Semiconductor Optoelectronics

Prof. M.R. Shenoy
Department of Physics
Indian Institute of Technology-Delhi

Lecture-19
Rates of Emission and Absorption

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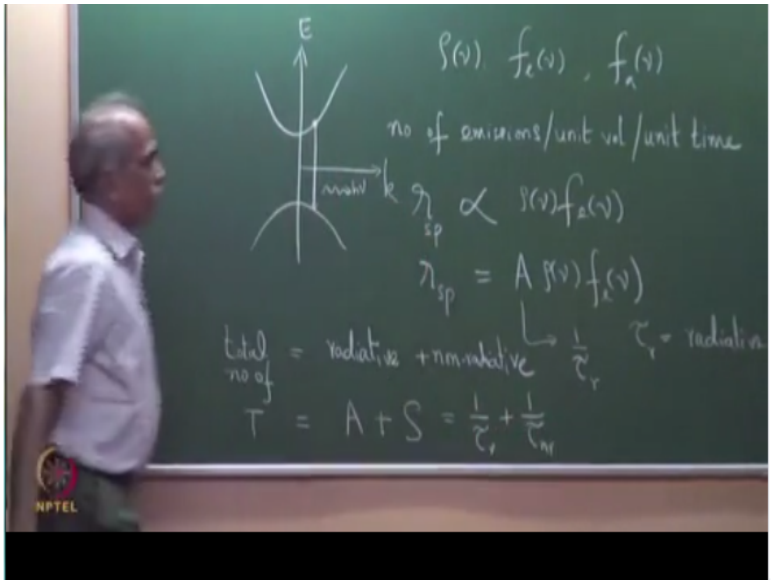

In the last class we discussed about optical joined density of states row of mu and the probability of emission f e of mu or PE of mu and probability of absorption f a of mu we want it to find out the number of emission finally our objective is today in this class we will discuss about rates of emission and absorption. And our objective is rates of emission and absorption refers to number of emission or absorptions number of emission or absorptions per unit volume of the material per unit time.

Because once we know this number then multiplied by h mu will give us the power energy per unit time gives us the power output from this device or material. So, the number of unit emissions per unit volume this is the density of optical joined density of states that is number of states evaluable for photons of frequency mu to interact with so row mu is the number of states. This is the emission probability and therefore number of states multiplied by the emission probability will give you number of emissions.

So, row of mu into f e of mu will give you number of emissions per unit volume. Because this is per unit volume and therefore the rate of spontaneous emission row rate of emission or spontaneous emission or a speed is proportional to row mu into fe of mu or a sp standing for spontaneous emission why it is spontaneous emission I will come in a minute. So, this I can write as equal to a rate constant A into row of mu into f e of mu.

A is the rate constant and you can show that this is nothing but one over tau or where tau or is the radiative the combination lifetime. So, this is the radiative recombination time or recombination time for radiative transitions recombination time radiative recombination time is recombination time for radiative transitions why I mean radiative transitions is the following. If you see the EK diagram as you can see I keep drawing this many times.

So, EK diagram a photon an electron which is in the conduction band can make a downward transition in energy to the valence band giving out spontaneously on its own. A photon of energy h mu it is also possible that this recombination may not result in the emission of a photon at all. This recombination is also possible through phone on phone on emission vertical transitions are also possible with phone on emissions.

And therefore in general or this may be sitting here it makes an oblique transition. But the point is a transition from conduction band to valence band may take place with emission of a photon or without emission of a photon. Therefore total number of spontaneous downward transitions comprise of radiative+non-radiative transition. So, radiative+non-radiative transition so, total number here total number of spontaneous transitions spontaneous.

Because it is on its own it is coming down spontaneous transitions would comprise of radiative and non-radiative components. And therefore correspondingly the total rate constant if I denote it as T. Then this will comprise of a radiative constant A+a non-radiative constant which I denote as S total rate constant equal to radiative constant+non-radiative constant here. The rate constant therefore this is inverse of time it is rate constant therefore it is inverse of time. So, this is written as one over tau or +one over tau nr.

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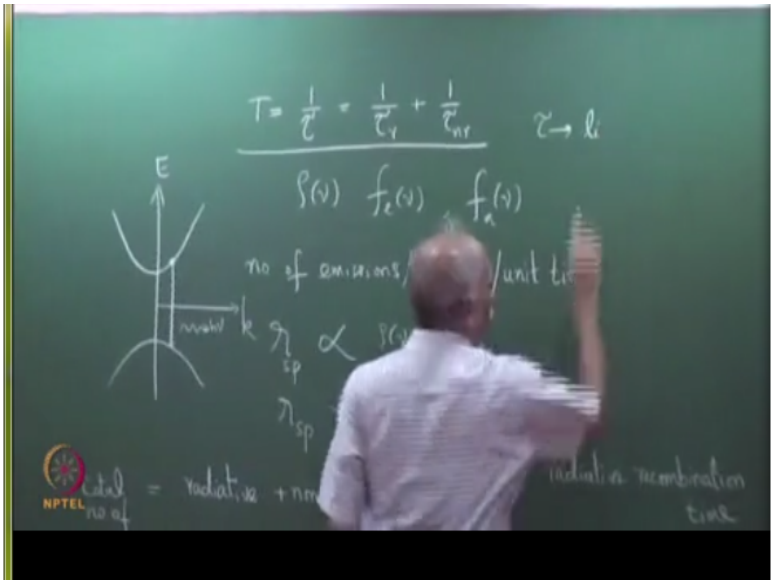

In other words T is equal to 1/tau =1/taur+1/tau nr where tau is the life time of the transition or recombination time spontaneous recombination time so, this is recombination time.

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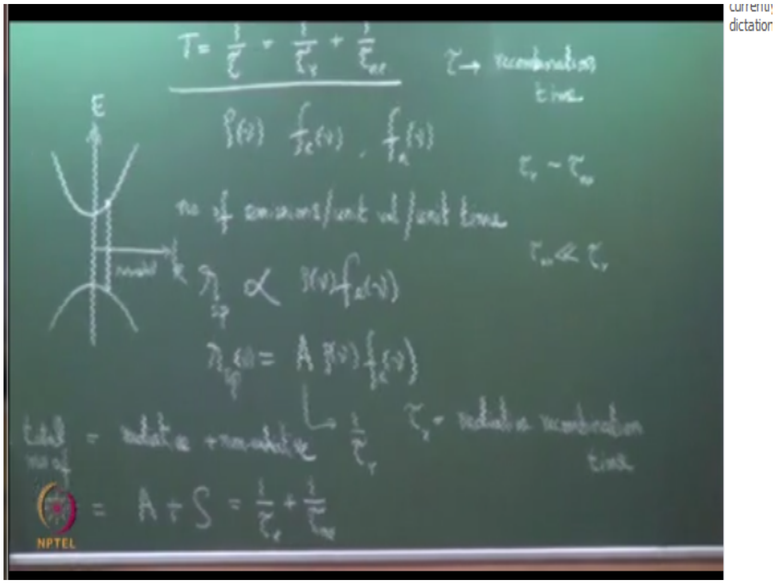

This is radiative recombination time this is non-radiative recombination time the relation is 1 over tau=1 over taur+1 over tau nr. We will discuss about this little bit more a little later but at this point I will just make a statement that in direct band gap semiconductors tau are is of the same order as tau nr which means the rate constant A is of the same order as the rata constant S which means the spontaneous emission.

This is spontaneous emission rate of spontaneous emission of photons or sp of mu of frequency mu. If this is equal to this it means out of the total recombination 50% of them result in emission of photon. And 50% of them result in emission of no photon that is no emission of photons. If A equal to S if taur equal to tau nr it means that A equal to S total rate constant is this which comprises of these two radiative non-radiative.

Therefore in a direct band gap semiconductor the probability of radiative transitions is equal to nearly equal to probability of non-radiative transitions almost every electron combining with a hole 50% probability of emission of a photon. And 50% probability of no emission of photon but in the case of indirect band semiconductors tau nr is much smaller than taur. And therefore S is much greater compare to A.

We will put some numbers a little later when we go to materials therefore non-radiative transitions dominant much more over radiative transitions. It is the that is the important of taur and tau nr. So, I want to bring at this state and an knowledge with atomic systems because most of you would an elementary theory of lasers. Most of you are familiar with a elementary theory of lasers. You will see that there is a complete one to one a knowledging and then I would like to proceed further see atomic system.

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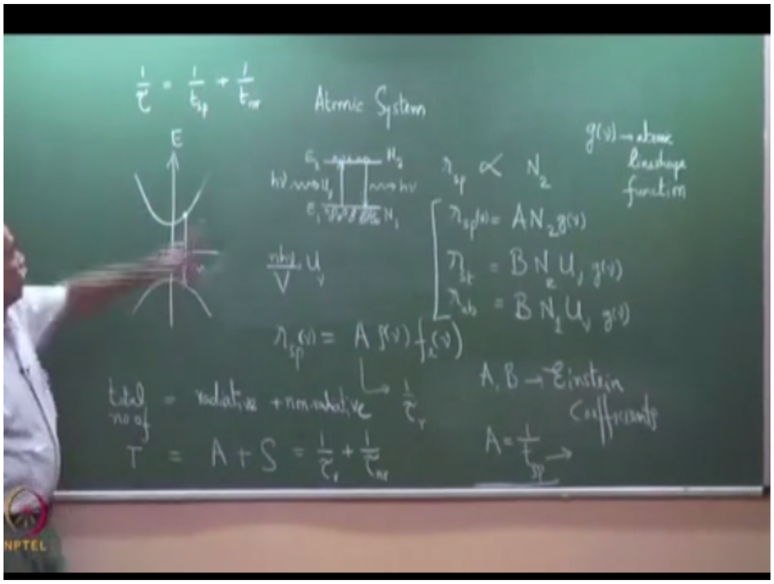

If you consider a two level atomic system with N1 number of atoms in the ground state and N2 number of atoms in the excited state with energy event and energyE2 and energyE1. Then there are number of atoms large number of atoms in the ground state which is given by the borgement distribution. There are few are number of atoms here the three basic atomic processes are the same interaction processes.

An atom sitting here can make spontaneously a downward transition giving out an emission of photon h mu. The number of transitions N2 is the number of atoms per unit volume of the material the number of transitions downward transitions spontaneous transitions is proportional to the number of atoms which are sitting here. And therefore the rate of transition spontaneous transitions is proportional to if I am looking at spontaneous emissions.

Then this is proportional to N2 or rate of spontaneous emission is equal to A times N2 there can be stimulated emission in the presence of photons of energy h mu. If there is a energy density u mu of photons of energy h mu u mu is the energy density which means number of photons per unit volume of frequency mu that is if you have N number of photons incident. Then nh mu/volume of the material V is unit.

So, this is equal to u energy density so, if u mu is the energy density then rate of stimulated emission is equal to a constant B into N2 into unit. And rate of absorption is proportional to the number of atoms in the ground sate and therefore rate of absorption is equal to B into N1 into unit. Please see when there is an energy density when there are photons of energy h mu here.

They can stimulate downward transition but they can also cause absorption upward transition. So, the absorption depends on the energy density u mu and the number of atoms in the ground state. Emission depends on the number of atoms N2 here but also on the energy density u mu that is why we have this kind of expression to make it to incorporate to more features into this actually atom interacts with radiation over a certain range of frequencies.

So, there is a line shape function g mu atomic line shape function rows of few have studied basic laser physics line shape function. If you have not studied does not matter g mu and therefore every where this rsp will be a function of mu and N2 into g mu, g mu gives as the strength of interaction at the frequency mu that is any atomic system interacts with radiation over a range of frequencies**.**

Because of various mechanisms which are called line broadening mechanisms this g mu is the line shape function which determines the strength of interactions at a particular frequency. So, every where you will have an additional term g mu that is N2 into g mu N1 into g mu so, spontaneous emission at a frequency mu is proportional to N2 into g mu and A is the proportionality constant.

So, these are the three expressions that will have for a non-degenerate system this for absorption and emission B is the same otherwise you have to write here B21 and B12, for non-degenerate system it is the same. And these coefficients A and B are called what are they known as these are known as the Einstein coefficient. So, A, B are the Einstein coefficient A is equal to one over tsp where tsp is the spontaneous emission life time.

You can write this as dN2/dt=–AN2 into g mu and simplify and you can show that A is equal to one over tsp where tsp is the spontaneous emission life time. In atomic physics also there is a relation that is life time tau is equal to one over tsp+1/tnr non-radiative transitions. So, tsp here request to spontaneous emission but also there can be an atom come down without even giving A photon.

And they are called non-radiative transitions, so it is the same relation that we have in a semiconductor that one over tau lifetime is equal to one over radiative life time+one over non-radiative life time. So, A is given one over tsp this is called spontaneous emission life time.

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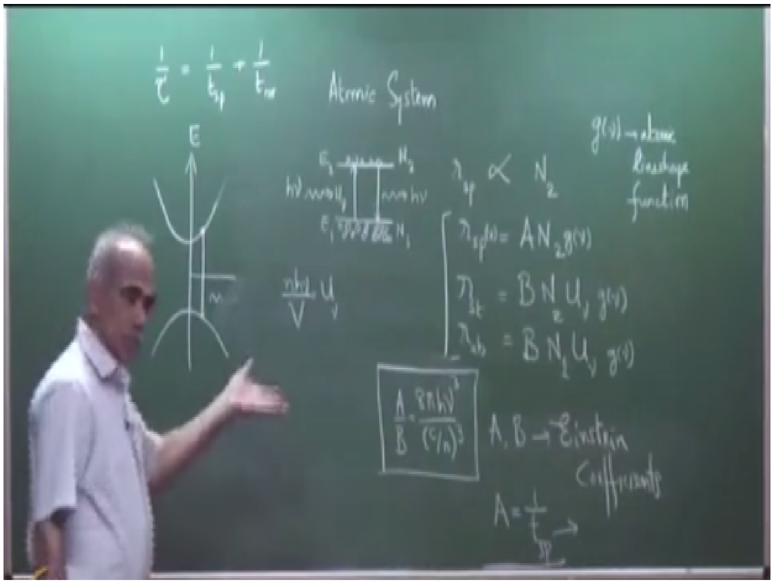

And the relation between A and B relation between the Einstein coefficients is given by A/B is equal to 8pye h mu 2/cq or in a medium of refractive in the same it is (c/n) cube this is the relation between the Einstein co efficient A and B. And A is equal to 1 over tsp why I am writing this is because the coefficient that we have in semi conductor A and d also have the same relation.

So the rate of therefore this I have written for the atomic system and now I write for in the semi conductors and then you can see what is the comparison from here in an atomic system and in semi conductors. Here we are writing N1, N2 are number of atoms, here we are dealing with electrons and holes.

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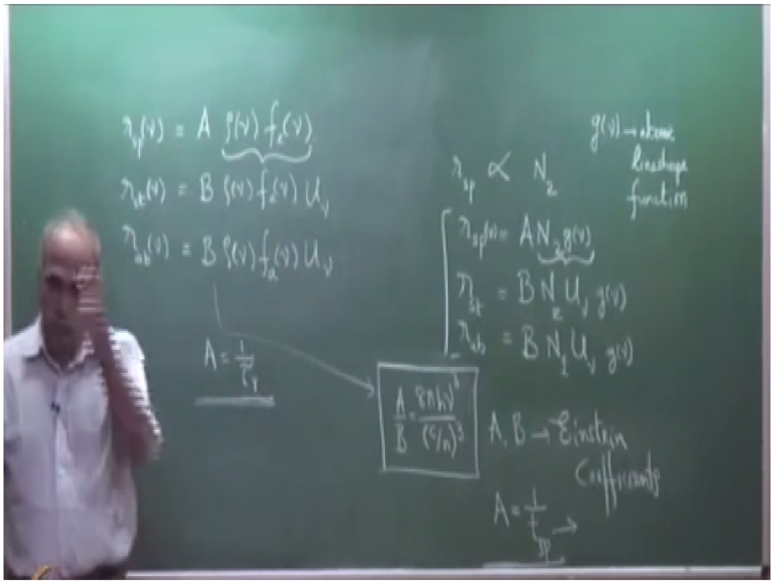

But you see the similarity and the relations, so for semi conductors R sp of mu is equal to the constant A into row of mu into f e of mu is this spontaneous emission. Stimulated emission rst of mu is equal to constant B actually it is A dash and B dash because the numerical value is not the same but the relation is the same. So, B into row of mu into f e of mu into energy density u of mu and r absorption of mu is equal to B into row of mu and now f a of mu into u.

And A and B have the same relation here in this case A is the spontaneous emission life time here A with A =1 over tau r radiative recombination lifetime. There are techniques these are measurable parameters you will see ultimately that everything will be in terms of lifetimes because lifetime is a measurable parameter. If time permits later on I will discuss the measurement technique how one can measure the lifetime.

So, A is equal to it is completely 1 to 1 correspondence and what do you see that if you replace n2 into g mu this tells you effective number of atoms available for the transition and this tells you the same thing. So, it is row mu into f e of mu is replaced by this otherwise it is the same. So, these are the rate of spontaneous emission stimulated emission and absorption. The difference between stimulated and spontaneous is the energy density.

Spontaneous emission does require any radiation to be present it is spontaneously makes the downward transition. In stimulated emission you need radiation to be present and therefore u mu here refer the density of states. So, u mu here refers to the energy density in the medium.

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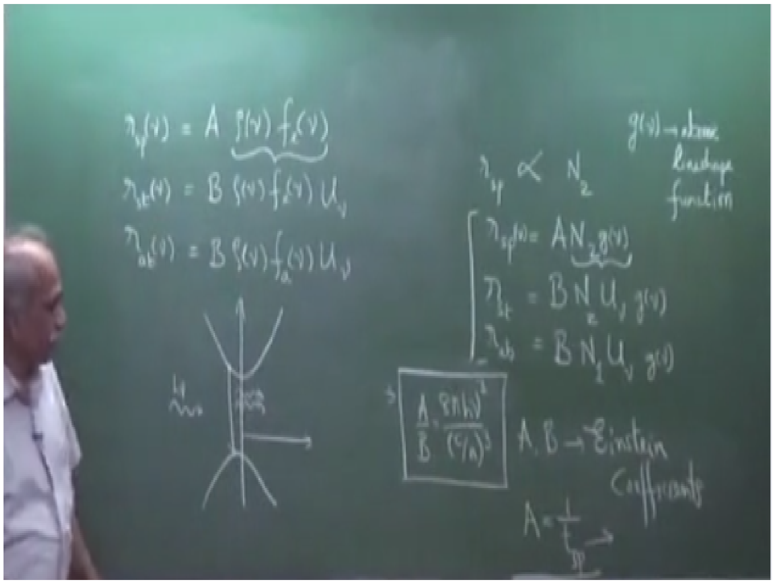

So, in stimulated emission if I want to illustrate stimulated emission then I have photons coming here of energy density u mu and electron which is here making a downward transition and leads to, so if there are more number of photons coming here it can induce more number of downward transition that is why stimulated emission rate is proportional to u of mu larger the number of photons which are incident here larger will be the possibility of stimulated emission.

And the coefficient here is the, so these are the Einstein coefficient they are the same as this not in magnitude but in nature and the relation is the same in semi conductors. So, let us find out first the rate of spontaneous emission first we will discuss spontaneous emission because as I have already mentioned spontaneous emission is the basis of operation of LEDs. So, we will discuss the rate of spontaneous emission.

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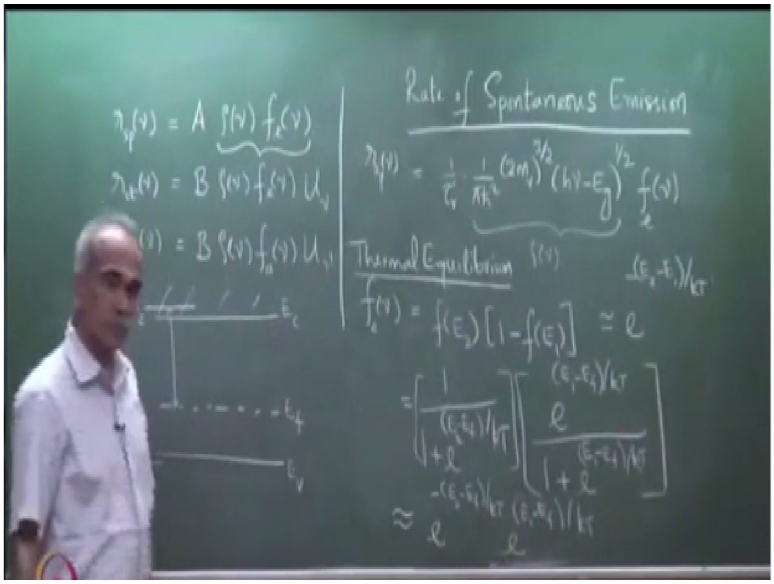

We will find out what is this rate we have written an expression here but let us see what is that rate. So, rate of spontaneous emission rsp at the frequency mu because the density optical joint density depends on the frequency mu rsp of mu is equal to 1/tau r into row of mu density optical joint density of states 1/pie h cross square into twice mr to the power 3/2 please check the expression into h mu- eg to the power half.

So, this is row of mu, so a this is row of mu multiplied by f e of mu probability of emission. Let us see the probability of emission f e of mu is equal to if you recall first at thermal equilibrium let us see a thermal equilibrium. I want to find f e of mu, so we have a semi conductor the Fermi level is somewhere I do not know P type or N type I am just taking semi conductor not taking a P and A junction yet.

And this is Ec a semi conductor in thermal equilibrium f e of mu is equal to f of E2 that is probability of having an electron at state E2 into probability of having a hole at state E1 1- f of E1. So, substitute f E2 is 1/1+E to the power E-f so E2-Ef/k into this one will give me E to the power E1-Ef/Kt in numerator/1+E to the power E1-Ef/ kT. Let us approximate a little bit Ef is here E1 remember that E2 is a energy level here in the conduction in the valance band in the conduction band, so E2 is an energy level in the conduction band E1 is an energy level in the valance band here.

So, you can see that this gap that is E2-here is much greater than kT at room temperature this is the bores 1 approximation E2-Ef is much greater than kT because is about 0.025 ev therefore just like we did bores 1 approximation for carrier concentration under the bores 1 approximation E2-Ef is much greater than kT therefore the exponent is a very large number, so I can neglect to 1 with respect to E to the power large exponent which means this second term here is in first term here.

The first term this one is simply approximately that you write approximately I want to see what kind of numbers are there approximately equal to E to the power so only this one now it is – goes to the numerator E2-Ef/kT. The second term here is approximately equal to E to the power E2-Ef/kT. What about that this term this is a large number E1-Ef/kT is a large number, so you can neglect but it is a large number but E1if less than Ef please see E1 here Ef is here.

So, E1-Ef is a negative number, so it is a large but negative number therefore this is close to 0 and therefore I can write this term is close to 0, so I neglect this, so what we are left with is only E2 to the power E1-Ef/kT. And what is that you see +Ef and –Ef/kT that goes off, so you are left with E2 to the power of –E2-E1, so this is therefore this is approximately equal to E to the power –E2-E1/kT. E2-E1 is what h mu, so this is equal to okay let me erase this.

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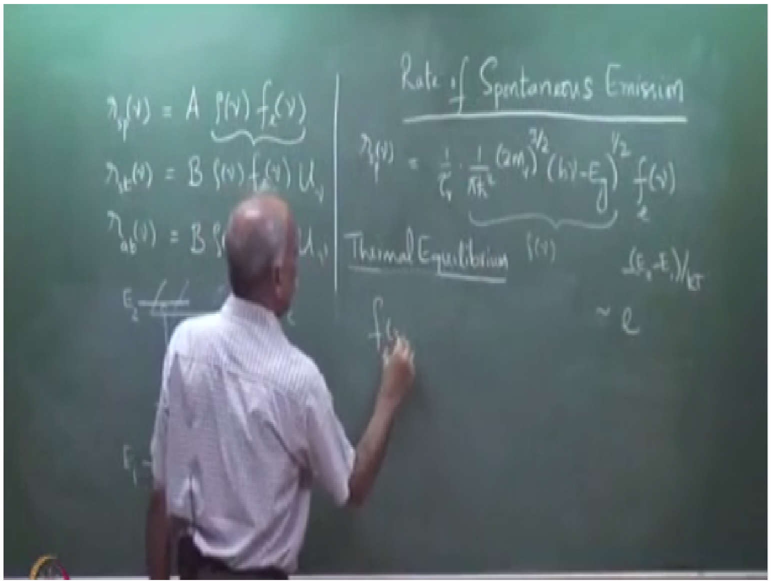

So what we have got is f e of mu is equal to E to the power I should not have erased E to the power- let me write again E2-Ef/kT into E to the power E1-Ef/kT which is equal to f e this term is approximately equal to because I have made approximation here this is equal to Ef Ef goes and we have e to the power – h mu/kT. So, f e of mu is e to the power –h mu/kT.

You can put some number if you are talking of visible photons corresponding to visible light h mu is about 1.5 or 2 ev and kT is extremely small compare to that therefore this is a very large number e to the power –this, so f e of mu is an extremely small number probability of a mission at thermal equilibrium is extremely small. Suppose I was in quasi equilibrium we were in a state of quasi equilibrium.

So, we will substitute this here in a minute but just before I proceed suppose we were in quazy equilibrium which means let us say we have pump.

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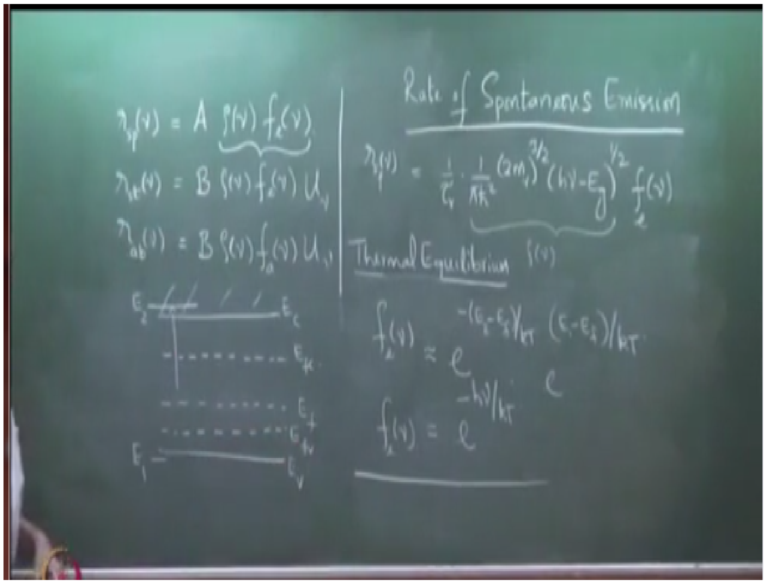

So, this was our original here we excite this medium by maybe by illumination maybe by current injection whatever reason so that the Ef separate out 2 efs come now, so we have 1 e f for the conduction band Efc and Ef for the valance band, so Efv, so I erase this thermal equilibrium case, so this was thermal equilibrium.

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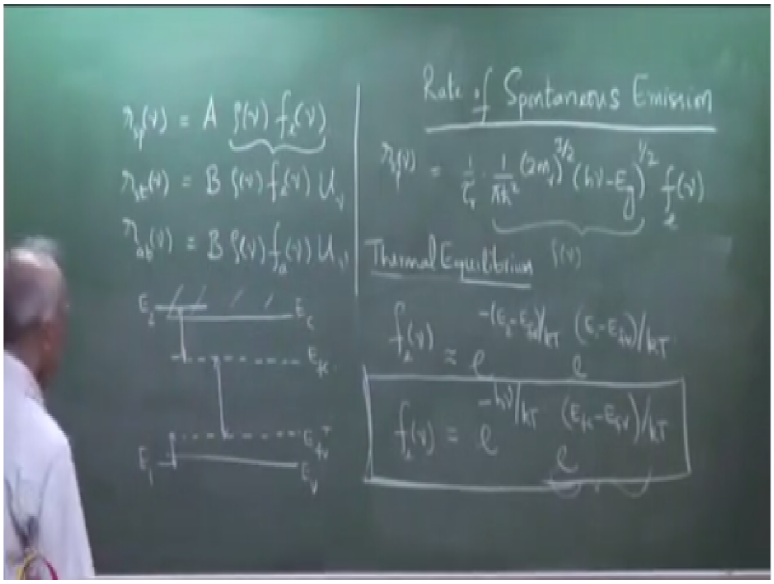

Now this is in quasi equilibrium. So, what difference would we see please see I have it is not large enough that Ef and Ec have merged here still this gap is sufficiently large between E1 and Ef, so that I can still apply (()) (30:03) approximation. So, what difference in this expression will come, in this we will have Efc and here we have Efv that is the only difference Efc, E2-Efc/kT.

And E1-Efv/kT, please write this expression again, what do we then have earlier Ef Ef was it cancelled each other, now we have this multiplied by e to the power Efc-Efv/kT. So, the probability of emission in quasi equilibrium has changed to this expression where this was in thermal equilibrium that is now multiplied by a term Efc-Efv/kT seeing this diagram that I have drawn Efc-Efv is this gap it is much larger compare kT 0.025ev.

And therefore this exponent here is a very large number now this is very large no doubt but this is also large the point is by pushing the semi conductor into quasi equilibrium you have additional multiplication factor which will increase Fe of mu by orders of magnitude a semi conductor in thermal equilibrium had Fe of mu approximately equal to E to the power –h mu/kT which means the probability was very small by pushing the semi conductor into quasi equilibrium maybe by injection of carriers maybe by illumination different means.

You have an additional factor which is also a very large factor in other words you can change the probability of emission by orders of magnitude we will see the this implication later that is how in an LED, LED before base there is nothing 0 output you just forward biasing light starts coming out what are you doing by forward biasing you are simply raising the probability of emission alright we will discuss more about this a little later.

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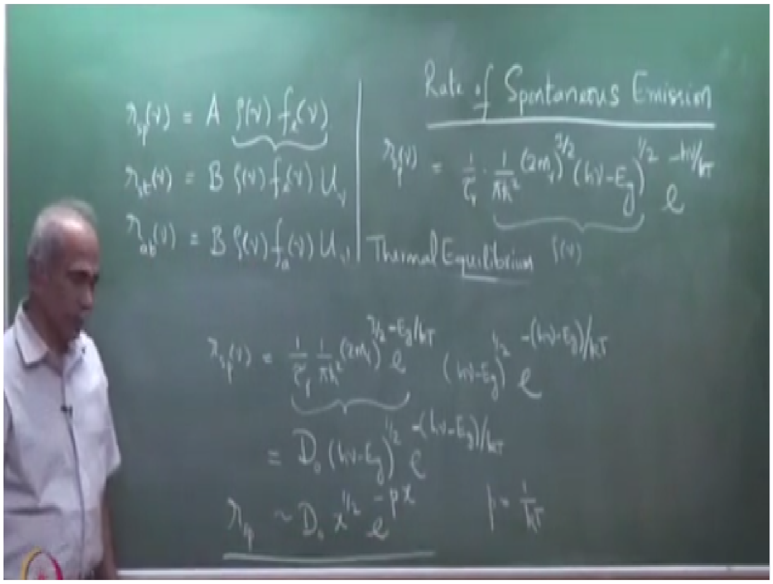

So, let me continue with the thermal equilibrium case, so I erase this, so in thermal equilibrium the rate of spontaneous emission is given by so we have the expression here Fe of mu is please write you could write this again e to the power-h mu/kT, Fe of mu is replaced by e to the power –h mu/kT. We want to simply this to a form if you multiply by this expression by e to the power Eg/kT and e to the power –Eg/kT**.**

So, Rsp is equal to 1/tau r, 1/pie h cross square twice mr to the power 3/2, e to the power Eg/k Tinto h mu-Eg to the power half I have simply multiplied by e to the power Eg/kT and e to the power –Eg/kT e to the power h mu-Eg/kT, e to the power –Eg/kT is here and e to the Eg/kT is here is just to write it in the form of h to the purpose. So, this therefore this constant here if we call it as some constant D or D0 then this into h mu-Eg to the power half into E to the power h mu oh this is minus, e to the power –h mu/kT.

In which case so minus, minus plus, so I should have – here please correct it, so e to the power –h mu-Eg because h mu is greater than Eg therefore –h mu- so please correct this. So, this is of the form some constant D0 into X to the power half into e to the power –ax where a is some constant a or p let me write p because a we have used for lattice constant, so e to the power –px where is equal to 1/kT is that okay.

This is off the form x to the power half that is x is h mu-Eg I am calling as x, h to the power half into e to the power–px where p is 1/kT, so this is the rate of spontaneous emission. I want to see what kind of variation I will get that is why I have written it that form.

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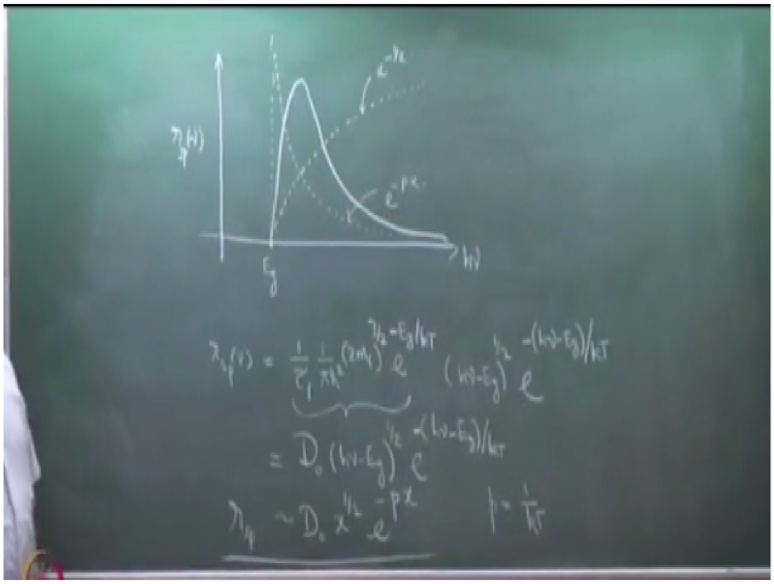

$$r_{sp}(\nu) = \underbrace{\frac{1}{\tau_r}\frac{1}{\pi\hbar^2}(2m_r)^{3/2} e^{-E_g/kT}} (h\nu-E_g)^{1/2} e^{-(h\nu-E_g)/kT}$$

$$= D_0 (h\nu-E_g)^{1/2} e^{-(h\nu-E_g)/kT}$$

$$r_{sp} \sim D_0 x^{1/2} e^{-px} \qquad p = \frac{1}{kT}$$

So, how does this vary we want to find out rate of spontaneous emission, so this is h mu photon energy h mu and here I am plotting Rsp of mu, so where will it start h mu equal to Eg because all density of states everything is valid for h mu greater than Eg, so this is Eg h mu equal to Eg is x equal to 0, so from here as h mu increases this goes as h to the power half, so h to the power half variation goes like this h to the power half variation.

E to the power –px variation is exponential drop okay, so e power –px let x equals to 0 it has the value 1 at x equals to 0 and then it drops down rapidly, so it is an exponential drop depending on the value of p is p is very large as you can see here it is a large quantity 1/kT, kT is 0.025 that means 1/0.025 40, so p is about 40 it is dropping like this, so this is e power –px variation and this is x power half variation and D0 is a constant.

So, D0 does not involve mu so it is a constant, so how would the net result look like so the net result would like initially this function is very, so it is a product of functions product of 2 functions so the product is 0 here but suddenly this is a large value, so the product jumps it goes up to some maximum because later on exponential takes over please remember that x power half ho I am sorry why did I write x power–half, x power half please correct this x power half.

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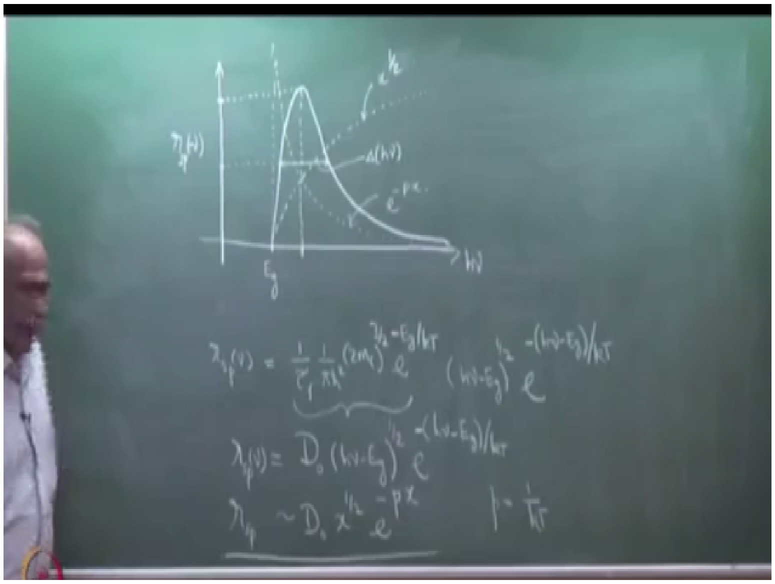

X power half is a much slower variation exponential dependence is much faster so exponential takes over and then the net drops down, so what I have plotted is rate of spontaneous emission as a function of h mu which means it is maximum at some value here so Rsp max corresponds to some value here, so this is Rsp max and it also has a certain line width line width in the sense where Fwhm from here if you come down to half it is value approximately half it is value.

So, this is delta lamda, it is delta actually here in this case but you can find out what is the corresponding of in this case it is delta h mu the full width at half maximum gives you the line width of this source, full width at half maximum. How do we find out what is the maximum where does the maximum occur we have an expression here for Rsp mu is equal to D0 with does not have mu h mu-Eg here to the power half into this.

We want to find out the maxima how to find out the maxima, so how do we find the maximum you differentiate this function with respect to h mu and find out where is the maxima.

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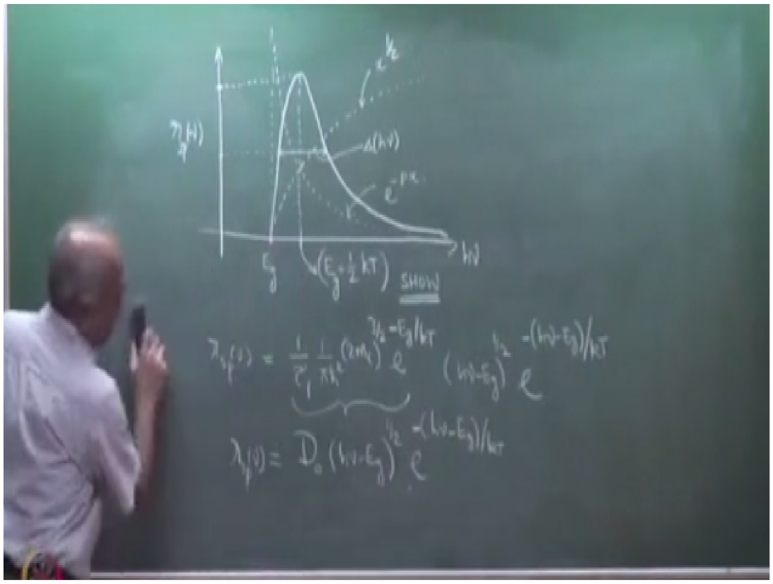

So, please do this as an exercise and see where do you get this maxima this maxima corresponds to Eg+half kT please show this that the maxima appears at Eg+half do this as an exercise and once you know that the maxima is at Eg+half kT simply substitute Eg+half kT here and you get what is the maximum value Rsp of mu if you know that it occurs the maxima occurs at Eg+half kT substitute this value for h mu.

So, Eg is goes away you are left with half kT to the power half and here again half kT so kT kT cancelled see what you get. So, you know what is the maximum value here and we know every parameter in this for a given material we know radiative recombination lifetime, constant, constant, the reduced mass all parameters are known. So, I take given temperature you can you know what is D0.

And therefore you can find out what is the maximum rate of spontaneous emission remember that rate of spontaneous emission is number of emissions per unit time per unit volume if you multiply this by the energy of the photon h mu it will give you power output from that material power per unit volume.

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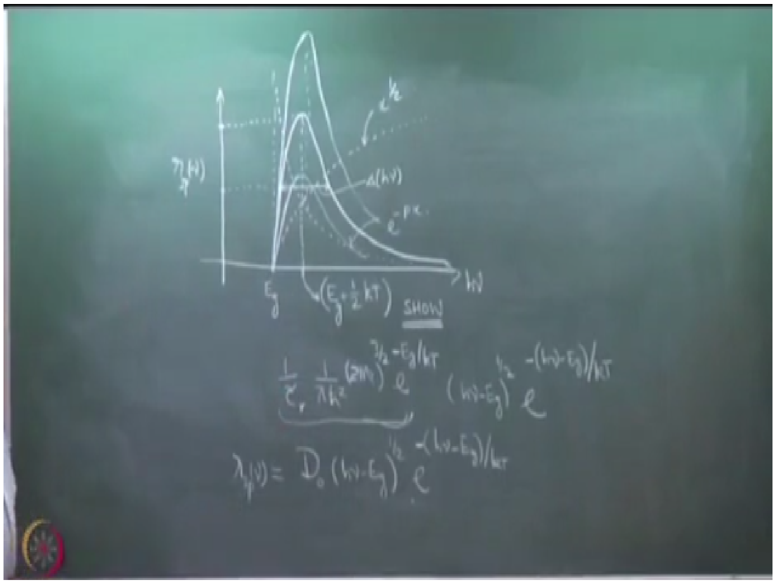

One other thing that note that the D0 alright let me write twice mr 1/pie h cross square what else did I have 1/tau r the D0 the Do value of D0 tells you how far it goes it is an amplitude multiplying constant here, so whether if it this variation goes like this whether this variation does like this is determined by D0 and what is determined by what determines D0, D0 is determined by temperature all are constants for a given material, so 2 things 1 is tau r radiative recombination time and here T.

So, larger if the temperature is larger than this exponential negative quantity becomes smaller and therefore D0 will be higher. So, it is instructive to see that you put different temperatures for example you put 250 k here, 300 k, 350 k you see that it changes. Similarly when you put T different values the peak shift because this is Eg+half kT larger kT is the peak shift, so deliberately I have shown the peak shifting like we see.

So, this peak shifting because, higher temperature will have larger spontaneous emission coming out and also the peak shifted by a small value. So, this is this tells us the rate of spontaneous emission from which we can calculate the spontaneously emitted power at nay frequency greater than each. So, please do this as an exercise to find out the h mu corresponding to the peak value and this one corresponding to the full width at half maximum.

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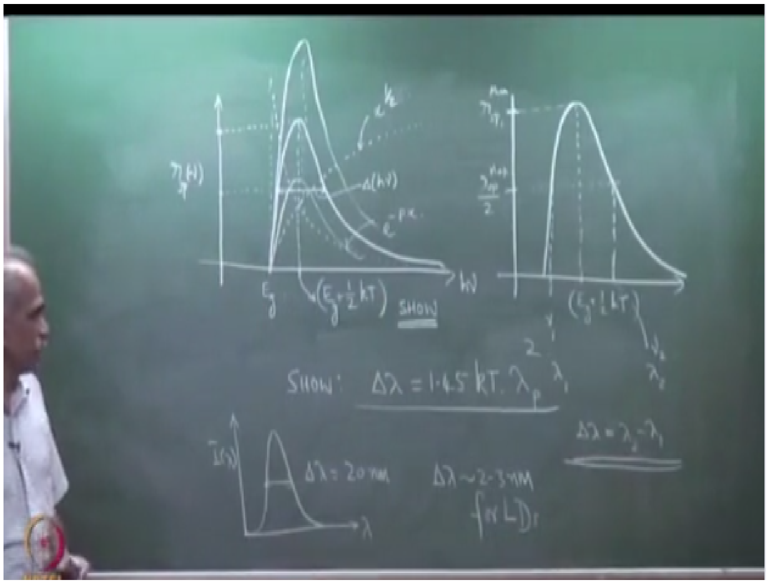

So, you can show that in terms of lamda, delta lamda I think is nearly equal to 1.45 into kT delta lamda comes out to be I think it is 1.45 into kT multiplied by lamda p square. So, 2 exercises please work this out delta lamda is equal to so delta lamda here refers to so let me just explain this how you would go about it and then I will stop, so first for a given temperature find out this.

So, you know Eg+half kT substituting this value you know what is the maximum value Rsp maximum, so this is max, so where Rsp becomes max half, so this is the value where this is Rsp max by 2 there are 2 values of h mu and hence mu you will get 2 different values for which you will have the value Rsp max by 2 at 2 different h mu value. So, correspondingly you will get 2 frequencies 1 here for mu 1 and 1 here mu 2 h mu 1 and h mu 2.

So, you know mu 1 and mu 2 if you know mu 1 and mu 2 you can find out what is lamda 1 and lamda 2 simply see by mu 1 and see by mu 2 and therefore delta lamda is equal to lamda 2-lamda 1. Why we want in delta lamda is normally the light, the line width of the source is expressed in terms of nano meters wavelength delta lamda. You may have heard or studied that an LED has a line width delta lamda equal to 20 nano meters what I have plotted I have plotted intensity output i lamda as a function of lamda.

Optical measurements are done at wavelength, so you change the wavelength and measure the spectrum of the an LED output and you have intensity variation as a function of lamda the full width at half maximum is delta lamda which is typically 20 to 30 nano meter for a light emitting diode. If you measure the same thing for a laser diode normal laser diode this delta lamda will be of the order of 2 to 3 nano meter for laser diodes for normal laser diodes fabricator laser diodes.

We will study about specialised laser diodes, special laser diodes which have extremely small delta lamda single frequency laser diodes. So, that is why please calculate delta lamda in terms of nano meters, lamda p here corresponds to the peak value, the peak wavelength here that is lamda p alright. So, I will stop here and in the next class we will discuss about rates of stimulated emission and absorption and see the condition for gain under what condition we can have gain from the semi conductor.

Spontaneous emission there is no gain but you have an emission spectrum but stimulated emission can lead to gain under certain circumstance certain conditions and we will see that under what condition we will get gain.