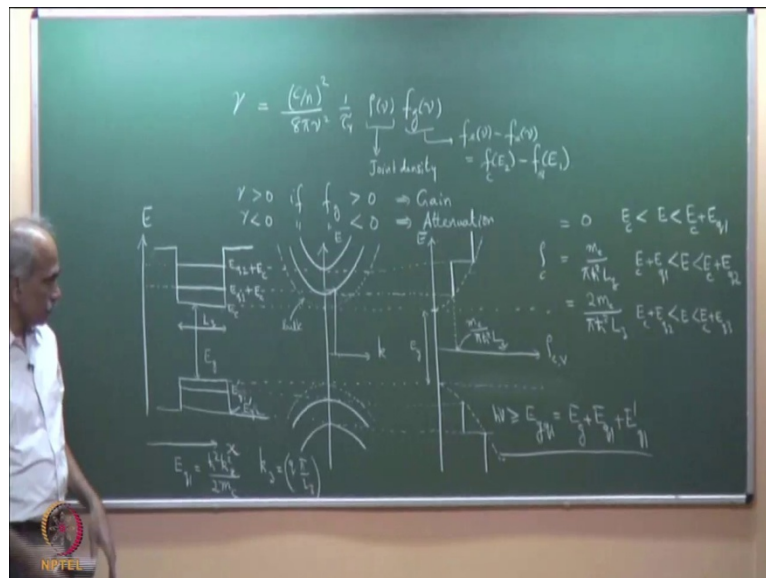


Semiconductor Optoelectronics
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Lecture - 23
Gain and Absorption Spectrum of Quantum Well Structures

In the last couple of classes, we discussed about amplification in a semiconductor and also the absorption spectrum. We discussed these issues with respect to bulk semiconductors. And today, we will see the absorption spectrum and gain spectrum in a quantum well structure.

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So, recall that we have an expression for gain coefficient gamma is equal to c by n square divided by 8 pi nu square 1 divided by tau r into rho of nu into f g of nu. This is the fermi inversion factor, which is essentially f e minus f e of nu minus f a of nu. Probability of emission minus probability of absorption which is also equal to f of E 2 f c of if it is in quasi equilibrium. Then it is c of E 2 minus f c f v of E 1. This is the optical joint density of states rho nu.

So, joint density of states; this is the radiative recombination life time. And this c by nu can also be written as lambda. But then we have to remember the, that is wavelength in the medium, lambda in the medium c by ns c by n by nu is nothing but wavelength

λ , so λ^2 . So, I prefer to write it in this fashion. So, this is the gain coefficient. This is $\gamma > 0$ if f_g of ν is greater than 0. And then we have gain implies gain, and $\gamma < 0$ if f_g of ν is less than 0 and we have attenuation or loss.

It is the joint density of states which determine the gain profile, γ of ν is primarily determined by ρ of ν . So, today we will see the gain and loss in a quantum well structure. So, recall that in a quantum well structure. So, let me draw the quantum well structure here I will draw three figures. So, you will need space here this is I would like you to draw three parallel figures. So, this is energy axis E versus x . This is E_c of the low band gap material and this is E_v . So, this is E_g .

So, this is E_g of the low band gap material. And corresponding to this I would like to draw the corresponding E_k diagram, and the density of states that we have already discussed. But let we draw this. So, the E_k diagrams if I plot the E_k diagram this is E versus k . Then you know that at E equal to E_c we do not have any density of any states there. And therefore, in a quantum well if this quantum well supports energy E_1 here and. So, E_{q1} , E_{q1} equal to 1.

So, E_{q1} let we call this is E_{q1} and E_{q2} . Similarly, for the holes we have states here E_{q1} and E_{q2} . The dash is just to say that it is basically q equal to 1 and q equal to 2 the dash that have written is to distinguish between. So, let me write E_{q1} and E_{q2} that is for the holes two state here. And two states here have I taken quantum well suppose it is take two states.

So, the corresponding E_k diagram if you plot we have energy sub bands. And the sub bands look like this, they are they have the same k square kind of the dependent. But only the transverse k_t and there is no allowed state at E_c the earlier E_c . This was E_c and this was E_v . Similarly, for the valance band we have if m_c is not equal to m_v the shape would be slightly different.

So, these are the energy sub bands. This is k axis for different what it tells is for a given value of k , the allowed value of E gives you the E_k diagrams. So, this is the quantum well. We have also got the density of states if I had taken the bulk material; this material which was bulk. Then I would had E_k diagram going like this, please remember right from E_c .

So, this is for the bulk the dotted line is showing for the bulk. Let me write here bulk. And these are for the energy sub band for the quantum well structure. We also want to plot E versus the density of states. So, ρ_c comma v of v in just recalling what we have already studied the density of states. The density of states in the case of bulk had a dependence which is E to the power half dependence. This is for the bulk and in the case of quantum well it started the density of states will start at E_{q1} this value here.

So, this is E_{q1} actually E_c plus E_{q1} . Please remember that the total energy here is E_{q1} plus E_c . Please add this E_c here because the way, we have define E_{q1} . So, let me write here E_{q1} was $\frac{\hbar^2 k^2}{2m^*}$ here $\frac{\hbar^2 k^2}{2m^*}$ k_z^2 square $\hbar^2 k^2$ square divided by $2m^*$. So, this is k_z^2 square which means it is only above E_c . Therefore, if I am considering the total energy here it is E_c plus E_{q1} this is E_c plus E_{q2} .

So, this case at just reminding you that this case at was q times π by l_z , q times π by l_z . If you put q equal to 1 it is k_{q1} is it. And the corresponding energy we called has E_{q1} just to remind you that this was k_z in this expression. So, the density of states they also derived the density of states up to this there is no density of states. But at E_{q1} there is a constant density of states here. And then again we do not have any states constant density of there is a sudden jump from here to here this is energy axis. So, this value is E_c this is E_c ; this is E_c plus E_{q1} . Exactly like that here also at E_v we do not have any density of states. But we have density of states at this value here. You see this value here is the density of states; this axis is now density of states.

So, density of states is constant this is constant for all energy values here up to E_c plus E_{q1} we have no density of states which means there are no states at E_c plus E_{q1} . We have suddenly the density of states here and that density of states remaining constant. Because this is the axes of the density of states for all energies up to E_c plus E_{q2} that is this value E_c plus E_{q2} . And then suddenly there is a jump which is double. If you recall that this number here was $m^* c$ divided by $\pi \hbar^2$ cross square into L_z $m^* c$ divided by $\pi \hbar^2$ cross square into L_z . L_z is the dimension of this.

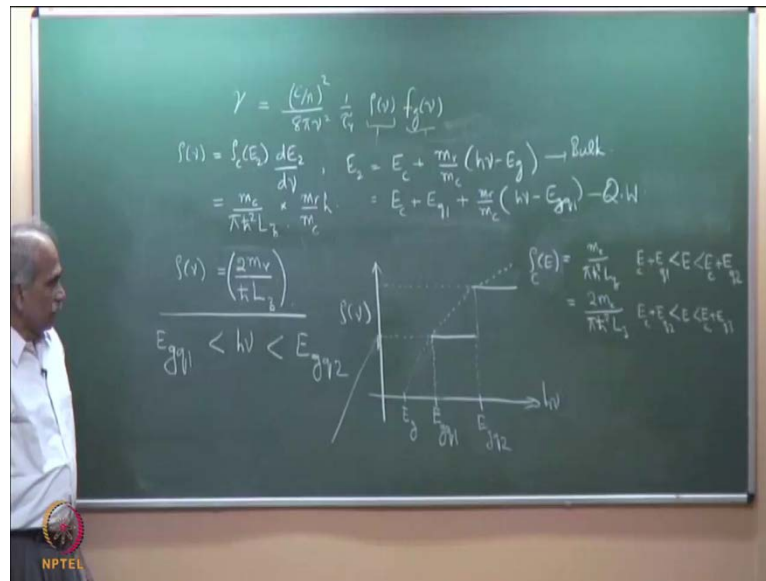
So, this dimension I had taken as l_z in the z direction width in the z direction. And this will be double of that two times $m^* c$. So, if I write if you ρ_c was equal to $m^* c$ divided by $\pi \hbar^2$ cross square L_z up to E_c plus E_{q1} beyond E_c plus E_{q1} up to E_c plus E_{q2}

up to this it is constant. And from here it suddenly jumps to. So, this is for $E_c + E_{q1}$ less than $E_c + E_{q2}$. I have come to the end of the board $E_c + E_{q2}$. And this is equal to 0 up to this value equal to 0 for between E_c and E_c less than E_c for all energy values $E_c + E_q$, which was not the case earlier in a bulk, and this is equal to 2 times it doubles 2 times $m c$. You recall the discussion that we had with two discs when we get a second disc. And this for $E_c + E_{q2}$ onwards for E greater than $E_c + E_{q1}$. Let me write that otherwise.

So, $E_c + E_{q2}$ less than $E_c + E_{q3}$ and so on. And similarly, ρ_v will be $m c$ is replaced by $m v$ that is all. So the point is you have a step function density of states is a step function. Same thing is true here density of states is the step function. So from here you have the next step. So, photons; this is E_g . In this diagram also you can see this is E_g . In the case of quantum well photons of energy equal E_g energy $h \nu$ equal to E_g does not have any state to interact. The first the lowest energy photon would correspond to this different which I call as E_{gq1} E_{gq1} is $E_c + E_{q1} + E_{dq1}$.

This is the first photon which can act which can interact which means $E_g + E_{gq1}$ is equal to $E_g + E_{q1}$. This is in the conduction band plus E_{dq1} in the valance band. So, interaction can takes place for photons of energy $h \nu$ greater than or equal to E_{gq1} . This is the first point to remember, you can find out the corresponding optical joint density of states keep this picture in mind. Let me erase this, the optical joint density of states can be found out exactly right what we have done earlier.

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If you recall that we had return ρ_{ν} equal to ρ_c of E_2 into dE_2 . Therefore, ρ_{ν} is equal to dE_2 by $d\nu$, E_2 we had an expression for E_2 , E_2 in terms of ν was E_c plus m_r by m_c into $h\nu$ minus E_g this for bulk. Now, E_2 is an energy level in the conduction band which is E_c plus E_{g1} and then this difference. So, this will be equal to this is for bulk. You work out exactly similarly you will get this is equal to E_{g1} plus m_r by m_c into $h\nu$ minus E_{g1} . Let me erase little bit here. This is for quantum well.

Therefore, dE_2 by $d\nu$ will be of course, these are constants will be as before m_r by m_c into h , dE_2 by $d\nu$ is m_r by m_c into h all others are constants m_r by m_c into h . Therefore, you substitute here ρ_c of E_2 . So, this is ρ_c , ρ_c of E here when I am finding ρ of ν , ρ_c of E was this expression. So, you substitute for the first one. So, this m_c by πh cross square into L_z into dE_2 by $d\nu$, which is into m_r by m_c into h that is all.

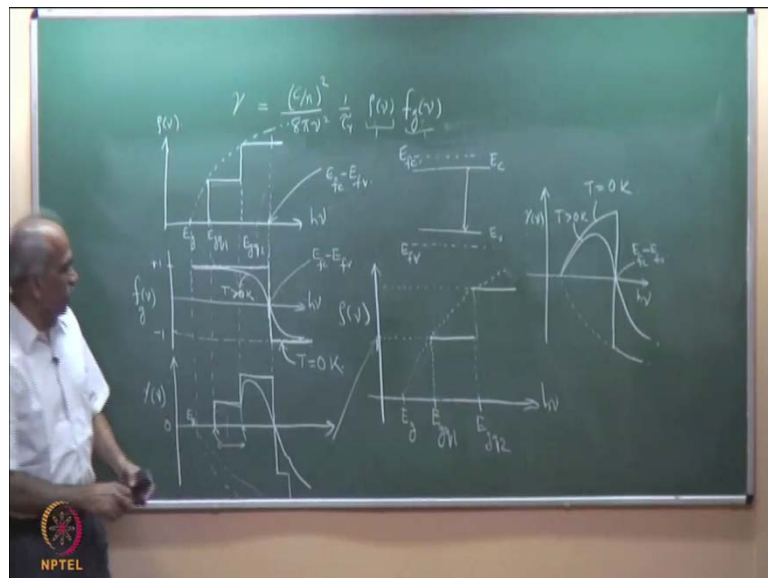
So, ρ_{ν} is again constant just like here it was constant it is again constant ρ_{ν} is equal to m_c m_c goes here m_r one h would go here by 2 , π^2 will come up. So, 2 divided by π , no let me keep πh cross. So, $1 h$ cross let me keep here $1 h$ goes with this the 2π is up. So, 2 is there π is there. So, this is h cross into L_z that is all which is a constant which is independent of ν . So, for $h\nu < E_{g1}$ less than $h\nu$ less than E_g

q 2, what is E_g ? E_g is this difference E_g band gap from $q=1$ here to $q=2$ is this difference.

So, the density of states are constant here, and you get the same thing density of states is constant. So, if I plot this density of states that is optical joint density of states. How did it look like? The optical joint density of states in a quantum well. So, I plot $\rho(\nu)$ in a quantum well structure E_g is here E_g $q=1$ E_g $q=2$. So, this is $h\nu$ and $\rho(\nu)$ for the bulk x to the power of half dependence. So, we have a constant here. So, the density of states is constant up to E_g $q=2$. And then jumps up again remain constant up to E_g $q=3$ the value here. This value here is $2 m_r$ into h cross L_z .

So, this value here is $2 m_r h$ cross L_z and what would this ν ? 2 times $2 m_r h$ cross L_z double the density of state simply doubles. So, the optical joint density of states in the case of bulk was varying like this. In the case of a quantum well it varies like a step function. This is important because we want to get the gain coefficient, and loss coefficient, because the profile the gain profile and absorption spectrum is determined by $\rho(\nu)$. This is just in the derivation which is just extension of derivation are for density of states.

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So, let us now, find the gain probe gain spectrum. So, let we draw it here, let we draw a fresh I want to draw in the same line three curves one below another now. So, first one is density of states. So, the bulk is here. So, this is $\rho(\nu)$ versus $h\nu$ the variation for a

quantum well structure. This is E_g this is E_{g1} and E_{g2} I want to plot also f_g of ν . Let us recall the part experiment that we have done at 0 k. At 0 k up to $E_{fc} - E_{fv}$ f_g greater than 0 up to an energy which is equal to. Let us say we are here; this is $h\nu$; this is E_g and this is $E_{fc} - E_{fv}$. Recall what we had done $E_{fc} - E_{fv}$ that is the separation between the fermi levels.

So, I am now looking the semiconductor which is in quasi equilibrium; this is E_v ; this is E_c . And E_{fc} has entered the band E_{fv} has entered the band. So, this is E_g and this is $E_{fc} - E_{fv}$. So, $E_{fc} - E_{fv}$, what we had seen was for all $h\nu$ between this, and this if I take 0 k experiment at 0 k. Then f_g of ν was plus 1 and for all others if it is minus 1. If it exceeds it is minus 1 which means if I plot f_g of ν here f_g of ν . Then it is this is plus 1 plus 1, and this is minus 1. So, it remained plus 1. This is experiment at 0 k up to this, and then it went down to minus 1 the step function. This is $E_{fc} - E_{fv}$ f_g of the probability of emission minus the probability of absorption. The fermi inversion factor was plus 1 up to this and minus 1 up to this. And accordingly if you recall that the gain if I had the $E_{fc} - E_{fv}$ let us say $E_{fc} - E_{fv}$ was here $E_{fc} - E_{fv}$.

So, I could show it right here on the same over this or let me on the same line. Let me show this here because all the three figures which will get one below another. So, this is $E_{fc} - E_{fv}$. So, the product of $\rho\nu$ into f_g of ν for the bulk we had a gain profile which went up to this and then it short down. So, I will draw it here just to remind you how it was there for the bulk. So, the gain profile went like this it followed exactly ρ of ν . And then it came down, because beyond this value we had $h\nu$ versus gain. So, γ of ν and at 0 k it did this.

So, again it was positive, negative beyond $E_{fc} - E_{fv}$. This was for t equal to 0 at a finite temperature this changed to this why did this changed? Because f_g of ν was no more a step function f_g of ν was varying like this that is why this change over to this. So, this is for t equal to 0 k. And this is for t greater than 0 k at a finite temperature that is for the bulk I want to draw the same thing for quantum well structure. So, if I want to plot the same thing for quantum well structure. Let me plot it here below. So, this is ρ of ν I want to plot γ of ν the third figure. So, how do we go about this, first let me plot for 0 k. So, it is plus 1 or minus 1.

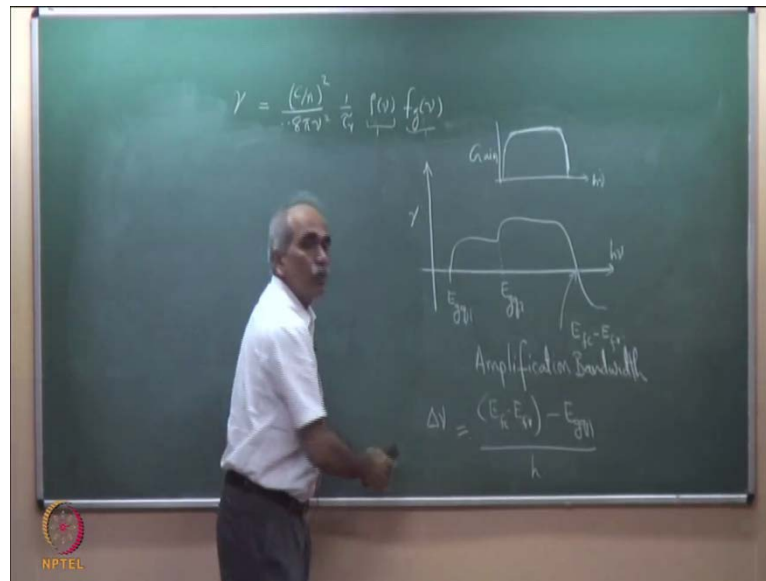
So, the gain will be this into this the density of states starts only here from here. Therefore, the gain will also from there. And the gain will remain constant like this, same as is this multiplied by this, this multiplied by this two functions. So, first function ρ of ν is varying like this and f_g of ν is also varying like this. So, I have let me draw this fully why draw. Because there are so many dash lines that you might have confusion, let me draw it fully. So, that is f_g of ν . So, the product of these two is constant. When you come at this value $E_g q_2$ there is a jump. So, the gain would also just becomes double, because the density of states becomes double at this value here.

So, this has gone up to this, but the next $E_g q_3$ has not come. But at this value we have f_g of q becoming negative minus 1; this is minus 1 here and therefore, minus into this. So, we have the gain dropping down like this and this will become like this. So, we have gain profile which is a step function at $0 k$. So, what I have plotted is the gain profile. Please remember at E_g there is no gain this value is $E_g q_1$ and this value is $E_g q_2$. So, what is the point? Wavelengths or frequencies which correspond to within this band from here to here the gain is constant flat is a flat gain. Similarly, for frequencies from here to here gain is flat, but a different value it is a step function this is at $0 k$. If I go for a finite temperature f_E of ν will get modified.

So, this will get modified and this will cross here and go in this fashion. So, this is f_g of ν for t greater than 0 greater than $0 k$ the step function was for $0 k$. So, this was for T equal to $0 k$, F_g of ν dose this just as before. Therefore, the gain here would also it is a product of these two, this remains constant. But this is a product and therefore, if you multiplied initially it will remain the same. But slowly the difference will start coming up. So, it separating out is coming down here there is a step here no doubt. But it separating out and then when this become 0 the product is 0 .

So, this will pass through like this till the next step there will be a next step here. And you will have a step here, and then a second step which is coming. But that is the absorption part, γ is positive, γ is 0 γ is gain. And this is loss, this loss part we will discuss separately. Now, you see that in a practical amplifier quantum well amplifier the gain will have the spectrum like this. You can see in the literature just open it up little bit you can see actual calculated. And measured gain profiles in the literature and you will see them indeed there are step like variations. So, let me just widen this range.

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And show how you in practice the gain of a quantum well amplifier look likes. So, typically it looks like. So, what I have plotted is gain coefficient versus energy. So, this is the value where gain starts at E_{g1} please remember E_{g1} not E_g . And this is jump here is at E_{g2} . And obviously, this cross over point is always E_{fc} minus E_{fv} which determines the amplification bandwidth. The amplification bandwidth in this case is always E_{fc} to E_{g1} . In the case of a bulk amplifier it would be E_{fc} to E_g , E_g to E_{fc} is the amplification bandwidth I am not calling it as a gain bandwidth. Because gain bandwidth is normally defined at the frequencies at which the gain falls to 3 db that is falls to half.

So, this I am calling as amplification bandwidth, because in this o for all frequencies in this range you have gain. So, we call it has amplification bandwidth in the last class when we had written the expression it was for amplification bandwidth. So, in this case amplification bandwidth is determined by E_{fc} minus E_{fv} minus E_{g1} , E_{g1} divided by h . So, this is the amplification bandwidth $\Delta\nu$, $h\Delta\nu$ is the energy difference. Therefore, $h\Delta\nu$ the bandwidth is given by this for a quantum well structure. If you design properly it is possible to get this cross over right here E_{g2} because I need to pump only that much.

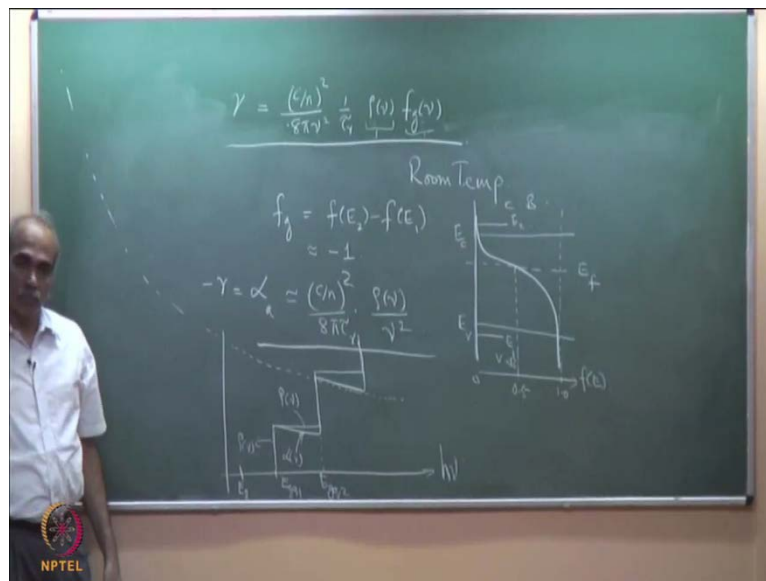
So, that the separation between E_{fc} minus E_{fv} is here. And you will have a nice amplifier like this almost flat it almost like a box type is an ideal amplifier characteristic

is you want the gain to be flat like a box. So, if you properly design and shift this here of course, the range is small, but you will have a box. This is gain versus frequency or energy a flat band amplifier quantum well amplifier. We will discuss more about this quantum (()) well at a later stage.

Let me now come to attenuate spectrum, loss spectrum identical discussion. And but you see only difference when we discuss about attenuation. And gain is remember that the gain bandwidth amplification bandwidth is very small in the frequency range about 10 to the power of 13 Hertz. Whereas, when we discuss about loss we are discussing over a very wide a spectrum. We discuss the gain spectrum over a loss spectrum over wide energy range, because if you take silicon detector. For example, you it can detect anywhere from 0.4 micro meter that is visible down to 1.1 micro meter. So, it is a large energy $h\nu$ range.

Whereas, the gain bandwidth is much smaller compare to the absorption range. And therefore, what is the difference we have to also include the effect of this ν in discussion of the gain we take this has constant. Because the range is very narrow therefore, we say that over the range of interest $1/\nu^2$ variation is neglected. But in the case of loss spectrum we would like to keep the $1/\nu^2$ dependence.

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So, let us discuss I am still keeping the same expression here if f_g of ν is negative. Let us take for example, room temperature, room temperature in normal semiconductor at

thermal equilibrium how would it have the? So, this is E_v this is E_c , and the E_f is somewhere where ever you want the normal. So, E_f is somewhere here which means if you plot the fermi distribution.

So, what I have plotted on this axis is also $f(E)$ thermal equilibrium, normal semiconductors, absorption spectrum I want to determine. So, this is E_f , and this is $f(E)$ which is 1.0 here and 0 here. And how we determine the fermi energy? Here it is 0.5 and therefore, $f(E_2)$ here $f(E_1)$ or $f(E)$. So, is equal to $f(E_2) - f(E_1)$, $f(E_2)$ here equal to $f(E_2) - f(E_2)$ is an any energy value here.

So, this is E_2 and E_1 is some value here in the valance band. So, this is the valance band; this is the conduction band, $f(E_2)$ is a very small value here. If you put actual numbers as I already mentioned it will be very, very 0.0001 or something like that. So, very small value and $f(E_1)$ is very, very close to 1.9999 which means this is 0 minus 0.99. So, it is approximately equal to minus 1 approximately a little less than minus 1, but approximately equal to minus 1.

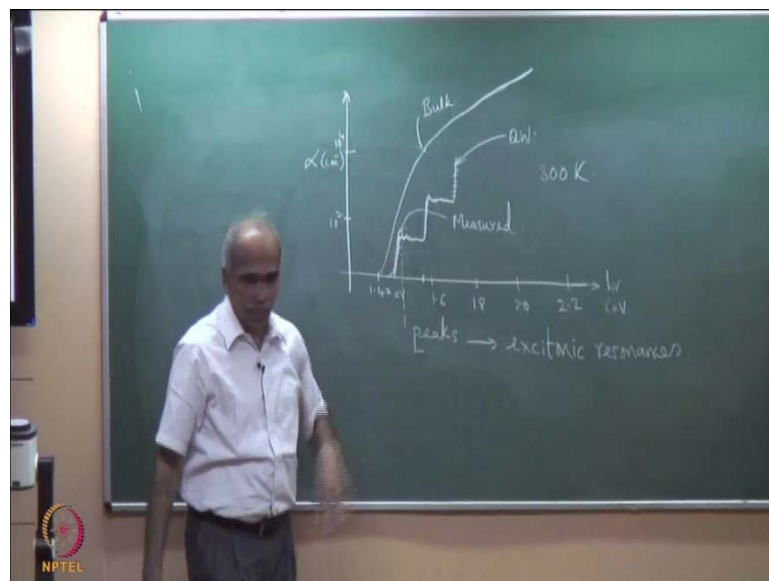
So, I can safely write $\alpha - \gamma = \alpha$ equal to is approximately equal to $c \cdot n^2 \cdot \pi \cdot \tau \cdot \rho$ divided by n^2 , $f(E)$ I have written have as minus 1. So, minus sign was there and that is the attenuation coefficient. For a quantum well structure ρ is a constant it is a step function up to certain energy. So, what kind of attenuation spectrum would you observed? So, let me plot the attenuation spectrum what is expected.

So, this is $h\nu$ 1 over n^2 is a monotonically dropping function 1 over x^2 dependence. Remember that we are coming from a along frequency long range. So, this is continuous, this is actually going like this and going down to 0. So, in our optical region this is continuously dropping like this 1 over n^2 . What is ρ doing? ρ is the a step function.

So, ρ is constant; constant it is a stair case like function. So, this is ρ where does it start? This is E_{g1} E_{g1} E_{g2} is somewhere here E_{g1} and this is E_{g2} . Remember again what is E_{g1} is the difference between the first sub band in the conduction band to the first sub band in the valance band. And E_{g2} is the difference in the second to the second. So, this is E_{g2} E_{g2} and so on.

So, the product this is the very small variation, but nevertheless if I multiply this and this. What you expect is a function which is the product expected. So, this is the upper one is ρ of ν and the lower one is α of ν into some constants are there. But the variation I am showing only the variation. So, this is α of ν the important point you see in a quantum well structure is a step like variation. So, if I now forget all these expressions. And compare the attenuation spectrum of bulk and quantum well structures, what do we expect and where is the difference? A bulk semiconductor let me take a direct band gap semiconductor like gallium arsenide.

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So, this is $h\nu$, and this is E_g 1.42 eV. And let me draw some values I do not know let me spread 1.42. This is let me say 1.6, 1.8, 2.0, 2.2 and. So, on E_g $h\nu$ E_g . And this is α axis α centimeter inverse. And typical numbers 10^2 10^4 10^5 and. So, on centimeter inverse in the bulk case we would get a dependence which would be something like this. Compared to what I had drawn earlier I have treated this you remember earlier when I drawn almost short vertically. And then but remember my scale was different it was going to 3×10^3 eV s.

Now I just expanded the scale that is why it is slightly celeting is slightly going at an angle. Because of the expansion in the scale I had to expand because these differences are not 1 2 3 it is a small difference. Because it is corresponding to the energy difference

here in the quantum well it is only 50 to 100 meV difference it is not difference. That means, 0.1 eV is the difference between these two not a 1 eV. That is why I had to expand this to be realistic. E_g q₁ will be somewhere here. And E_g q₂ may be somewhere here 1.58 approximately 1 point 1.4 6 5 2 something like that approximately I am just giving you an idea. But how do we expect? We expect that this should go up like a step function. Now, not like in quantum it is almost a step function. And then it becomes flat here, and then again a step function a little bit downward. And you indeed see this when you measure I will show you the measured spectrum how does that look like.

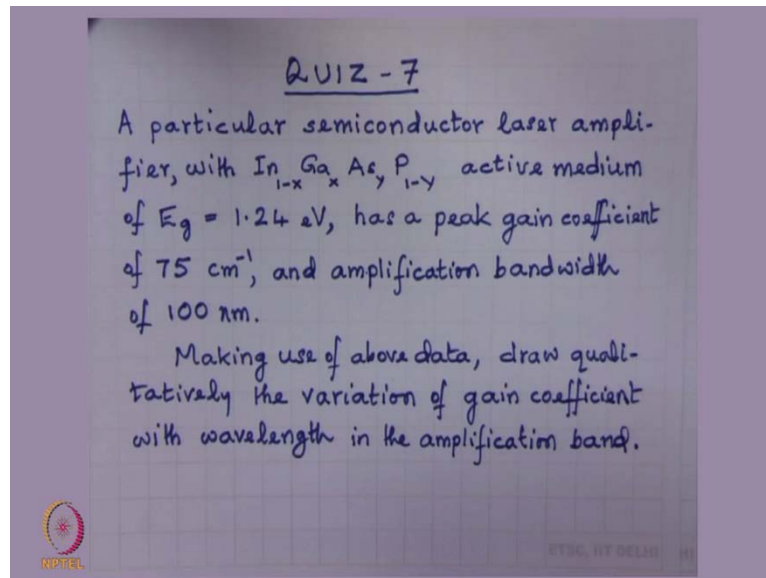
So, you see you expect step like variation. A measured spectrum almost shows this a measured spectrum if I plot on the same. So, let me just for differentiation. Let me make this; this is theoretically what you expect, if you expect this theoretically for a quantum well. This is for the bulk, bulk gallium arsenide starts from E_g first difference you see is this starts at E_g q₁ and then it is a step function.

So, this is for step function in quantum well. What is a measured shows a spectrum, a little bit earlier its starts it goes like this. And then it shows a little bit of peaking like this. And then of course, it continues there again it comes up a little bit of peaking. And then it goes like this. So, on the same thing there is a little bit of peaking which we see in a quantum well absorption spectrum of a quantum well. So, this is the measured one this is measure. This is theoretically expected and measured.

So, one thing is sure it shows the step like variation. But there are some peaks there, what do we think is the explanation for these peaks? These peaks correspond to excitonic peaks. So, the peaks correspond to excitonic resonances. This plays a very important role in the device that we will discuss quantum well modulators or electro absorption modulators, the peaks correspond to excitonic resonances. In the bulk case we do not see anything. All these measurements are at room temperature.

So, this is at 300 K, we are able to see excitonic resonances in the case of quantum well structures. Why do not we see that in the case of bulk? Let us stop here this discussion. And continue in the next class because we have to take a quiz now is a very interesting explanations and discussions follow. But I will continue in the next class.

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So, please let us take a quiz a simple quiz all quizzes are very simple. Let we read it for you. A particular semiconductor laser amplifier with indium, gallium, arsenide, phosphate active medium of E_g equal to 1.24 eV , has a peak gain coefficient of 75 cm^{-1} peak gain coefficient that is peak of the gain spectrum. And amplification bandwidth of 100 nm it is in wavelength. Now, 100 nm is the amplification bandwidth.

Making use of the above data, draw qualitatively the variation of gain spectrum with wavelength in the amplification band. Variation of gain spectrum with wavelength in the amplification band I have drawn this many times with the energy. So, you have been asked because in practice normally discuss in terms of wavelengths in optics. So, plot the variation of gain coefficient of qualitatively with wavelength γ of λ versus λ qualitatively gain variation with wavelength should not take more than 2 to 3 minutes, if you have completed you can leave.