# Semiconductor Optoelectronics

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## Lecture - 22

## Absorption Spectrum of Semiconductors

We start with the absorption spectrum of semiconductors.

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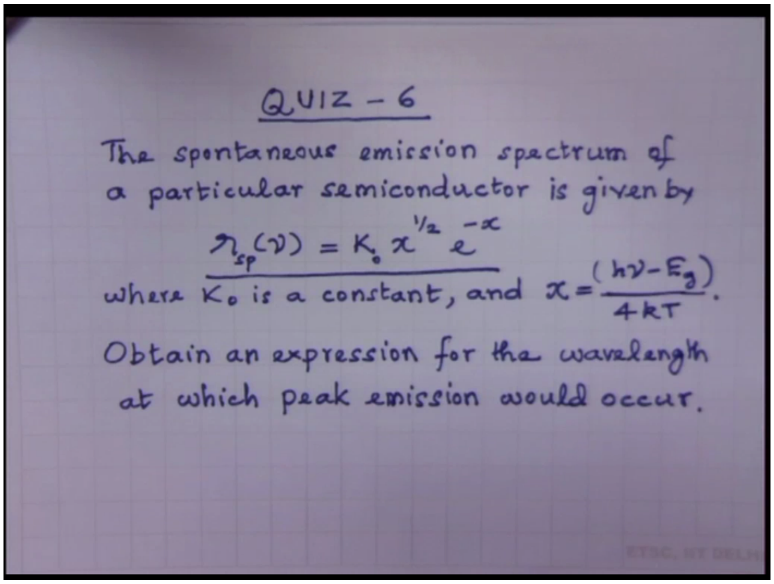

QUIZ - 6

The spontaneous emission spectrum of a particular semiconductor is given by

$$r_{sp}(\nu) = K_0 x^{1/2} e^{-x}$$

where $K_0$ is a constant, and $x = \frac{(h\nu - E_g)}{4kT}$.

Obtain an expression for the wavelength at which peak emission would occur.

Before I begin with the lecture, let me recall the quiz that I had given in an earlier class. Already most of you have done it rightly, but just to show the correct answer. The spontaneous emission spectrum of a particular semiconductor is given by so an expression is given. rsp of nu = k0 x to the power 1/2 - x where ko is a constant and x is given. Obtain an expression for the wavelength at which peak emission would occur.

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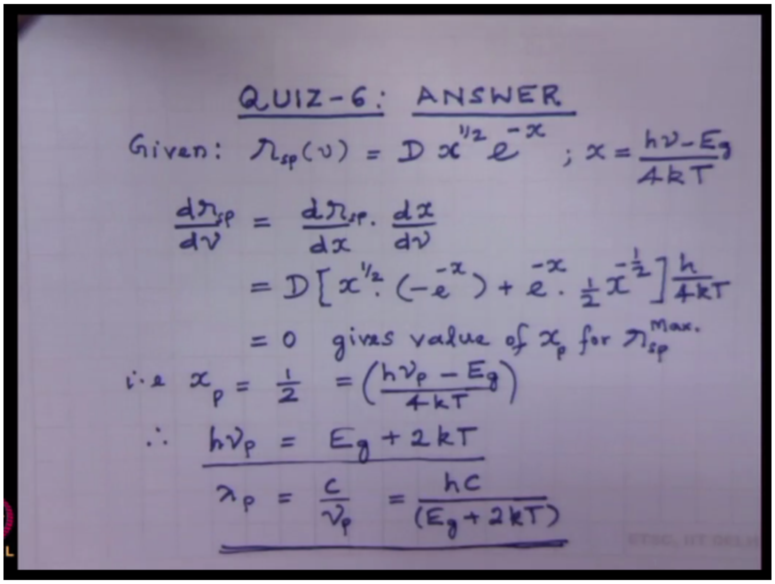

QUIZ-6: ANSWER

Given: $r_{sp}(\nu) = D x^{1/2} e^{-x}$ ; $x = \frac{h\nu - E_g}{4kT}$

$$\frac{dr_{sp}}{d\nu} = \frac{dr_{sp}}{dx}\cdot\frac{dx}{d\nu}$$

$$= D\left[x^{1/2}\cdot(-e^{-x}) + e^{-x}\cdot\frac{1}{2}x^{-\frac{1}{2}}\right]\frac{h}{4kT}$$

$= 0$ gives value of $x_p$ for $r_{sp}^{Max.}$

i.e $x_p = \frac{1}{2} = \left(\frac{h\nu_p - E_g}{4kT}\right)$

$$\therefore \underline{h\nu_p = E_g + 2kT}$$

$$\underline{\lambda_p = \frac{c}{\nu_p} = \frac{hc}{(E_g + 2kT)}}$$

So the answer, while writing the answer by mistake I wrote ko as D, but that is the ko which I have given in the question so this was by mistake I wrote it as D, but it is a constant. so drsp/d nu you can split like this instead of trying to have a complex derivation there if you split the advantage is you first differentiate with respect to x and then you differentiate x with respect to nu, put that = 0.

And u get xp = and then since xp is given by this expression here we have h nu p = Eg + 2kt and lambda p = c/nu p = hc/Eg + 2kt as I have seen most of you have done it correctly.

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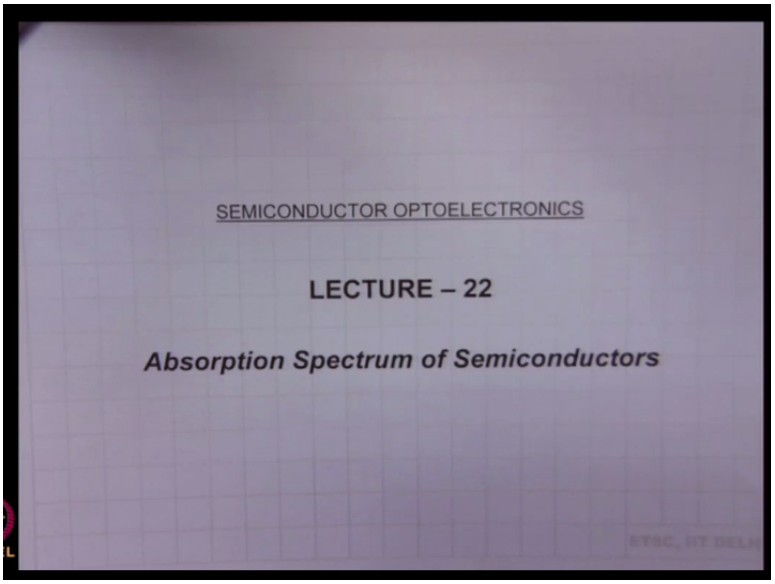

SEMICONDUCTOR OPTOELECTRONICS

LECTURE – 22

*Absorption Spectrum of Semiconductors*

In the last class we saw about the gain and amplification by stimulated emission and the condition for amplification by stimulated emission and the laser amplifier.

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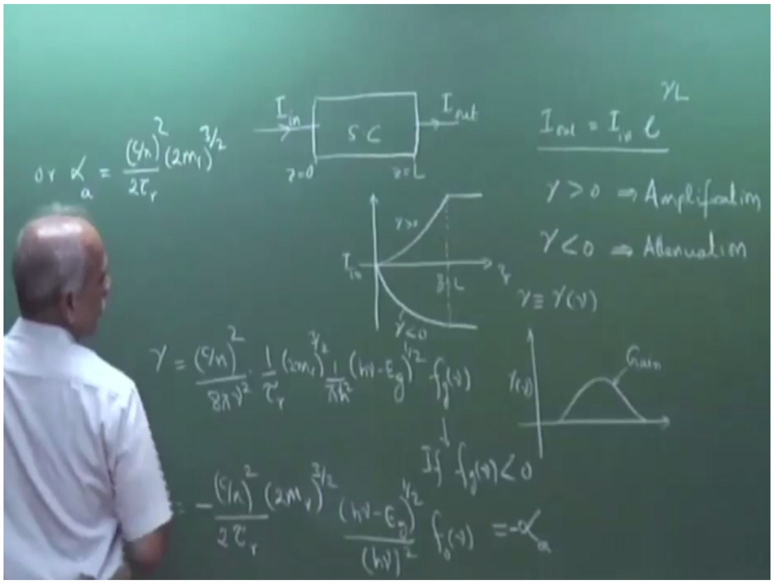

If we recall the same expression I had our objective as I said was to find out if beam of light is entering a semiconductor of Iin intensity, then what is the output Iout. So this is the semiconductor and we have got an expression that Iout = Iin * e to the power gamma * l if l is the length of the reconductor then so this is z = 0 and z = L. e to the power gamma and we had said that gamma if it is greater than zero it implies amplification and gamma < 0 implies attenuation.

If you were to plot the intensity here versus length so in the z direction, then if I0 is the intensity here that is Iin is the intensity at z = 0, then for gamma = gamma > 0, the intensity will build up exponentially up to z, z = l and after that of course outside it is. So if I had to plot the intensity versus z we would get an expression get a curve like this. This is for gamma > 0. So gamma < 0 it will exponentially drop down here and then and this is for gamma < 0 that is attenuation.

We know also that gamma is a function of frequency, gamma is identically is a function of frequency therefore the absorption spectrum or gain spectrum refers to dependence of gamma on nu and we have seen that the gain spectrum so if you plot gain versus nu it approximately behaves something like this so this is gamma of nu gain, so these are gain. So today we will see how would the absorption spectrum be?

And there are many more processes because this gain that we have calculated is gain due to stimulated emission. So stimulated emission taking place in the presence of an incoming photon and therefore we have worked out that this is the gain spectrum of a semiconductor laser amplifier. So we will see how would the loss spectrum look like. So recall that we have expression for gamma which is c/n whole square, n pi nu square, 1/tau r, 2mr to the power 3/2.

There was 1/pi h square also, so let me write it here, because this is part of the density of states and then h nu - Eg to the power half. This was the expression that we have for gain coefficient * fg, the Fermi inversion factor. So if Fg of nu we first started with the 0k experiment thought experiment where Fg of nu was either + 1 or - 1 and we obtained the gain profile and then we took a finite value and saw that the gain profile looks like this.

If Fg of nu were negative, so if Fg of nu is less than 0, gamma is negative. Gamma is negative means we have an expression where it is E * - gamma * l. So if Fg of nu = 0, then gamma = -c/n square I will write the same expression again maybe I can club some of these here so c/n square and there is a pi here and there is a pi here so pi square and h cross square has h square/2pi the whole square so 4 pi square so h cross square = h square/4 pi square.

So this pi square cancels with these two and four cancels with two here eight to give you 2 and h nu I am taking together so I have 2 tau r * if I make a mistake please point out twice mr to the power 3/2 * h nu - Eg/to the power half and h nu whole square * this quantity is negative so I have taken the negative sign here and I write this as F0 of nu where F0 of nu is mode of Fg of nu. sign I have kept here and then I want to call this as = alpha a - this quantity = - alpha a or alpha a = c/n whole square let me write again into twice tau r 2mr to the power 3/2 *F0 of nu.

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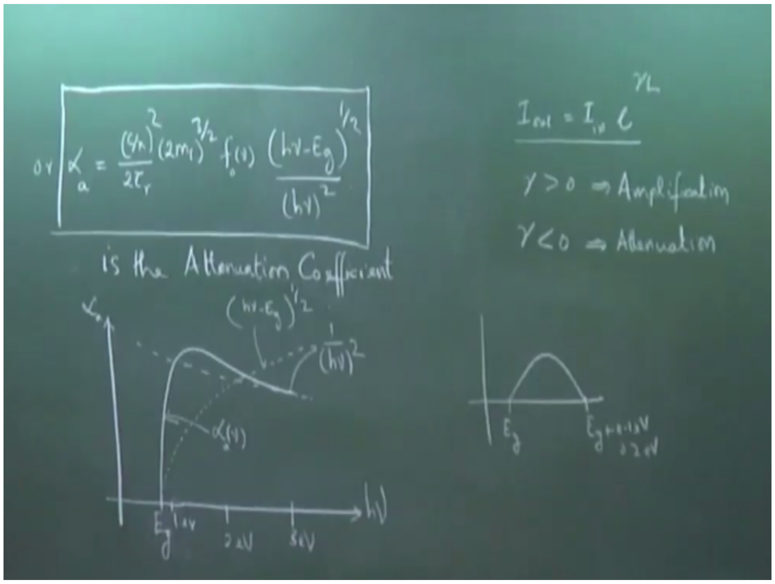

I am writing Fo of nu to indicate that this is the positive number F0 of nu is basically mode F g of nu * h nu - g to the power half divided by h nu whole square so is the attenuation coefficient. IF I plot the frequency dependence of this attenuation coefficient I will see the attenuation spectrum. So how would this look like. So alpha a is basically - gamma so alpha is a positive number is the attenuation coefficient.

If you see again that of course there is a small frequency dependence here if x of nu what we have missed we not missed, there is a frequency dependence here F0 of nu otherwise it primarily dependence of h nu - Eg to the power half and h nu whole square so if I plot this earlier also we had a similar expression here and I had said that h nu square variation is very small in the range of interest.

The range of interest was the bandwidth of the amplifier which was only 10 to the pwoer of 13 Hz and therefore that is a small fracture of light frequency, but when we measure the absorption spectrum absorption takes place over a wide range and therefore now I would like to consider this also if you remember I had showed this as a variation because it is 1/x square variation so this is h nu 1/h nu whole square variation.

Now we want to plot it over a wide range for example from Eg here to a wide range, what do I mean by wide range is let us say this is one electron volt this is 2 ev so 1 ev, 2 ev, 3 ev. Earlier where did we plot it from Eg to Eg + 0.1 ev or 0.2 ev that is all because that was the amplification band and therefore it was only a small fracture of this and that is why I had assumed it to be constant.

Now I am not assuming it to be constant because I want to see for example as we will see later the absorption coefficient is important when you want to use the material for photo detection. So a photo detector can detect over a wide range of wave length and therefore we have to consider the absorption spectrum over a wide range of energies. The plot which I had made earlier this amplification band corresponds to Eg to approximately Eg + 0.1 ev or 02 ev.

It is a very small range of this spectrum that I am going to plot now and therefore I had assumed this dependence as constant, but now we want to plot it over a wide range and therefore I have to consider that. The other part is as you know it starts from Eg h nu - Eg to the power half goes the same way x to the power half dependence so this is h nu - Eg to the power half. So the net result is so the absorption coefficient so this is variation of alpha n.

So what I have plotted here is alpha a. Any question? is it alright? **"Professor - student conversation starts"** Sir (()) (14:18) I am sorry I had showed that as 1 this Eg. Thank you actually it should start from Eg because this is valid for h nu > Eg and the density it starts from Eg Yes, I am sorry I wrote there 1 and so let the 1 be here it does not matter because there are materials with Eg < 1.

If you take germanium it is 0.66 ev, so from Eg this starts from Eg so this is 1 ev here, please correct it that it starts from Eg goes up and then has a variation like this. **"Professor - student conversation ends"**. This is what we expect to get in a semiconductor, the absorption spectrum of a semiconductor, but in practice if you see the absorption spectrum I will draw typical absorption spectrum just by the side here.

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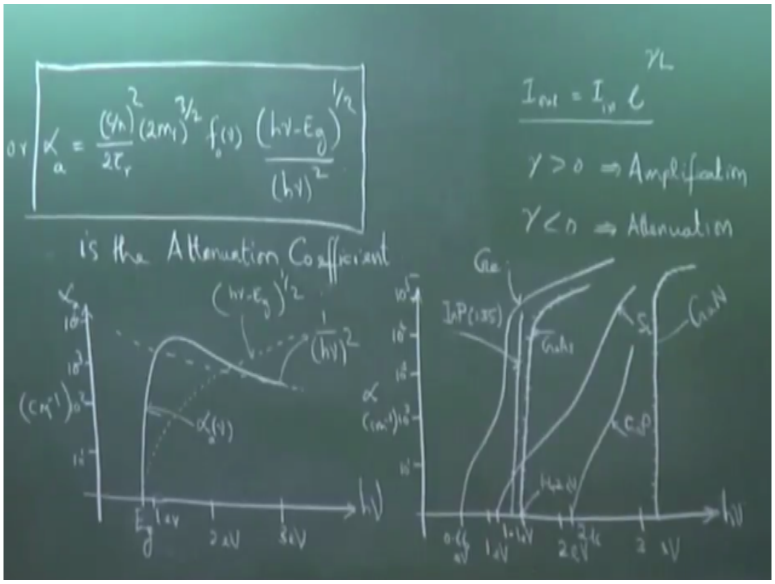

So, let me again put 1 ev, 2 ev so this is h nu photon energy versus gallium arsenide. Let us start with gallium arsenide. Alright let us start with the lowest one, so germanium is 0.66 ev, this is germanium, let me plot typical numbers here. So this is 10 to the power 1, 10 power 2 and 10 to power 3, 10 to the power 4, or 5 alpha in centimeter inverse. You can see gamma or alpha has to have inverse length dependence to centimeter inverse.

This number is typical of gallium arsenide that I have plotted. So typical numbers are 10 to the power of 2, 10 power 3, 10 power 4, 10 power 5 and so on. So if I start with germanium this has a behaviour, this is for germanium it starts with Eg and it goes like this. We will see we will discuss why it is behaving like this. If you take silicone the next one that is 1.1 ev so you know that silicone has a band gap of 1.1 ev and this does this, this is for silicone.

The numbers and this are typical it is all right little bit variation so next comes gallium arsenide so I have to gallium arsenide so gallium arsenide is here 1.42, so 1.42 so gallium arsenide does this. You see the difference I am just for differentiation this is what I have plotted for gallium arsenide. The numbers are very typical if you see in a literature you will see that the numbers are typical gallium arsenide. 2.16 gallium phosphide again, gallium phosphide does this.

So this is gallium phosphide, let me plot this and then we have something to discuss gallium phosphide. If you take gallium nitrite by band gap semiconductor 3.3 ev here it is also a direct band gap material and this goes like this. I am showing a mark here this is gallium nitrite, these are direct band gap, let me draw one more, this one here is indium phosphide whose band gap is 1.35 ev. We have something to discuss 1.35 ev, but if you just look at the graph. You can all the ones which are dashed marked here are direct band gap materials and the other ones are indirect band gap materials.

So just looking at the absorption spectrum you can see something that for direct band gap materials the absorption curve shoots up as soon as you cross the Eg, as soon as you cross the Eg it simply shoots up almost vertically whereas for indirect band gap materials it goes like this, but for none of these, it appears that none of them seem to be coming down here we expected to come down, but it does not seem to come down. There are several explanations that are due. This graph that we have obtained by considering a direct band gap semiconductor.

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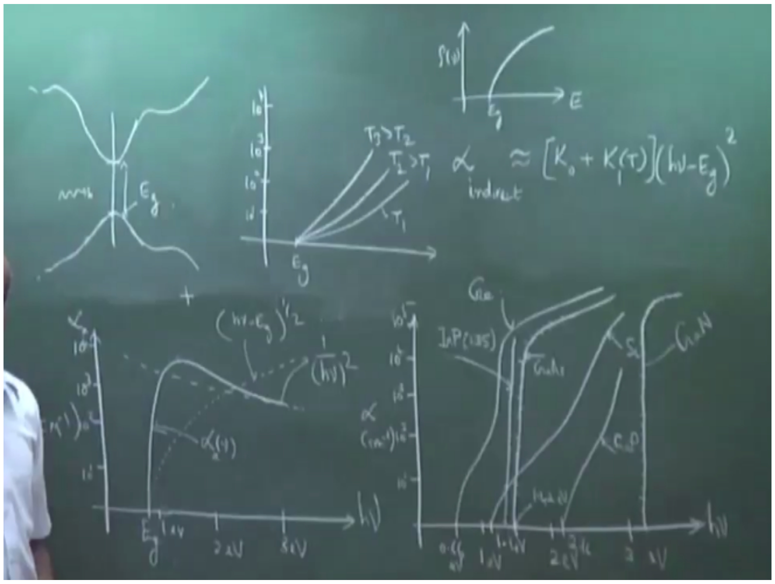

We have considered a direct band gap semiconductor and we have concentrated only on the parabolic approximation close to the band H. This is close to the band H, but these bands do have behaviours which are very different. Close to the band edge if you are seeing let us see Eg + 0.2 ev, 0.1 ev, 0.2 ev parabolic approximation is a very good approximation and based on that we have derived these expressions for a direct band gap semiconductor.

But beyond that this is not true so this curves are only valid close to the band edge. point number 1, second there is also almost all materials also have an indirect band gap and therefore if you consider the theory of an indirect band gap semiconductor and find out the absorption phonon assisted absorption then you have a variation which looks like this, alpha indirect so this alpha that we have is alpha direct band gap.

Alpha indirect we are not going to this derivation is approximately empirically this is given that approximately you can show that this variation is at the form K0 some constant + k1 of T * e to power h nu - Eg whole square you are going to derive this. These are constants K0 and K1, K1 is a temperature dependent constant, H nu - Eg whole square, so this variation looks like this typically.

What I am plotting is the variation at different temperatures, T1, T2 >T1 and T3 > T2 that is because of this term k1 of T, the second constant K1 is a function of temperature. The numbers here are slightly smaller so 10 power 1, 10 power 2, 10 power 3, 10 power 4. So they are little smaller you can see these have already reached 10 power 4, the direct gap has come to 10 power 4. This particular graph is approximately calculated for gallium arsenide so it should have been 1.42 ev.

Because if you use the parameters of gallium arsenide, then approximately 10 power 4 is the peak. therefore, the net absorption is this plus this, absorption due to direct and absorption due to indirect plus there are other components, but this plus this will give you reasonably it explains this why gallium arsenide beyond this continues like this because if you add this and this you will get a variation which is approximately like this.

So that is why this portion is going up, both for germanium, gallium arsenide, almost all of them, this is the first reason why this is not coming down, second why indirect band gap it goes like this whereas in direct band gap it shoots up like this. Let us see the ek diagram as soon as h nu reaches Eg, so this gap is Eg, it gets density of states for interaction. rho nu if you recall the density of state, rho nu let me plot it here.

So rho nu versus e, we have already plotted this then we have seen that it starts from Eg and goes up. So as soon as photons reach this energy there are plenty of states which are available and absorption starts immediately before this there is no absorption and therefore absorption is literally zero. There are other components we will come to that, but right now absorptions shoot up like this that is why absorption shoots up in direct band gap semiconductors.

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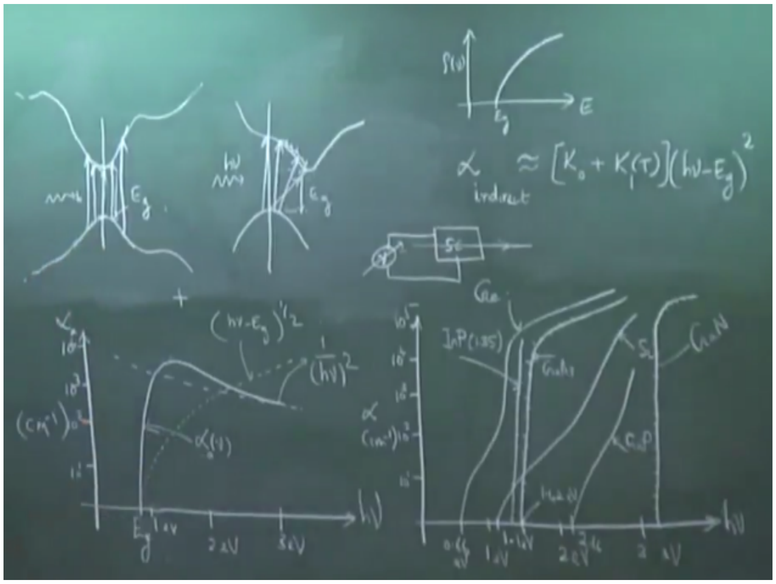

What about indirect band gap semiconductors? So if I want to plot the Ek diagram for a indirect band gap semiconductor, an indirect band gap semiconductor. We see that this Eg, so this gap is Eg. There are as soon as you cross there are states yes available states Eg, but they do not any absorption going at an angle like this in the EE diagram in the energy space is an oblique transition violates the law of conservation of momentum.

And therefore although the state in this case the moment the energy exceeds Eg all absorptions can take place because all are vertical transitions which are highly allowed transitions. So the probability of this transition is very high in the case of a direct band gap semiconductor, whereas in the case of an indirect band gap semiconductor the probability is low because of this phonon assisted transition because of violation of conservation of momentum and therefore the graph slowly starts increasing.

Absorption can take place, but it increases relatively less rapidly as compared to direct band gap semiconductors, however as the energy increases please see if I have a slightly higher energy then this make a less oblique transition. If I have more higher energy, this will make almost vertical transitions. If I ask h nu increases as the frequency of the energy of the photon increases, it can make vertical transitions and that probability is very high that is why you see germanium.

Afterwards again it does this rapid increase because now vertical transitions can also take place. You have higher energy, band gap is the minimum energy so if you have photons incident with higher energy they can cause vertical transitions which are highly probable and therefore the absorption coefficient shoots up same here with silicone you see it was going like this then again it started increasing same here with gallium phosphide.

So it is as the energy increases the probability of vertical transitions increase and once of course the carrier comes here then it will thermalize and it will come down here like this, this is thermalization which we have discussed due to phonon transitions and therefore the graph again shoots up. So two things, just by looking at the absorption spectrum almost you can make out whether the material is direct band gap or indirect band gap.

You just measure the absorption spectrum and you can make out whether it is direct band gap or indirect band gap, right at the beginning when we started with direct and indirect how would one know which is direct band gap and which is indirect band gap? if you are given a piece of semiconductor how do you know? Now here is a simple test so you have a energy source so here is the semiconductor let us say you have a tunable source here that is nu, tunable source which is frequency variable and you pass light through this.

This is light I am talking of light it is not electrical current tunable source and you measure the absorption output as a function of the frequency and simply plot this you can identify which is the direct band gap material and which is a indirect band gap material. So this term here explains for the transition which is going up there are many more effects these are not the only effects what we have discussed is inter-band transitions, conduction band to valance band or valance band to conduction band this is inter-band transition.

There are also intra-band transitions which are possible. What is intra-band transition? and there are phonon transitions which are possible and there are excitonic resonances are possible so the absorption comprises of several effects not just direct band gap or vertical transitions. So what we have discussed is one.

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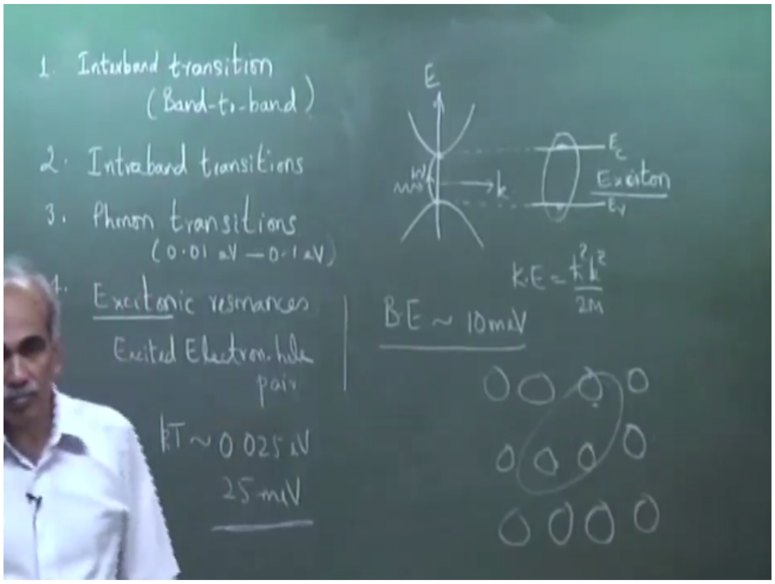

Inter-band transition that is band to band valence band to conduction band, inter-band or band to band transition. There is intra-band transition within the band. These are generally low frequency or low energy transitions if you look at the Ek diagram every time you see this is the Ek diagram which tells about all the physics which I mentioned in one of the first classes. An electron which is sitting here can make an upward transition to a position here an electron which is here makes an upward transition to a higher vacant state within the band by absorption of photons.

Of course this will require phonon assistance because there is a change in momentum, but an electron can make an upward transition here by absorption of photon and this is called intra-band transition within the band, but the carrier which is here subsequently we will thermalize and come down through phonon transitions. This is intra-band transition. So, when photons of a broad spectrum of energy is incident so if you are finding the energy spectrum.

If you assume that the source as all these wavelengths then some of the wavelengths particularly here at the lower energy will result in an electron gaining energy and going to a higher vacant position and this becomes a hot electron, a higher kinetic energy electron is called a hot electron, but subsequently it will come down by thermalization. This is intra-band transition same thing can happen with a hole.

A hole for example a high energy photon lifts an electron from here to here. Then it leaves a hole here, a high energy photon incident here can make a vertical transition so this electron goes here. There is a hole here, but hole here is a high energy state and electrons which are sitting here will immediately come down to this. An electron from low energy can immediately come down there. So what happens then the hole moves here. an electron comes to that vacant state leaves the hole moves up.

This is also within the band. So intra-band transitions or vice versa. A hole which is sitting here can absorb energy and go to this place. So these are intra-band transitions. Generally, the energy of intra-band transitions are less than band gap transitions. A third type of transitions are phonon transition, but we know that phonon energies are typically in the range 0.01 ev to 0.1 ev. Phonons can also absorb energy; low energy photons can be directly absorbed by phonons or low energy photons can directly generate phonons within the semiconductor.

And therefore the absorption of low energy photon to given rise to phonons will also have absorption somewhere in this region particularly low energy region and a fourth type of absorption which is also possible are excitonic absorption, excitonic resonances. So some of you may not be knowing what is an exciton, exciton here refers to an excited electron pole pair what it means is exited electron hole pair. Let us see this is fully an introductory.

I write the diagram erase it, explain something, and erase it, again you need Ek diagram so, so many times you need to draw the Ek diagram. This is our traditional band diagram. An electron which is sitting at the bottom here or a hole which is sitting at the top of this band are low energy states, very low energy state therefore the K is almost 0 which means the kinetic energy is very close to zero.

Very small kinetic energy, because kinetic energy is h cross square, so h cross square k square/2m and therefore if k is close to zero that means they are low kinetic energy carriers close to the edge of the band. An electron sitting at this place can absorb a photon of energy which is just near to Eg and make an upward transition if there is a vacant state. So there is a hole which is positively charged and there is an electron which is negatively charged so there is a hole which is close to the edge of the band and there is an electron which is just gone up to this Eg.

So the negatively charged, positively charged hole and negatively charged build up Coulomb attraction. If they are close in space this is energy axis, but if they are close in space physically then they form a hydrogen atom like band like an hydrogen atom, they behave like an hydrogen atom which is held by Coulomb interaction between a positively charged nucleus and a negatively charged electron.

So if you see the semiconductor lattice there may be large number of atoms which are sitting may be in one of the atoms there is a hole energy there may be a electron somewhere here and they form there is a Coulombic attraction in addition to all the bonds which you can covalent bands there is a Coulombic attraction between them. So the electron and the hole are localized. So this localized particle it is not a particle it is localized entity is called an exciton, the localized entity is exciton.

The binding energy though between these is extremely small. Typically, the binding energy of this binding energy is of the order of 10 mev, of the order typically 8, 10, 12, 15 binding area of this exciton is very small. In a hydrogen atom what is the binding energy of hydrogen atom more than 10 ev, more than 10 ev is the binding energy of hydrogen, but this is not actually like a hydrogen atom, but it behaves they are weekly couple there is some positive charge there or positive field and there is an negative field which tries to hole together with a very small binding energy and this is called exciton.

Normally at room temperature you do not see excitons in semiconductors because the room temperature average kinetic energy is 0.24 ev which is 25 mev. So binding energy that is kinetic energy that is KT is of the order of 0.25 ev or 25 mev. So this much average thermal energy is present in the material and therefore the exciton gets dissociated. So the binding energy is only 10 mev so if already so much energy is present the exciton is dissociated.

Dissociated means they are no more holding together, so they are free carriers. So they behave like free carriers. So at room temperature you would not see this, but due to this binding due to this positive potential the energy of this is slightly lower, energy of this pack is slightly lowered than Eg this was at Eg, but the net state here exciton has a slightly lower energy and therefore this leads to states which are here close to the band edge, but little less than band edge.

So photons of energy slightly less than Eg can also be observed and these are exciton sticks. So there is a density of states due to excitons. So photons close to the band edge even a little less than actual Eg can also get observed by in the formation of excitons. This is excitonic transition and therefore now we have if I take the total spectrum let us look at the total spectrum we will discuss a little bit more about exciton a little later because we are going to discuss the quantum confined stark effect and I will discuss about it probably in the later class.

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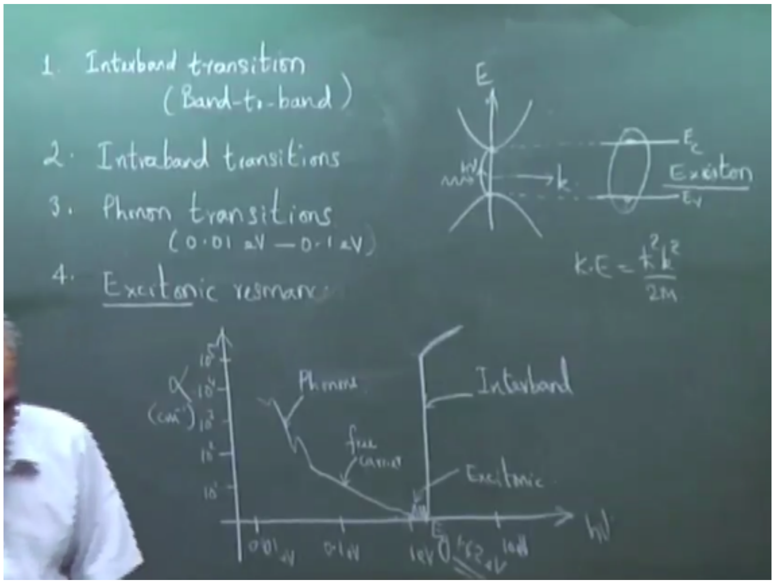

1. Interband transition (Band-to-band)
2. Intraband transitions
3. Phonon transitions (0.01 eV — 0.1 eV)
4. Excitonic resonance

If you now see the total spectrum h nu we are starting now from 0.01 not 0.1, so 0.01 ev and let us 0.1 ev and then 10 ev any how 10 ev is too much we do not need. So the whole absorption spectrum earlier I had plotted 1, 2, 3, 4, but we are looking at the entire absorption spectrum alpha of a semiconductor. I will plot this for a typical gallium arsenide numbers so you have 10 to the power of 2, 10 power 4, so this is 10 to power 1, 10 power 2, 10 power 4, 10 power 4, 10 power 5 centimeter inverse.

All the inter-band transitions which we discussed were from here 1.42 let us say this was Eg for gallium arsenide 1.4 is here. Now I want to plot how it would look qualitatively that there is absorption taking place at lower photon energy cell. So this goes of course shooting up like this around 10 power 4 it will start turning. Below band gap there are some excitonic resonances. These are normally peaks like this and not below 1 ev here.

So excitonic resonances and then there are free carrier absorption which looks something like this just showing you the overall. This is free carrier absorption and there are phonon absorptions. So this part here is primarily due to free carrier. Free carrier you remember within the band intra-band transition. So this was free carrier portion. This is excitonic resonances just below the band gap. Excitonic and then this is due to phonons because the energy range here corresponds to that of phonons that is creating vibrations in the lattice.

So energy is spent in creating lattice vibrations directly so this is due to inter-band. So in this complete spectrum we see all the four points which I have written. So inter-band excitonic, free carrier, and phonon transition. This is the spectrum of absorption spectrum of a typical semiconductor what I have plotted is for gallium arsenide of course. I hope we understand why the behavior of spectrum comes. Before we stop I want to just talk one step here and that is absorption look like in a quantum.

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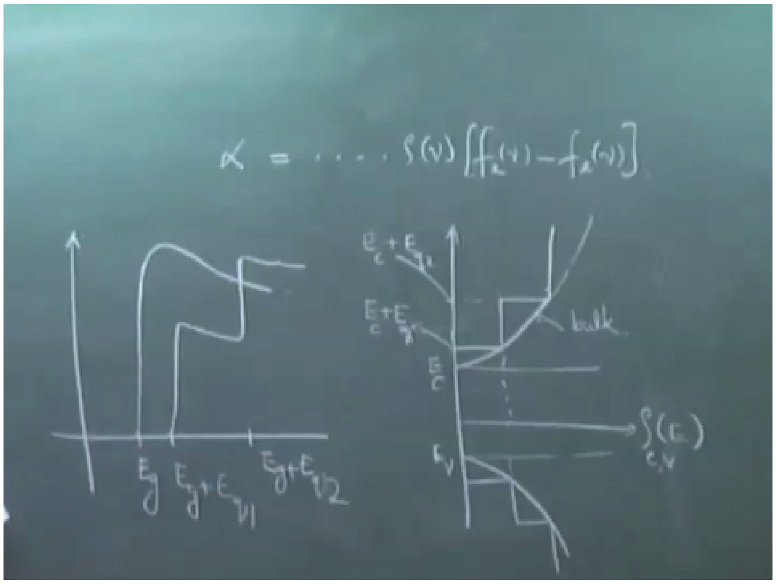

We have an expression for last coefficient alpha which had alpha = so many constants there was rho of nu, the density fo states. It is the density of states which gave and then Fg of nu or F0 here that is Fa probability of absorption. F0 which is Fa - Fe let me write it fully so this is Fa of nu - Fe of nu. Some constants here rho of nu and this and this rho of nu is the one which gave us the term h nu - Eg to the power half.

The density of states and recall that the density of states the first time when we plot it E and this was ev so this is ev, ec, and then we saw that the density of states goes like this density of states variation. From this we derived all the absorption coefficients and then we got this absorption coefficient which doing this band to band trans. In photonics we are interested in band to band transition because we are looking at photon energies which are normally in the visible or IR so most of the times we will focus on band to band transition.

So the absorption coefficient was like this. Rho nu was varying like this. In a quantum well structure we know that rho nu is a (()) (50:33) function. so it starts from somewhere here more than Eg and then it is constant up to a certain energy and then it remains constant here so this was Ec + Eq1 that q = one state. The density of states was constant we have some values for this here then when the energy reached Ec + Eq2 so Ec + Eq2 there was a second step varying like this and then continuing like this.

So this is for the bulk and this is for the quantum, this is for the bulk and it is a step function same thing here so if I assume that Mc = MV then I will have a similar variation. What does this mean? This means that for all photon energies from this gap here, Eg up to this the density of states is constant therefore the absorption should remain constant. Density of states is going it is no where varying, this is variation, but here density of states is remaining constant.

This is I have also plotted here. This is not k. Why did I plot k here? This is rho c and rho v, so rho of e. Let me write rho of e c, b. This is for c and this is for b. Rho c, b. For all energy from Eg up to this the density of states, number of available states is constant and therefore the absorption coefficient must also remain constant because absorption remains this almost constant here.

And then suddenly the density of states increases and therefore absorption coefficient should suddenly increase and in need quantum well structure show an absorption coefficient variation which is like this. So if I want to plot in the same graph then this is Eg. There is no absorption at Eg. First absorption starts at Eg + Eq1. So in the same graph if I want to plot it shows absorption and then the absorption remains constant almost constant except for the 1/ h nu square dependence almost constant till Eg + Eq 2.

Since then the absorption again jumps vertically almost vertically and then it remains constant. A step like variation of absorption coefficient and if you connect this to the last class a step like variation in the gain coefficient as well and therefore it is the density of states which determines what is the absorption spectrum or gain spectrum of a semiconductor. That is why we spent so much time on density of states.

So you get a step-like variation and this has very important applications. So we will discuss about this in one of the later classes, but remember that a quantum well structure will show step like absorption spectrum. We will stop here and continue in the next class.