Fermi energy. This is required in several design characteristics so, we will discuss the relation between this.

(Refer Slide Time: 00:47)



Today we recall to recall what we had seen in the last class the carrier concentration n is given by n of e dE which is equal to row if it is n. It corresponds to carrier concentration of electron which means the conduction band. So, row c of e*f of e the probability of occupation dE, similarly if you are calculating the carrier concentration of holes in the valence band. Then this is equal to p of e dE which is equal to row v of e, f of e dE.

It is interesting to see what will be the integrand and it is integrated over the limits of energy. So, if it is conduction band. Then Ec to infinity and if it is valence band it is infinity to–infinity to or –infinity to Ev from somewhere from below to Ev. The integrand here, so let us have a graphical representation see the graphical representations it gives you graphical representation of the carrier density**.**

So, n of e here is the integrand which is equal to row c of e into f of e how does it look like it is a product of two functions. This is the energy axis row c of e so, if this is Ec, and this is Ev then row c of e varies like this, and row v of e varies like this. So, along the x axis we have row c of n row v. So, this is the lower one is row v and this is row c. On the same graph this is the product of two functions in the same graph.

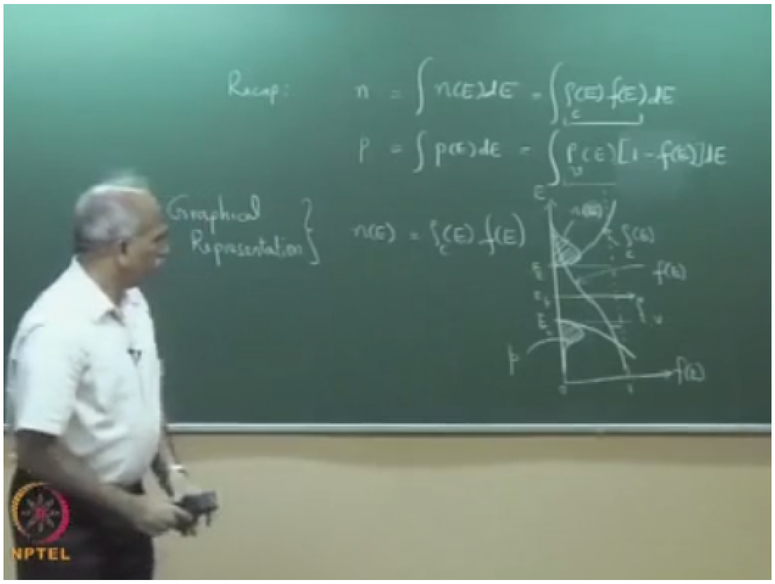
I want to plot also f of e so, if this is 1 this is 0 f of e then f of e if I take for example an intrinsic semiconductor. Then Ef remains somewhere in between somewhere near mid way between Ec and Ev and therefore the if I plot f of v it would look something like this. I should probably have brought a colour chalk met in yours suppose you follow this. This is f of e so, where ever you have 0.5 so, this is Ef.

For an intrinsic semiconductor Ef remains somewhere in between somewhere near midway so, for e greater than Ec for every value of e here. The integrand is product of this function and this function f of e multiplied by row of e, f of e is maximum near Ec and it goes down to 0 almost 0 as you as e increases this of course increases a square root of e. So, how would the product look like here it is 0 therefore if you find out the product of these two it would vary something like this.

It will initially increase and then it will come down so, this third variation that I have plotted is n of e. This third variation is the product of this and this which means for every given value of e multiplied by f of e/row of e you get this value. So, it goes up to a maximum but then starts decreasing because f of e is decreasing that is n of e integrate over e which means area under this curve gives you n. So, the carrier density the carrier concentration is area under this curve.

Because here integrated over e dE integrating similarly if you see here for the valence band this is the density of states and in yes I made a small mistake please correct this.

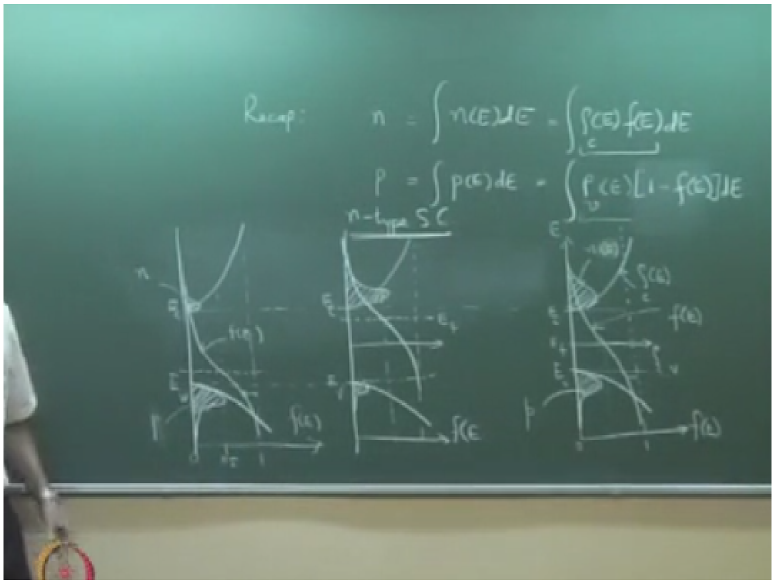
(Refer Slide Time: 06:44)



Because the probability of occupation is 1-f of e you can point out it earlier if you so, 1-f of e again you see 1-f of e is this part. Because from here to here it is f of e and this small length here magnitude corresponds to 1-f of e that is very large near Ev but decreases down to 0. The density of state is 0 at Ev and continuously increases. So, if you take a product of these two here also you will get a curve which goes up to a maximum and comes down.

And area under this curve your integrating over e, area under this curve gives you the carrier concentration p area here means n this is n integrated the curve is a n of e. But the area the curve is of course n of e, but the area under that gives you p. Let me quickly go to how did look like in the case of n type and p type. You see that this areas are equal in a intrinsic semiconductor the area under this curve here will be equal to the area because n is equal to p.

(Refer Slide Time: 08:34)



If you use doped semiconductor so, let me draw at the same scale almost near the same height. So, this is Ev, Ec but this time the Fermi level in the case of n type semiconductor here somewhere here. The Fermi level is up this now for n type I am plotting f the density of states varies the same way is the same material. But now n doped Fermi level is up which means the probability of probability f of e is half here at this value.

It is half which means the curve would look like this and the density of states here is f of e please see the f of e curve has been shifted look at this compared to this. It s shifted up because Ef is up therefore what do we expect now the product of this into this. This quantity is now large compared to this you see the magnitude here is very small here the magnitude is large. Because the curve has shifted up f of e curve as shifted up.

So, in this case you will see if you take a graphical represent if you take a product it will look something like this where as in the second case. You see the magnitude 1- f of e is very small although the density of states increases. But that is very small, and therefore if you this will come out to be very small. And therefore the area under the curves clearly tells you that in n type semiconductor n is much larger compared to p.

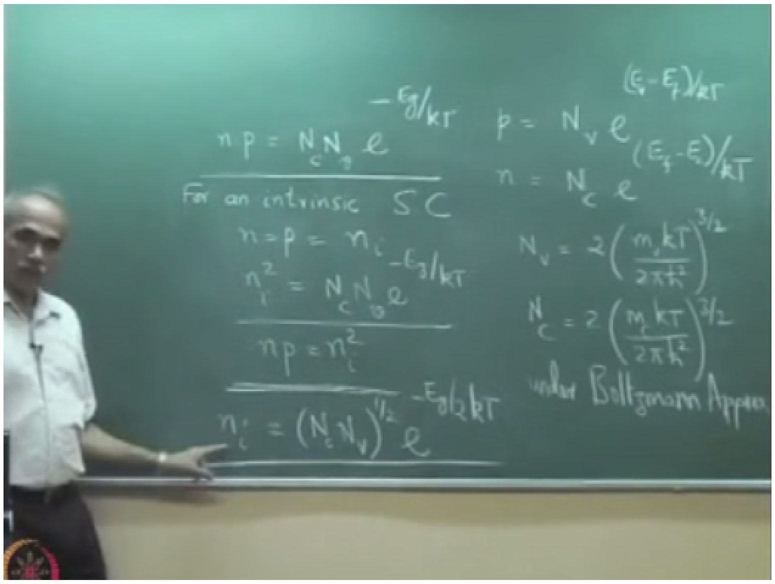
So, this is for n type this is graphical representations qualitative of course qualitatively I am drawing but if you can indeed multiply the 2 functions and see you will get a this kind of variation. And if you go for the p type so, first draw the density of states and p type so, Fermi level is now below. So, this is Ec, Ev Fermi level is below which means if probability 1, 0 so, half is here which means the curve is now.

So, this is my f of e now and you see that the magnitude 1-f of e that you see here 1- f of e is much larger compared to the 1-f of e that you see here. At the same energy if you go here the 1-f of e because f of e itself to be about 0.99 also. So, 1-f of e is the difference very small but here now 1-f of e is large whereas here f of e is very small. Please remember f of e is now along the x-axis which means this distance gives you f of e.

This magnitude of the length is proportional to f of e so, here f of e is now very small density of states is the same. Because that does not depend on the doping concentration or very little dependents it depends on the material only. And therefore if you take the product this time you will get n of e varying like this, whereas p of e varies. And the area under the curves give you p and so, you can clearly see from this graphical illustration also that.

What we have plotted is carrier density verses E the area under n of e and p of e tells you the total carrier concentration. So, this is for the p type semiconductor is it alright so, we have also derived a mathematical expression for p and n under the Boltzmann approximation.

(Refer Slide Time: 14:35)



$np = N_c N_v e^{-E_g/kT}$

For an intrinsic SC

$n = p = n_i$

$n_i^2 = N_c N_v e^{-E_g/kT}$

$np = n_i^2$

$n_i = (N_c N_v)^{1/2} e^{-E_g/2kT}$

$p = N_v e^{(E_v - E_f)/kT}$

$n = N_c e^{(E_f - E_c)/kT}$

$N_v = 2\left(\frac{m_v kT}{2\pi\hbar^2}\right)^{3/2}$

$N_c = 2\left(\frac{m_c kT}{2\pi\hbar^2}\right)^{3/2}$

under Boltzmann Appro

We have p is equal to nv into e to the power e-Ev-Ef/kT and n is equal to nc into e to the power Ef-Ec/kT. These are under the Boltzmann approximation where nv is 2 into mvkT/2pye h cross square to the power3/2 all of them are constant except mv. There is effective mass of holes in the valence band and Nc is equal to 2 into mc kT/2 phy/h cross square to the power 3/2 this is under Boltzmann approximation.

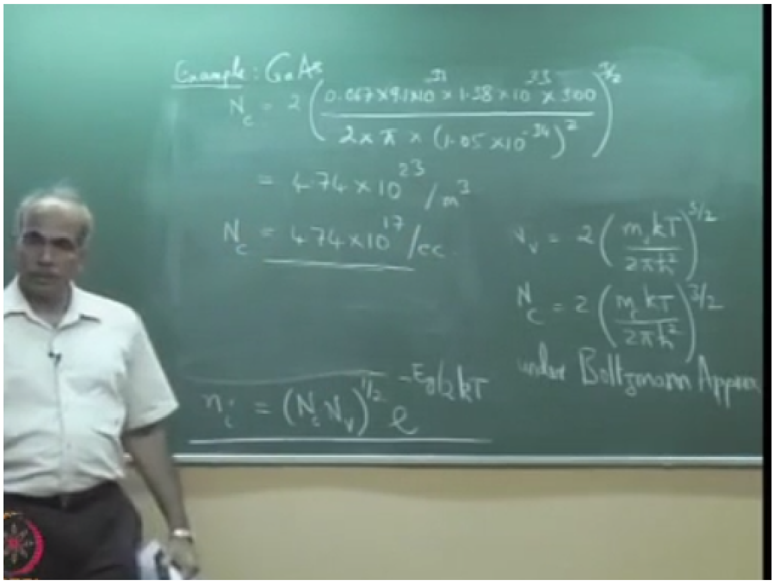
And we have discuss Boltzmann approximation which simply says that if Ef- Ec is much greater than qt that is when the Fermi level lies well within the band gap. Then you can use Boltzmann approximation so, under Boltzmann approximation n into p therefore is equal to Nc into Nv into e n into p, Nc into Nv into Ef there is one + here for – here. So, we have e to the power-Ec and Ev so, it is –Eg/kT n into p is equal to Nc into Nv-Eg/kT.

For an intrinsic semiconductor n is equal to p is equal to ni and if you substitute here ni square is equal to Nc into Nv e to the power-Eg/kT. Please see that this expression is independent of Ef, this expression is the same as this it is the independent of Ef. Therefore whether you take an intrinsic material or an n type material or a p type material the right hand side is the same.

It means that n into p is equal to ni square for all semiconductors under Boltzmann approximation. The class always remains but n into p is equal to ni square is this sometimes for the law of mass action np equal to ni square. Therefore carrier concentration ni the intrinsic carrier concentration ni is Nc into Nv to the power half into e to the power-Eg/kT. Please see the by 2k I am sorry by 2k Eg/-Eg/2kT.

Expression for Nc and Nv are here therefore given a material if you know the effective mass of holes and effective mass of electrons rest of them are all constants. You can calculate Nc, Nv and therefore the intrinsic carrier concentration simply depends on the band gap each. Please put some numbers let me for example calculate Nc what is Nc is equal to let us take for example gallium arsenate Nc is equal to 2 into mc.

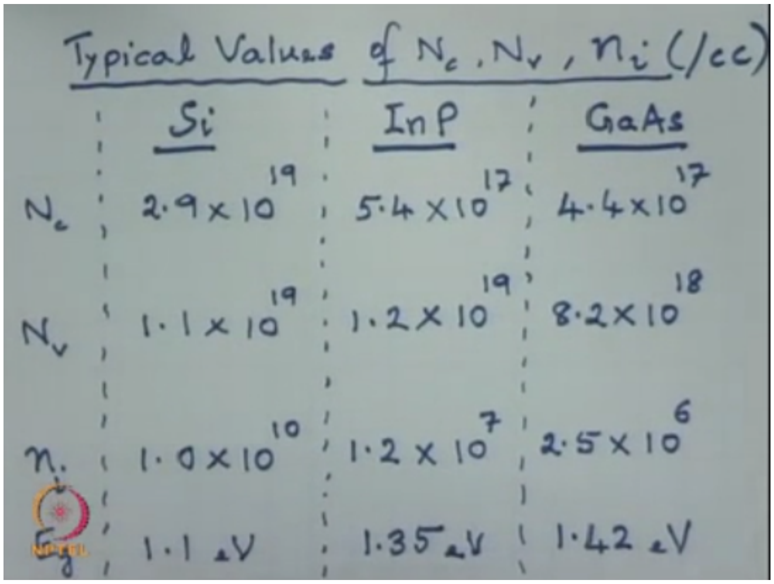
(Refer Slide Time: 19:31)



That is effective mass of electrons at 300k is 0.067 into n0 into 9.1 into 10 the power of -31kg mc multiplied by kT 1.38 that is k Boltzmann constant k is 1.38 into 10 to the power of -23 T300k/2 into phy*h cross 1.05 into 10 to the power of -34 square and whole to the power 3/2. So, this will come out I will give you some values that I have calculated is comes out to be 4.74 into 10 to the power of 23 per metre cube.

Because all these are a SI units therefore this came out per metres cube are 4.74 into 10 to the power of 17 per 6 Nc. Similarly you can calculate Nv and I want to give you these numbers some numbers I have calculated. So, let me give you typical numbers.

(Refer Slide Time: 21:28)



Typical Values of N_c, N_v, n_i (/cc)

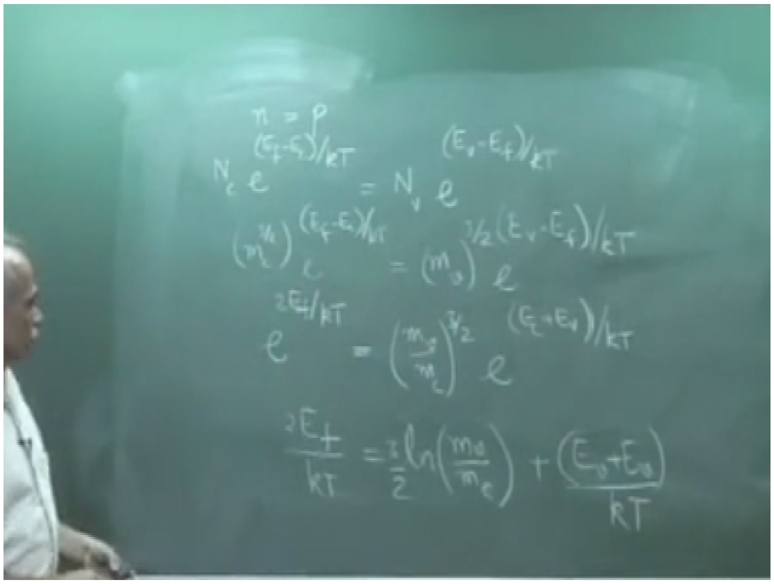
	Si	InP	GaAs
N_c	2.9×10^{19}	5.4×10^{17}	4.4×10^{17}
N_v	1.1×10^{19}	1.2×10^{19}	8.2×10^{18}
n_i	1.0×10^{10}	1.2×10^{7}	2.5×10^{6}
E_g	1.1 eV	1.35 eV	1.42 eV

Okay all of it is visible so, typical values of Nc, Nv, ni per cc so, is the material silicon band gap is 1.1uv and you can see and see Nc and ni calculated by taking constants. So, it is advisable to take some constants from books or literature and calculate these Nc, Nv and ni. This is indium phosphide I have written for 3 very important substrates for opto-electronics the most important substrates are gallium arsenide and indium phosphide.

So, indium phosphide has a band gap of 1.35ev and you can see the important point to see is for smaller Eg as you have seen in this expression for ni. It depends on Eg ni is equal to Nc into Nv to the power half. So, smaller the Eg larger will be carrier concentration ni and as the band gap increases you can see that the intrinsic carrier concentration decreases. If you take gallium nitride why band gap semiconductor which has a band gap of 3,.4 electron volt.

The intrinsic carrier concentration is 10 power-10 it is of the order of 10 power-10 to a strong dependents on the band gap. It was a exponential dependents, and of course it has a temperature dependents these numbers are calculated at 300k. So, please calculate Nc whether you get such numbers. For an intrinsic semiconductor n is equal to p which implies Nc into e to the power Ef-Ec/kT is equal to Nv into e to the power Ev-Ef/kT.

(Refer Slide Time: 24:22)



$$n = p$$

$$N_c e^{(E_f-E_c)/kT} = N_v e^{(E_v-E_f)/kT}$$

$$(m_c^{3/2}) e^{(E_f-E_c)/kT} = (m_v)^{3/2} e^{(E_v-E_f)/kT}$$

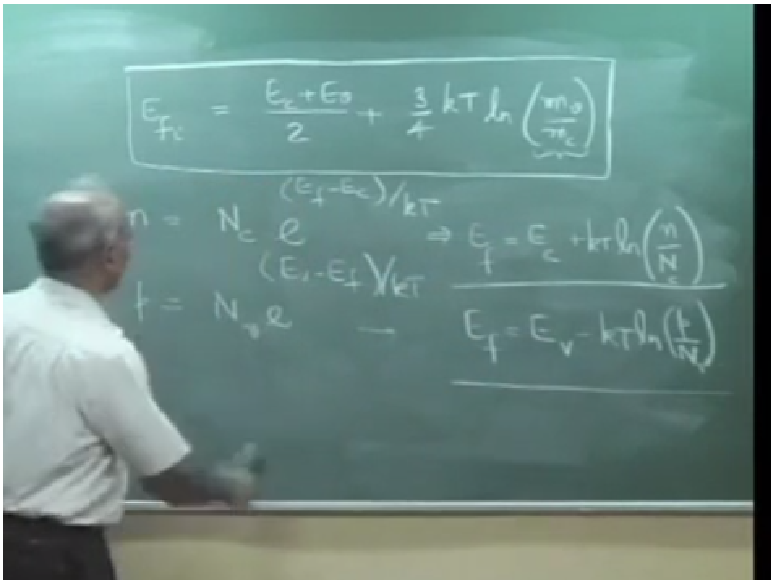
$$e^{2E_f/kT} = \left(\frac{m_v}{m_c}\right)^{3/2} e^{(E_c+E_v)/kT}$$

$$\frac{2E_f}{kT} = \frac{3}{2}\ln\left(\frac{m_v}{m_c}\right) + \frac{(E_c+E_v)}{kT}$$

What is different in Nv and Nc only this is the difference effective mass rest of them is the same therefore this means m to the power 3/2 mc to the power 3/2 into e to the power Ef-Ec simple algebra by kT is equal to mv to the power 3/2 e to the power Ev-Ef/kT. I can bring that Ef here so, I have e to the power twice Ef /kT is equal to mv/mc to the power 3/2*E to the power what do we have this Ef I have brought here.

Ec goes there so it is e to the power Eg/kT Ec+Ev/kT is that alright okay therefore Ef we can simplify this so, take ln on both sides. So, you have 2Ef/kT is equal to ln3/2 ln mv/mc mc+Ec+Ev/kT or multiply by kT on both the sides. And take this to other side so, you have Efi for intrinsic semiconductor I was writing this expression.

(Refer Slide Time: 27:19)



So i standing for Efi is equal Ec+Ev/2+3/4kT ln mv/mc the expression for Fermi level of a intrinsic semiconductor. You can see that it is exactly in between Ec+Ev/2+a term here only when mv is equal to mc ln1 is 0. Therefore the Fermi level will lie exactly midway between Ec and Ev if the effective mass of hole is equal to effective mass of electrons. Otherwise there will be a small difference e.

Now how small is the difference, the difference is really small because you can put some the corresponding values for mv and mc. And you can see that this difference is going to be very small for example if this comes out to be a factor of 10. If you take gallium arsenide m vi s0.45 and 0.067 this approximately 8. So, if I take 10 ln10 is how much ln10 is 1/2.303 which is 0.4 approximately 0.4.

So, this term is a approximately 0.4 multiplied by so, there is a 3/4 here approximately estimating what is this what is the typical number here 3/4 into kT is approximately 0.025. So, the number is really very small so, this is 10 times and this is 0.075/10 which is equal to 0.0075e approximately estimated just to find what is the order. So, what is the point e value if mv and mc are very different as in the case of practical materials?.

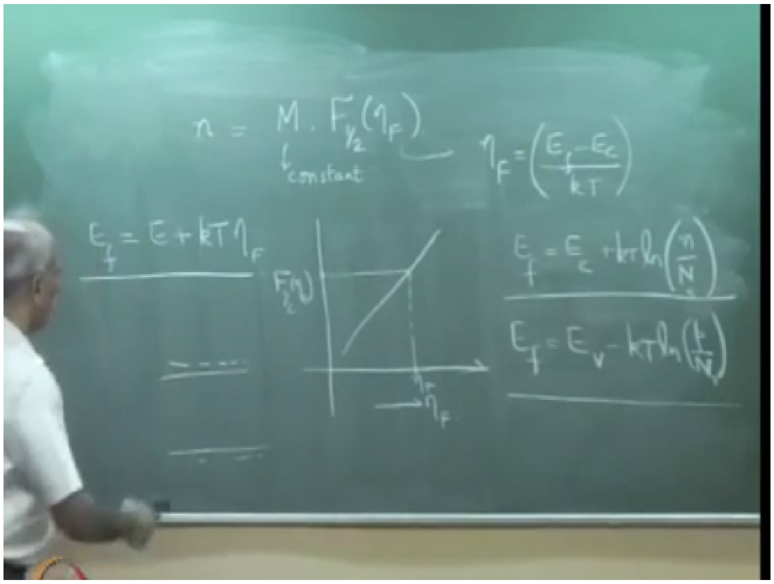
The second term is very small and therefore with some approximation we can say that the Fermi level for an intrinsic material lies midway between Ec and Ev. It is not exactly but almost midway between Ec and Ev. Because Efi is equal to Ec+Ev/2 what about the Fermi level of other materials that is p doped and n doped. We had an expression for carrier concentration n is equal to Nc into e to the power Ef-Ec/kT.

We have determine this expression already p is equal to Nv e to the power Ev-Ef/kT. I want to find out Ef from here so, you can bring this here take log and you get f. So, this gives you f is equal to **Ec** Ec+kT into ln/Nc n/Nc then take ln so, this will come down multiply by kT you will be left with Ef-Ec take Ec to the other side. So, you have got expression for ef from this expression you get here Ef is equal to Ev-kT into lnp/Nv.

You must be wondering Fermi energy Ef Ec+this which means greater than Ec is that true does not because this n is much smaller compare to Nc. That is why I gave you these numbers Nc is much larger compare to n and therefore this is a fraction log of a fraction is a negative number. And therefore the second term here is negative so, actually it is easy-something. And therefore it lies within the band gap so, this is under Boltzmann approximation.

Therefore Ef has to lie within the band gap, otherwise this expressions are not valued. If Boltzmann approximation is not satisfy this is fine for a likely doped materials where Boltzmann approximation was a good approximation. If Boltzmann approximation is not satisfied for do it to carrier concentration and Fermi level which means if you know the carrier concentration Nc is known for the given material kT, Ec.

(Refer Slide Time: 33:20)



So, if you know the carrier concentration you can determine Fermi level what was the carrier concentration in the case of without any Boltzmann approximation this was through Fermi half integrals. So, Fermi half integrals there was some constant m some constant you can see what is the constant 1/2pye square into 2mc kT/h cross square to the power 3/2 into the Fermi integral f half of eta.

This is the, that constant which is outside and what is eta f, eta f is Ef-Ec/kT and recall what we had done in the last class. We had plotted eta f here verses the Fermi half integral f of half of eta if you know the carrier concentration it means you know the value of this Fermi half integral that is you know the value here. You had I just showed a qualitative variation like this. So, you know the value of f half of Fermi half integral of eta f that means if you know that the value here.

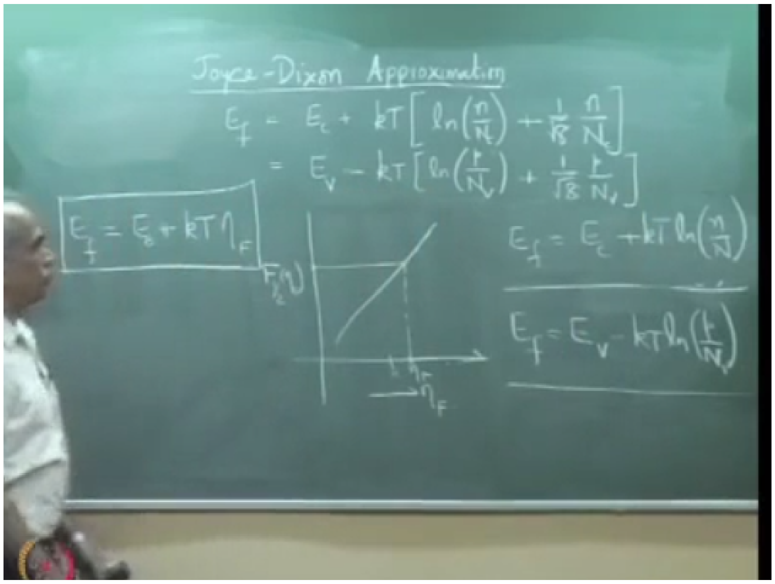
Then you can find out what is eta f the value eta f corresponding to this so, m is equal to therefore n corresponding to this n I know this value. And therefore we know eta f so, once you know eta f, eta f we know and therefore Ef is equal to Ec+kT into eta f because eta f has been numerically obtain from this graph. Because you have found out this for a given carrier concentration you know this and from this numerical plot.

You know what is eta f, and if you know the value of eta f then you can find out what is the Fermi energy. This is the way to exactly get the Fermi energy for and without any approximation no Boltzmann approximation and this important. Because in opto-electronics we will see that most of the devices are degenerate semiconductors or the Fermi level is entering at the conduction band or valence band or both the bands.

How is that possible Fermi level entering valence band and conduction band, yes it is possible and that is our next topic. We will see how it enters valence band and conduction, so Ef is equal to Ec Ec+kT into eta f. You can see here eta f value yesterday I have written eta f values 0 to 4, -2, 4, 6, -6 so, this eta f could be positive or negative. Therefore Ef could be greater than Ec or less than Ec.

So, it will give you Ef value even in the Fermi level enters the band this is the exact expression where you have to numerically evaluate the Fermi half integral. However it is sometimes difficult and there is a very important approximation for oh let me not erase this very important approximation for the Fermi level Ef is equal to Ec+kT into ln n/N. And that is problem the joysticks approximation this approximation you want find in older books.

(Refer Slide Time: 37:39)



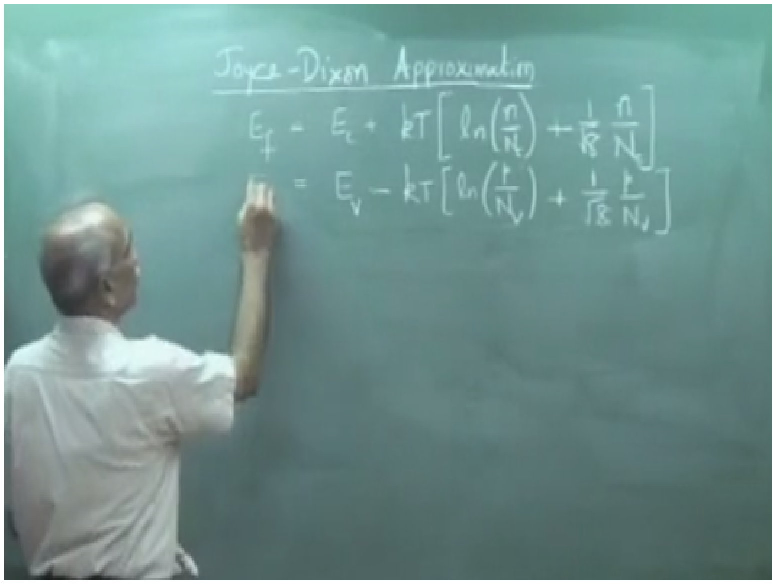
Joyce Dixon approximate for the Fermi level this is very good approximation this skills tag the Fermi level Ef is equal to same expression Ec+kT ln n/N same term+ an additional term 1 over root 8 n/N this is an empirical formula or this is also equal to Ev-kT into ln because you may be knowing whole concentration p/this is n/Nc. Please see the subscript n/Nc and this is Nv+1over root 8*p/Nv.

So, any of these formula could be used because in highly p doped material you know p where as in highly n doped material you know n. Because when you highly doped n is nearly equal to the donor ion concentration that is the doping concentration. Because intrinsic concentration is negligible and similarly if you highly p doped material.

The p is essentially equal to the acceptor ion concentration so, in some materials you will know p in some materials you will know n. And you can find out the Fermi level without any Boltzmann approximation, this is a very useful formula. We will need this later when we pass current through the device for a given carrier concentration you will have to estimate the Fermi level. The Fermi level will be required as you will see later on to for example.

To determine the band width of an amplifier this is Joyce Dixon approximation for normal calculation it is sufficient to use this approximation. It may be difficult to get the exact numerical solution.

(Refer Slide Time: 40:32)



Joyce-Dixon Approximation

$$E_f = E_c + kT\left[\ln\left(\frac{n}{N_c}\right) + \frac{1}{\sqrt{8}}\frac{n}{N_c}\right]$$

$$= E_v - kT\left[\ln\left(\frac{p}{N_v}\right) + \frac{1}{\sqrt{8}}\frac{p}{N_v}\right]$$

So, we have we know for a given carrier concentration how to determine the Fermi level whether it is highly doped or not. Our next topic will be so, long we have been discussing about Fermi level the next topic will be quasi Fermi level**.**