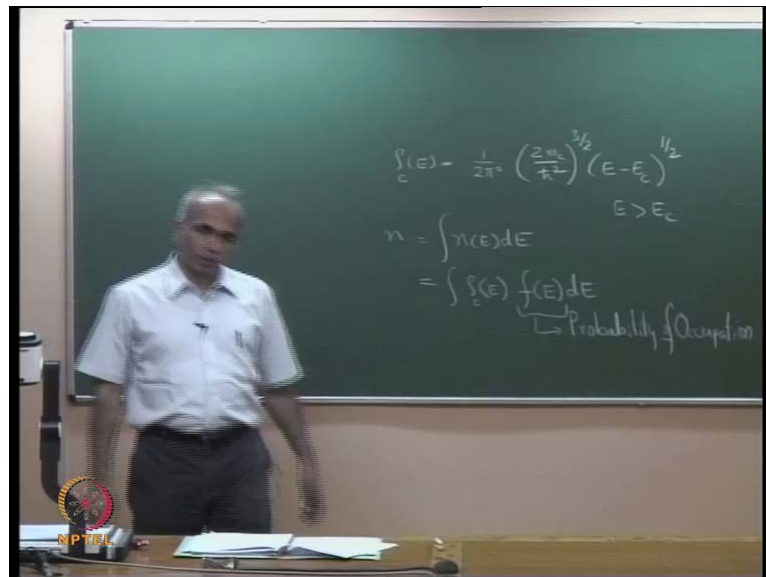


Semiconductor Optoelectronics
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Lecture - 7
Occupation Probability and Carrier Concentration

Continuing with the course, today we will discuss occupational probability and carrier concentration.

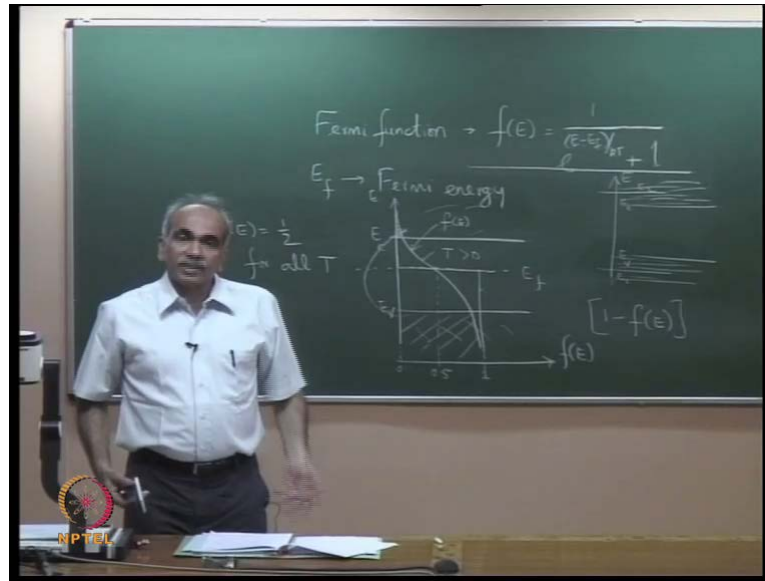
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In the last class we have discussed about the density of states ρ of e it was given by an expression $\frac{1}{2\pi^2} \left(\frac{2m_c}{\hbar^2} \right)^{3/2} (E - E_c)^{1/2}$ for valence band of for the conduction band to the power $3/2$ into $E - E_c$ to the power $1/2$ for E greater than E_c . This was the density of states and as I mentioned the importance of density of states is to determine the carrier concentration and we need to determine the carrier concentration to know the current. And carrier concentration is given by $n = \int n(E) dE$ and where, n of E is the density of carriers and this is equal to density of carriers is equal to available density of states available density of states multiplied by the probability of occupation the probability of occupation. So, f of E here is the probability of occupation.

We will discuss this probability of occupation in this class and determine the carrier concentration.

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The probability of occupation for electrons is given by statistical mechanics is given by the fermi function: f of E is equal to 1 divided by E to the power E minus E_f by kT plus 1. The probability of occupation of electrons in an energy level E . So, electrons in the energy band this is E always the vertical axis is energy, this is E_c , this is E_v . Anywhere if you take an energy level in the valence band let us say E equal to E_1 or if you take in energy level in the conduction band let us say E is equal to E_2 . Then, if you want to find out the probability of occupation both these bands have number of states, large number of states. What is the probability of occupation of these allowed states?

The probability of occupation for valence band as well as for the conduction band, probability of occupation by electrons is given by the fermi function. So, if you substitute if you want to find out the probability of occupation here E_1 then, you substitute E_1 here. If you want to find out the probability of occupation E_2 at the level E_2 then, you substitute E_2 in this expression where, E_f is a constant for the fermi energy.

So, fermi function or it is also called fermi dirac distribution because it tells us it indicates the distribution of electrons in the energy bands. So, if you want to plot this f of E . As you know in the valence band the probability of occupation is very high, valence band is completely full at 0 k. At 0k if so, this is E_v and this is E_c and here is E_f . So, this is the conduction band and this is the valence band. If you consider at 0 k, I want to plot

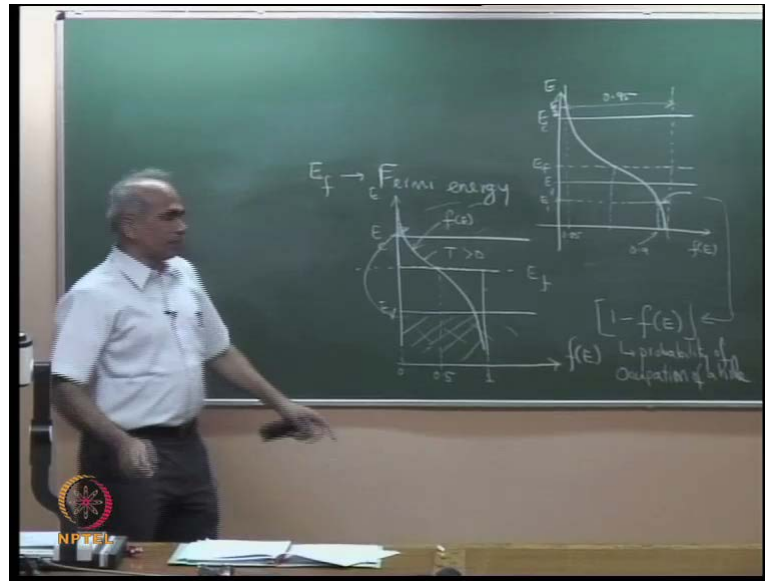
the probability here on this axis f of E in any time solid state physics f of E is plotted here and energy in this axis but, here for uniformity will always keep vertical axis E . So, if you plot f of E here at 0 K , how it be? That the valence band is completely full which gives the probability of occupation of every state is 1 and the probability of occupation here is 0 and as you know that if E f remains somewhere here, it can be anywhere but, if E f is here then, the probability of occupation is 0 up to E f . And for E greater than E f , the probability of occupation is 1. So, in this axis this is 0 f of E is equal to 1.

So, what I have plotted is the f of E as a function of f of E as a function of E . For E greater than f probability is 0 and E less than. There may not be a level if there were the level at f or anywhere here for example, probability equal to 1 here what does it mean? There is no level, there is no state permit. If there were to be a state then, the probability would have been 1. But, all the level here are completely full, completely occupation of, probability of all. If you take a finite temperature, this is what you already know. If you take for any temperature at E equal to f that is said this level, E is equal to this level for any temperature that 0 and therefore, it is upon 0 is 1, 1 plus 1 or the probability is half. So, f of E equal to half for all temperature f of E equal to half.

So, if I indicate the half here so, this is 0.5 . See this I am plotting the probability of f of E if this is half. Then, if you take any finite temperature we know that the electrons from the valence band it can upward transition in finite probability of finding electron in conduction band and the probability function is still described by the probability distribution is still described by fermi function and the fermi function gets smeared, this step function gets near but, passes through the same point, this is f of E . We can actually plot this numerically you can plot this numerically calculate and plot this for any temperature.

So, this is for a finite temperature T , T greater than 0 . What this indicates is this the finite probability of occupation of these states in the conduction band. This function is f of E . What is 1 minus f of E ? If f of E is the probability that a state is occupied then 1 minus f of E is the probability that the state is vacant unoccupied, the state is vacant. In the valence band if 1 minus f of E please see, it is f of E is going like this. Therefore, if I take an energy value E 1 here, let me draw a fresh. f of E is the probability of occupation therefore, 1 minus f of E is the probability that the state is vacant.

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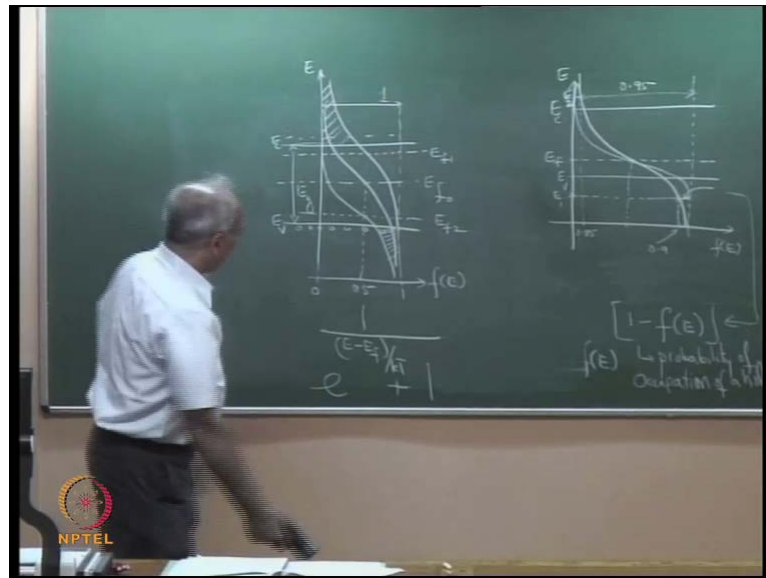
So, if I plot again the same graph but, I am plotting again. This is E_v and this is E_c . The fermi function is at a finite temperature is varying like this. So, this is the line f of E this is f of E , on the x axis I have f of E , vertical axis is E energy and 0.5 is somewhere here therefore, E_f is somewhere here E_f .

At any given energy value E_1 , I find out what is f of E ? This is the value f of E here. Same for example, 0.9 say f of E is 0.9 this means the energy the occupation probability of the energy level given is 0.9 which means the probability that the states at E_1 is vacant is 0.1, 1 minus 0.9 is 0.1. Is this 1, see this magnitude here if these magnitude of these length is 0.9 then, this is 0.1. This is the probability that the state is vacant. A vacant state in the valance band represent what it is the hole. Therefore, 1 minus f of E represents the probability of occupation of a hole, 1 minus f of E is the probability that the state is vacant which means as far as valance band, we do not have the hole in the conduction band.

For example, let me take here at some value E_2 , E_2 is a value in the conduction band energy value in the conduction band some value E_2 . This may be some number let us say 0.05 for example. So, probability of occupation here is 0.05. What is the probability that there is no occupation? The probability that there is no occupation that is this one 1 minus f of E is 0.95 because this is 0.05 therefore, this is 0.95. That is not the probability of occupation of the hole but, that is the probability that the state is vacant. So, please

remember f of E is the probability of elect occupation of electron whether in the conduction band or the valance band. But, 1 minus f of E is the probability of occupation of the hole if you are talking of valance band. Otherwise, the probability of state is unoccupied. So, 1 minus f of E is the probability of occupation of a hole in the valance band obviously. At a finite temperature there is always a probability that there are some electrons in the conduction band and there are some holes in the valance band. Of course, depends on the band gap E_g .

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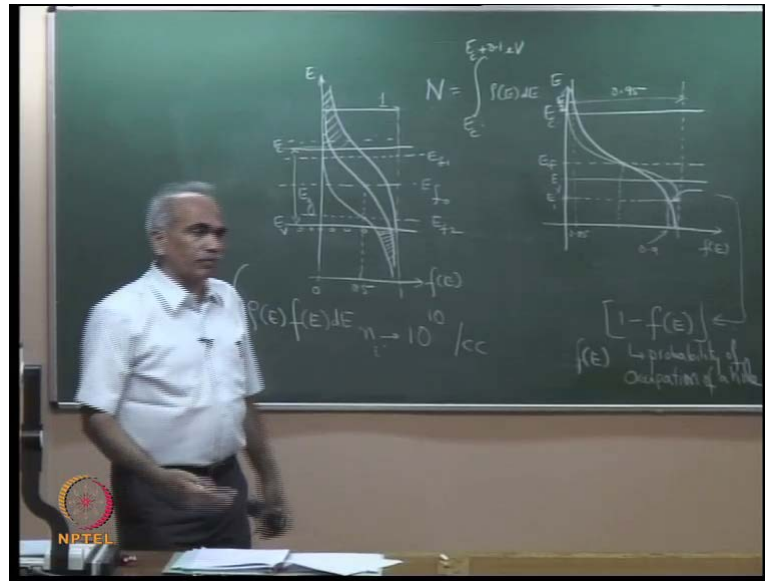
If E_g is very large that is a 5 electron holes then, the probability of finding the electron may be 10 to the power of 20 which means literally null. But, for normal semi conductor E_g is the order of 1 electron volt which means a probability of finding the electron here. So, the second thing that you have to see is this is E_f then the curve looks something like these, this let me indicate first graphically because graphically becomes very clear. This axis is f of E and this is 1 probability and this is 0 and E is equal to E_f this is 0.5 . If E_f is towards E_c , this is E_v . If E is shift over E_c that means the probability function because this point at E is equal to E_f . Let me draw a second value of E_f this is E_{f0} let me call this as E_{f1} . For another material if fermi function is E_{f1} it means this point of probability of half should come here that means this should shift for half.

So, I will have the curve will remain parallel please see that the cure will remain parallel this one but, it is shifted up. What is that mean? And clearly that curve shows that now

the probability of occupation of electrons in the conduction band is more and probability here has become very small, $1 - f(E)$, number of holes here will be very small, probability of occupation. On the other hand if the Fermi level were to be here let say E is equal to $E_f + 2kT$ that means, this curve would shift down because there is nothing else in the Fermi function other than E_f that is simply $E - E_f$ by kT plus 1, 1 divided by. So, that means that the curve is now shifted downwards.

So, you have the 0.5 coming here and you see now the area under here is quite large. It means the larger number of holes in the valence band. So, graphically it is very clear of course, in mathematics it will show you the numbers. One thing which I would like you to do is, if you shift E_f here I have shown it like this it appears as the probability is very high because please see this is 1 because it is 0 to 1 here 1. This appears as see this is 0.2 or may be this is 0.4. But, just put numbers in this Fermi function for practical p type material, practical n material and you see what is the kind of number said you get. $E - E_f$ by kT plus 1. Choose some suitable temperature say 300 and k and put number and try to plot this. You will be surprised that here I shown it is a 6.4×10^{-6} this will be very very small number. The probability here is extremely small result among 10^{-4} , 10^{-6} that kind of number literally you cannot see any think like this, it will be actually in next graph it will be literally here almost 0 it will be almost 0 and then it goes like this and almost 0 everywhere because the probability of occurrence here is extremely small at normal temperature. But, we have larger number of electron. How is it possible? At room temperature if you take silicon then, the number of electron the carrier concentration there is of the order of for the silicon.

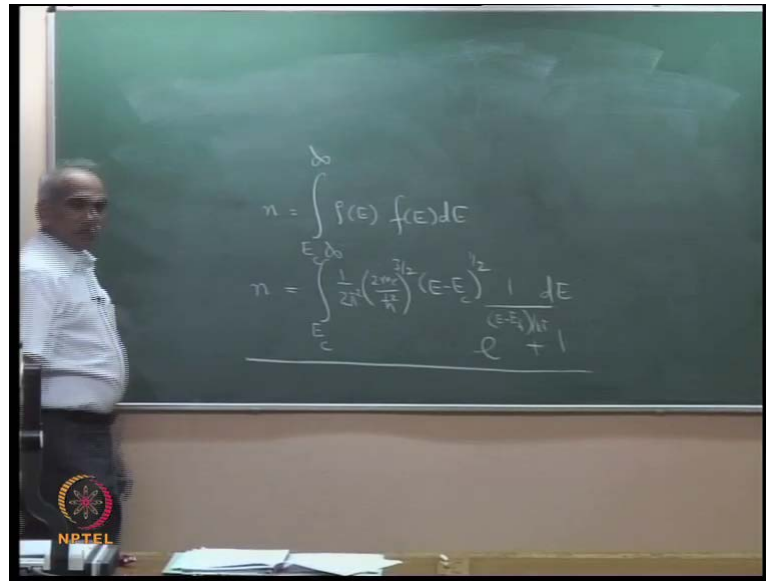
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It is of that graph 10 to the power of 10 per cc concentration n i intrinsic carrier concentration at room temperature is this. But, I am saying the probability is very very small how is it possible because n is equal to ρ of E into f of E . The density of states is very large. In the last class I had calculated and the total num of states from E_c to E_c plus $0.1 E_v$. Has anybody try to complete that integral? So, E_c to E_c plus $0.1 E_v$. This I called as n and ask you to calculate what is the number of states? Number of states was of the order of 10 to the power of 18 , I think 2.5 10 to the power of 18 is the number.

Please do numerical, it is very important because there will be numerical in the exam and more importantly you are handling 10 to the power of minus 34 , 10 to the power of minus 19 , 10 to the power of minus 23 . So, unless you parties you will not be able to do the numerical calculation. Every one of you will get a different answer. So, please practices numerical because there are huge problem you have to handle. So, the number of states it is all fussed very large and therefore, even though the probability here is very small, probability actually number is very small, multiplied by the number of states, it is significant, it is still of the order of 10 to the power of 10 . So, this is the distribution of occupation probability of course, we had aware of this I am just recalling all the concept that is essential.

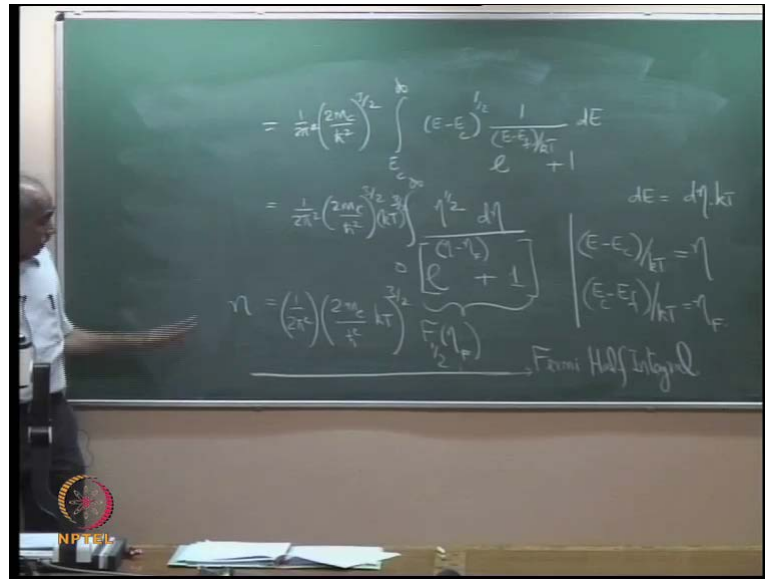
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Therefore, the carrier concentration the carrier concentration is equal to rho of E into f of E d E. Let me consider n is a carrier concentration in the conduction band. So, this is E c to infinity I am writing E c to infinity actually, E c to certain values some values of band but, I am writing it up to infinity because as I mention to you the numbers here are extremely small. Larger the difference E c minus E here, smaller will be the probability that is, this term goes to 0 it does not matter you write E c this may be up to E c plus 2 E v or 1 E v but, f of E already goes to 0.

Therefore, it does not matter whether you integrate from E c to E c plus 1 E v or E c to infinity it is 1 and the same. That is why I am writing the total number of carriers in the conduction band as n is equal to E c to infinity. So, substitute here rho of E integral one among 2 pi square 2 m c by h cross square to the power 3 by 2 into E minus E c to the power of half into f of E. So, 1 divided by E to the power E minus E f by E t plus 1 d E, E c to infinity. This gives as the carrier concentration. Before I carry out the actual integration here this is actually not possible to integral analytical, all right. Let me continue with this.

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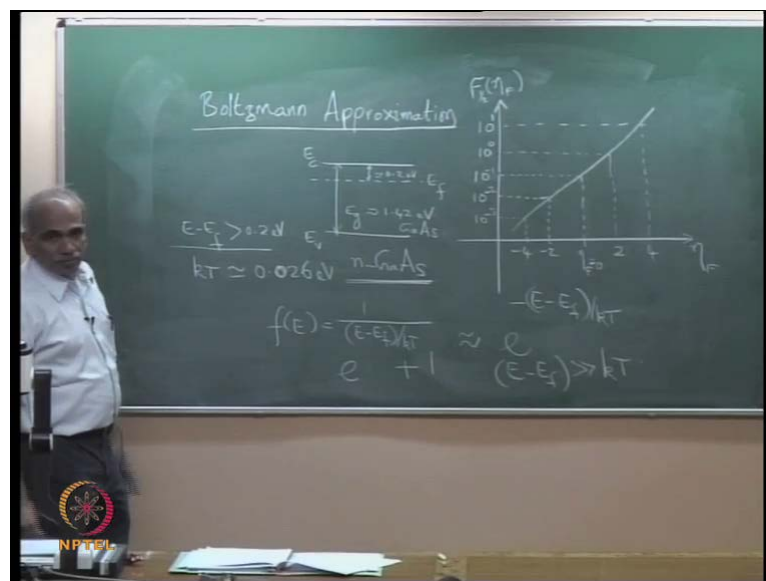


So let me take out 1 over 2 pi square into 2 m c by h cross square to the power v by 2. Constants I have taken out into integral E c to infinity E minus E c to the power of half 1 divided by plus 1. This can be integral only numerically or we can make some simplification. You add and subtract E c here in the denominator and this multiply divided by k t. So, this is equal to 1 over 2 pi square twice m c by h cross square to the power 3 by 2. So, I add and subtract E c and then, we put E minus E c E minus E c by k t equal to eta and E c minus E f by k t equal to eta f in the lower. So, this is d E, d E is equal to d eta into k t and substitute in the expression.

Further, when E is equal to E c this is 0 E c, E is equal to E c then the lower limit is 0 and when E is equal to infinity it is still remains infinity. So, I have 0 to infinity eta to the power of half into d eta divided by E to the power eta minus eta of by k t eta minus eta f plus 1. So, eta is already dimension less. These are the substitution it make this substitution and you see that you get this expression. And outside you will have 1 k t coming from here and 1 root k t coming from this so, you will have k t to the power 3 by 2. Please do this substitution and see. So, you will have this equal to 1 over 2 pi square into twice m c by h cross square into k t to the power 3 by 2 into this integral here is called the fermi half integral, f half of eta. f half of eta this is called this last step is called the fermi half integral.

I will come to the simpler Boltzmann approximation but, because I had written the complete expression I thought I will give you the vigorously the correct solution. So, this is n . There is no approximation in this so fermi half integral. This has to be evaluated numerically you cannot there is no analytical solution this has to be evaluated numerically. So, we have 1 over 2π square to $m c$ by h cross to the power 3 by 2 , $k t$ to the power 3 by 2 , we see $1 k t$ coming for here, square root of $k t$ and $1 k t$ coming from here. So, that is why $k t$ to the power 3 by 2 . η and η are dimensionless that is why it is E to the power η minus ηf . This energy dimension energy dimension so η is dimension. So, this is called the fermi half integral. So, if you have to actually calculate the carrier concentration for any value of E_f whether it is highly or doped or low dope does not matter this is the rigorously correct solution fermi half integral. There are the approximation which are available. The fermi half integral varies. I will just show you what kind of value we talk of for the fermi half integral. Just to get an idea whether we are talking of 100s or 1000s million, what kind of number is this?

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So, this fermi half integral varies approximately just showing you approximately. So, this is fermi half integral half ηf versus ηf . Because it is already integrating over energy therefore, it simply depends on position of ηf where is η and ηf is $E_c - E_f$ by $k t$ which means it simply depends on the position E_f . So, at ηf is equal to 0 so, this is ηf is equal to 0 this are approximate number just to get you are idea so this is approximately 0.1 which gives you some idea that what kind of number we are talking

of. So, this is 0, this is 10. So, $0.1 \cdot 10^{\text{power minus } 1}$, $10^{\text{power minus } 2}$, $10^{\text{power minus } 3}$, the numbers are 0 here, this is 0 around η_f equal to 0.24, this is minus 2 minus 4 and so on. Typical numbers for the Fermi. η_f is equal to 0, means what? E_f is at E_c , the fermi level entered the conduction band. At E_f equal to E_c η_f is 0. The fermi integral is approximation point approximation numbers please, it just get you get me an idea what kind of numbers are here. So, let us say the fermi function just entered the conduction band. So, this is your E_c and E_v in a highly doped in a doped semiconductor. So, E_f has also come here then η_f is equal to 0 and this is approximate 0.1. So, you substitute 0.1 and put all that numbers here and you get the carried concentration. So, this is the exact way of calculating. But, we are also familiar with the boltzman approximation it is by simpler I come to the boltzman approximate. So, what is the boltzman approximation? Boltzmann's approximation. Any doubts? Which one? This one. Yes. So, let me write in terms of 10^0 , 10^1 . Yes, that is right. Yes right because E will become 0 then, carrier concentration will become 0 if it is 10^0 . So, please make a correction there if fermi integral is 0 then, a carrier concentration is 0 and this value at this such a high value of intent that becomes 0 than, yes. So, the boltzman approximation is correct that normal doped semiconductor, if you take an n type doped semiconductor then, E_f may be somewhere here. You take gallium arsenide or silicon. So, this is E_g is approximately at the order of 1.42 for gallium arsenide and it as already more toward the conduction band.

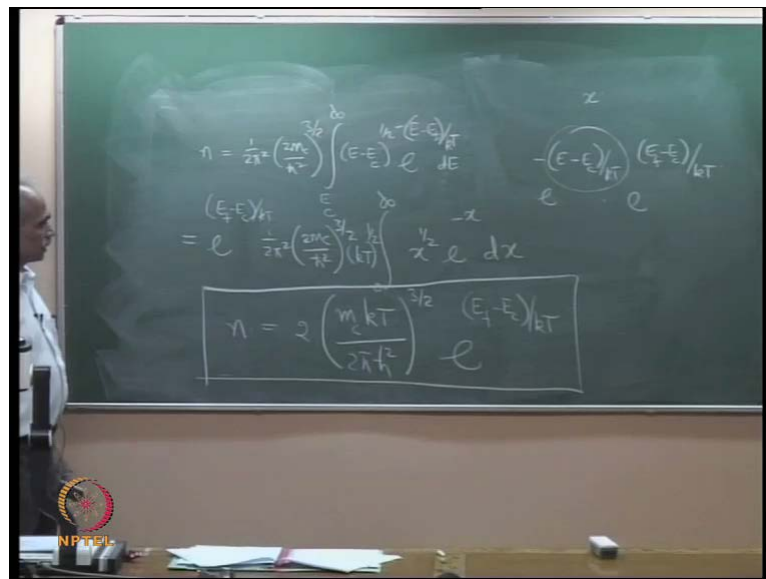
Let us see this difference here is about 0.2, $0.2 E_v$. I put some numbers because numbers make it clearer that is wise use numbers. So, $0.2 E_v$. Let us say the different here is it is always the move toward the conduction band because it is a n type material. So, n gallium arsenide. The probability of occupation f of E is given by 1 divided by $e^{E - E_f / kT} + 1$ in the conduction band.

So, in the conduction band E is greater than E_c , E is greater than E_c for conduction band. This is forward of both conduction band and valance band but, for conduction band E is greater than E_c . So, if you substitute E here then, $E - E_f$ is always greater than for this particular example that I have taken, $E - E_f$ is greater than point 2 E_v . At room temperature so, $E - E_f$ is greater than point 2 E_v . I am writing this for this particular example that I have taken $E - E_f$ is greater than $0.2 E_v$. And kT , at room temperature is nearly equal to 0.026 electron volts 0.26 electron volts.

Which means this exponent here is 0.2 divided by 0.026 which is approximately 8 is the exponent. So, you have E to the power 8 plus 1, E to the power 8 is a very very large number compared to 1. E to the power 8 is much larger then compare to 1 and you can neglect this 1 compared to E power 8. And therefore, this I can write approximately equal to for E minus E f much greater than E t, I can neglect to 1 in comparison with this and I can write this is equal to E minus E f by k t. This is the boltzman approximation.

In Boltzman approximation, the Boltzmann's approximation is valid when E minus E f is much E f is much greater than k t. When E minus f is much greater than k t then 1 is neglected in comparison to this number and therefore, we write this as approximately equal to E minus E f by k t. If you substitute this, then that integral is easily integrated. This is true for both conduction band and valance band. The only requirement is E minus f should be much greater or E f minus E for the valance band E f minus E should be much greater then k t.

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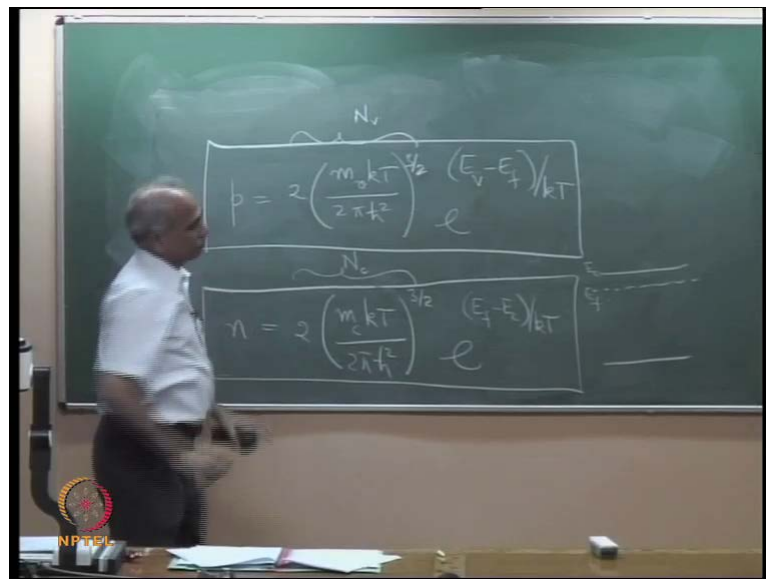


Then, n is equal to all those constant 1 over 2 pi square 2 m c by h cross square to the power 3 by 2 into integral E c to infinity, we had rho of E which is E minus E c to the power half into e to the power E minus E f, minus E minus E f, here by E t. Let me erase this let me rub this off. This is now integrable, this is easily integrable. So, E minus E f by k t. So, how do we integrate this? Follow the same procedure add and subtract the E c, some of you would have done already so please. So, E minus this will become E minus

minus E_c by kT into E_f minus E_c by E_t . Added and subtracted for this term. And then, we can put this as x there I will have this equal to 1 over 2π square $2m_c$ by h cross square. Let me complete the algebra here and integral 0 to infinity as before I am putting this equal to x . Therefore, at E is equal to E_c lower limit it is 0 so, 0 to infinity this will be x to the power half and E to the power minus x dx into this term outside.

Let me keep it here, E to the power E_f minus E_c by kT , any other term? Any other term? kT to the power so, dE will contain $1/kT$, $1/kT$ to the power half. So, kT to the power half kT to the power half. So, simplify this, you should be getting an expression like this $m_c kT$ by $2\pi h$ cross square to the power $3/2$ into e to the power E_f minus E_c by kT . Please, simplify this and check the n is equal to 2 into $m_c kT$ by $2\pi h$ cross square to the power $3/2$ into e to the power E_f by kT . So, this is under the boltzman approximation.

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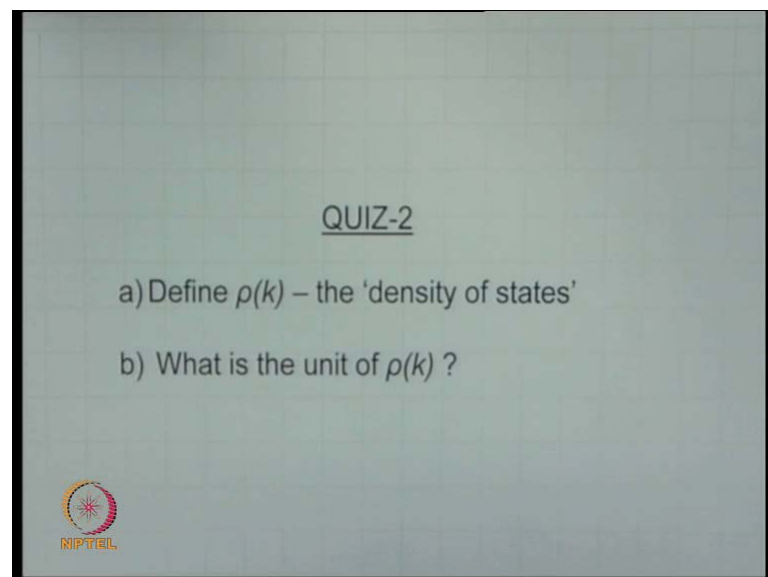


Exactly like this if you follow for the valance band you get expression for p is equal to 2 into $m_v kT$ by $2\pi h$ cross square to the power $3/2$ into e to the power E_f , e to the power E_v minus E_f by kT . This is sometimes denoted as n_c for the conduction band and this is denoted as n_v . You see that it contains every think is concept except the effective mass, effect mass m_c in mass m_c and effect mass n_v in the case of m_v in the case of n_v . If m_c equals to m_v both constant have the same. This is a very large

number, if you put numbers you will see that this will be 10 to the power of 18, 10 to the power of 19 or very large number this part. Sometimes you are getting doubt whether it is E_f minus, may be you had made mistake E_f minus E_c or E_c minus E_f . How do you verify? E_f minus E_c so, this is for the conduction band under boltzman approximation which means my E_f is somewhere here, this is E_c , this is E_f under boltzman approximation. So, E_f minus E_c is negative E_f is smaller E_c is larger. So, this is negative therefore, this qualities will be e to the power negative which means factor less than 1.

And the factor has to be the less than 1 because the carrier concentrations I am talking about order of 10 to the power of 10, 10 to the power of 12 and if this number is very larger 10 to the power 19 naturally, this number should be smaller much smaller than 1 and therefore, it is correct this correct E_f minus E_c . A cross check, a quick cross check, sometimes were by using a negative sign or something you would have plot as E_c minus E_f . If this becomes E_c minus E_f this the positive number much greater then kT so, this will also give e to the power of 8, e to the power of 10 which is a very larger number. This is correct, all right. I will stop here because we have to take a quiz today, a very simple quiz.

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So, please take a sheet. This is a very simple one. And please write your name first before you forget. Define rho of k, the density of states and 'b' what is the unit of rho of k? Define prescribedly rho of k. In the last week I think we would have defined atleast

10 times. So, I am sure you will be there with you. Those who finished you just give it to me and you can leave.