Semiconductor Optoelectronics
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Lecture-7
Occupation Probability and Carrier Concentration

Good morning continuing with the course today we will discuss occupation probability and carrier concentration.

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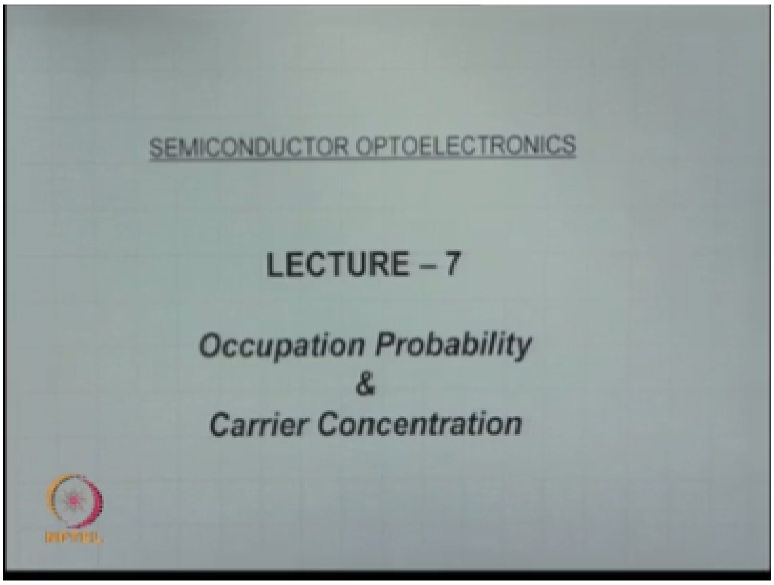

SEMICONDUCTOR OPTOELECTRONICS

LECTURE – 7

Occupation Probability
&
Carrier Concentration

In the last class we have discussed about the density of states, row of E, it was given by an expression 1/2phy square*2mc/h cross square for valence band and conduction band to the power 3/2*E-Ec to the power 1/2 for t greater than the density of states and as I mentioned the importance of density of states is to determine the carrier concentration and we need to determine the carrier concentration to know the current and carrier concentration is given by N of E de.

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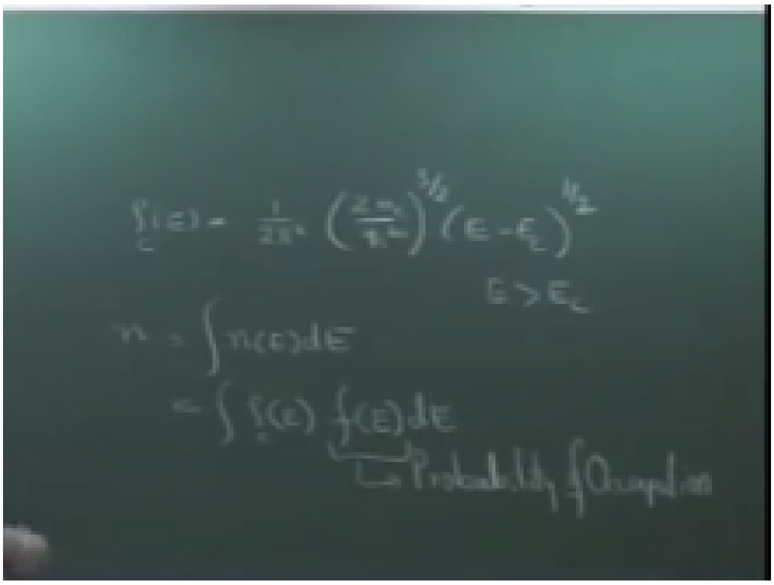

$$g_c(E) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(E-E_c)^{1/2}$$

$E > E_c$

$$n = \int n(E)\,dE$$

$$= \int g_c(E)\,\underbrace{f(E)}\,dE$$

↳ Probability of Occupation

And where N of E is the density of careers and this is equal to density of carriers is equal to available density of states multiplied by the probability of occupation**.** So f of E here is the probability of occupation. We will discuss this probability of occupation in this class and determine the carrier concentration.

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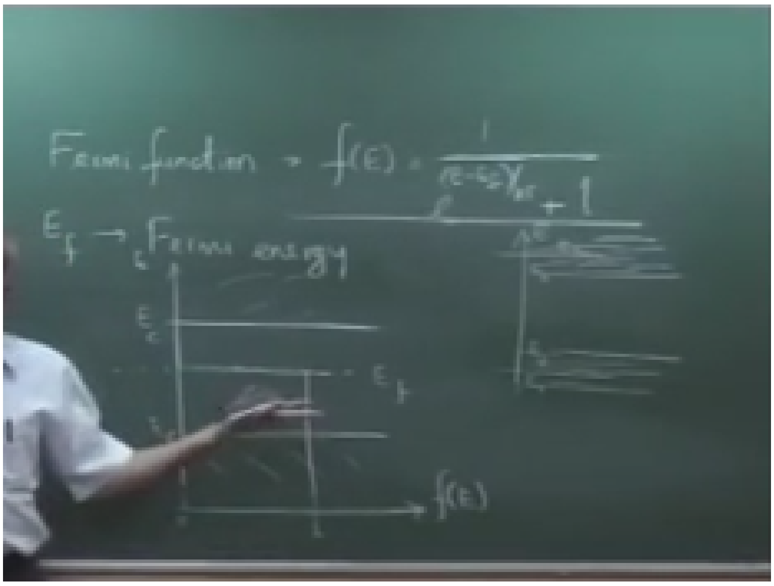

Fermi function → $f(E) = \frac{1}{e^{(E-E_F)/kT}+1}$

E_f → Fermi energy

The probability of occupation for electrons is given by the Fermi function f of E=1/E to the power E-Ex/dt+1. The probability of occupation of electrons in an energy level E, so electrons in energy band this is E call is the vertical axis is energy, this is Ec, this is Ev, anywhere if you take an energy level in the valence band let us say E=E1 or if you take an energy level in the conduction band let us say E=E2.

Then if you want to find out the probability the occupation both these bands have number of states allowed number of states what is the probability of occupation of these allowed states. The probability of occupation for valence band as well as the conduction band probability of occupation by electrons is given by the Fermi function. So if you substitute if you want to find out probability of occupation here E1.

Then you substitute E1 here, if you want to find out the probability of occupation E2 at the level E2 then you substitute E2 in this equation. That Ef is the constant or Fermi energy here. Fermi function is also called Fermi graph distribution because it tell us it indicates the distribution of electrons in the energy bands. So if you want to plot this f of Environment

As you know in the valance band the probability of occupation is very high valance band is completely full at 0 times, so this is Ev and this is Ec and here is E, this is the conduction band and this is the valance band, if you consider a 0k I want to plot here on this axis f of E many times in statistic physics f of E is further here and energy in this axis. But here this always give the vertical axis E, so if you plot f of E here at 0k I would take valance band is completely full.

And the probability of occupation here is 0 and as you know that if Ef remains somewhere here can anywhere, but if you have s here then the probability of occupation is 0 up to Ef and for Er here greater than Ef, the probability of occupation is 1. So in this axis this is 0 f of E=1. So I plotted as f of E as a function of f of E as a function or F of E as a function of E where E greater than Ef probability 0 and E less than.

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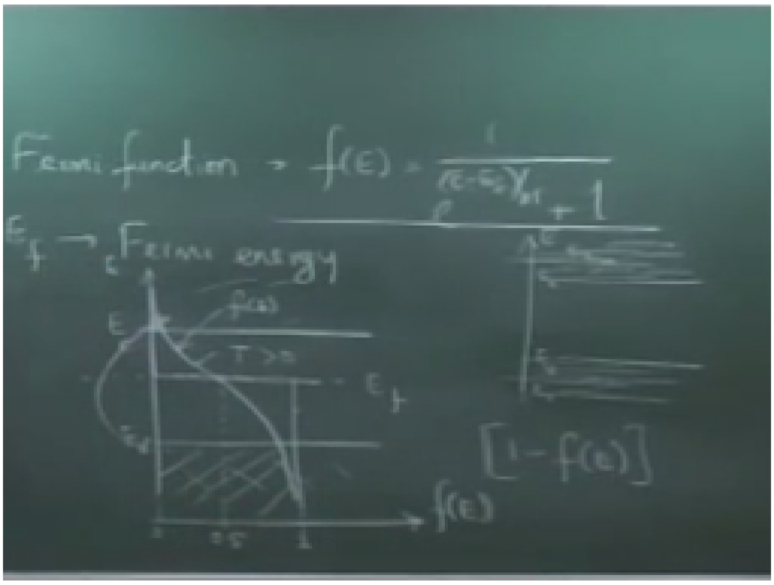

There will be a level if the level of f or anywhere for example probability=1 here, what does it mean, there is no level, there is no state permitted. If there were to be a state then the probability would have been 1, but all the levels here are completely full completely probability is 1. If you take a finite temperature this is what you already know that if you take for any temperature at E=Ef that is at this level.

E=Ef for any temperature the numerator is 0 and therefore E, 0is 1, 1+1 or the probability is half, so f of E=half for all temperature f of E=1/2, so if I indicates that half here this is 0.5 and see I am plotting probability on this axis f of E, if this is 1/2 then if you take any finite temperature we know that the electrons from the valance band make an upward transition here.

This is a final probability of finding electrons in the conduction band and probability function is still described by the property distribution is still destroyed by the Fermi function and the Fermi function gets smear, this step function now get smear here, but passes thorough the same point this is f of E then actually block this numerical you plot this and numerically you calculate and plot this for any temperature.

So this is for a finite temperature t, t greater than 0, what it indicates if there is a finite probability here of occupation of these states in the conduction band. This function is f of E what is 1-f of E if f of E is the probability that a state is occupied wher 1-f of E is the probability that the state is vacant and occupied the state is vacant. In the valence band if 1-f of E you see it is a f of E is going like this.

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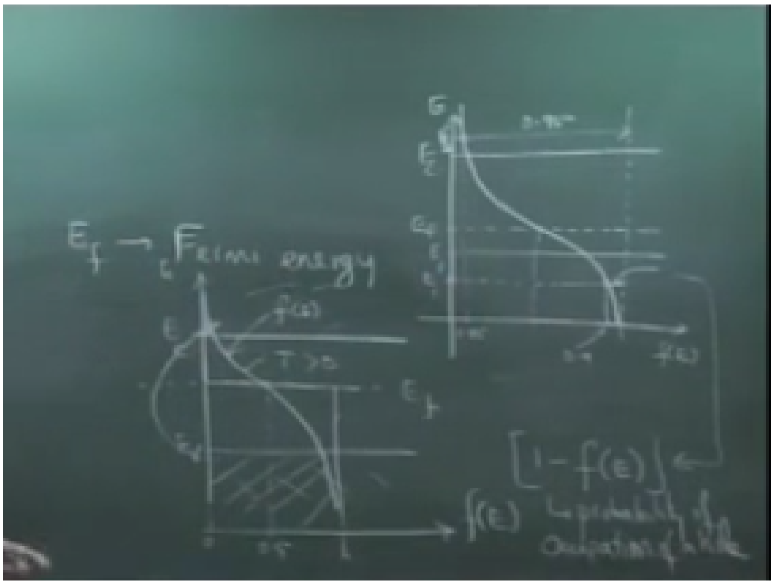

Therefore f of E is the probability of occupation therefore 1-f of E is the probability that the state is vacant, if I plot again the same I am plotting again this is Ev and this is Ec, the Fermi function is at a finite temperature is varying like this. So this is the line f of E this is f of E or on the x-axis I have f of E vertical axis is the energy and find somewhere here therefore BF is somewhere here Ef.

At any given energy value E1 I find out what is a f of E, this is the value f of E say for example 0.9 0.9 say f of E 0.9 it means the energy the occupation probability of the energy level E1 is 0.9 which means the probability that the state at even is vacant is 0.1 1-0.9 is 0.1 that is this one, this magnitude here it is magnitude of this length is 0.9 then this is 0.1. This is the probability that the state is vacant.

The vacant state in the valance band represents what it is a code therefore 1-f of E represents the probability of occupation of a hole 1-f of E is the probability that the state is vacant which means as far as valence band you do not thought the hole in the conduction band for example let me take here at some value E2, E2 is the value in the conduction band energy value in the conduction band somewhere E2, this maybe for number let us 0.05.

For example so probability of occupation here 0.05, what is the probability that there is no occupation the probability that there is no occupation that is this one 1-f of E is 0.95 because this is 0.05 therefore this is 0.95, that is not the probability of occupation of a hole but that is the probability that the state is vacant. So please remember f of E is the probability of occupation of electrons weather in the conduction band or the valance balance.

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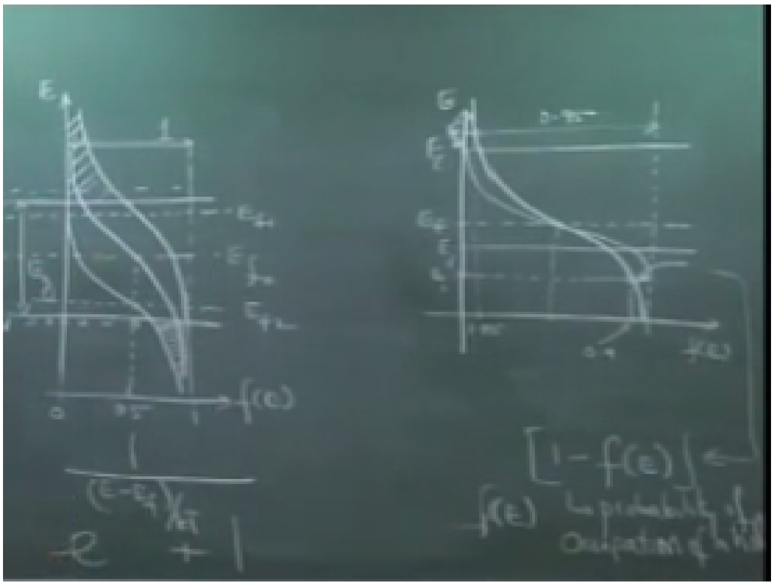

But 1-f of E is the probability of occupation of a hole if you are talking of the valance band. Otherwise it is the probability that the state is unoccupied, so 1-f of E is the probability of occupation of a hole in the valance band obviously. At a final temperature there is always a probability that there are some electrons in the conduction band and there are some holes in the valance band.

Of course depend on band Eg, if Eg is very large that is a 5 electron volts then the probability of finding electron here maybe 10 to the power -20 which means literally none, but for normal semiconductors Eg is of the order of 1 electron volt which is the probability of finding electrons here. So that the second thing that you have to see is this is Ef then the curve look something like this.

Please let me indicate first graphically because graphically becomes very clear this axis is f of E and this is 1 probability and this is 0 and E=Ef this is 0.5. If Ef is towards Ec, this is Ev, if here shifts towards Ec that means the probability function because this point at E=Ef. So let me draw a second value of Ef, this is Ef 0, let me call this as Ef1. So another material if Fermi function is Ef1 it means this point of probability of half should come here.

That means this should shift up for the up so I will have the curve will be parallel please see that the curve remains parallel this one but it is shifted up, what does that mean and clearly the curve shows that now the probability of occupation of electrons in the conduction band is known and probability here has become very small 1-f of E number of holes here will be very small probability of occupation.

When the other band if the Fermi function where to be here let us say E=E2 that means this curve would shift down because there is nothing else in the Fermi function other than Ef, it is simply E to the power E-Ef/kt+1/. So this is means that the curve is now shifted downwards you have a 0.5 coming here and you see now the area under here is 5 plots it means there are larger number of holes in the valence band.

So graphically it is very clear of course in mathematic you put it will show you the numbers, one thing which I would like to do is if you shifted this Ef here I have shown it like this it appears as if the probability is very large because please see this is one because it is 0-1 here 1, this appears as this is 0.2 or maybe this is 0.4 but just put numbers in this Fermi functions for practical B type material practical N type material.

And you see what is the kind of numbers that you get E-Ef/kt+1 choose some suitable temperature set 300k and put numbers and try to plot this, you will be surprised that here I have shown as it 0.4. This will be very very small number, the probability here is extremely small which will come out 10 to the power -4, -6 that kind of number little you cannot see anything like this it will be actually in an actual graph it will be as it.

It is almost 0 everywhere almost 0 and then it goes like this and almost 0 because of occupation here is extremely small at normal temperature. But we have large number of electrons how is it possible. At room temperature if you take silicon then the number of electrons carrier concentration that is of the order of for Silicon it is of the order of 10 to the power 10/cc concentration Ni intrinsic carrier concentration.

At room temperature is of this order, but I am saying that the probability is very very small, how is it possible because n-row of E*f of E, the density of states is very large. In the last class I had calculated n the total number of states from Ec-Ec+0.1Ev as anybody try to complete that integral so Ec-Ec+0.1Ev. This I call as N and I would ask you to calculate what is the number of states.

Number of states was total 10 to the power of 18 I think 2.5*10 to the power of 18 is the number, please do numerical is very important because there will be numerical in the exam and more importantly you are handling 10 to the power of -34, 10 to the power of -19, 10 to the power of – 23, so unless you practice we will not able to do the numerical correctly, every one of you will get a different answer.

So please practice numerical because there are huge powers you have to handle. So the number of states itself is very large and therefore even though the probability here is very small, probability actual number is very small multiplied by the number of states it is significant did you still of the order of 10 to the power of 10. So this is the distribution of occupation probability of course we are aware of this.

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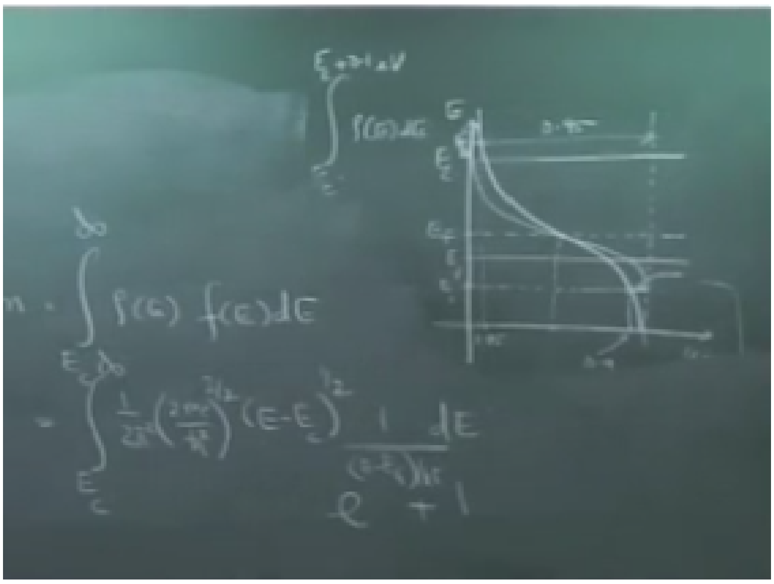

I am just recalling on the concept that is essential therefore the carrier concentration the carrier concentration N=row of E*f of E dE let me come N is the carrier concentration in the conduction band, so this is Ec to infinity I am writing Ec-infinity actually Ec to certain values some value of the band but I am writing it up to infinity because as I mention to give the numbers here are extremely small, larger the difference Ec-E here.

Smaller will be the probability that is this term goes to 0, it does not matter you write Ec is it may be up to 2Ec+2Ev or 1Ev but if already goes to 0 it does not matter whether integrate from Ec to EC+1Ev or Ec to infinity, it is one and the same that is why I am writing the total number of career in the conduction band as N=EC to infinity, so substitute here row of E integral 1/2phy square 2mc/h cross to the power 3/2*E-Ec to the power 1/2*f of Environment

So 1/E to the power E-Ef/kt+1Et, Ec to infinity, this gives us the career concentration before I carry out the actual integration here this is actually not possible to integrate analytically, alright let me continue with this.

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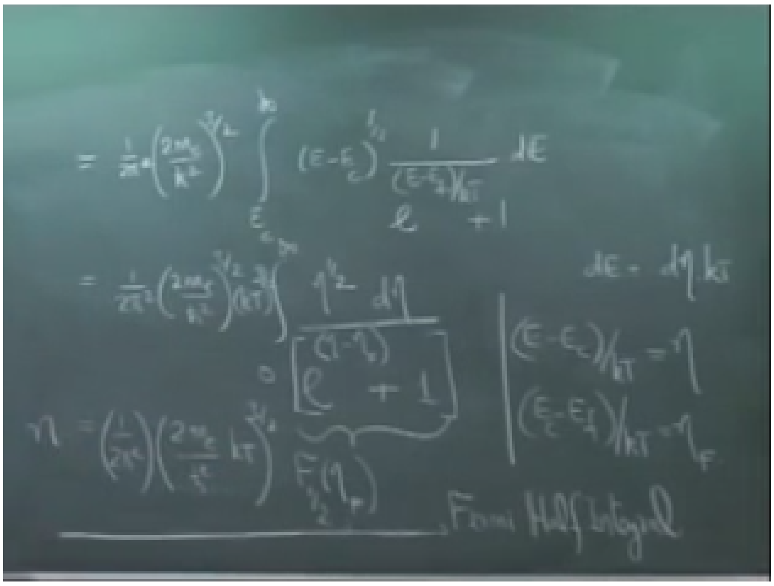

$$= \frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2}\int_{E_c}^{\infty}(E-E_c)^{1/2}\frac{1}{e^{(E-E_f)/kT}+1}\,dE$$

$$= \frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right)^{3/2}(kT)^{3/2}\int_0^{\infty}\frac{\eta^{1/2}\,d\eta}{\underbrace{\boxed{e^{(\eta-\eta_F)}+1}}}$$

$dE = d\eta\,kT$

$(E-E_c)/kT = \eta$

$(E_c-E_f)/kT = \eta_F$

$$n = \left(\frac{1}{2\pi^2}\right)\left(\frac{2m_c}{\hbar^2}kT\right)^{3/2}F_{1/2}(\eta_F)$$

Fermi Half Integral

So let me take out 1/2phy square*2mc/h cross to the power 3/2 constant I have taken out*integral Ec to infinity E-Ec to the power 1/2 1/+1. This can be integrated only numerically, but we can make some simplification you add and subtract Ec here in the denominator and this multiply divide kt, so this is equal to 1/phy/square twice mc/h cross squared to the power 3/2.

So I add and subtract Ec and then we put E-Ec E-Ec/kt=theta and Ec-Ef/kt=theta F in the lower. So this is dt dt=d theta*kT and substitute in the expression further when E=Ec this is 0, Ec E=Ec then the lower limit is 0 and when is equal to infinity it still remains infinity. So I have 0 to infinity theta the power of ½* D theta/E to the power theta-theta F/dT theta-thetaF+1.

So theta is already dimensionless, these are the substitutions to make this substitution and you see that you get this expression and outside you have 1 kT coming from here and one root kT coming from this. So you will have kT to the power 3/2. These gives the substitution as E, so you may have this equal to 1/2phy square82mc/h cross square*kT to the power 3/2* this integral here is called the Fermi of integral f 1/2 of theta F.

F of theta this is called this last term is called the Fermi of integrity, the simpler Bozeman approximation but because I have returned the complete expression I thought I will give you the rigorously correct solution for this is N, there is no approximation in this, so Fermi 1/2 integral. This has to be elevated numerical we cannot this no analytical solution this has to be evaluated numerical.

So we have 1/2phy square 2mc/h cross to the power 3/2 kT to the power 3/2 is E 1 kT coming from here, square root of kT and 1 kT is coming from here. So that is why kT to the power 3/2 ok, theta and theta has dimensionless that is why this E power theta-theta is energy dimension energy dimension theta is dimension less. So this is called the Fermi half integral. So if you have to actually calculate the carrier concentration for any value of a Ef.

Whether it is highly doubt or lowly doubt does not matter this is the rigorously correct solution Fermi half integrity. There are which approximation which are available the Fermi half integral theories, I will just show you what kind of values are we talking for the Fermi of integral, just to get an idea we are talking of hundreds or thousands, millions what kind of number is this.

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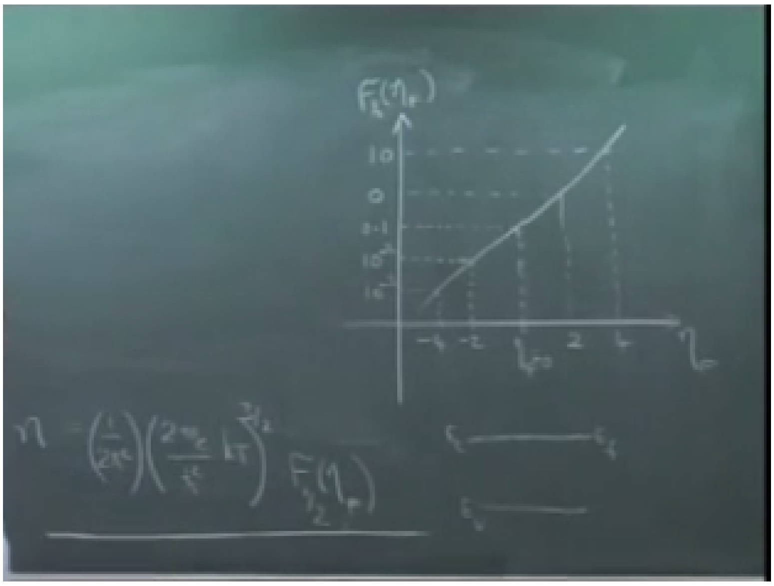

This can be half integral where is approximately this showing you approximately. So this is for me of ½ integral of theta F verses theta, because it is already indicated over energy method simply depends on the position of theta F where is the theta and theta is Ec-Ef/kT which means it simply depends on the position here, so yet Ef=0, this is thetaF=0 this is approximate number.

Just to get you an idea this is approximately 0.1 give you some idea that what kind of numbers are we talking. So this is 0, this is 10, so 0.1 10 power -1, 10 power -2, 10 power -3. So the numbers are 0, 0 here this is 0 around thetaF=0 around thetaF=0 around 2, 4, this is -2, 4 and so on typical numbers but for the for me thetaF=0 means what Ef is that Ec the Fermi level has entered the conduction band.

That Ef=Ec thetaF is 0 the Fermi interval is approximately point approximate numbers be this just to get you numbers are we talking here. So let us say the Fermi function has have just entered the conduction band. So this is your Ec and Ev and highly dot in a semiconductor so Ef has also come here then thetaF=0 and this is approximately 0.1. So we substitute 0.2 and put all the other number here and you get the carrier concentration.

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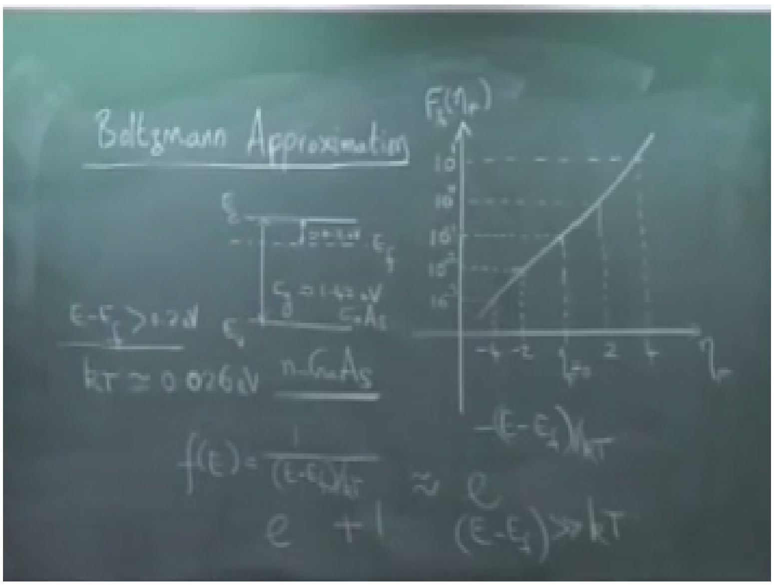

So this is the exact way of calculating familiar with the Bozeman approximation which is quite simpler I come to the Bozeman approximation. So what is the Bozeman approximation any doubts which one this one yes let it write in terms of 10 power 0, 10 power 1, yes that is right, become it become 0 when carrier concentration will become 0.

This 10 power 0, so please make a correction there, the Fermi integral is 0then carrier concentration is 0 and this value at this such a high value of theta that becomes 0 yes, so the Bozeman approximation please correct that normal doped semiconductor if you take an N type doped semiconductor then Ef may be somewhere here, you take gallium arsenide or silicon, so this is Eg is approximately of the order of 1.42 for gallium arsenide.

And it has already moved towards the conduction band, let us see the difference here is is about 0.2, 0.2E I put some numbers because numbers make it more clear that is why use number. So 0.2e let us see the difference here is because is already moved closer to conduction band because it is a N type material. So N gallium arsenide**.** The probability of occupation F of E is given by 1/E E-F/kT+y in the conduction band.

So in the conduction band E is greater than Ec, E is greater than Ec for conduction. This is valid for both conduction band and valence band, but for conduction band E is greater than Ec. So if you substitute E here then E-Ef is always greater than for this particular example that have taken E-Ef is greater than 0.2Ev. At room temperature so E-Ef is greater than 0.2Ev I am writing this for this particular example that I have taken E-Ef is greater than 02Ev.

And kT at room temperate si nearly equal to 0.026 electron holes, 0.26 electron holes which means this exponent here is 0.2/0.26 which is approximately 8 is the exponent. So you have E to the power 8+1 E to the power 8 is a very very large number compared to 1 E to the power 8 is much larger compared to 1 and you can neglect this 1 compared to E power 8 and therefore this I can approximately equal for E-Ef much greater than kT.

I can neglect 1 in comparison with this and I can write this is equal to E-Ef/k. This is Bozeman approximation. In Bozeman approximation the Bozeman approximation is valid when E-Ef is much greater than k, when the when E-Ef is much greater than kT the 1 is neglected in comparison to this number and therefore we write this has approximately equal to E-Ef/kT.

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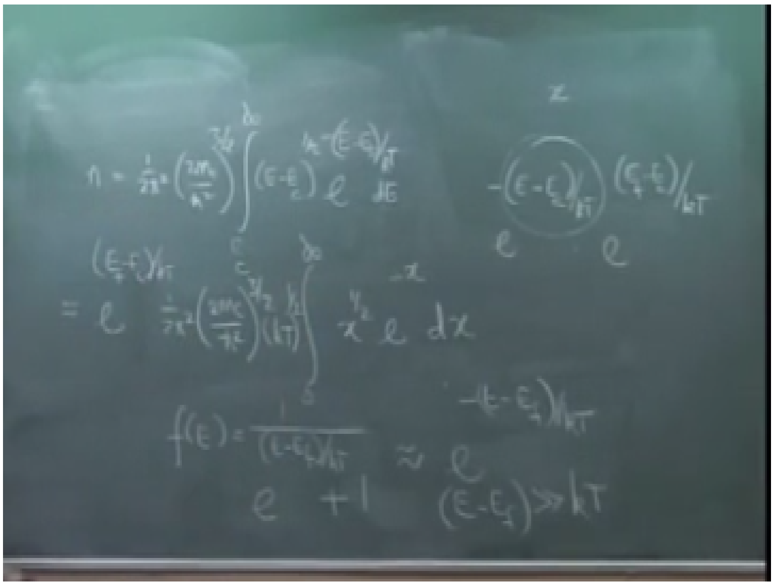

If you substitute this then that integral is easily integrable this is true for both conduction band and valence band the only requirement is E-Ef should be much greater for Ef-E for the valance band Ef-E should be much later than kt. Then N=all those constants 1/2phy square 2mc/h cross square to the power 3/2*integral Ec to infinity, we had row of E which is E-Ec to the power 1/2* E to the power E-Ef-E-Ef/kT.

Let me erase this is not required reviews of the top, this is now integrable is easily integrable so E-Ef/kT. So how to integrate this for the same procedure add and subtract Ec some of you would have done this already, so please so E- this will become E- -E-Ec/kT*E-Ef-Ec/kT added and subtracted for this term and then we can put this as x that I will have this is equal to 1/2phy square 2mc/h cross square let me complete the algebra here and integral 0 to infinity.

As before I am putting this equal to X therefore at E=EC lower limit it is 0, so 0-2 infinity this will be x to the power 1/2 and V to the power –x Vx* this term outside ok, let me keep it here E to the power Ef-Ec/kT any other term kT, kT to the power, so dE will contain 1 kT, 1kT to the power 1/2, so kT to the power 1/2 right. So kT to the power 1/2.

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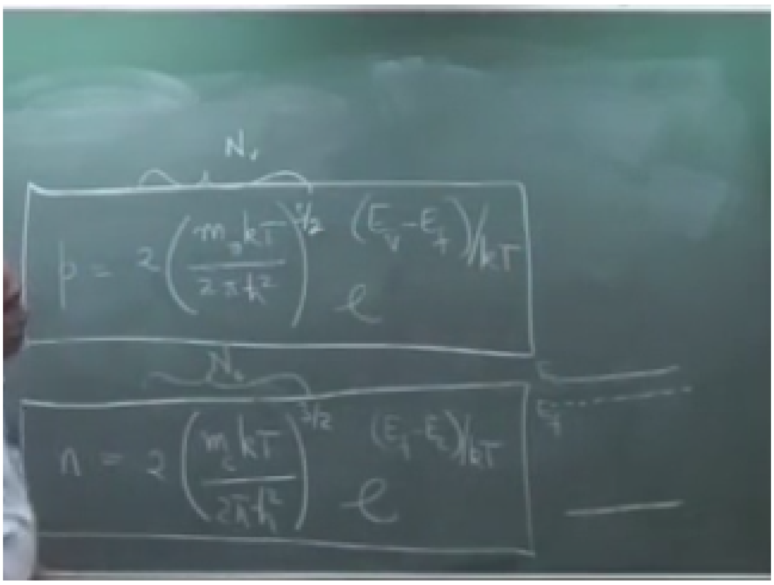

So simply this you should be getting an expression like this mckT/2phyh cross square to the 3/2*E to the power E2F-Ec/kT. Please simplify this and check that N=2*mckT/2phy h cross squared to the power 3/2*E to the power F/kT. So this is under the Bozeman approximation. Exactly like this if you follow for the valence band you get expression for P=2*mVkT/2phy h cross square to the power 3/2*E to the power Es E to the power Ev-Ef/kT. This is sometimes denoted as mc for the conduction band and this is denoted as NV, please see that it contains everything is constant except the effective mass.

Effect mass Nc in the case of Nc and effective mass NV in the case of NV in the case of nv if Nc=1Mv both constant of the same. This is a very large number if you put numbers you will see that this will be 10 to the power of 18, 10 to the power of 19 for very large number this part, sometime you get into doubt whether it is here Ef-maybe you are made a mistake and Ef-Ec or Ec-Ef.

How do you verify Ef-Ec so this is called the conduction band under Bozeman approximation which means my Ef is somewhere here this is Ec, this is Ef under Bozeman approximation. So Ef-Ec is negative Ef is smaller, Ec is larger, so this is negative. Therefore this quantity will be E to the power negative which means a factor less than 1 and the factors to be less than 1 because the carrier concentration that I am talking about the order of 10 to the power of 10, 10 to the power of 12.

And if this number is very large 10 to the power of 19 naturally this number should be smaller much smaller than 1 and therefore it is correct that Ef-Ec a cross check a quick cross check sometimes by using a negative sign or something you would have plotted this as Ec-Ef. If this become Ec-Ef is a positive number much greater than kT. So this you also give E to the power of 8, E to the power of 10 which is a very large number. This is correct alright. I will stop here.