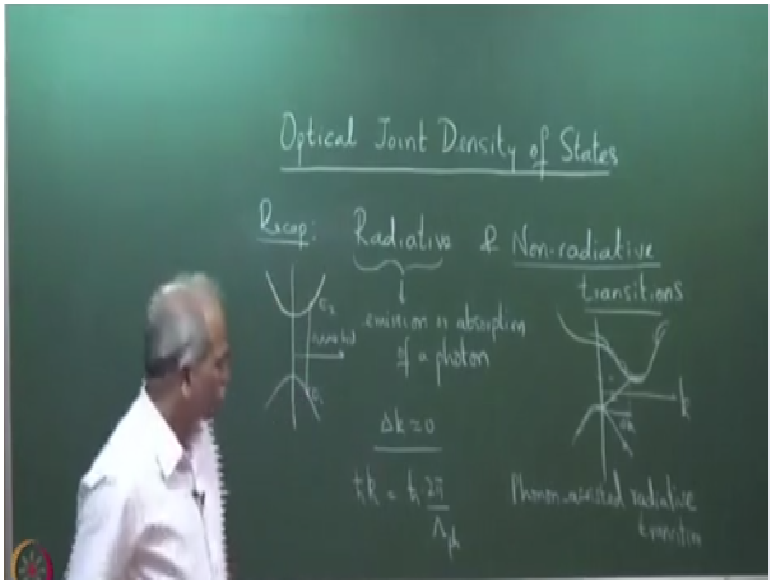
Semiconductor Optoelectronics
Prof. M.R. Shenoy
Department of Physics
Indian Institute of Technology-Delhi

Lecture-18
Optical Joint Density of States, and Probabilities of Emission and Absorption

(Refer Slide Time: 00:23)



So, today we will discuss the concept of optical drawing density of states, so before that we will recall what we had studied in the last class that is we discussed about radiative and non-radiative recombination, radiative and non-radiative transitions as a name indicates radiative transition involves emission or absorption of a photon, and non-radiative transition does not involve emission or absorption of a photon.

It is quite clear when we look at this that an electron it sits in the conduction band if it is makes a downward transition to the valance band, the difference in energy is given in the form of h mew, so this is E2 this is E1, E1 is an energy level in the valance band and E2 is some energy level in the conduction band. So, it involves emission of a photon or it could be incident photon leading to a transition of electron from the valance band to the conduction band.

In the case of non-radiative transition the energy difference is given in the form of phonons. The energy difference is this is usually the dominant forces in the case of indirect band gap semi conductors. So, for example if I show an indirect band gap semi conductor like this whatever be the process of excitation whether it is optical process of excitation or current injection it essentially leads to injection of carriers in to this band which then come down by thermalisation band accumulate at the bottom of the band.

Similarly here whatever hole generated wherever it is generated whether a hole is generated here or here or here it will start moving to the top because the top of the band corresponds to low energy for holes or electrons move down to lower levels and holes accumulate here. Now if an electron wants to make recombination with hole then it has to make an oblique transition and as we have seen by this is not permitted by the case selection rule.

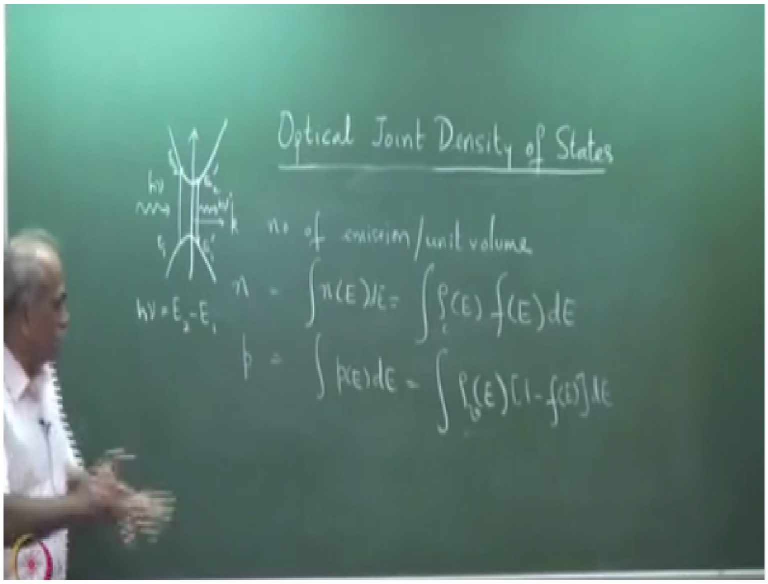
Case selection rule requires that the transition need to be vertical that is delta k should be nearly equal to 0 but this is not permissible by case selection rule. However as we have seen phonons can have large momenta because the momenta is the momentum is given by h cross k which is equal to h cross into 2 pie/lamda here, so this is lamda of phonon, lamda of phonons are of the same order as that of electrons and therefore these this different in delta k.

So, please see this axis is k, so the difference delta k here is made up by phonons they are also another class of transitions which are called phonon assisted radiation, phonon assisted radiative transition. In this case an oblique transition can also lead to the emission of photon but it will be assisted by a phonon a 1 or more than 1 phonons, this is called phonon assisted radiative transition.

This is whereas there is no phonon pure photon transition, radiative transition it can be the energy difference can be given out and momentum difference can be made out by number of phonons participating and there is also possibility of 1 photon+phonon leading to this transition which is called phonon assisted radiative transition. The energy difference is primarily made up of photon but the momentum difference is primarily made up of phonons and that is called phonon assisted radiative transition.

So, keeping this in mind let us discuss the topic of optical joint density of space, now what is the need for defining optical joint density of states we have see in density of states we had a reasonable detailed discussion on density of states tell us the number of states available per unit volume and if you recall that our objective here is now to realise sources and detectors. So, we are interested in emission and absorption means we want to see what is the number of transitions or number of emissions.

(Refer Slide Time: 05:59)



Because number of emissions will tell us number of photons emitted and therefore we can estimate what is the power that is emitted, so number of emissions per unit volume is given by the number of density the density of states multiplied by the probability of occupation what is this let me recall what we had seen the carrier density of electrons in the conduction band was given by density of states row c of E into probability of a occupation.

You recall this n of E and n the carrier concentration was equal to integrated over this, so integrated over this, this was total number of carrier or carrier concentration equal to n of E into this, so n of E is equal to density of states multiplied by probability of occupation. Similarly P of E, P was given by P of E dE which was equal to row v of E into 1-f of e the probably of occupation of holes is 1-f of E dE.

If you keep this at the background then if I consider let us consider a radiation which is incident of energy h mu and consider a level E1 here and a level E2, a level E2 in the conduction band for a given photon of energy h mew or frequency mew if you fix E2, E1 is automatically fixed by the condition that h mew equal to E2-E1. An emission or absorption involves for example absorption, absorption involves an electron state in the E1 at E1 in the valance band.

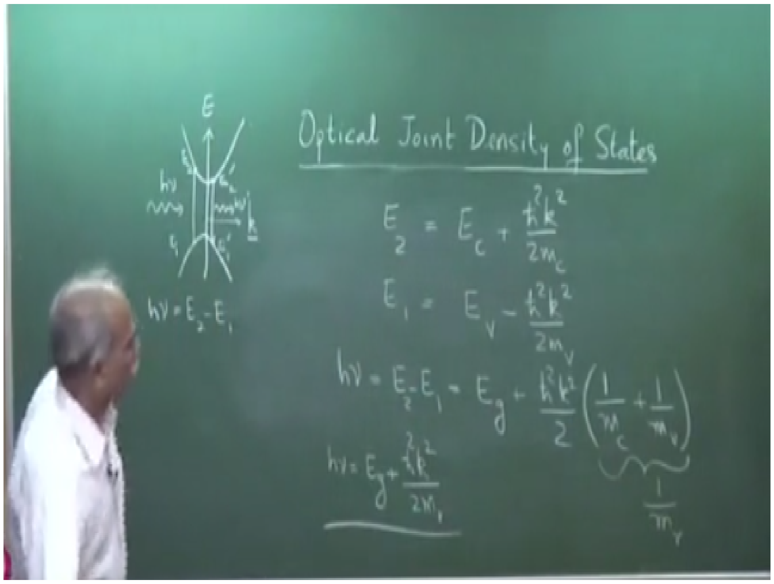
And a valance state E2 in the conduction band, similarly if you are looking at emission then this involves an electron sitting here at some other value let us say E2 dash and a vacant state at E1 dash this would lead to emission of a photon of energy h mew dash. So, emission and absorption process simultaneously involves the a state in the valance band and a state in the conduction band and therefore any process involves states in the valance band and states in the conduction band.

Normally in this case we had density of states in the conduction band, density of states in the valance band because hole concentration does not depend on this hole concentration depends on the density of states in the valance band but in photon interactions you have 1 state here and 1 state here and therefore we define an optical joint density of states which simultaneously takes care of the number of states for a given energy h mew please see.

If an incident energy h mew photon of energy h mew is incident, if you fix E2, E1 is fixed because the difference has to be same I am considering only allow transition I am not considering oblique transitions we consider the allowed transitions are those transitions which follow the case selection rule. So, when E2 is fixed E1 is also fixed and then it is possible that V define a joint density of states.

Let me proceed with a derivation and then it will become clear to you that the joint density of states does not depend on the actual value of E2 and E1 it always depends on E2-E1, so we will just see the derivation and then it will become clear, so the named for defining an optical joint density of states is because the transition involves states in the valance band and conduction band and there can be several combinations of E2 and E1 which will be equal to h mew.

(Refer Slide Time: 10:59)



Optical Joint Density of States

$$E_2 = E_c + \frac{\hbar^2 k^2}{2m_c}$$

$$E_1 = E_v - \frac{\hbar^2 k^2}{2m_v}$$

$$h\nu = E_2 - E_1 = E_g + \frac{\hbar^2 k^2}{2}\underbrace{\left(\frac{1}{m_c} + \frac{1}{m_v}\right)}_{\frac{1}{m_r}}$$

$$h\nu = E_2 - E_1$$

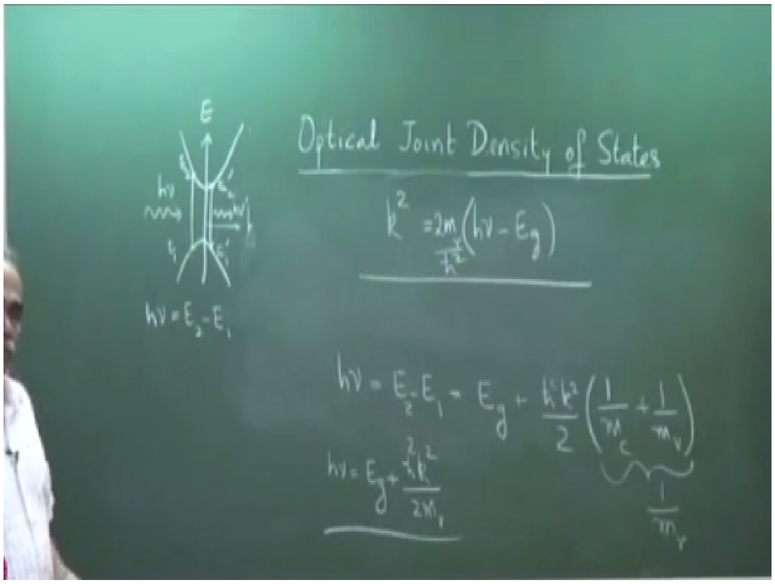
$$h\nu = E_g + \frac{\hbar^2 k^2}{2m_r}$$

We see this is E versus k, k is a vector depending on the direction of propagation of the electron beam the EK diagram continuously changes if you are taking a direction 1,0,0 the EK diagram will be slightly different if you are following along 1, 1 direction then EK diagram will be slightly different which means for a given energy h mew there are several pairs of E2-E1 and therefore there is a density of state that is number of states number of pairs of states available for a photon of energy h mew to interact and this is given by the density of states.

We will see it will become more clear as we proceed for the so E2 is equal to Ec+h cross square k square/ 2mc this is the parabolic approximation which is valid near the bottom of the band parabolic approximation and E1 is equal to Ev-h cross square k square/2mv we already have this. So, E2-E1 h mew is equal to E2-E1 is equal to Ec-Ev is Eg+h cross square k square/2 into 1/mc+1/mv, this is actually 1 over mr, mr is the reduced mass.

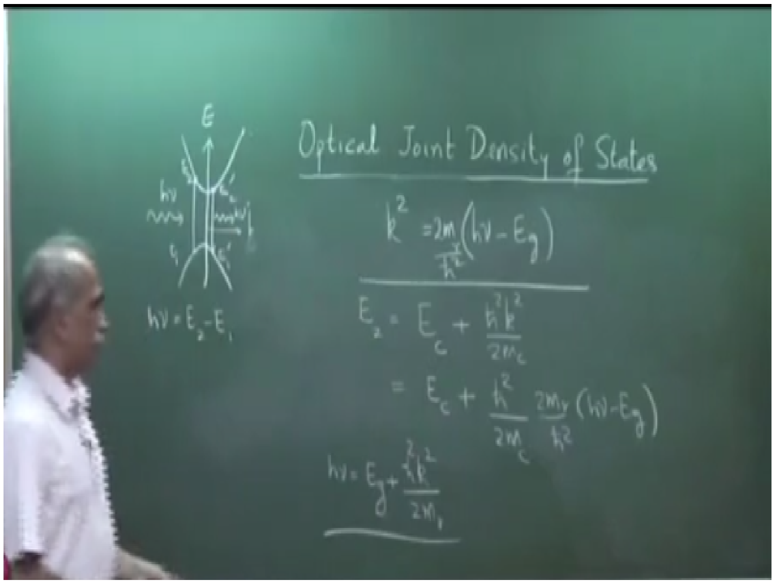
So, we can write h mew is equal to Eg+h cross square k square/2mr where mr is the reduced mass 1/mc+1/mv equal to 1/mr. So, let me remove these and therefore from this expression we have k square is equal to h mew- Eg.

(Refer Slide Time: 13:32)



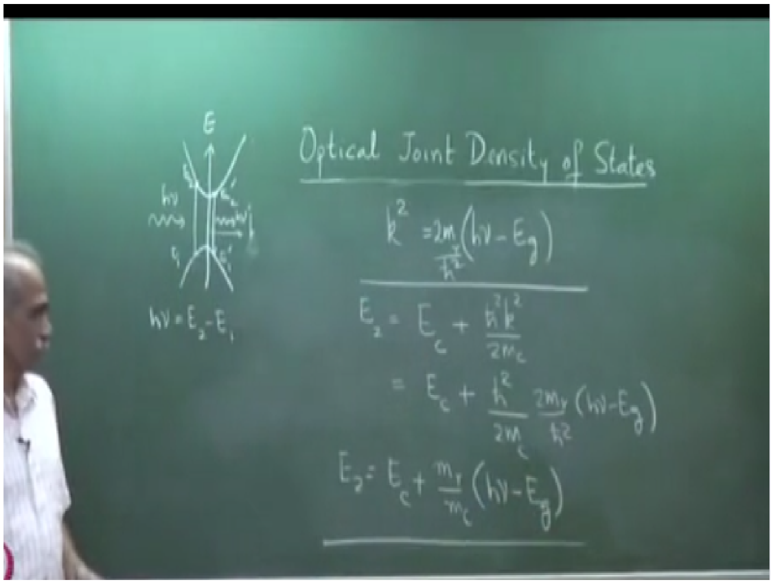
So, h mew-Eg into 2mr/h cross square twice mr/h cross square, so k square is equal to h mew-Eg is this okay.

(Refer Slide Time: 14:03)



And if you substitute in the expression we had E2 is equal to Ec+h cross square k square/2 mc if you substitute foe k square from here we have Ec E2-Ec+h cross square/2 mc maybe write it fully 2mc into this one substitute here so for there is k square, k square is here, so I substitute 2mr/h cross square into h mu-Eg a substituted form.

(Refer Slide Time: 14:52)



Optical Joint Density of States

$$k^2 = \frac{2m_r}{\hbar^2}(h\nu - E_g)$$

$$E_2 = E_c + \frac{\hbar^2 k^2}{2m_c}$$

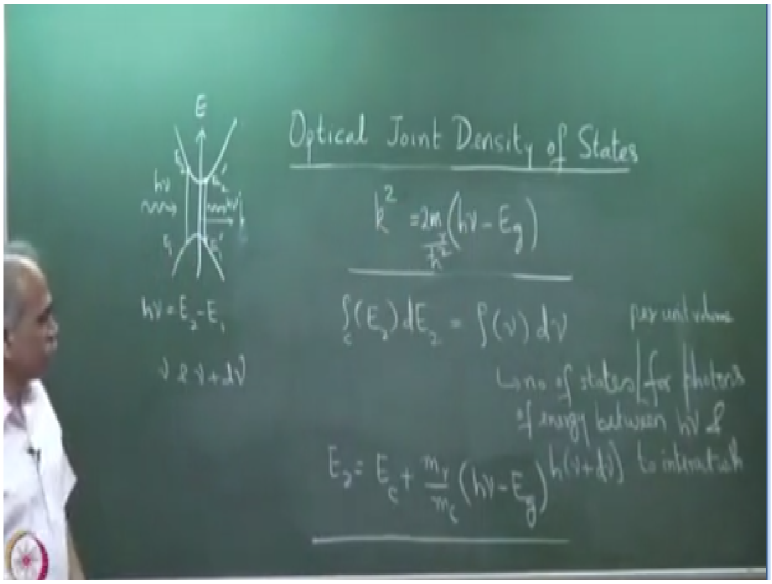
$$= E_c + \frac{\hbar^2}{2m_c}\frac{2m_r}{\hbar^2}(h\nu - E_g)$$

$$E_2 = E_c + \frac{m_r}{m_c}(h\nu - E_g)$$

$h\nu = E_2 - E_1$

So, E2 is equal to Ec+mr/mc into h mew-Eg what do we see for every value of E2 there is a value of mew, mew and E2 there is no E1 in this because E2, E1 automatically gets fixed once you say h mew and therefore there is a 1 to 1 correspondence this a constant these are all constant only E2 is any energy level here which means E2 is a variable and mew is a variable and therefore there is a 1 to 1 correspondence between E2 and E1 exactly like this you can also write the in terms of E1 and then you see that there is 1 to 1 correspondence between E1 and mu.

(Refer Slide Time: 16:00)



Optical Joint Density of States

$$k^2 = \frac{2m_r}{\hbar^2}(h\nu - E_g)$$

$$\rho_c(E_2)\,dE_2 = \rho(\nu)\,d\nu$$ per unit volume

↳ no. of states/for photons of energy between $h\nu$ & $h(\nu + d\nu)$ to interact with

$$E_2 = E_c + \frac{m_r}{m_c}(h\nu - E_g)$$

$h\nu = E_2 - E_1$

ν & $\nu + d\nu$

This means that row of E2, DE2 must be equal to row of mew d row there is a 1 to 1 correspondence between mu and E2 therefore the number of states between E2 and E2+dE2 row E2, dE2 is the number of states between E2 and dE2 this row is actually row c conduction band density of states row c of E2 is dE2 is equal to row mew d mew what is row mew d mew is the number of states for photons of energy between h mew and h mu+delta mew, h mu+d mew to interact with row mud.

Mu is the number of states per unit volume number of states please include per unit volume because you remember this was per unit volume per unit volume of the material per unit volume number of states per unit volume evaluable for the photons to interact to interact means what to emit or absorb. If photons are incident then if photons are incident with frequencies between mu and mu+d mu**.**

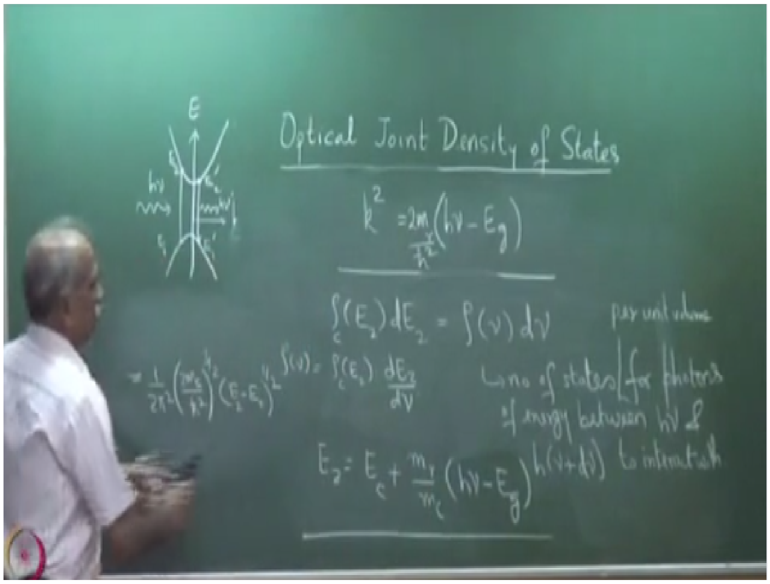
Then the number of states evaluable for these photons to interact per unit volume is given by row mew d mew. And this row mew d mew is related to row of E2 d E2 because of this one to one relation. And therefore our interest is to get row mew why we are interested in row mew because once you know the density of states evaluable for interaction means emission or absorption in this case.

So, number of states evaluable for interaction multiplied by the probability of that interaction probability of emission or probability of absorption will give you total number of emissions or total number of absorptions per unit volume. So that is our interest to find out how many photons are emitted per unit volume of the material. And then of course if you multiply by the rate constant it will give you rate of emission.

Finally we are interested in the rates of emission and absorption so, optical joint density of states the advantage of this is it is simultaneously takes care of number of states in the valence band and the conduction band. It does not know which is E1 and which is E2 it knows that if h mew is this energy gap how many pairs are possible where are those pairs corresponding to which the energy difference is so, how many states are available for a photon of energy h mew to interact that is given by optical joined density of states.

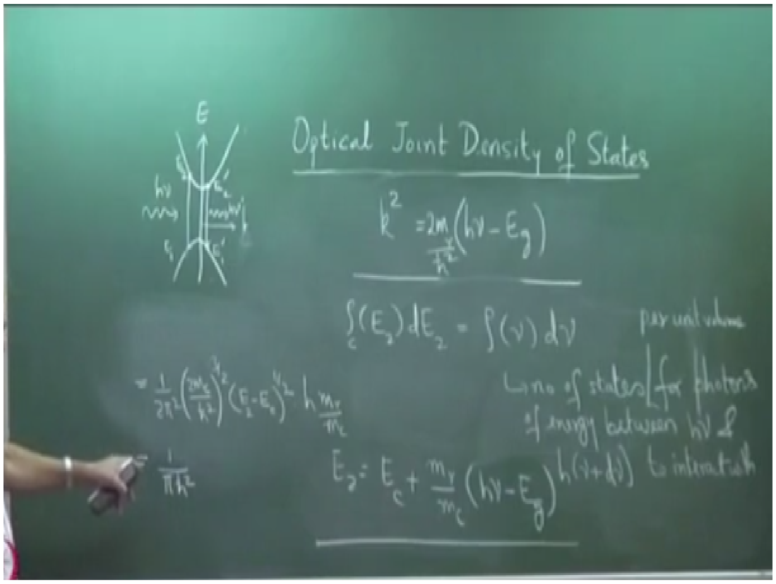
It many books re-teaches the also write this is cv because row c is for the conduction band row v is for the valence band. And this is cv or some books also write this as row op optical so, but we can just drop that v I do not think we have any other where we have row mew and therefore I would simply keep as row mew d mew, row mew d mew is the number of states for unit volume for photons of energy between h mew and h mew mu+d mu to interact with alright.

(Refer Slide Time: 20:28)



And therefore so simplify this therefore row of mew is equal to row c of E2 multiplied by dE2/d2. We can simplify this for the row E2 we know the expression for row E2 that is equal to I will write it here 1/2pie square into 2 mc/h cross square to the 3/2 recall the definition of a row c of E2 into E-Ec E2-Ec to the power half this was the first term row c of E2 multiplied by dE2/d mew what is dE2/dmu, dE2/dmu this is a constant, this is a constant.

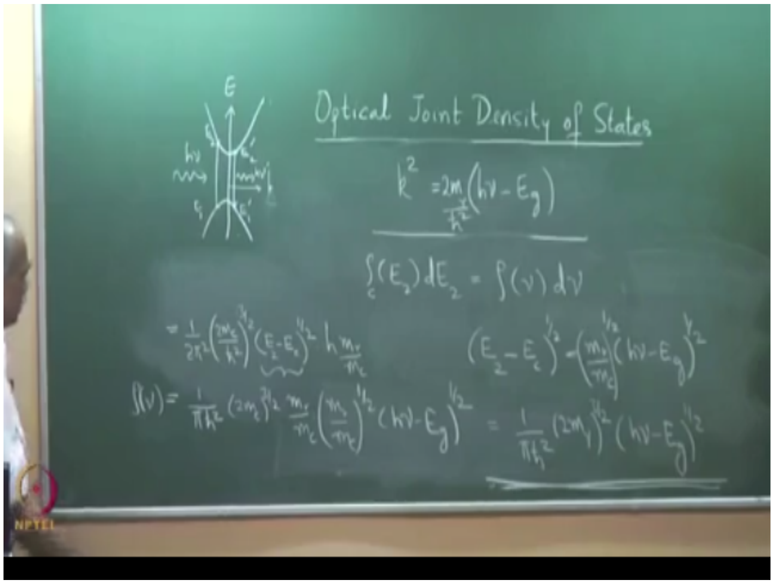
(Refer Slide Time: 21:31)



So, we simply have h into mr/mc, so multiplied by, so let me erase this into h into mr/x this simplifies for the 1, 2 pie from this we can take with this and say h cross 2 pie from here I bring here h/2 pie h cross 1 h cross this is h cross square to the 3/2 is h cross q and it cancels and so, I am left with 1 divided by pie h cross square into 2 mc to the power 3/2 that is 1 mc here is that something else that I have missed.

This is alright mc to the power 3/2 what I have missed dE2/d mew I should have a term mr/mc here wait a minute a row of mew is equal to row E2 into d E2/dmu so dE2/ there was a was there a power half here h mew-Eg did I miss something 1/pie s square that is correct mr, mc is here mr what have I missed that is correct what I have written is correct there is nothing wrong okay it is perfectly fine.

(Refer Slide Time: 23:54)

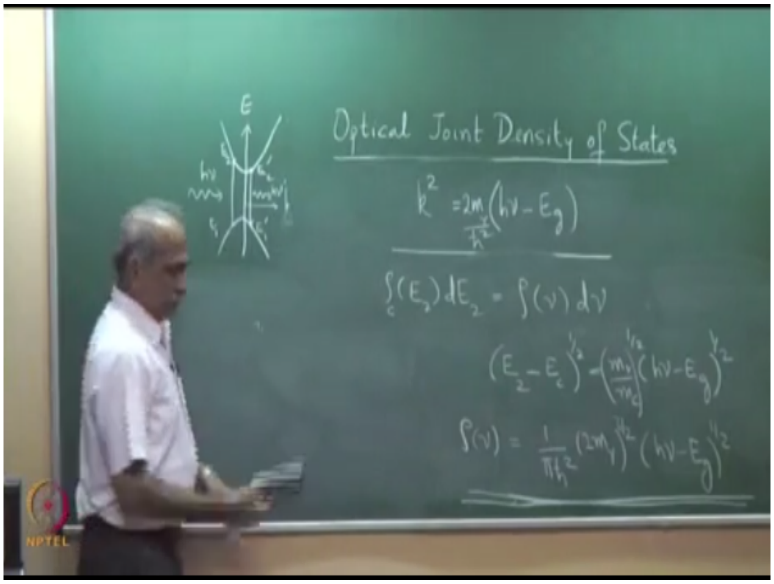


I want to substitute for E2-Ec I just now erased that I am sorry, so E2-Ec E2 was equal to E2+Ec and then +mr/mc so this is equal to mr/mc into h mew-Eg, E2-Ec, so E2-Ec to the power half equal to this multiplied by the power half I am sorry and for this term here I want to substitute this is that alright there is no confusion I suppose that for this also I wanted to substitute E2-Ec to the power half is this.

So, I will let me write all of this 1/pie h cross square 2 mc to the power 3/2 here and into this h is already taken, so mr/mc into all of this mr/mc to the power half into h mew-Eg to the power it is simple algebra you can do yourself but I thought I will finish it. So, now you can see mc to the power 3/2 cancels with mc to the power 3/2. But mr to the power 3/2 will remain so, the final answer will be one over pie h cross square into twice mr to the power 3/2 into h mew –Eg to the power half.

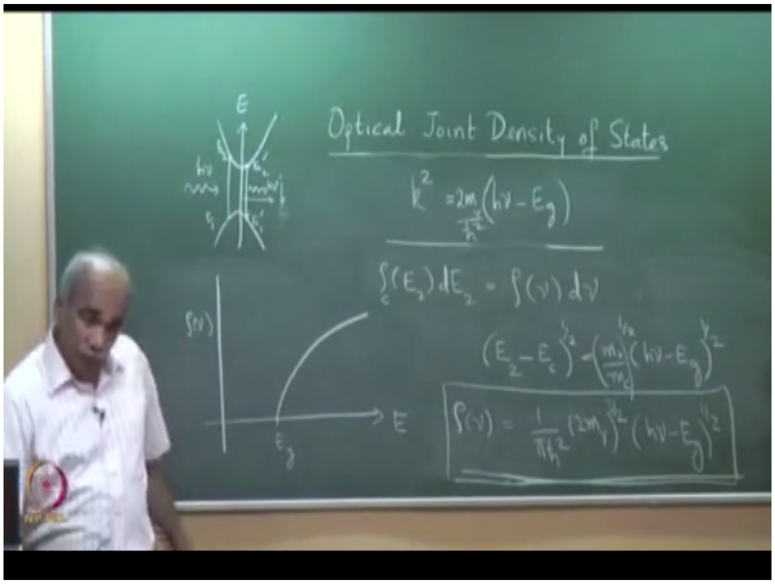
This is expression for row of mew so, let me erase all these unwanted things and directly write the expression.

(Refer Slide Time: 26:00)



So, row of mew here is equal to one over pie h cross square 2 mr to the power 3/2 into h mu–Eg to the power half is it okay. It is a simple algebra therefore, how does row mew vary.

(Refer Slide Time: 26:24)

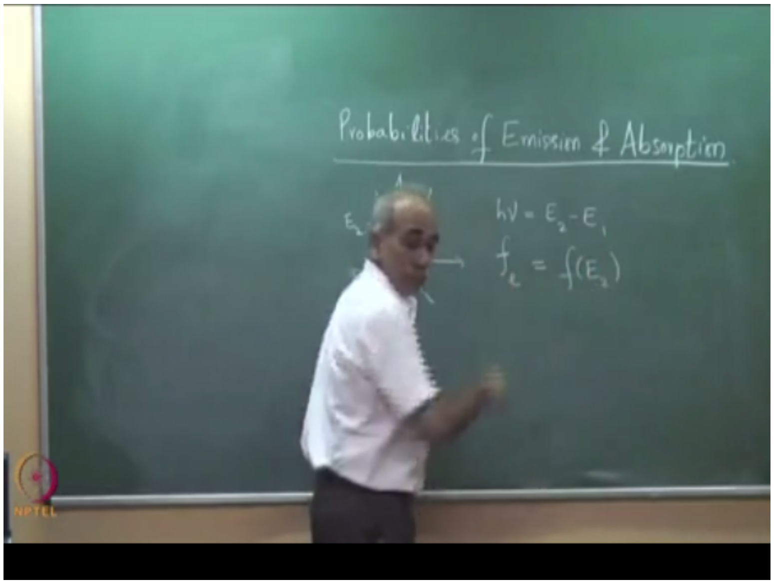


If I want to plot row of mew verses E so, row of mew verses E for normally we plot E here will plot E there also. Please see this for h mew greater than Eg it up to Eg there is because this expression all the expressions are valued for h mew greater than Eg any emission absorption takes place. We are looking at inter band transitions there also intra band processes we will discuss a little later.

Now we are looking at inter band transitions which are possible for h mew greater than Eg therefore from Eg starts as h mu–Eg to the power half variation of row mew. So, we have a mathematical expression for row mew this is very important. Because next we will multiplied by the probability of occupation probability of emission and probability of absorption and we will get an expression for gain coefficient which will contain all these terms that is why we need this derivation.

So, optical joined density of states tells us the number of states evaluable for photons to interact with it is simultaneously takes care of states in the valence band and the conduction band. So, in all optical processes it is the joined density of states which are important. In electronic processes we always deal conduction band and valence band separately but photon emission and absorption involves a state in the valence band and state in the conduction band. Simultaneously and therefore we need to take optical joined density of states alright.

(Refer Slide Time: 28:28)



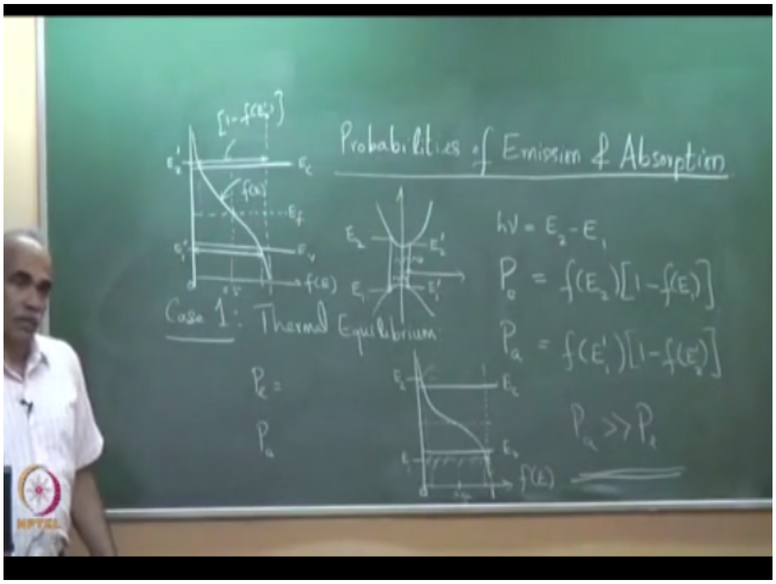
So, let us go to probability of because if we multiply this by the probability of emission will get number of emissions. If we multiply by probability of absorption will get number of absorption. So, probability of probabilities of emission and absorption. So, all the while you see that we are discussing with EK diagram one of the early classes I would said everything can be seen from the EK diagram.

The normal band and energy band diagram does not tell you anything and I am considering direct band gap semiconductors. And we are looking at vertical transitions that is allow transitions other effects we are not taking at the moment. We will see that we will have to take those into a account to explain actually practically absorbed spectrum. So, probability of emission let us for emission.

You need an electron sitting in state E2 which makes a downward transition and therefore you need a hole or a vacant state at E1. So, this is emission what is the probability of emission, probability of emission therefore requires probability of emission of a photon of a energy, h mew equal to E2-E1 requires that the state E2 is occupied by an electron and the state E1 is a vacant state in the valance band which is a hole which means this is probability of occupation of hole.

The probability of emission therefore probability of emission f of e is product of the probability of occupation of electron here and probability of occupation of hole. So, that is equal to f of E2 this f e is the Fermi function you can write this as P you wish we can write this as P probability.

(Refer Slide Time: 31:09)



Because f we may so P of e this is Fermi function f of E2 into 1-f of E exactly like that probability of emission will be given by for probability absorption is given by if you have a state here an electron is sitting at energy E1 dash. And if you have a vacant state here at E2 dash then a photon can get absorbed so, the probability of occurrence of this processes or probability of absorption is probability of occupation f of E1 dash.

And here 1-f of E2 dash as shown two different energy levels otherwise it is f of E1 into this. Let us see graphically because therefore the net probability of net emission will depend on which is greater a probability of emission is greater than probability of absorption than you will have net emission and vice versa. And therefore which one is greater of the two or which one can be greater of the two and how can you make one greater than the other.

If I want to realise the source then I need probability of emission greater than probability of absorption. How to make the probability of emission greater than probability of absorption let us see this. But first there are two cases here case1 thermal equilibrium and case2 will be quasi equilibrium. So, thermal equilibrium means the probability of occupation in the valence band and in the conduction band is simultaneously described by one single Fermi function.

A thermal equilibrium one Fermi function describes the probability of occupation of electrons in the valence band as well as in the conduction band. In quasi equilibrium you will required two different Fermi functions one for the valence band and one for the conduction band. So, let us see first case1 thermal equilibrium probability f of E2 PE is equal to f of E2 into 1-f of E1. Let us see graphically what is this so, we have this is Ev is we do not of Ec I am sorry Ec.

And this Ev and if I want to plot a typical Fermi function we have plotted this earlier also. So, this is f of E this function is a f of E it is 0 at this end 0 and 1 here 1. And 0.5 is here approximately so, I go here and in this semiconductor Ef remains here it need not the intrinsic or anything. I would just taken from semiconductor, and this is the valence band and this is the conduction band and the upper one is the conduction band.

E2 is some value here so, E2 is here and E1 is some value here in the valence band. It could be close to the h so it is some value there and some value. What is f of E2 f of E2 is this at E2 the value of the function here. So, it is this value here which means it is simply this width this width here this magnitude may be 0.1 or 0.2 whatever in my graph actually as you know the numbers are extremely small.

But in my graph it looks like say point because this is 0.5 and therefore this as if 0.1 approximately. This is f of E2 probability of emission is f of E2 into 1-f of E1 what is f of E1 E1 is here so, this value here up to this. This value approximately 0.9 is f of E1 1-f of E1 is this difference here is this clear maybe I should have drawn a little big bigger figure. But I hope you can find out this is 1-f of E1 this is f of E2.

So, f of E2 into f of E1 is this, this product this number whatever it is multiplied by this. So, that is probability of emission so, we have this into this and probability of absorption. Suppose I take E1 and E2 I could have taken let we draw another graph it would be better rather than making complications there. So, this is Ec this is Ed here is the probability f of E and I draw a fresh probability function Fermi function here.

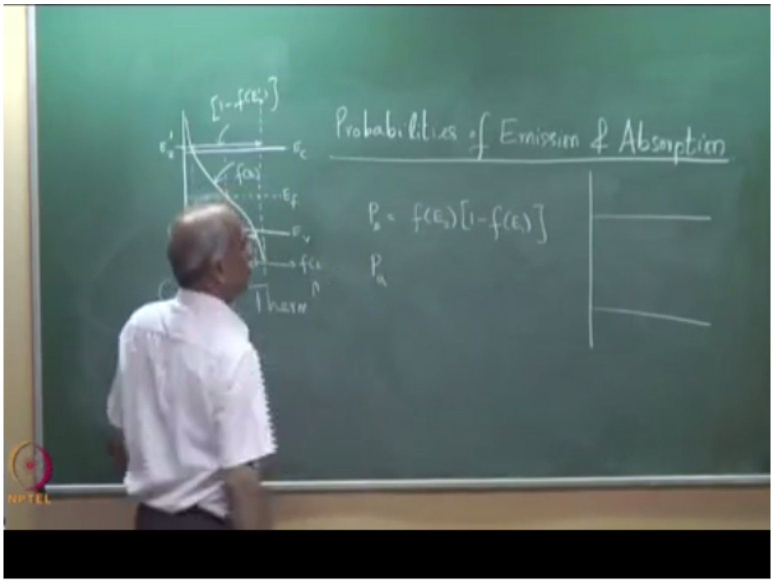
And this is 1 this is 0 0.5 I gave you a question in the minor to graph because I am drawing in front of you regularly. If you should also make practice to draw so, this is approximately 0.5 so, Ef easier for me energies here. So, we now have some value E2 dash here E2 dash and some value E1 dash. I could have please remember I could have taken the same value E2 and E1. Now you see probability f of absorption f of is a product of f of E1 which is this.

This is f of E1 okay because this function is a f of E probability Fermi function therefore at E1 the value of the function is f of E1. So, this is f of E1 this value which is here is f of E1 and 1- f of E2 what is f of E2, E2 is here so, E2 is this that is this value 1 this is 1. Therefore this difference is 1-f of E2 this one is 1-f of E2 dash. So, what do you see do you need to do any mathematics here no need of any mathematics probability of emission is f of E2.

A small quantity multiplied by a small quantity and Pa probability of absorption is f of E1 a big quantity multiplied by 1-f of E2 dash another big quantity. So, it is obvious that Pa is greater than Pe or Pa is in general much greater than Pe. Because the numbers involved here or 0.001 and this will be 0.999 so the product here Pa is much greater than P. In fact in thermal equilibrium it is impossible to have Pa Pe greater than Pa that means probability of emission is always less than probability of absorption.

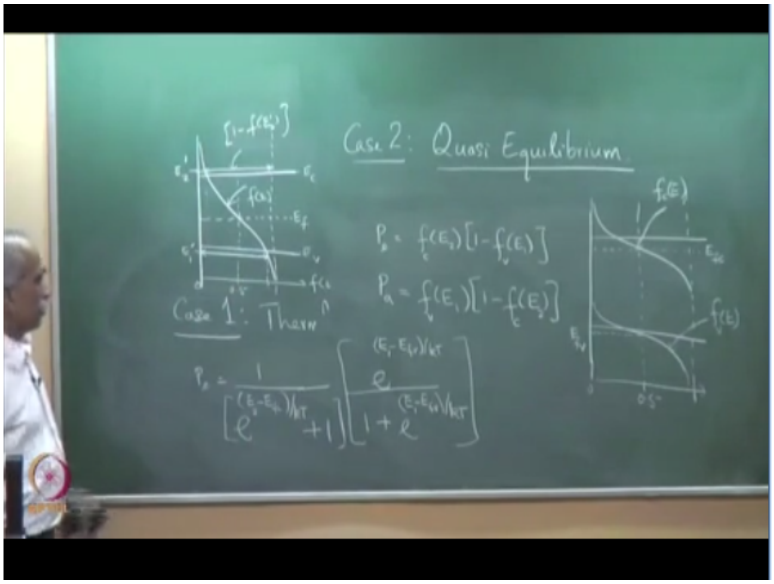
That means no semiconductor will emit on its own at thermal equilibrium you have to do something. So, that it can which means we have to move over to quasi equilibrium we know how to get quasi equilibrium there are different ways of pumping the semiconductor and the most convenient is using a pn junction. So, let us just see this is quasi equilibrium what happens you can see that also graphically but mathematically also you can.

(Refer Slide Time: 40:29)



So, in quasi equilibrium let me draw the graphs Pe probability of emission is equal to f of E2 into 1-f of E1. And Pa so, case2 now I am discussing case2.

(Refer Slide Time: 41:12)



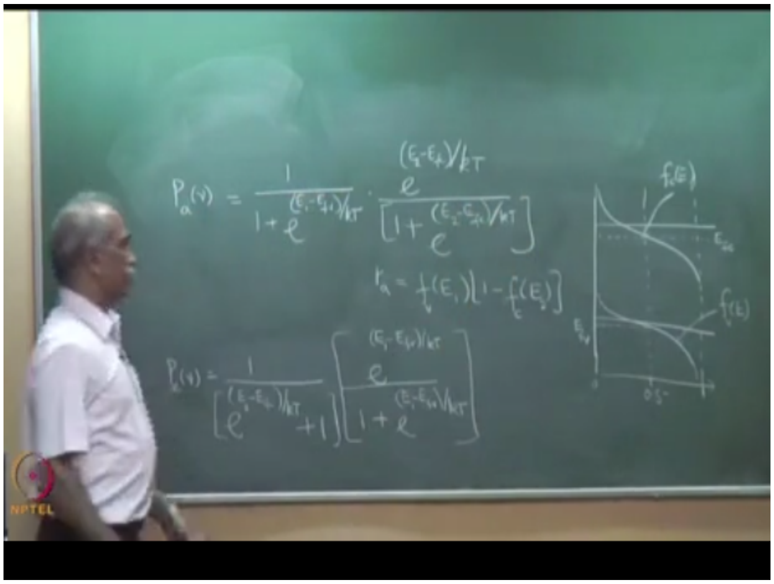
So, case2 quasi equilibrium Pa is equal to probability of absorption is equal to f of E1 into 1- f of E2 one single function for thermal equilibrium. But for quasi equilibrium this is fc of E2 a Fermi function for the conduction band. And fv of E1 this is fv of E1 and this is fc of E2 recall the graphical picture that we had one Fermi function like this. And a second Fermi function like this so, there were two Fermi functions this is fc of E for the conduction band.

And this is fv of E where is the Fermi energy this is 0 and this is 1 then 0.5 is a so, 0.5 draw a vertical line. So, this intersect here so, we have Efv the Fermi energy of Efv for the valence band sitting here Efv here and Efc is sitting here Efc is the Fermi energy corresponding to the conduction band function. This function and Efv is the there are two Fermi functions now you can immediately see that there is a possibility**.**

If we push this sufficiently high or if you push this sufficiently low there is a possibility of having. So, fc of E2 into this so, let us substitute mathematically and see what do we get this therefore Pe equal to fc of E2 there is 1/E to the power E2-Efc/kt+1 first term multiplied by 1-f of E1. So, 1-f of E1 I can take a common denominator so, I get E to the power E-Efv/kt/1+E to the power E-Efv/kt.

This is 1-f of E this is f of E2 and this is f of E1 1- f of E1 okay 1-this so, then I multiplied by this 1 1 cancelled. So, I am left with only this term in the numerator okay. So, this is Pe of E write for Pa of E so, P of e is fv of E1 so, instead of this you will get so, we have Pa of E.

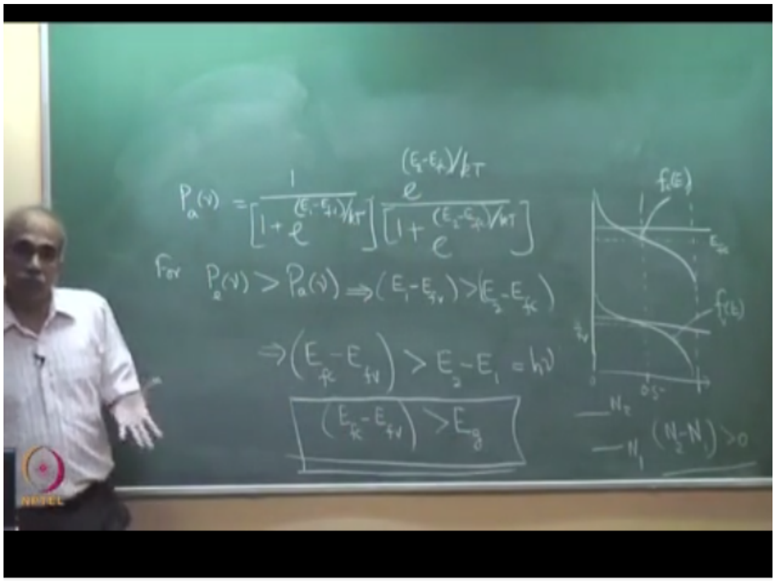
(Refer Slide Time: 45:10)



Pa is equal to probability of absorption all of this is for mew Pe of mew and pe of mew. Because E2-E1 in this expression is fixed h mew so, Pa of mew is equal to fv. So, 1/1+e to the power E1-Efv only thing is we need to add an suffix c or v. Otherwise it is the same Fermi function so, because we have two Fermi functions we have to add this into 1-fc of E2. So, in the numerator I will get e-e to the power E2-efc/kt/1+e to the power E2-Efc.

I just jumped one step that is all what do you see the denominator is common see the denominator is common 1+this 1+here. And this is here 1+E2-Efc is here denominator is common.

(Refer Slide Time: 46:45)



So, Pe of mew and Pa of mew for Pe of mew we want Pe of mew to be greater than Pa of when can this happen. In thermal equilibrium it is not possible, when can this happen when this numerator term is greater than this numerator term denominator is the same. This numerator is greater than this numerator which it means it is e to the power therefore this power is greater than this power kt is also common that means E1-Efv is greater than E2-Efc.

So, in twice E1-Efv is greater than E2-Efc a very, very important condition which is now coming so, I can take E1 there I can take Efc here this implies Efc-Efv is greater than E2-E1. And h mew is always greater than Eg this is the condition for emission to be greater than absorption probability of emission to be greater than probability of absorption that is the difference in the quasi Fermi levels must be greater than the band.

The difference in the quasi Fermi levels must be greater than the band gap energy Eg. This is a very important condition this is equivalent condition to the condition of population inversion. In atomic systems you know when for amplification to take place N2-N1 should be positive or N2 should be greater than N1 or N2-N1 should be greater there 0. If you have two atomic energy levels and N2 is the number of atoms in the upper level.

And N1 then if N2-N1 is greater than 0 then you can have amplification for net amplification this we call population inversion in laser physics. This is an equivalent condition as we will see further for semiconductor lasers. In semiconductor lasers this is not the condition in semiconductor lasers this is the condition that the difference between the quasi Fermi levels must be greater than Eg will stop here for today.