Semiconductor Optoelectronics
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Lecture - 23
Gain and Absorption Spectrum of Quantum Well Structures

In the last couple of classes, we discussed about amplification in a semiconductor and also the absorption spectrum. He discussed these issues with respect to bulk semiconductors and today we will see the absorption spectrum and gain spectrum in a quantum well structure. So, recall that we have an expression for expression for gain coefficient gamma.

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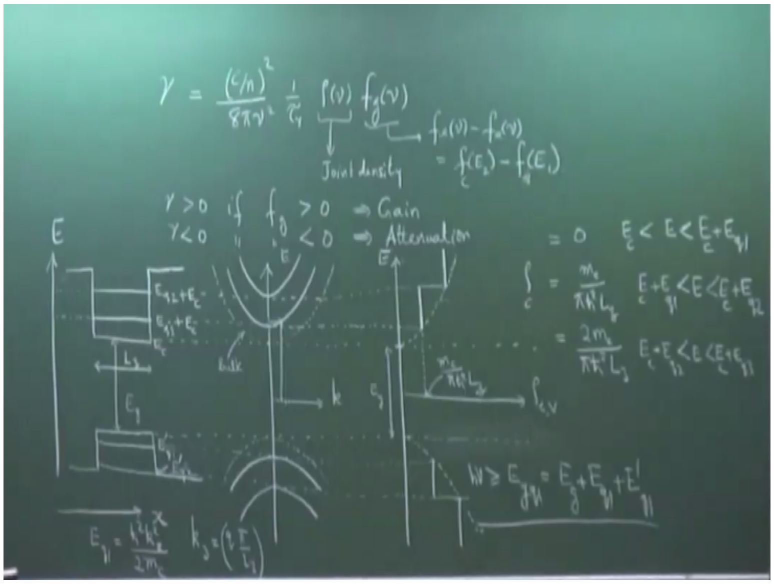

Gamma = c/n square/8 p I nu square 1/tau r * rho of nu * fg of nu. This is the Fermi inversion factor which is essentially fe - fe(nu) - fa(nu) probability of emission - probability of absorption which is also equal to f(e) to fc of it is in quasi equilibrium then it is c(e2) - fcfe of e1. This is the optical joint density of states rho nu so joint density of states. This is the radioactive recombination life time and this c/nu can also be written as lambda.

But then you have to remember that that is wavelength in the medium, lambda in the medium. c/n/nu is nothing but wavelength lambda so lambda square so I prefer to write it in this fashion. So this is the gain coefficient. This is gamma greater than 0 if g(nu) is > 0 and then we have gain implies gain and gamma < 0 if fg(nu) is < 0 and we have attenuation or loss. It is the joint density of states which determine the gain profile gamma of nu is primarily determined by rho of nu.

So today we will see the gain and loss in a quantum well structure. So recall that in a quantum well structure so let me draw the quantum well structure here. I will draw three figures so you will need space here. This is I would like you to draw three parallel figures. So this is energy axis E versus x. this is Ec of the low band gap material and this is ev so this is eg1 so this is eg of the low band gap material and corresponding to this.

I would like to draw the corresponding EK diagram and the density of states that we have already discussed but let me draw this. so the EK diagram, if I plot the ek diagram this is E versus k, then you know that E = ec we do not have any states there and therefore in a quantum well if this quantum well supports energy E1 here and so Eq1, Eq = 1, so Eq1 let me call this as Eq1 and Eq2. Similarly, for the holes we have states here Eq - 1 an Eq - 2, Eq - the - is just to say that it is basically q = 1 and q = 2 the – that.

I have written is to distinguish between so let me write E - q1 and E - q2 that is for the holes. Two states here and two states are there. I have taken a quantum well which supports two states. So the corresponding EK diagram if you plot you have energy sub bands and the sub bands look like this. They have the same K square kind of dependence, but only the transverse k KT and there is no allowed state at EC the earlier EC so this was EC.

And this was Ev similarly for the valence band we have if MC is not = MV the shape would be slightly different. So these are the energy sub bands. This is K axis. Fo different what it tells is for a given value of K the allowed value of V gives you the EK diagram. So this is the quantum well. We have also got the density of states if I had taken the bulk material this material which was bulk then I would have had EK diagram going like this.

Please remember right from EC so this is for the bulk. The dotted line is showing for the bulk. So let me write here bulk and these are for the energy sub bands for the quantum well structure. We also want to plot e versus the density of states so tau p, v just recalling what we have already studied the density of states. The density of states in the case of bulk had a dependence which is e to the power half dependence this is for the bulk and in the case of quantum well it started the density of states will start at Eq1 this value here so this is Eq1 actually Ec + Eq1.

Please remember that the total energy here is Eq1 + Ec please add this Ec here because the way we have defined is Eq1 so let me write here Eq1 was h cross square, kt square here k square/2 mc. So this is kz square which means that it is only above Ec therefore if I am considering the total energy here it is Ec + Eq1. This is Ec + Eq2. So this kz just reminding you that this kz was q times pi/lz q * pi by lz. If you put q = 1 it is kq1 z and the corresponding energy, we called as Eq1.

Just to remind you that this was kz in this expression. So the density of states they also derive the density of states up to these there is no density of states, but at Eq1 there is a constant density of states here and then again we do not have any states constant density. There is a sudden jump from here to here. This is energy axis. So this value is Ec. This is Ec + Eq1. Exactly like that here also at ev we do not have any density of states, but we have density of states at this value here. Please see this value here is the density of states.

This axis is now density of states. So density of states is constant. This is constant for all energy values here up to Ec + Eq1 we have no density of states which means there are no states at Ec + Eq1 we have suddenly density of states here and that density of states is remaining constant, because this is the axis, density of states. For all energies up to Ec + Eq2. So that is this value Ec + Eq2 and then suddenly there is a jump which is double.

If you recall that this number here was mc/pi h cross square * lz, mc/pi h cross square * Lz. Lz is the dimension of this. So this dimension I had taken as Lz. In the z direction width in the z direction and this will be double of that two times mc so if I write Eq rho c = mc/pi h cross square z up to Ec + Eq1 beyond Ec + Eq1 up to Ec + Eq2 up to this it is constant and from here it suddenly jumps to so this is for Ec plus Eq1 < E < Ec + Eq2.

I have come to the end of the board Ec + Eq2 and this is equal to 0 up to this value = 0 for between Ec and Ec < Ec for all energy values Ec + Eq which was not the case earlier in a bulk and this is equal to two times it doubles two times Mc. Recall the discussions that we had with two discs when we get a second disc and for Ec + Eq2 onwards for E > Ec plus E. Let me write that otherwise Ec + Eq2 < E < Ec plus Eq3 and so on and similarly rho v will be mc is replaced by mv that is all.

So the point is you have a step function. Density of states is a step function. Same thing is true here. Density of states is a step function. So from here you have the next step. So photons this is Eg. In this diagram also you can see that this is Eg. In the case of quantum well photons of energy = Eg energy h nu = Eg. It does not have any state to interact. The first the lowest energy photon would correspond to this difference which I call as Eg q1. Eg q1 is Ec + Eq1 + E dash q1.

This is the first photon which can interact which means Eg q1 = Eg + Eq1. This is in the conduction band + E dash q1 in the valence band. So interaction can take place for photons of energy h nu >= Egq1. This is the first point to remember. You can find out the corresponding optical joint density of states. Keep this picture in mind. Let me erase this. The optical joint density of states can be found out exactly like what we have done earlier.

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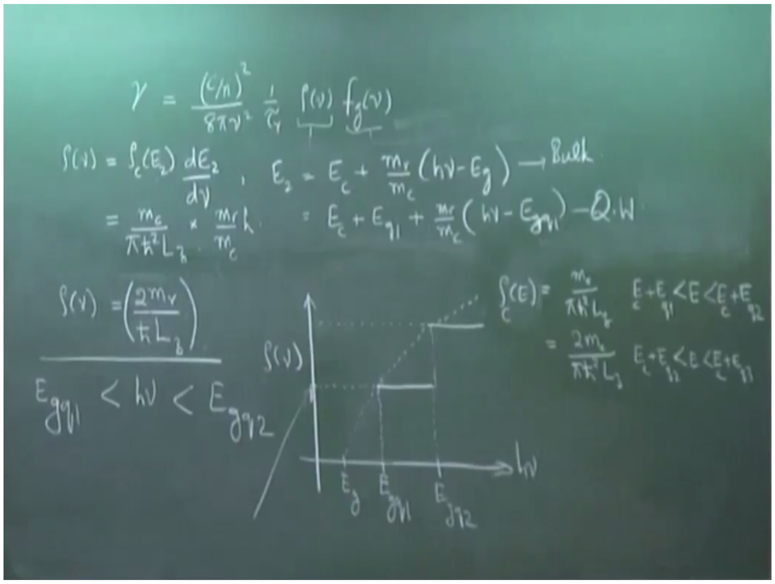

If you recall that we had written rho nu, d nu = rho c of E2 into dE2 therefore rho nu = dE2/d nu. E2 we had an expression for E2, E2 in terms of nu was Ec + mr/mc * h nu - Eg. This for bulk. Now E2 is an energy level in the conduction band which is Ec + Eg1 and then this difference. So this will be equal to so this is for bulk. If you work out exactly similarly you will get this equal to Eq1 + mr/mc * h nu - Eg q1. Let me erase a little bit here. This is for quantum well.

Therefore, dE2/d nu will be of course these are constants will be as before mr/mc * h. DE2/D nu is mr/mc * h all others are constants. Mr/mc * h. therefore you substitute here. Rho c of E2 so this is rho c of E here because I am finding of nu. Rho c of E was this expression. So you substitute for the first one so this is mc/pi h cross square into Lz * DE2/d nu which is into mr/mc * h that is all. So rho nu is again constant just like here it was constant it is again constant.

Rho nu = mc mc goes here so you have mr one h should go here/2 pi 2 will come up so 2/pi. Let me keep pi h cross. So one h cross let me keep here. One h goes with this. The 2 pi is up so 2 is there so pi is there. So this is h cross into L sector that is also which is a constant which is independent of nu so for h nu Eg q1< h nu < Eg q2. What is Eg q1. Eg q1 is this difference. Egq1 band gap from q = 1 here to q =1. Egq2 is this difference.

So the density of states are constant here and you get the same thing density of states is constant. So if I plot this density of states that is optical joint density of states how would it look like. The optical joint density of states in a quantum well so I plot rho of nu for a quantum well structure Eg is here. Eg q1, Egq2. So this is h nu and rho of nu. For the bulk h to the power of half dependence so we have a constant here so the density of states is constant up to Egq2 and then jumps up again remains constant up to Eg q3.

The value here this value here is 2 mr * h cross Lz. So this value here is 2 mr h cross Lz and what would this be two times 2 mr h cross Lz double the density of state simply doubles. So the optical joint density of states in the case of bulk was varying like this. In the case of a quantum well it varies like a step function. This is important because we want to get the gain coefficient and loss coefficient.

Because the profile, the gain profile and the absorption spectrum is determined by rho of nu. This is just the derivation which is just an extension of the derivation for density of states. So let us now find the gain spectrum. So let me draw it here.

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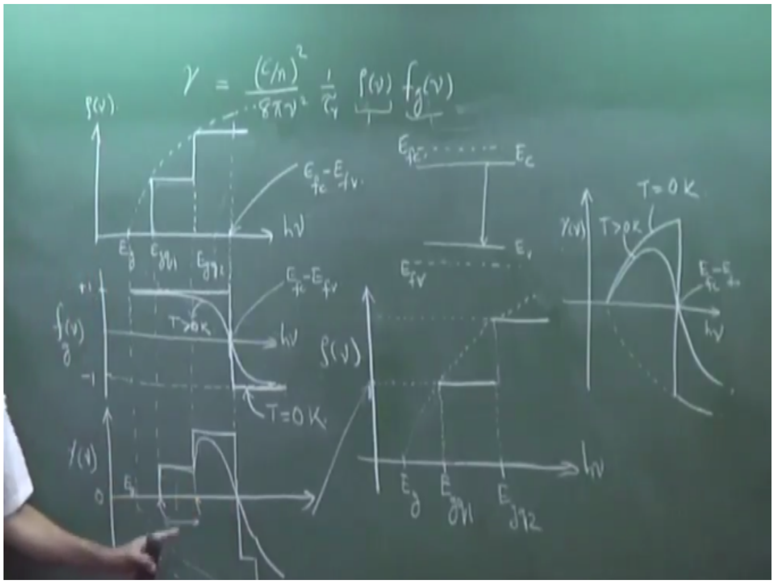

Let me draw a fresh. I want to draw in the same line three curves one below another now so first one is density of states so the bulk is here. So this is rho of nu versus h nu, the variation for a quantum well structure. This is Eg, this is Eg q1, Eg q2. I want to plot also Fg of nu. Let us recall the thought experiment that we had done at 0k. At 0k up to Efc - Efv > 0 up to an energy which is equal to let us say we are here. This is h nu, this is Eg and this is Efc - Efv recall what we had done Efc - Efv that is the separation between the Fermi levels.

I am now looking at the semiconductor which is in quasi equilibrium. This is Ev, this is Ec and Efc has entered the band, Efv has entered the band so this is Eg and this is Efc - Efv. What we had seen was for all h nu between this and this if I take 0k experimented 0k then Fg of nu was + 1 and for all others it is - 1. If it exceeds it is - 1 which means if I plot Fg of nu here, then it is this is + 1 and this is - 1. So it remained + 1.

This is experimented 0k up to this and then it went down to -1. The step function, this is Efc - Efk, Fg of nu. The probability of emission - probability of absorption. The Fermi inversion factor was + 1 up to this and - 1 up to this and accordingly if you recall that the gain if I had the EFc -

Efv was here. Efc - Efv. I could show it right here on the same over this or let me on the same line let me show this here because all the three figures which I will get one below another so this is Efc - Efv.

So the product of rho nu * Fg of nu. For the bulk we had a gain profile which went up to this and then it short down so I will draw it here just to remind you how it was for the bulk so the gain profile went like this. It followed exactly rho of nu and then it came down because beyond this value we had h nu versus gain so gamma of nu and at 0k it did this. So gain it was positive, negative beyond Efc - Ef. This was for T = 0.

At finite temperature this changed to this why did this change because Fg of nu was no more a step function. Fg of nu was varying like this that is why this change over to this. So this is for T = 0k and this is for T > 0 at a finite temperature. That is for the bulk. I want to draw the same thing for quantum well structure. So if I want to plot the same thing for quantum well structure, let me plot it here below so this is rho of nu I want to plot gamma of nu. The third figure.

So how do we go about this. First let me plot for 0k so it is + 1 or - 1. So the gain will be this into this the density of state starts only here up here therefore the gain will also start from there and the gain will remain constant like this. Same as this, this multiplied by this. This multiplied by this two functions. So first function rho of nu is varying like this and Fg of nu is also varying like this so I have.

Let me draw this fully why to draw because there are so many dash lines that you might have confusions. So let me draw it fully. So that is Fg of nu. So the product of these two is constant. When you come at this value Egq2 there is a jump so the gain will also just become double because of the density of states become double. At this value here so this has gone up to this, but the next Egq3 has not come, but at this value we have Fg of nu becoming negative - 1.

This is - 1 here and therefore - into this so we have the gain dropping down like this and this will become like this. So we have gain profile which is a step function at 0k. So what I have plotted is the gain profile. Please remember at Eg there is no gain. This value is Eg q1 and this value is

Egq2. So what is the point. Wavelengths are frequencies which correspond to within this band from here to here the gain is constant flat, it is a flat gain.

Similarly, for frequencies from here to here gain is flat, but a different value. It is a step function. This is at 0k. If I go for a finite temperature Fg of nu will get modified so this will get modified and this will cross here and go in this fashion. So this is Fg of nu for T > 0k the step function was for 0K so this was for T = 0k. Fg of nu does this just as before. Therefore, the gain here would also it is a product of these two.

This remains constant, but this is a product and therefore if you multiply initially it will remain the same, but slowly the difference will start coming up so it is separating out it is coming down here there is a step here no doubt, but it is separating out and then when this becomes 0 the product is 0 so this will pass through like this till the next step. There will be a next step here and you will have a step here and then a second step which is coming.

But that is the absorption part. Gamma is positive. Gamma is 0, gamma is gain and this is loss. This loss part we will discuss separately. So now you will see that in a practical amplifier, quantum well amplifier the gain will have a spectrum like this. You can see in the literature let me just open it up a little bit. You can see actual calculated and measured gain profiles in the literature and you will see them in deed there are step like variations. So let me widen these range and show you how impact is the gain of a quantum well amplifier look likes.

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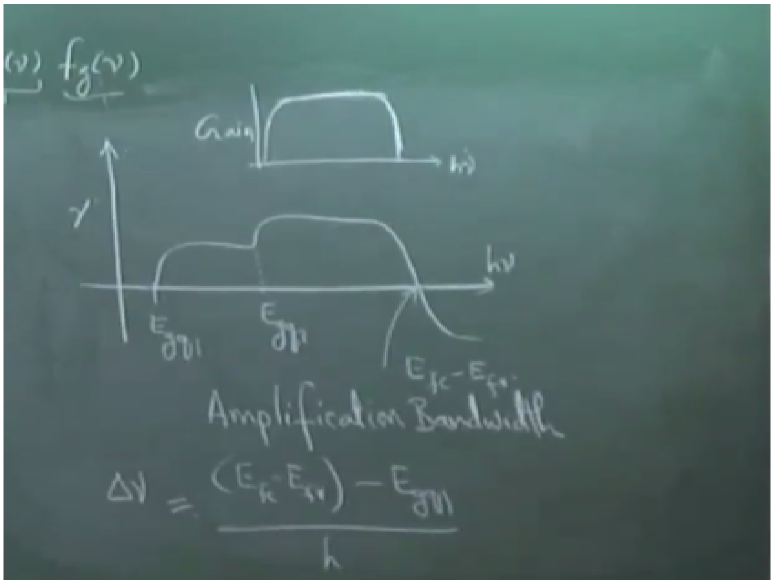

So typically it looks like so what I have plotted is gain coefficient versus energy. so this is the value where gain starts at Egq1 please remember Egq1 not Eg and this is the jump here is at Egq2 and obviously this cross over point is always Efc - Efv which determines the amplification band width. The amplification band width in this case is always Efc to Egq1. In the case of a bulk amplifier it would be Efc to Eg, Eg to EFc is the amplification band width.

I am not calling it as a gain band width because gain band width is normally defined at the frequencies at which the gain falls to 3DB that is falls to half. So this I am calling as amplification band width because for all frequencies in this range you have gain. So we call it as amplification band width. In the last class when we had written the expression it was for amplification band width. So in this case, amplification band width is determined by Efc - Efv - Egq1/h.

So this is the amplification band width delta. H delta nu is the energy difference so therefore h delta nu the band width is given by this for a quantum well structure. If you design properly it is possible to get this cross over right here Egq2 because I need to pump only that much so that the separation between Efc - Efv is here and you will have a nice amplifier like this almost flat it is almost like a box type is an ideal amplifier characteristic is you want the gain to be flat like a box.

So if you properly design and shift this here of course the range is small but you will have. So this is gain versus frequency or energy a flat band amplifier quantum well amplifier we will discuss more about these quantums well in a later stage. Let me now come to attenuate loss spectrum. Identical discussion but you see only difference when we discuss about attenuation and gain is remembered that the gain band width amplification band width is very small n.

The frequency ranges about 10 to the power of 13 Hertz whereas when we discussed about loss we are discussing over a very wide spectrum. We discussed the gain spectrum over a loss spectrum over wide energy range because if you take silicone detector for example nu it can detect anywhere from 0.4 micrometer that is visible down to 1.1 micrometer.

So it is a large energy h nu range whereas the gain band width is much smaller compared to the absorption range and therefore what is the difference we have to also include the effect of this nu in discussion of the gain we take this as constant because the range is very narrow therefore we say that okay over the range of interest 1/nu square variation is neglected, but in the case of loss spectrum you would like to heat the 1/nu square dependence.

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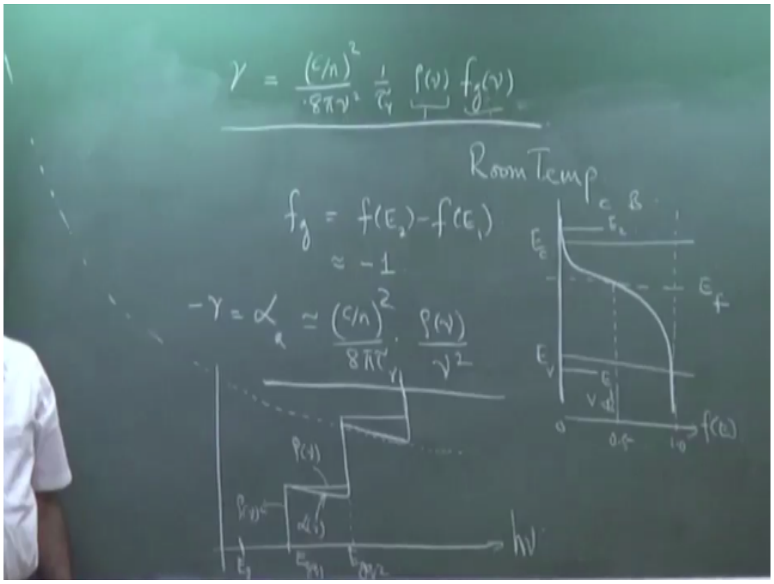

I am still keeping the same expression here. If Fg of nu is negative, let us take for example room temperature a normal semiconductor at thermal equilibrium how would it have the so this is Ev, this is Ec and the Ef is somewhere where you want a normal Ef is somewhere here which means if you plot the Fermi distribution so what I have plotted on this axis is also F of e thermal equilibrium normal semiconductors absorption spectrum I want to determine.

So this is Ef and this is F of e which is 1.0 here and 0 here and how do we determine the Fermi energy here it is 0.5 and therefore Fg of nu here or Fg of E so = F of e2 - F of e1, F of fg = F of e2 -, e2 is any energy value here. So this is e2 and E1 is some value here in the valence band. So this is the valence band, this is the conduction band. F of e2 is a very small value here if you put actual numbers.

I have already mentioned it will be very, very small 0.0001 or something like that so very small value and F of e1 is very, very close to 1 0.999 which means this is 0 - 0.999 so it is approximately equal to - 1 approximately a little less than - 1, but approximately = - 1. So I can safely write alpha - gamma = alpha a = is approximately = to c/n whole square 8 pi * tau r * rho of nu/nu square. Fg of nu I have written as - 1.

So - sign was there and that is the attenuation coefficient. For a quantum well structure rho nu is a constant. It is a step function up to certain energy. So what kind of attenuation spectrum would you observe so let me plot the attenuation spectrum. What is expected so this is h nu 1/nu square if monotonically dropping function 1/x square dependence that we are coming from a long range so this is continuously this is actually going like this and going down to 0.

So in our optical region this is continuously dropping like this 1/nu square. What is rho nu doing? Rho nu is a step function. Rho nu is constant it is a staircase like function, so this is rho of nu. Where does it start this is Eg q1, Eg is somewhere here and this is Egq2. Remember again what is Egq1 is the difference between the first sub band in the conduction band to the first sub band in the valence band and Egq2 is the difference in the second to the second.

So this is Eg q2 and so on. So the product this is a very small variation, but nevertheless if I multiply this and this what you expect is a function which is the product expected. So this is the upper one is rho of nu and the lower one is alpha of nu into some constants are there, but the variation I am showing only the variation. So this is alpha of nu. The important point you see in a quantum well structure is a step-like variation. If I now forget all these expressions and compare the attenuation spectrum of bulk and quantum well structures what do we expect and where is the difference.

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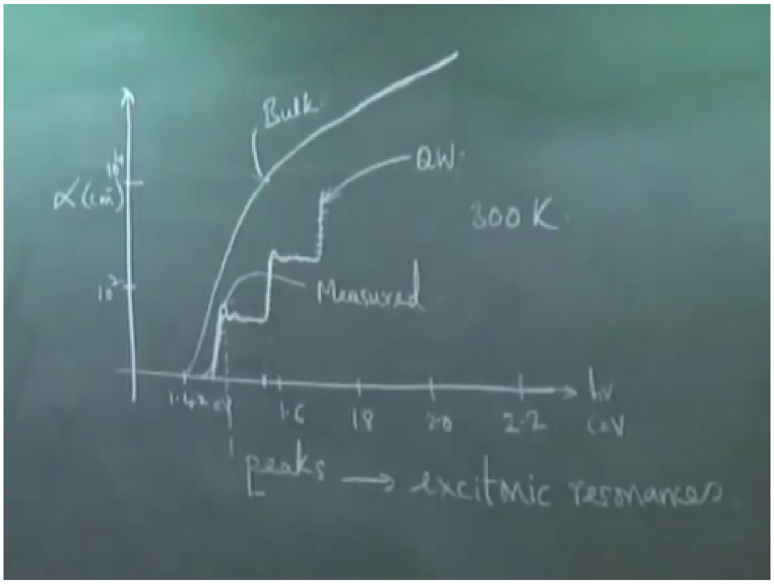

A bulk semiconductor let me take a direct band gap semiconductor like gallium arsenide so this is h nu and this is Eg 1.42 ev that is Eg and let me draw some values I do not know let me spread 1.42, this is let me say 1.6, 1.8, 2.0, 2.2 and so on h nu and ev and this is alpha axis. Alpha centimeter in words and typical numbers 10.2 10 to the power of 2, 10 to the power of 4, 10 to power of 5 and so on centimeter inverse.

In the bulk case we would get a dependence which would be something like this compared to what I had drawn earlier I have tilted this you remember earlier when I have drawn almost short vertically and then, but remember my scale was different it was going to 3, 1, 2, 3 ev. Now I have just expanded the scale so it is slightly tilting. It is slightly going at an angle because of the expansion in the scale.

I had to expand because these differences are not 1, 2, 3. It is a small difference because it is corresponding to the energy difference here in the quantum well. It is only 50 to 100 mev difference. It is not difference that means 0.1 ev is the difference between these two not a one ev that is why I had to expand this to be realistic. Egq1 will be somewhere here and Egq2 may be somewhere here 1.58 approximately 1.4652 something like that approximately I am just giving you an idea but how do we expect.

We expect that this should go up like a step function now not like in quantum it is almost a step function and then it becomes flat here and then again a step function a little bit downward and you indeed see this when you I will show you the measured spectrum how does that look like so you see you expect a step like variation. a measured spectrum almost shows this. A measured spectrum if I plot on the same so let me just for differentiation let me make this this is theoretically what you expect.

If you expect this theoretically for a quantum well this is for the bulk, bulk gallium arsenide starts from Eg first difference you see is this starts at Egq1 and then it is a step function so this is for a step function quantum function. What is measured shows a spectrum a little bit earlier it starts it goes like this and then it shows a little bit of peaking like this and then of course it continues there again it comes up a little bit of peaking and then it goes like this.

So on the same thing there is a little bit of peaking which we see in a absorption spectrum of a quantum well. So this is the measured one. This is measured. This is theoretically expected and measured. So one thing is sure it shows a step-like variation, but there are some peaks there. What do we think is the explanation for these peaks? These peaks correspond to excitonic peaks. So the peaks correspond to excitonic resonances. This play a very important role in the device that we will discuss quantum well modulators or electro absorption modulators.

The peaks correspond to excitonic resonances. In the bulk case we do not see anything. All these measurements are at room temperature. So this is at 300 k. We are able to see excitonic resonances in the case of quantum well structures. Why do not we see that in the case of bulk? Let me stop here this discussion and continue in the next class because we have to take a quiz now. This is a very interesting explanations and discussions follow, but I will continue in the next class. So please let us take a quiz. A simple quiz. All quizzes are very simple.

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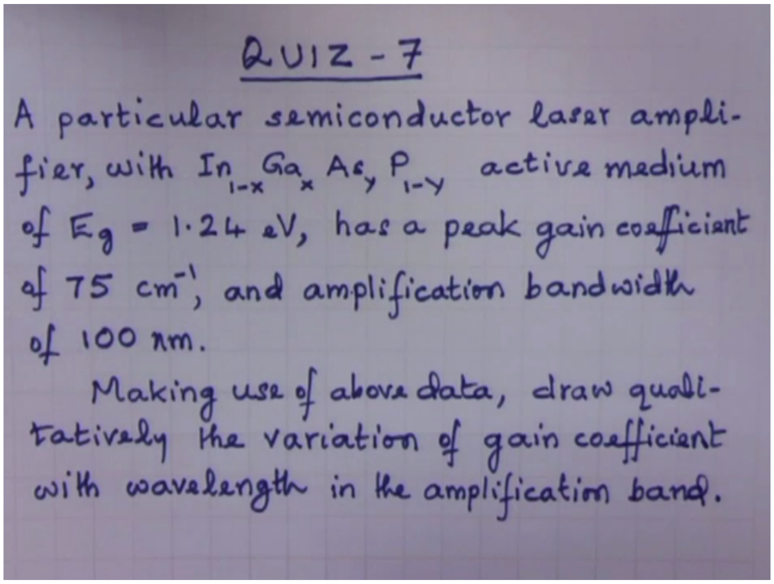

QUIZ - 7

A particular semiconductor laser amplifier, with $In_{1-x}Ga_xAs_yP_{1-y}$ active medium of E_g = 1.24 eV, has a peak gain coefficient of 75 cm^{-1}, and amplification bandwidth of 100 nm.

Making use of above data, draw qualitatively the variation of gain coefficient with wavelength in the amplification band.

Let me read it for you. A particular semiconductor laser amplifier with Indium, gallium arsenide, phosphide active medium of Eg = 1.24ev has a peak gain coefficient of 75 cm inverse. Peak gain coefficient that is peak of the gain spectrum and amplification bandwidth of 100 nanometer it is in wavelength. 100 nanometer is the amplification bandwidth. Making use of the above data, draw qualitatively the variation of gain spectrum with wavelength in the amplification band.

Variation of gain spectrum with wavelength in the amplification band. We have drawn this many times with the energy. So you have been asked because in practice normally we discuss in terms of wavelengths in optics so plot the variation of gain coefficient qualitatively with wavelength gamma of lambda versus lambda qualitatively gain variation with wavelength. It should not take more than two to three minutes. If you have completed, you can leave.