Semiconductor Optoelectronics

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Lecture - 08
Carrier Concentration and Fermi Level

Today, we will discuss carrier concentration and Fermi level. Given a carrier concentration or a current through the device you calculate the carrier concentration; and knowing the carrier concentration, it is possible to calculate the Fermi energy. This is required in several design characteristics. So, we will discuss the relation between this.

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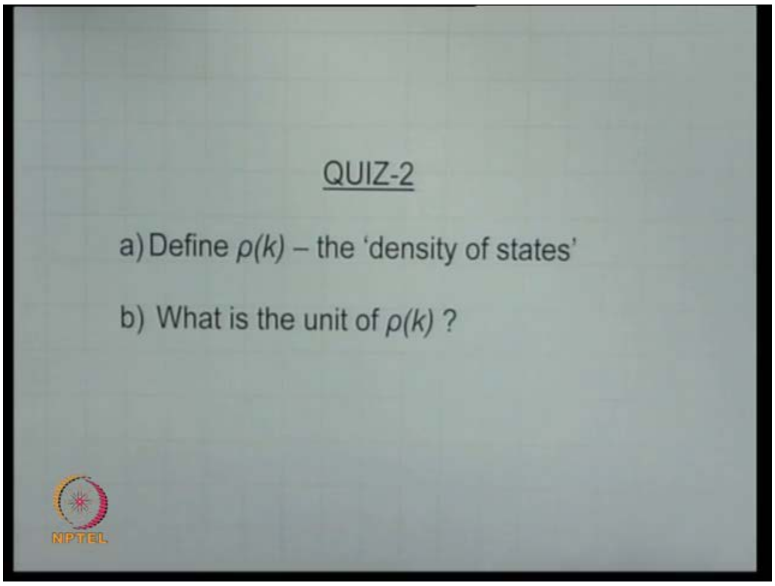

QUIZ-2

a) Define $\rho(k)$ – the 'density of states'

b) What is the unit of $\rho(k)$?

But before I start with the lecture, let me just discuss the quiz which we had taken in the last class. That is quiz 2; define rho of k, the density of states and what is the unit of rho of k?

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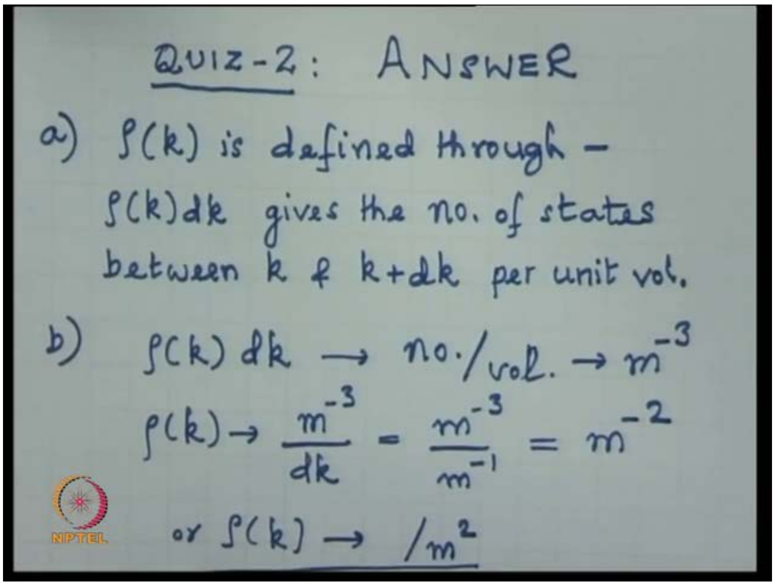

QUIZ-2: ANSWER

a) $\rho(k)$ is defined through –
$\rho(k)dk$ gives the no. of states between k & k+dk per unit vol.

b) $\rho(k)\, dk \rightarrow$ no./vol. $\rightarrow m^{-3}$

$$\rho(k) \rightarrow \frac{m^{-3}}{dk} = \frac{m^{-3}}{m^{-1}} = m^{-2}$$

or $\rho(k) \rightarrow /m^{2}$

The answer is rho of k is defined through rho of k d k rho of k d k gives the number of states, rho of k d k gives the number of states between k and k plus d k per unit volume. You can see your notebook, we have written this definition many times or if it was rho of e, we would have written rho of e d e is the number of states between energy e and e plus d e per unit volume of the material. If you write the correct definition, then part b that is to determine the unit, part b which is to determine the unit rho of k d k is the number per unit volume therefore, its dimension is meter minus 3 or per c c is the unit.

Therefore, rho of k has units of meter cube by d k. d k the dimension of k is 2 pi by lambda which means it is length inverse. So, meter inverse and therefore, the correct unit of rho of k is per meter square. Exactly you can see what is the unit of rho of e. rho of e will be number per unit volume per unit energy, that will be the unit of rho of e. Today we recall, to recall what we had seen in the last class, the carrier concentration n is given by n of E d E, it is equal to rho, if it is n it corresponds to carrier concentration of electron which means the conduction band; so rho c of E into f of E the probability of equation d E.

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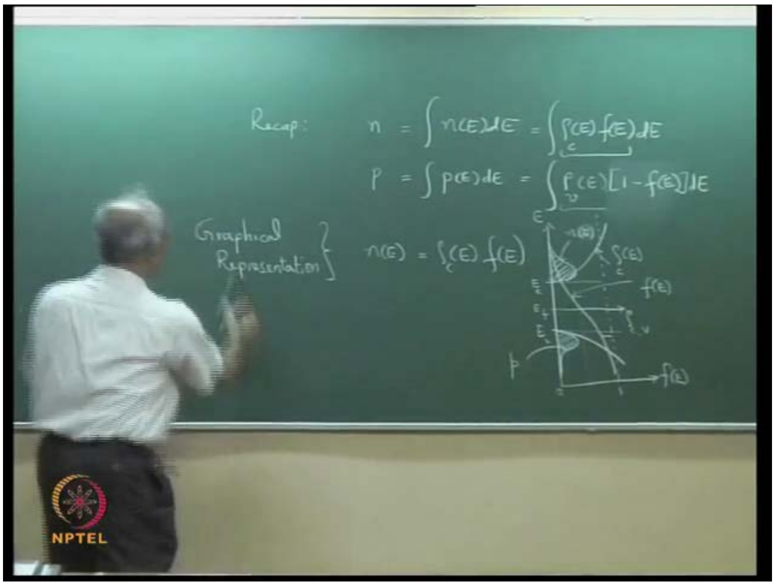

Similarly, if you are calculating the carrier concentration of holes in the valance band then this is equal to p of E d E which is equal to rho v of E f of E d E. It is interesting to see what will be integrant, integrated over the limits of energy. So, if it is conduction band then E c to infinity and if it is valance band it is infinity to minus infinity to or minus infinity to E v or somewhere below to E v. The integrant here, so, let us have a graphical representation. See the graphical representation, it gives you graphical representation of the carrier density of the carrier density. So, n of E is the integrant which is equal to rho c of E into f of E.

How does it look like? It is the product of two functions. This is the energy axis, rho c of E. So, this is E c and this is E v then rho c of E varies like this and rho v of E varies like this. So, along the x axis we have rho c of and rho v. So, this is the lower one is rho v and this is rho c. On the same graph, product of two functions, in the same graph I want to plot also f of E if this is 1 this is 0 f of E then f of E if I take for example, an intrinsic semi conductor then E f remains somewhere in between, somewhere midway between E c and E v and therefore, the if I plot f of E it would look something like this. I should probably have brought a color chalk, but in your eyes suppose if you follow this is f of E.

So, wherever you have 0.5, so this is E f. For an intrinsic semiconductor E f remains somewhere in between, somewhere near midway so far E greater than E c for every value of E here, the integrant is product of this function and this function. f of E multiplied by rho of E. f of E is maximum near to c and it goes down to 0 almost 0 as you as e increases. This of course, increases as square root of E. So, how would the product look like? Here it is 0 therefore, if you find out the product of these two, it would vary something like this. It will initially increase and then it will come down.

So, this third variation that I have plotted is n of E, this third variation is the product of this and this which means for every given value of E multiply f of E by rho of E you get this value. So, it goes up to a maximum, but then starts decreasing because f of E is decreasing. That is n of E integrate over E which means area under this curve gives you n. So, the carrier density, the carrier concentration is area under this curve because we are integrated over E d E integrated. Similarly, if you see here for the valance band this is the density of stage and in yes I made a small mistake, please correct this.

Because the probability of occupation is 1 minus f of E you can point out it earlier if you, so 1 minus f of E. So, again you see 1 minus f of E is this part because from here to here it is f of E and this small length here magnitude corresponds to 1 minus f of E that is very large near E v, but decreases down to 0. The density of state is 0 at E v and continuously increases. So, if you take a product of these two, here also you will get a curve which goes up to a maximum and comes down.

An area under this curve you are integrating over E, area under this curve gives you carrier concentration p, area here is n, this is n. Integrate the curve is n of E, but the area, the curve is of course n of E, but the area under that is u of t. Let me quickly go to how it look like in case of n type and p type. You see that this areas are equal. In a intrinsic semiconductor the area under this curve here will be equal to the area because n is equal to p.

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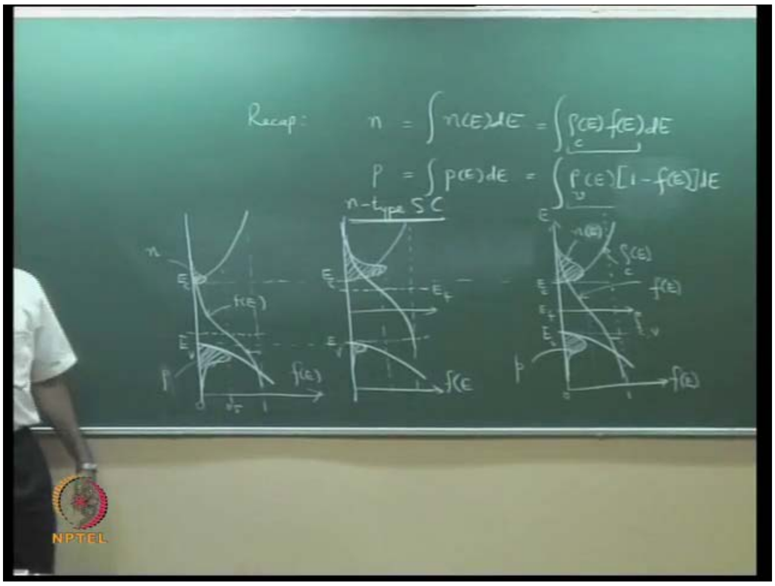

If you use a doped semiconductor, so, let me draw the same scale almost near the same height. So, this is E v E c, but this time the Fermi level in the case of n type semiconductor is somewhere here. The Femi level is up, this is now for n type I am plotting f, the density of states varies the same way, is the same material, but now n (( )) Fermi level is up which means the probability of probability f of E is half here and this value its half which means the curve would look like this and the density of states is here is f of E.

Please, see the f of E curve has been shifted look at this, compared to this it is shifted up because E f is up. Therefore, what do you expect now? The product of this into this, this quantity is now large compared to this you see the magnitude here is very small, here the magnitude is large because the curve has shifted up f of E curve has shifted up.

So, in this case you will see if you take a graphical representation, if you take a product it will look something like this. Whereas, in the second case you see a magnitude 1 minus f of E is very small, although the density of state increases, but that is very small and therefore, if you this will come out to be very small. And therefore, the area under the curves clearly tells you that in an n type semi conductor n is much larger compare to p. So, this is for n type.

This is graphical representation, qualitatively of course, qualitatively I am drawing, but if you can indeed multiply the 2 functions and see you will get a, this find of variation and if you go for the p type variation, first draw a density of states density of states and p type so Fermi level is now below and this is E c E v Fermi level is below which means if probability 1 0. So, half is here which means the curve is now, so, this is my f of E now. And you see that the magnitude 1 minus f of E that you see here 1 minus f of E is much larger compared to that 1 minus f of E that you saw here.

The same energy value should go here, the 1 minus f of E because f of E itself appears to be about 0.99 or so. So, 1 minus f of E is the difference very small, but here now 1 minus f of E is large. Whereas, here f of E is very small, please remember f of E is now along the x axis which means this distance gives you f of E, this magnitude of length is proportional to f of E. So, here f of E is now very small, density of states is the same because that does not depend on the doping concentration or whether it will depend on.

It depends on the material only therefore, if you take the product this time you will get n of E varying like this whereas, p of E varies and the area under the curves the area under the curves give you peak and so, can clearly see from this graphical illustration also that what we have plotted is carrier density versus E and the area under n of E and p of E tells you the total carrier concentration. So, this is for the p type semi conductor, is that alright? So, we have also derived a mathematical expression for p and n under the Boltzmann approximation we have p is equal to N v into e to the power E minus E v minus E f.

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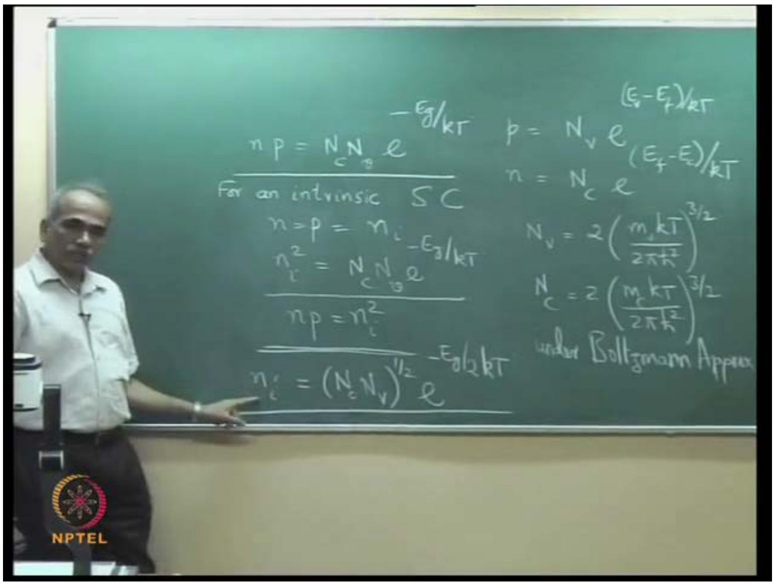

E v minus E f by k T and n is equal to N c into e to the power E f minus E c by k T, these are under Boltzmann approximation where N v is 2 into m v k T by 2 pie h cross square to the power 3 by 2. All of them are constant except m v, that is effective mass of holes in the valance band and c is equal to 2 into m c k T by 2 pie h cross square to the power 3 by 2. This is under Boltzmann approximation and we have discussed Boltzmann approximation which simply says that if is E f minus E c is much greater than E T that is when the Fermi level that lies well within the band gap, then you can use Boltzmann approximations.

So, under Boltzmann approximation n into p therefore, is equal to N c into N v into e. n into p N c into N v into E f, there is 1 plus E f 1 minus E f. So, we had e to the power minus E c and E v. So, it is minus E g by k T. n into p is equal to N c into N v minus E g by k T. For an intrinsic semiconductor for an intrinsic semiconductor n is equal to e is equal to n i and if you substitute here n i square is equal to N c into N v e to the power minus E g by k T. You see that this expression is independent of E f, this expression same as this expression. This is independent of E f therefore, whether you take intrinsic material or an n type material or a p type material, the right hand side is the same. It means the n into p is equal to n i square for all semiconductors under Boltzmann approximation. The clause always remains, but n into p is equal to n i square. This is sometimes called law of mass action, n p is equal to n i square.

Therefore, the carrier concentration n i, the intrinsic carrier concentration n i is N c into N v to the power half into e to the power minus E g by k T. You see that by 2 k T I am sorry by 2 k T E g by minus E g by 2 k T. Expression for N c and N v are here. Therefore, given a material if you know the effective mass of holes and effective mass of electrons, rest of them are all constants you can calculate N c N v and therefore, the intrinsic carrier concentration simply depends on the band gap E g.

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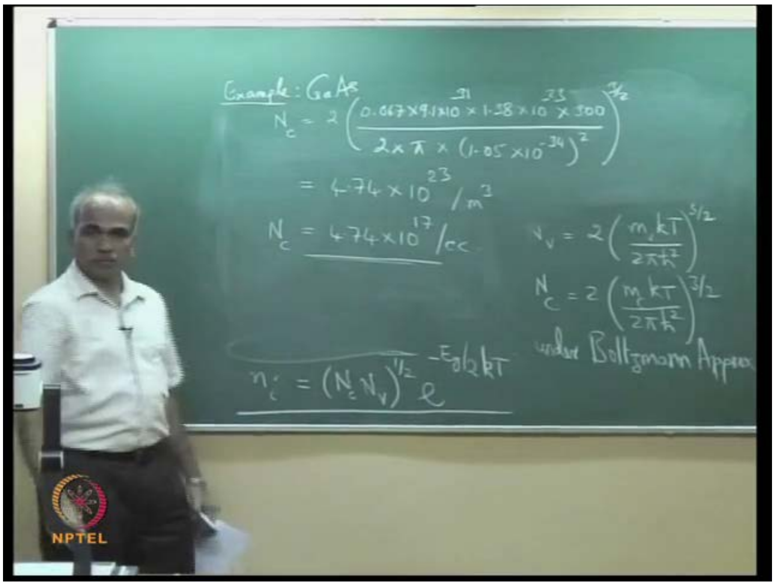

Please, put some numbers let me for example calculate N c N c N c is equal to let us take for example, gallium arsenide N c is equal to 2 into N c that is effective mass of electrons at 300 k is 0.037 into (( )) into 9.1 into 10 to the power of minus 31 kg. m c multiplied by k T 1.38 that is k Boltzmann constant k is a equal 1.38 into 10 to the power minus 23 300 k divided by 2 into pie into h cross into 1.05 into 10 to the power of minus 34 square and whole to the power of 3 by 2. So, this will come out, I will give you some value that I have calculated, just comes out to be 4.74 into 10 to the power of 23 per meter cube because all these are s r units, therefore, so this came out for meter cube or 4.74 into 10 to the power of 17 per c c, N c. Similarly, you can calculate N v and I want to give you these numbers, some numbers I have calculated, let me give you typical numbers.

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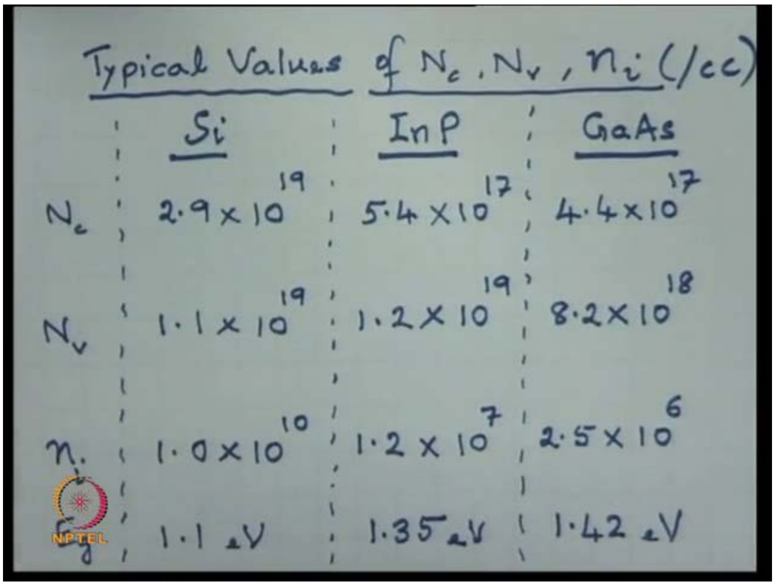

Typical Values of N_c, N_v, n_i (/cc)

	Si	InP	GaAs
N_c	2.9×10^{19}	5.4×10^{17}	4.4×10^{17}
N_v	1.1×10^{19}	1.2×10^{19}	8.2×10^{18}
n_i	1.0×10^{10}	1.2×10^{7}	2.5×10^{6}
E_g	1.1 eV	1.35 eV	1.42 eV

Okay, all of you it is visible, so, typical values of N c N v n i per cc is the material silicon band gap is 1.1 e V and you can see N c N v n i, calculate it by taking constants, it is advisable to take some constants from books or literature and calculate this N c N v and n i, this is Indium phosphide. I have written for three very important substrates for optoelectonics, the most important substrates are gallium arsenide and indium phosphide. So, indium phosphide has a band gap of 1.35 e V and you can see the important point to see is for smaller E g as you have seen in this expression for n i, it depends on E g, n i is equal to N c into N v to the power half.

So, smaller the E g, larger will be the carrier concentration N i and as the band gap increases you see the carrier concentration decreases. If you take gallium nitride wide band gap semiconductor which has the band gap of 3.4 electron volt, the intrinsic carrier concentration is 10 power minus 10, it is of the order of 10 power minus 10. To the strong dependence on band gap, there is a exponential dependence and of course it has a temperature dependence. These numbers are calculated at 300 k. So, please calculate and see whether you get such numbers.

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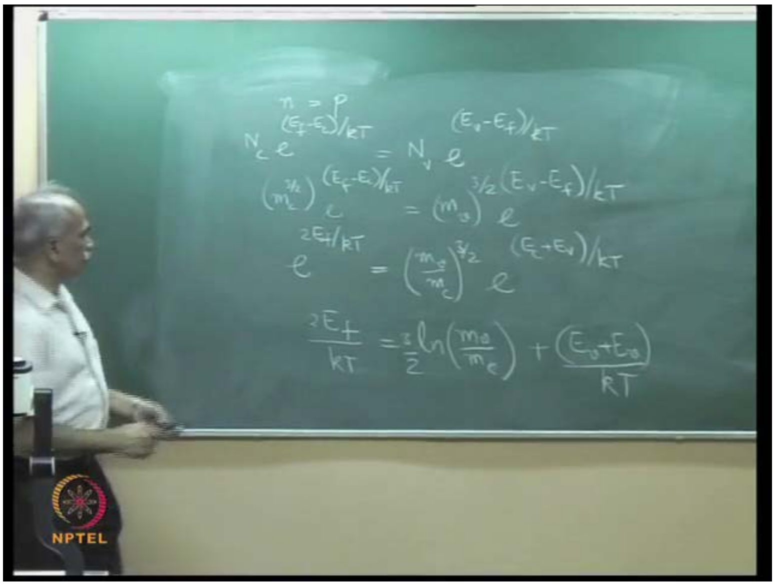

$$n = p$$

$$N_c e^{(E_f-E_c)/kT} = N_v e^{(E_v-E_f)/kT}$$

$$(m_c^{3/2}) e^{(E_f-E_c)/kT} = (m_v)^{3/2} e^{(E_v-E_f)/kT}$$

$$e^{2E_f/kT} = \left(\frac{m_v}{m_c}\right)^{3/2} e^{(E_c+E_v)/kT}$$

$$\frac{2E_f}{kT} = \frac{3}{2}\ln\left(\frac{m_v}{m_c}\right) + \frac{(E_c+E_v)}{kT}$$

For an intrinsic semiconductor for an intrinsic semiconductor n is equal to p, which implies N c into e to the power E f minus E c by k T is equal to N v into e to the power E v minus E f by k T. What is different in N v and N c? Only this is the difference effective mass, rest of them is same. Therefore, this means m to the power 3 by 2, m c to the power 3 by 2 into e to the power E f minus E c, simple algebra by k T is equal to then m v to the power 3 by 2 E to the power E v minus E f by k T. I can bring that E f here, so I have e to the power twice E f by k T is equal to m v by m c to the power 3 by 2, m v by mc to the power 3 by 2 into e to the power what do we have? This E f I have brought here, E c goes there. So, it is e to the power E g by k T. So, E c plus E v E c plus E v by k T, is that alright? Therefore, E f we can simplify this take l n on both sides. You have 2 E f by k T is equal to l n 3 by 2 l n m v by m c m c plus E c plus E v by k T or multiply by k T on both side.

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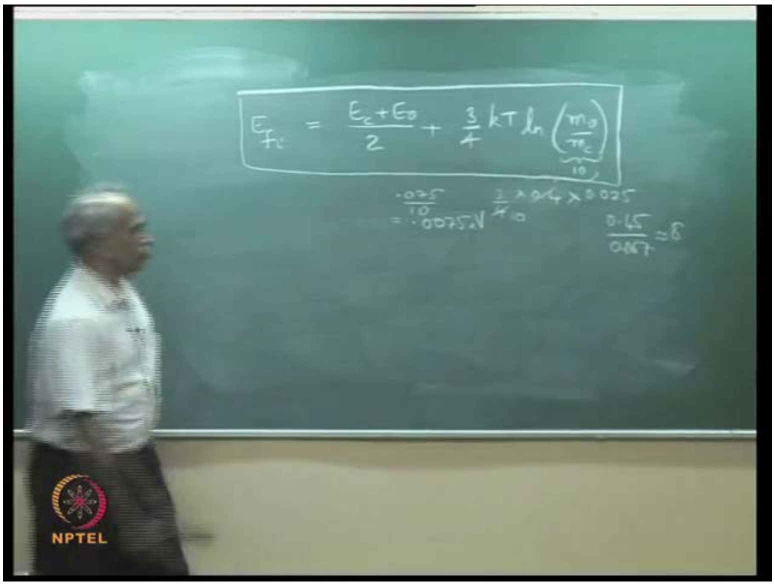

And take this to the other side. So, you have E f i for intrinsic semiconductor I was writing this expression. So, i standing for E f i is equal to E c plus E v by 2 plus 3 by 4 k T l n m v by m c, the expression for Fermi level of intrinsic semiconductor. You can see that it is exactly in between E c plus E v by 2 plus a term here. Only the m v is equal to m c l n 1 is 0 therefore, the Fermi will lie exactly midway between E c and E v if effective mass of hole is equal to effective mass of electrons. Otherwise, there will be a small difference here.

Now, how small is the difference? The difference is very small because you can put some the corresponding values for m v and mc and you can see the difference is going to be very small. For example, if this comes out to be a factor of 10, if you take gallium arsenide Fermi is 0.45 and 0.067. This is approximately 8. So, if I take 10, l n 10 is how much? l n 10 is 1 divided by 2.303 which is 0.4 approximately 0.4 so, this term is approximately 0.4 multiplied by there is a 3 by 4 here.

Approximately estimating what is this, what is a typical number here? 3 by 4 into k T is approximately 0.025 so, the number is really very small. So, this is 10 times and this is 0.075 divided by 10 which is equal to 0.0075 e V. Approximately, estimated just to find what the order. So, what is the point? Even m v and m c are very different as in the case of practical materials, the second term is very small and therefore, with some approximation we can say that the Fermi level for an intrinsic material lies midway between E c and E v, it is not exactly, but almost midway between E c and E v. Because, E f i is equal to E c plus E v by 2.

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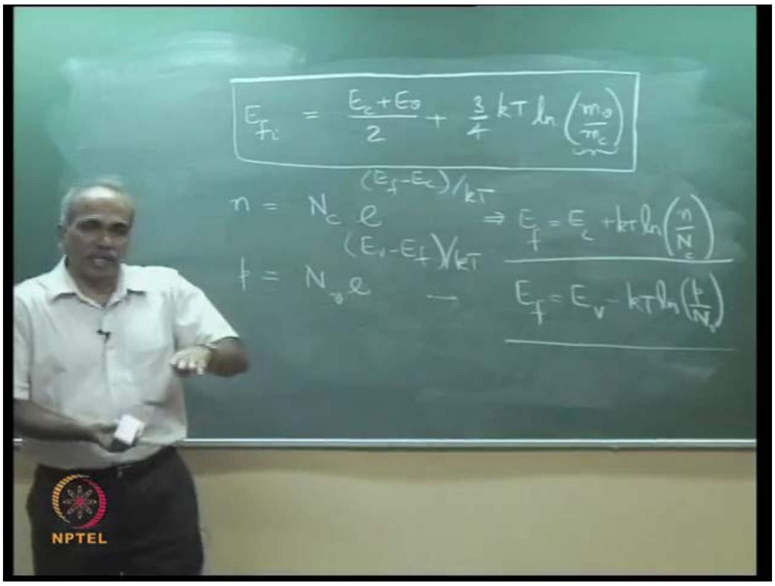

What about the Fermi level of other material that is p dope and n dope? We had a expression for carrier concentration n is equal to N c into e to the power E f minus E c by k T, we have written this equation already. p is equal to N v e to the power E v minus E f by k T. I want to find out E f from here. So, you can bring this here, take log and you get f so this gives you E f is equal to E c plus k T into l n n divided by N c. Then take N c, then take l n, this will come down, multiply by k T, you will be left with E f minus E c, take E c to the other side. So, you have got expression for E f. From this expression you get here E f is equal to E v minus k T into l n p divided by N v.

You must be wondering Fermi energy E f E c plus this which means greater than E c, is that true? is not because this n is much smaller compared to E c that is why I gave you these number N c is much larger compared to n and therefore, this is a fraction, log of a fraction is a negative number and therefore, the second term here is a negative. So, actually it is E c minus something and therefore, it lies within the band gap. So, this is under Boltzmann approximation.

Therefore, E f has to lie within the band gap otherwise these expressions are not valid. If Boltzmann approximation is not satisfied, this is fine for lightly doped materials where Boltzmann approximation was a good approximation. If Boltzmann approximation is not satisfied, what do we do? Carrier concentration and Fermi level, which means if you know the carrier concentration N c is known for the given material k T E c, if you know the carrier concentration you can determine Fermi level.

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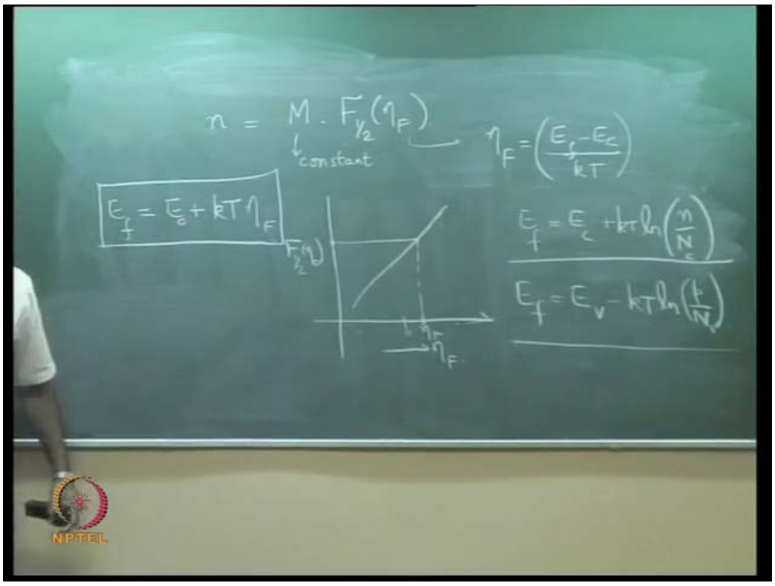

What was the carrier concentration in the case of without any Boltzmann approximation? This was through Fermi half integrals. So, Fermi half integrals there was some constant M, some constant, you can see what is the constant 1 by 2 pie square into 2 m c k T by h cross square to the power 3 by 2 into Fermi half integrals, F half of eta. This is the constant which is outside and what is eta F? Eta F is E f minus E c divided by k T. And recall what we had done in the last class, we have plotted eta F here versus the Fermi half integral F of half of eta F. If you know the carrier concentration, it means you know the value of this Fermi half integrals, that is you know the value here.

We had, I just showed the qualitative variation like this. So, you know the value of F half of Fermi half integrals of eta F. That means if you know that value here, then you can find out what is eta F, the value eta F corresponding to this. So, n is equal to, therefore, n corresponding to this n I know this value. Therefore, we known eta F. So, once you known eta F, eta F we know and therefore, E f is equal to E c plus k T into eta F, k T into eta F because eta F has been numerically obtained from this graph because you have found out this, for a given carrier concentration you know this and from this numerical plot you know what is eta F and if you know the value of eta F then you can find out what is the Fermi energy.

This is the way to exactly get the Fermi energy for without any approximation, no Boltzmann approximation and this is important because in optoelectronics we will see that most of the devices are degenerate semiconductor or the Fermi level is entering either conduction band or valance band or both the bands. How is that possible? Fermi level entering valance band and conduction band, yes, it is possible and that is our next topic. We will see how it enters valance band and conduction band. So, E f is equal to E c, E c plus k T in eta F, you can see here eta F value yesterday I had written eta F value 0 2 4 minus 2 4 6 minus 6. So, this eta F could be positive or negative. Therefore, E f could be greater than E c or less than E c. So, it will give you E f value even when the Fermi level enters the band. This is the exact expression where you have to numerically evaluate the Fermi half integral.

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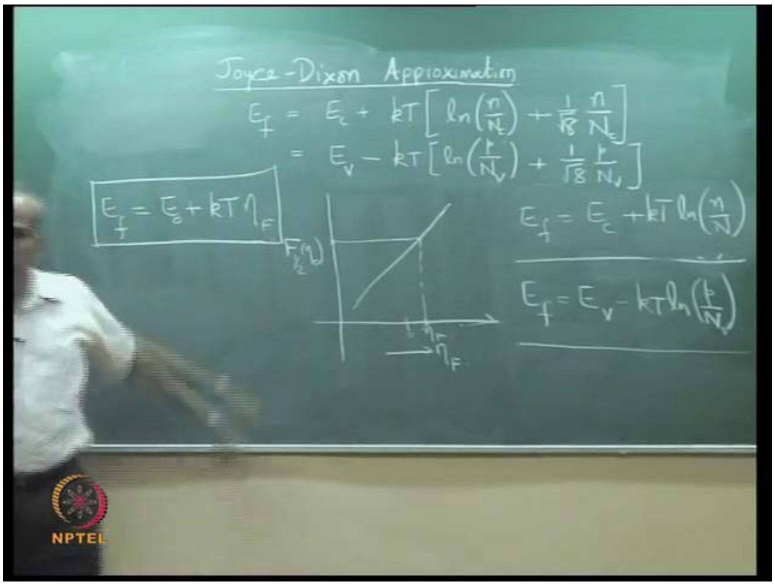

However, it is sometimes difficult and there is very important approximation for, let me not erase this, very important approximation for Fermi level E f is equal to E c plus k T into l n n by N and that is for the Joyce-Dixon approximation. This approximation you would not find in olden books. Joyce-Dixon approximation for the Fermi level, this is a very good approximation. This gives that the Fermi level E f is equal to same expression E c plus k T l n n by N same term plus an additional term 1 over root 8 n divided by N, this is an empirical formula or this is also equal to E v minus k T into l n because you may be knowing whole concentration p by this is n by N c, n by N c, we see the subscript n by N c and this is N v plus 1 over root 8 into p divided by N v.

So, any of these formula could be used because in highly p doped material you know p. Whereas, in highly doped n doped material you know n, because when you highly dope n is nearly equal to the donor iron concentration that is doping concentration because intrinsic concentration is negligible and similarly, if you highly p dope a material, the p is essentially equal to the acceptor ion concentration. So, in some material you know p and in some material you will know n and you can find out the Fermi level without any Boltzmann approximation. This is a very useful formula.

We will need this later, when we pass current through a device for a given carrier concentration you will have to estimate the Fermi level. The Fermi level will be required as you will see later on for example to determine the bandwidth of an amplifier. This is the Joyce-Dixon approximation. For normal calculations it is sufficient to use this approximation, it may be difficult to get the exact numerical solution of.

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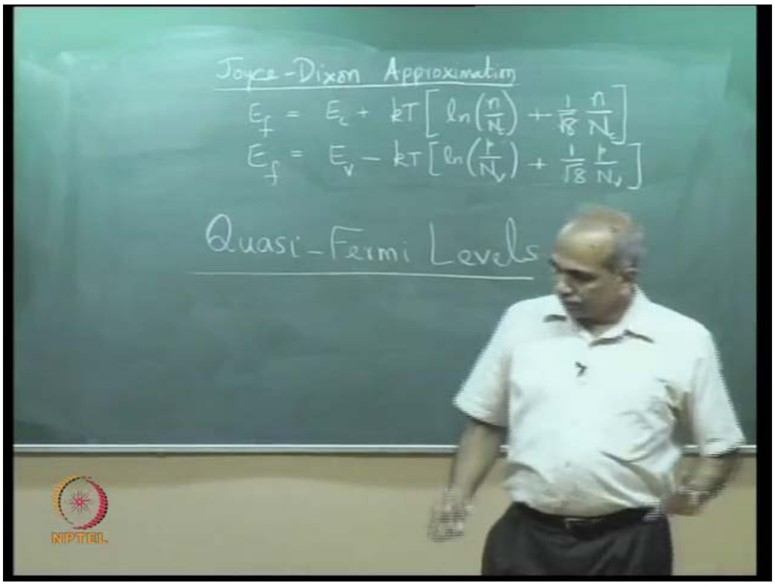

So, we have, we know for given carrier concentration, how to determine the Fermi level whether it is highly doped or not. Our next topic will be so, long we have been discussing about Fermi level. The next topic will be Quasi Fermi, Quasi Fermi levels. This is very important, Quasi Fermi levels. Normally, in electronic we do not, Quasi

Fermi levels are the every p n junction when you operate in the forward bias or reverse bias you have Quasi Fermi levels in the junction region.

There is no more Fermi level or Quasi Fermi levels and it is the difference between the Quasi Fermi levels which will determine the performance of the device. Therefore, we will have to determine Quasi Fermi levels. What are Quasi Fermi levels? Let me just give you a simple idea. In a normal semiconductor, we will discuss this in detail in the next class. However, let me just introduce what is the need to have these Quasi Fermi levels. Why do we need Quasi Fermi levels?

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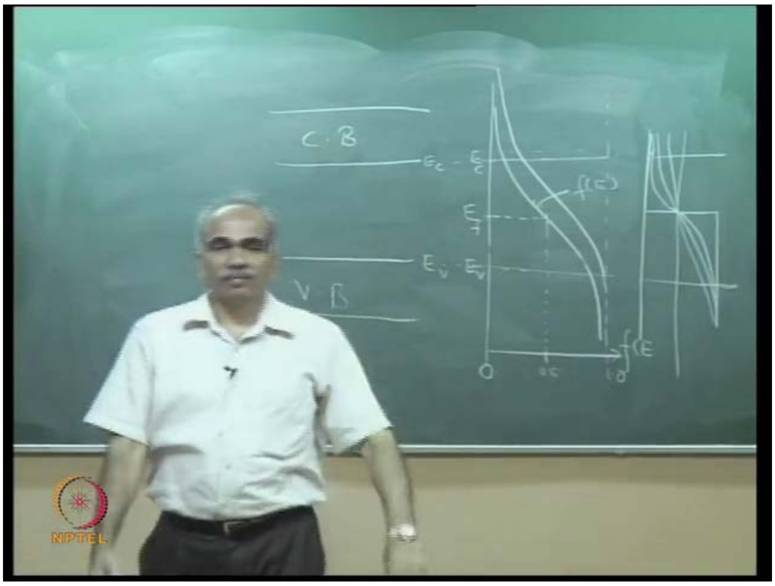

If you take our familiar band diagram, let us say valance band and conduction band. The basic picture of valance band and by this side, so this is E c and this is E v and this side let me also plot the Fermi function. The Fermi function E f c. So, this is f of E and this is 1 1.0, 0, this is E c, E v and E c. This is the f of E Fermi function and where is the Fermi energy at half way. So, 0.5 go upwards where it intersects and that is f. If you n dope this material this shifts up because there are more number of electrons here, which means the Fermi level will shift up because the probability of finding electrons in the conduction band increases.

If you p dope it, it will shift down. So, in n doped material you have large concentration of electrons, but very little holes and therefore, the shifting was very good because no problem I shift this up and now, I have parallel to this, but you see the probability of having holes is very small, it is very good, it is a one type material, very few holes. If you take p type material, very large number of holes so I shift this, the Fermi function is able to describe the occupation probability of electrons in valance band and the conduction band. Think of a situation where you have very large number of electrons and very large number of holes. What will you do now?

The same Fermi function cannot cannot express the probability of finding large number of holes and large number of electrons. Of course to some extent, it is possible because you smear this out further. If you, how did we have Fermi function at k at temperature t is equal to 0, it was like this. Step function, temperature increased, this started smearing because there are some electrons and some holes, temperature increase, this smears more, like this temperature increase this can smear further, that is the probability of finding electron is also larger, hole is also larger. How much can it smear?

At best it can become flat like this at best. Which means it is 50 percent probability of finding electrons and 50 percent of probability finding holes. Suppose, I have 70 percent electron here and 70 percent holes in both, can this function? So, we cannot describe the probability of finding electrons and holes by one single Fermi function and then we then we have to introduce what are called Quasi Fermi levels. We will discuss this in detail in the next class.