Semiconductor Optoelectronics

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Lecture-6
Density of States in a Quantum Well Structure

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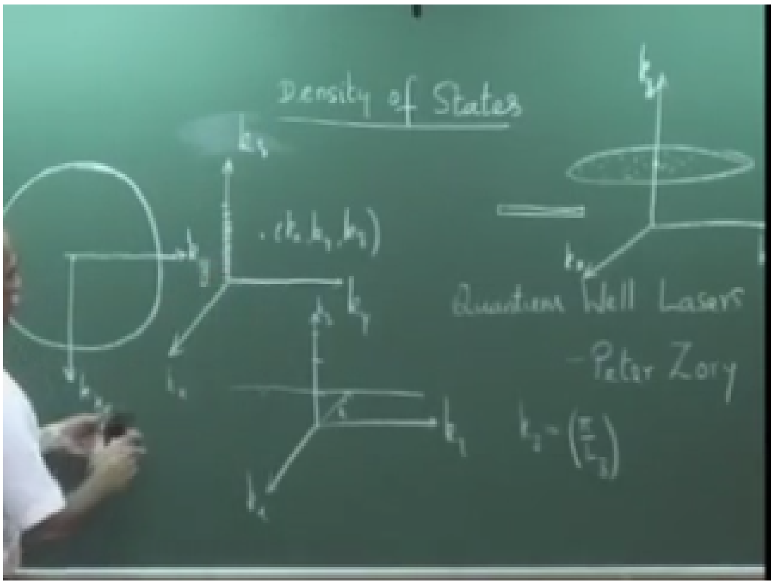

Ok so we continue with the density of states discussion and density of states in quantum well structure. In the last class we have discussed density of states for bulk semiconductors and in the k space in the case space I had shown that if this is kx, ky and kz. Then the k space that k components can take very large number of values in general for a bulk semiconductor and therefore the density of state and variable density of states is very large.

And if you every point in the k space corresponding to a kx, ky, kz represents a state that we have very large number of sub states, but if I reduce the dimension of the semiconductor in one direction let us say I reduce the dimension in the right direction to make it into a thin sheet then correspondingly the density of states change and what we have is kx and ky still having very large number of values.

However the allowed values of kz become highly discretized and since every k must have these three components in this sheet and electron wave propagating to the sheet has to have 3 of this. The first component of a k nonzero component of kz is here. So this is kz is here which means every k value that you can think of must have 1 value kz here which means the tip of the k vector in the k space must lie on a plane kz=constant.

So or the first value kz=constant is simply kz=phy/Lz*Q=1. So kz the first value is phy/Lz. Therefore all points director k can take large number of values k, but the tip of the vector must lie in this plane. Therefore we calculating the density of states, so we had this picture here where this is kz, kx and ky. We considered with kz=constant. The first one and in this because kx and ky take very large number of values or permitted to take large number of values.

Corresponding to each kx and ky will have a point a permitted point in this disk which means if I split that is I will show that this like this kz is here central axis kz. This is kx and this is ky, so kx, ky and kz, kx.

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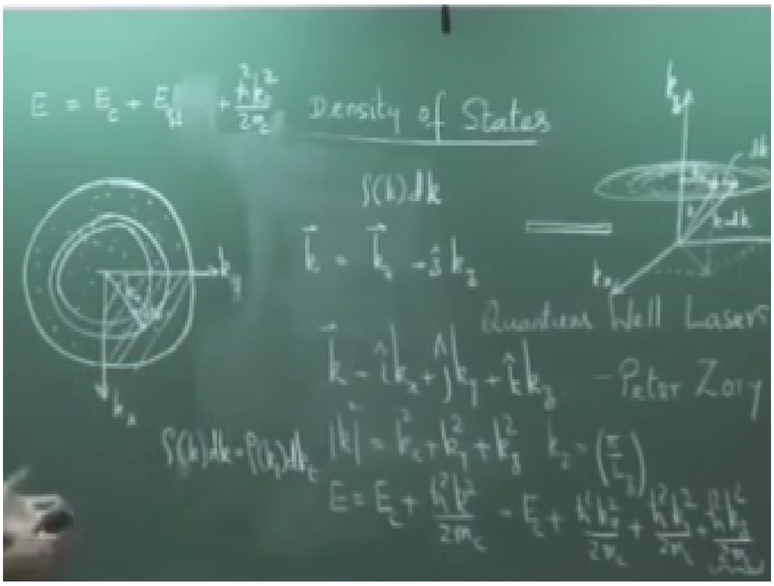

Kx, ky, kz that is this disk in the k space and showing like this, so kz is coming down, kz, ky, kz. In this you fill large number of allowed valance, our objective is to find out row of k which is defined row of kdk is the number of states between k and k+dk. So if this is somewhere dk and the next value k+dk if it stand here, so this is k, this is k+dk which means this is director dk here, this is dk.

Then our objective is to find the number of allowed States between k and k+dk. If you see in this the k will correspond to a value which is here please see this disk the k can take any way you lie between k and k+dk. SO this will corresponding to a circle here a circle corresponding to k and the next circle outside corresponding to k+dk. This transfer component here you see k=I kx will get I+J unit vector k*kz and more k square that is k square=kxsquare+ky square+kz square.

We have energy E=Ec+hsquare k square/twice mc which I can write as Ec+for this disc Q=1, the first value of kz corresponds to Q=1 therefore I write this as h cross square kx square/2mc and just substituting this and h cross square ky square/2mc+hcross square kz square/2mc. The vector here this vector k I can write as k=is a vector, I+Jky, I can write this as it transfer set of kv.

This is a transfer vector on this k without the z component+Z*kz is the transfers vector+Z*k is complete vector. So this part here kx square+ky square I can try it as kt square and I write energy E=Ec+E of Q=1+h cross square kt square/2mc where E of Q=1 is this h cross square kz square/2mc. If I put Q=1 kz takes the value kz=phy/Lz. So I can write this as Ec+E Q=1, this E states for that component only here or sometimes it denoted as Eq1.

Otherwise you can write this as Ec E at Q=1 or Eq1+h cross square kt square/2mc. Our objective is to find the number of points in this k, so this is kt and this additional vector here is dk, please see here this is k, this is k+dk corresponding to k there is a kt which has only 2 components x component and y component you can just drop down here and you find out what is the x component, what is y component and this at the top.

So what is kt, kt is just this that vector which has x component and y component, these 2 components makes this transfers vector. So this Ec is drop from here that is k and dkt is a intermittent volume interment vector, so that is dkt. So we have to find out a number of points therefore the number of points between k and k+dk is the same as number of points between kt and kt+dkt.

Because k can sit anywhere around the circle and k+dk can sit anywhere here, the number of points is the same between k and k+dk in fact it is row k row of k dk=row of kt dkt, the number of points as far as the number of points is concerned it is a say. So to find out the number of points point means every point represents an allowed state, number of points between k and dk+ and remember we have to take only positive kx, positive ky and positive kz values.

Because the negative values are already taken into an account in boundary condition and therefore I need to consider only this quadrant positive quadrant like in the case of the sphere I had to consider only the optent positive optent. So here of this circle I need to consider only the port, and therefore what is the density of state, the number of points always remember it has to come from definition.

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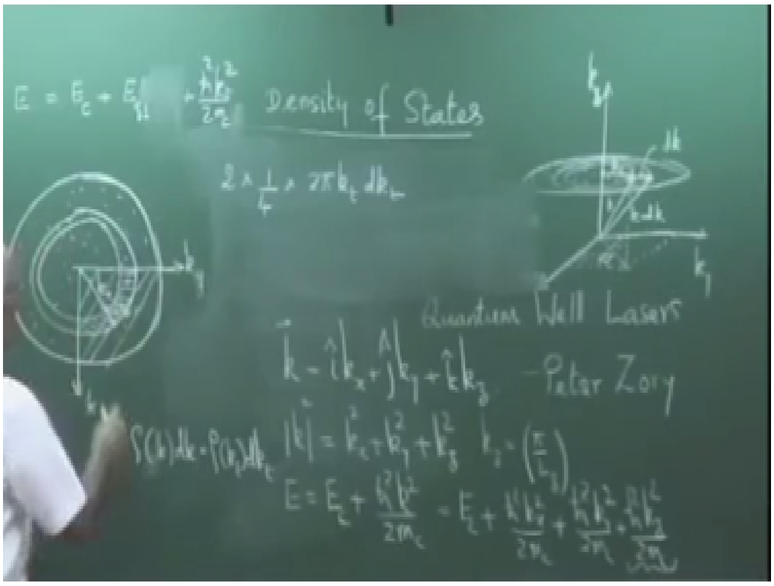

The number of points between k and k+dk is in the positive optent or quadrant, so 1/4 into what is the number of point to find out the number of points I have to find out that area here now it is disc not a sphere, the sphere what do we do, we found out the volume between that shell and then divided by the volume occupied by 1 state. Not it is the area is a disc therefore area between kt and kt+dkt.

What is the area ¼*2phykt*dkt 2phyr the area of that, so that real of this is 2phyr yeah, so 2phykt dkt ¼ of it and of course we need a factor 2 to take care of the electron spin this is that in terms of area. Now we need to know the number of points which means what is the area occupied by 1 point in this case what do you think is the area occupied by 1 point please see corresponding to every value here and every value here there is a point.

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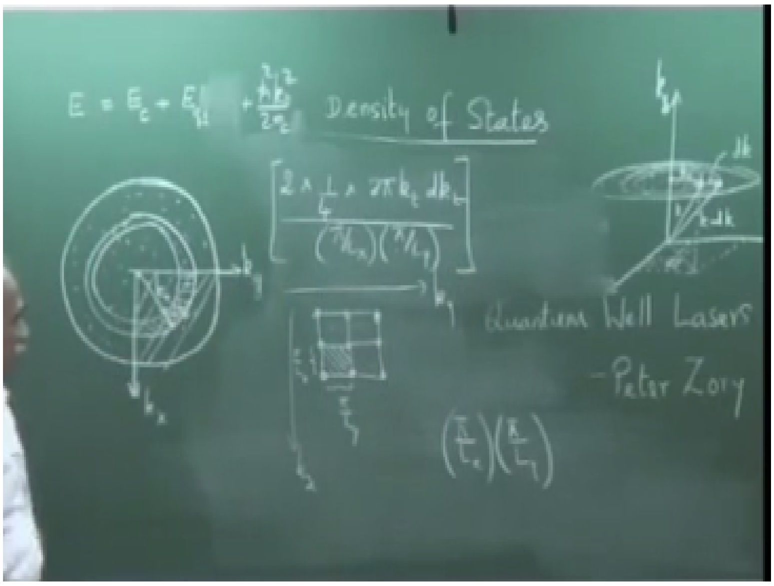

The next value here there is a point, next value here there is point, so what do you have you have a square actually rectangle if Lx is not equal to Ly it is a rectangle. So you have a rectangle with 4 points at the corner, but the every point is shared by four other rectangles. So if I enlarge the view we have a situation so we have points, so this is x direction, this is so this is kx direction and this is k in the k space.

These points are here where very closely top point, I just had 2 points, but actually it you will see that the points are sitting like this and what is this separation phy/Lx, what is this separation phy/Ly. So the area of a rectangle here is phy/Lx*phy/Ly. In the k space area of this rectangle where points represent k state allowed states, but you see that every point is surrounded by 4 separate ends which means number of times per rectangle is 1, 1/4 1/4 it has 1/4 to this.

And 1/4 to this and 1/4 to this like this therefore the number of points per rectangular are area per point is phy/Lx/phy/L1 area corresponding to 1 allowed state is this much in the k space. The area in the first quadrant here is this divided by area corresponding to one state will give me number of states, is this clear, the area in the k space divided by area occupied by one state will give me number of states.

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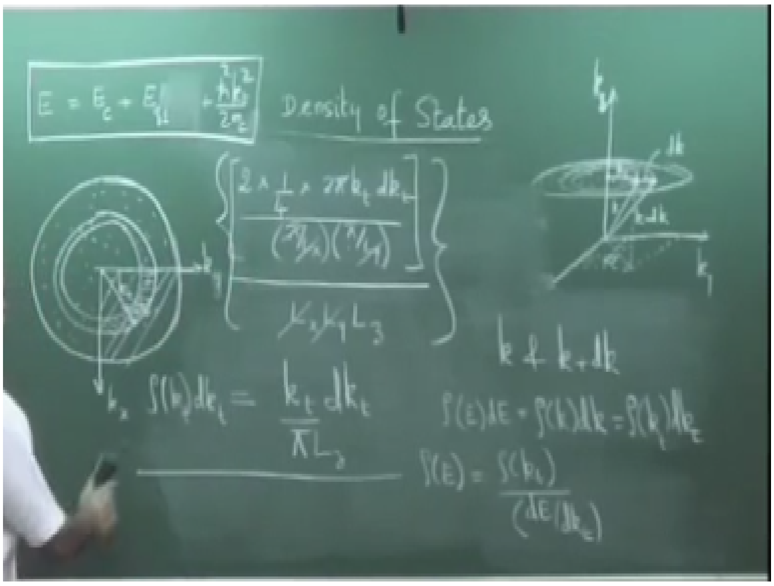

So this divided by phy/Lx*phy/L1. This is the number of points between k and k+dk. This per unit volume of the material will give me density of states. So this what is the volume of the material Lx*Ly*Lz. So this is the density of states. So we see it is just in one line for one expression the entire derivation is there, 2 standing for this thing one fourth of the quadrant of area 2phykt dkt/area occupied by one state will give me the numerator gives macro environment number of states between kt and dkt or ktk+row+k+dk between k and k+dk.

This is the number of points This is the number of points per unit volume that is the definition. So number of states per unit volume, so simply these what will you get just see what do you get, so Lx, Ly, Lz cancels and lz remains. So row of kt dkt+row of k dk=kt dkt/phyLz. We see this we can cancel these row one phy goes here, so one phy remains in the denominator Lz remains in the denominator here lz for unit volume is that ok phy/Lx/lz.

So in the k space we have got the density of states. Now as I indicated in the last class would like to know the density of state in the E space and how do we go for E space, we use the expression relation between kt and E and use the fact that row of E dE+row of k dk=row of kt dkt and therefore row of E and then we need the density of states in the E space, so row of E=row of kt/dE/dk are used this equal to this.

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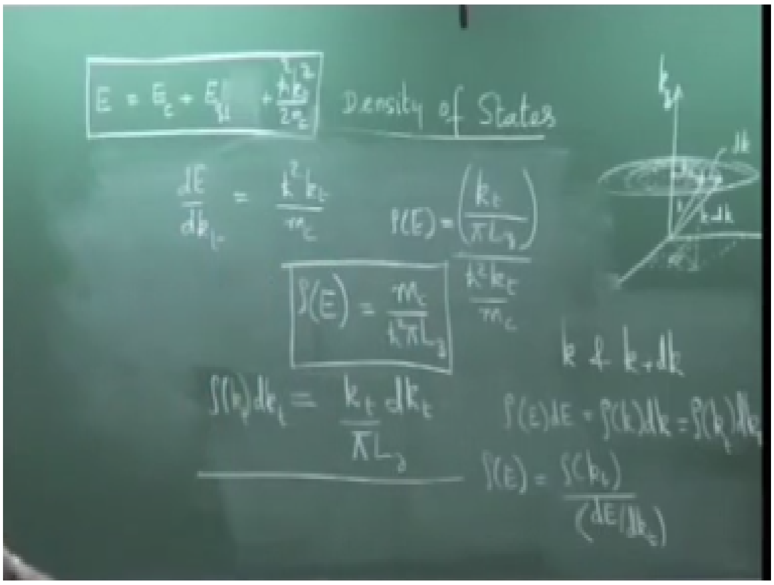

And we have the expression for row of kt already, so if I am erase these. So if you have a clear picture in one expression then entire derivation of the density of states is present. So dE/dkt that we do is simple algebra completely. So de/dkt, so this is the constant, this is the constant for that plane, this is the constant. Therefore is simply we have h cross square*kt/mc. 2, 2 cancels h cross square kt/mc. This is dE/dkt. So row of E therefore is equal to row of E=row of kt which is kt/phy*Lz.

That divided by de/dkt/h cross square kt row of kt/dE/dkt h cross square kt/mc. So mc goes to the top, kt kt cancels, so we have row of E, row of E=quantum well row of E=mc/h cross square phy*Lz. Where Lz is the thickness of the quantum well.

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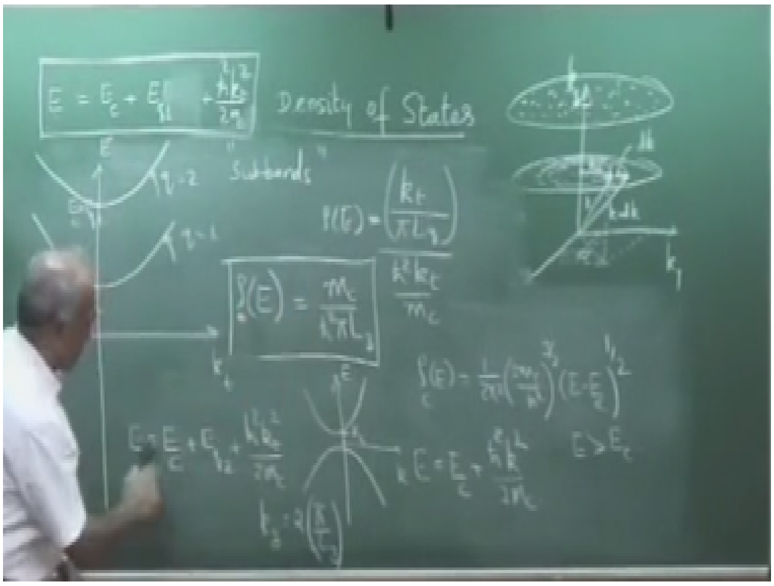

Something interesting that you see here that the density of states is independent of the energy right hand side there is no dependence on energy, recall the density of state in a bulk, row c of E this is for the conduction band actually therefore this is also will see row c of E=1/2phy square 2mc/h cross square to the power 3/2*E-Ec to the power of half conduction band which means E for E greater than this.

There is a energy dependence or there is no energy on the right hand side which means the density of state is constant first point, ok. Now let us come to some additional discussion about this energy, let us see what is this, this tells us that we call that E-k diagram, how did we have the E-k diagram, we had E, E varying parobollically, so E here, so E was equal to Ec+h cross square k square/2mc.

Right from at k=0, so this is k here E=Ec, so this is Ec, what are these, these are are simply nothing more points of allowed states here. So you have large number of allowed states for k greater than 0, it is varying parabolic. In this case you plot the density of states the E-k diagram kt versus E up to Ec you have no states up to kc if you put kt=0 I have Ec+Eq1.

E=what is the E, E is the allowed energy value of electron. So this is Ec, the next level here is Ec+Eq1, beyond this as kt almost takes continuous value it varies parabolic in other words the band varies like this parobolically ok for q=1, we have now plotted with respect to kt not the total k with respect to kt.

But what you need to see is earlier we had allowed states right from Ec, now there are most state above Ec up to Ec+eq1 and your band starts from here. So this is the parabolic variation of the band. What about when q=2 I have energy E=Ec+Eq2+h cross square kt square/2mc, if I put q=2 which means my kz is now 2 times phy/Lz originally for Q=1 kz was phy/Lz. Now q=2 two times phy/Lz.

That value is suddenly jump here, so this is Ec+Eq2 and Ec=Eq2 I again have a parabolic variation. So we have the next sub band these are called energy sub bands, sub bands, the E-k diagram in a bulk has a parabolic variation here. In the case of a quantum well you have energy sub bands, these are bands because almost continuum kt takes almost continuous values.

Because kx and ky take very large number of values, but kz has quantum jumps because phy/Lz is a very large number because Lz is a very small number. So it just comes here for one value of k, for this value of k you have large number of kx, ly, kz is permitted, next it goes to the next level and again large number of kx, ky value, that is why in the case of quantum wells the energy band E=k diagram is characterised by sub bands.

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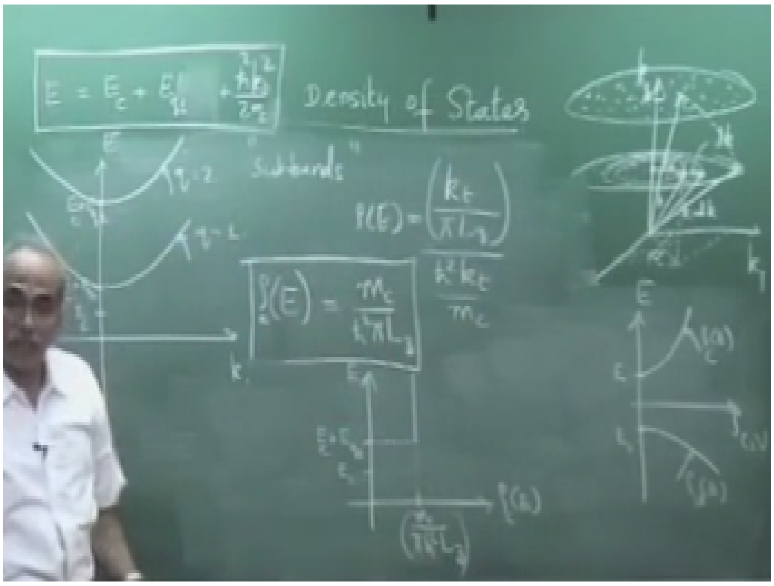

And corresponding to each sub band you have a density of states. Now why did I bring this here because if you see this, this was for q=1 and q=2 I have a second disc. Now a second disc is available very large number of points are present, if I change k energy changes, E changes. If I have to plot this what this how to make look like when you first plot this row of E, you you remember for the bulk I keep it here.

This is E and this is row, row of c,v we had Ec, E-Ec square root dependency here. So this was row c of Environment We had shown this in the last class, so Ec and similarly for Ev we had density of state varying like this where this variation is row v of E, you want to plot the corresponding variation here the density of states. Let me show first only the conduction band at E=Ec we have no state.

Because the first value of k starts E=Eq1, So Eq1 that is Ec+Eq1. So this is Ec+Eq1, Eq1 is only that components h cross square z square/2m. So Ec+Eq1 is the total energy at this value what is on this axis row c of E, so up to that know the density of states. When we can here there is density of states and can you tell me what is the value here mc/phy h cross square Lz. The density of states is constant this is the value.

What does that mean that means beyond E=Ec+Eq1 we have density of state, but density of state is const density of state continuously increases with energy here, density of state is constant repeatedly I keep telling that why am I density of states so much, many can think where after density of states for the last 3 class let me assure that it is the density of states which will determine the device characteristics and the device performance.

This is very important to understand the density of states. So it is constantly here, what happens when energy and energy exceeds Ec+Eq2 we have a second fed back equivalently we have a second disc. So we have another amount of area that is if my k value becomes large, so let me show this k like this, this length is large if k is large which means energy is large the tip of the vector can either lie in with this disc or it can also lie if we length the plot it can also lie here.

So when the energy is large you have the second disc available which means the density of state si doubles, the density of state per disc is what we have calculated here mc/phy h cross square*Lz. When you have a second disc available the density of states doubles and therefore the density of states here for E as E increases that we continue on the same diagram t increases to Ec+Eq2.

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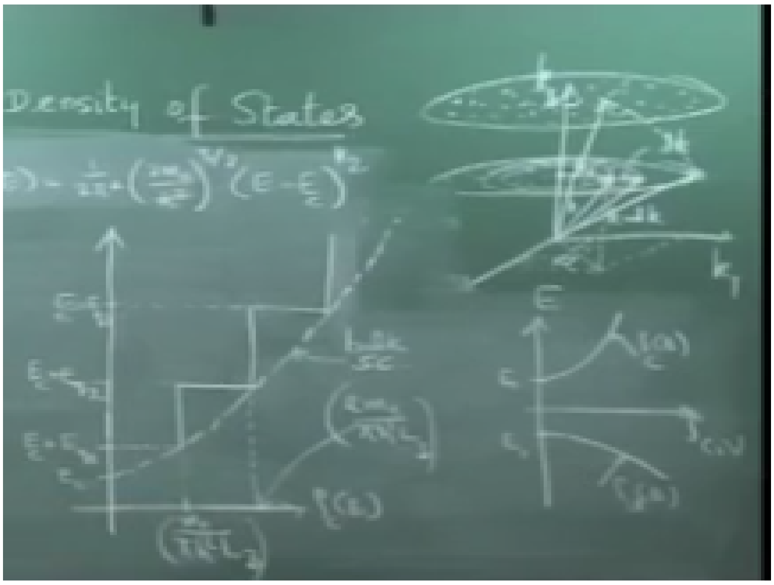

We have a second disc available and the density of state simply jumps to this value, what is this value here 2 times mc/phy h cross square Lz. So this value here is 2mc/phy h cross square Lz. When you reach you can extend this Ec+Eq3, so Ec+Eq3 let say there is third sub band available here which means that there is a third disc available and you have the density of states making the third jump.

What you have seen is a step like behaviour in the density of states, it so happened that if you draw the density of states my diagrams not very good. So what I have drawn here is the density of state corresponding to the bulk, bulk semiconductor this, if you draw the density of states corresponding to the bulk semiconductor you will get this step kind of variation. I need this an exercise to you to show that indeed in the bulk.

If you plot the density of state to quantum well and bulk corresponding to E=Ec+Eq1, the density of states that you get for the bulk is the same as this one and if you put Ec+Eq2 in the expression for this you will get 2 times mc/phy h cross square Lz, you just verify this which expression the expression for bulk where we will have de3nsity of states row of E=1/2phy square*2 mc/h cross square*3/2*E-Ec to the power half.

If you substitute for E this value here you will get this result as this, if you substitute for E this value Ec+Eq2 means Ec+4phy square h cross square, so what is Eq2, Eq2 is h cross square kz square/2mc. So Eq1=h cross squarephy square/Lz square/2mc physquere/Lz square/2mc. If you put Eq2 it will be 4 times because q=2. So q square will become 4, so 4 trims this.

If you substitute that value here you will get this number that is why the density of states is shown like that. Otherwise how do you know that this is touching this and you substitute and see it indeed touches that value. So what we have seen is the density of states as a step like variation. We will see its implication later when we go to the device. My discussion have been focused only on the conduction band.

Exactly similar discussion is applicable for the valence band and you will see the same stop like behaviour in the valance band. So let me give the final results for the density of states in a quantum well because the energy expression that I have written is for energy in the conduction band exactly similar discussions in the quantum in the valance band will give you similar results there.

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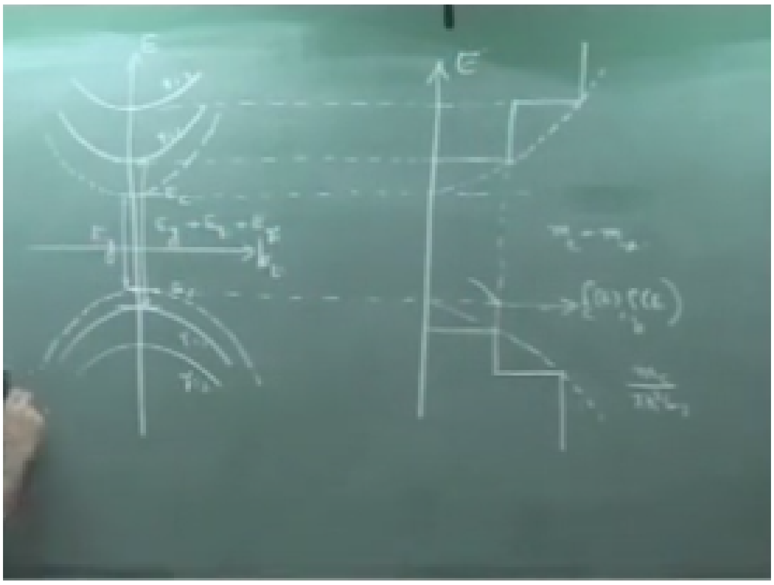

I want to write the results, so that E-k diagram or a bulk material for a quantum well, so this is q-=1q-=2, q- is the corresponding number for the valence band q=1, q=2, this E-k diagram, so E versus kt and here Ec and I have drawn this by dotted line because if you make a quantum well structure the first allowed state starts from here and immediate implication is the band gap of a semiconductor is Ec-EV, this is Eg.

If you make a quantum well structure the first allowed state is here and the first allowed whole state the lowest energy hole is here, but the lowest by effective band is this that is original Eg+Eq1+Eq-1, this is Eq-so that you see is by changing the structure into a quantum well structure you have effectively change the band here, you will discuss more discuss about this later when we discuss about band gap engineering and we will discuss more about this.

But this is the corresponding E-k diagram and you have a corresponding density of states diagram everywhere vertical axis is E, so the density of states for bulk and corresponding to, so we have this similar density of states, so this is row c of E and row v of Environment So the dotted line corresponds to bulk and solid line corresponds to the quantum well structure and I have assumed in showing that this point is the same as this point I have resumed that mc=mv.

Because recall that this value here was mc/phyh cross square*Lz. If I am showing this the same it means I have resumed mc=mv. If mc is not equal to mc both the curvature here and the first value corresponding to the density of states would be different in the valence band which is in general to in general where the density of states and the E-k diagram variation is different in the valance.

But if the effective masses are the same then they come at the same value, this si alright , we will discuss more about the quantum well and how to make etc. at a later stage. This is the E-k diagram and this is E versus the density of states, why do we need density of states, let us answer this questions why do we need density of states because as I already said that we want to know the carrier concentration. The carrier concentration is determined by the available states multiplied by the probability of occupation.

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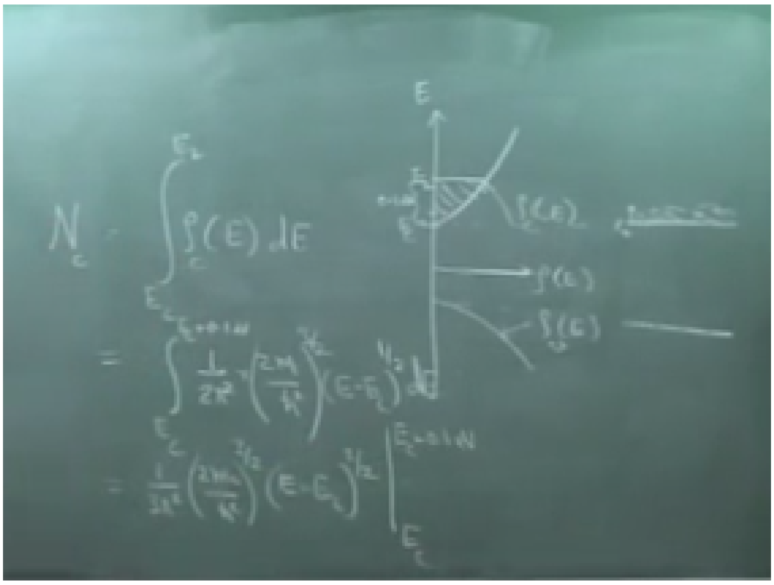

$$N_c = \int_{E_c}^{E_2} \rho_c(E)\,dE = \int_{E_c}^{E_c+0.1eV} \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}(E-E_c)^{1/2}\,dE = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(E-E_c)^{1/2}\Bigg|_{E_c}^{E_c+0.1eV}$$

How do we calculate the number of available states. The number of available states and is equal to row c of E number of available state in the conduction band is equal to row c of Ed integrate over Ec to some value, let us stay up to some value we need the number of stay up to a E2, what is this E2, so E2 for plotting again this is E, this is row of E, row c of E and row v of E, this tells me the density of states.

Therefore I want to calculate the actual number of states. So the number of states this is Ec and I want to know up to some energy E2, how many states are present here, that is the number of states is row c of E dE from Ec to E2. If you want to calculate the density of states in entire band, so you write from Ec up to the top of the band. So if you want to see up to some value E2 substitute here.

And let us see what do we get, so this si equal to Ec. For example E2=let us say this height is 0.1Ev let us put some numbers here, 0.1Ev I want to calculate from the your familiar old band diagram with does not tell anything except energy gap, so you want to calculate the density of state the number of states available here from Ec this bottom up to some I, how many states are available here.

So I can calculate only through the density of states. So now we are coming to number let us see from Ec up to this which means I calculate it from Ec+0.1Ev substitute the values very quick 1/2phy square*twice mc/ h cross square to the power 3/2*e-Ec to the power ½ dE. Integrate this what you get this is constant, this is constant E-Ec to the power ½ integrate you get 3/2 and a factor 2/3 and , 2 cancels.

So you are left we 1/3phy square*2mc/h cross square to the power 3/2*E-Ec to the power 3/2 from Ec to Ec+0.1Ev. If you put E=Ecit is 0, so it is simply Ec+0.1Ev which means substitute the value 1/3 physquare*2mc/h cross square to the power 3/2*E-Ec is simply 0.1 Ev to the power 3/2. So this si equal to 1/3*phy square substitute value for phy*2*mc let us say gallium arsenide mc=0.67 times m0 where m0=9.1*10 to the power of -31 kg.

So substitute mc=0.067*9.1*10 to the power of now we are like engineers calculate base divided by h cross square h cross square is at how much is h cross 1.01010505 alright so 1.05*10 to the power of -34, h cross this is H cross, so h cross square and the whole to the power 3/2*0.1 EV it is in electron volts whereas all the others are si unit. So you have to convert this into joules.

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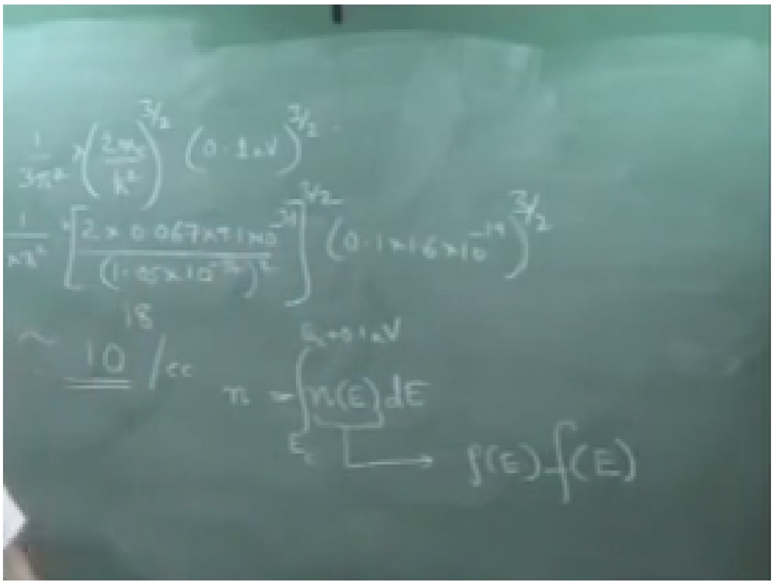

$$\frac{1}{3\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(0.1eV)^{3/2}$$

$$\frac{1}{3\pi^2}\left[\frac{2\times 0.067\times 9.1\times 10^{-31}}{(1.05\times 10^{-34})^2}\right]^{3/2}(0.1\times 1.6\times 10^{-19})^{3/2}$$

$$\sim \underline{\underline{10^{18}}}/cc$$

$$n = \int_{E_c}^{E_c+0.1eV} \underbrace{n(E)}\,dE \longrightarrow \rho(E)f(E)$$

So it is 0.1*1.6*10 to the power of -19 to the power 3/2 simplify this and find out what is the answer I think this will be of the order of 10to the power of 18, the number is the order of 10 top the power of 18/cc. So we have found out how many states are available from the bottom of the conduction band up to some height 0.1 Ev. This is available states, but if you want to know the carrier concentration.

Then we have to have N of E dE I want to get the carrier concentration n then it is n of Edv to integrate again from Ec to Ec+0.1Ev, but what is n of E, density of carriers, carrier density. Carrier density is given row of E density of states multiplied by the probability of occupation. The probability of occupation is given by the Fermi function, density of states multiply by the probability.

So now to know the carrier concentration we need probability of occupation, occupation probability. This will be our next topic is occupation probability. If you substitute the occupation probability now define the density of state and you integrate you will get carrier concentration. Now what will get the numbers, so once the basic physics and the picture is clear then calculating numbers is not a problem at all ok, so our next topic will be probability of occupation, so we stop here.