Semiconductor Optoelectronics
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Lecture - 20
Amplification by Stimulated Emission

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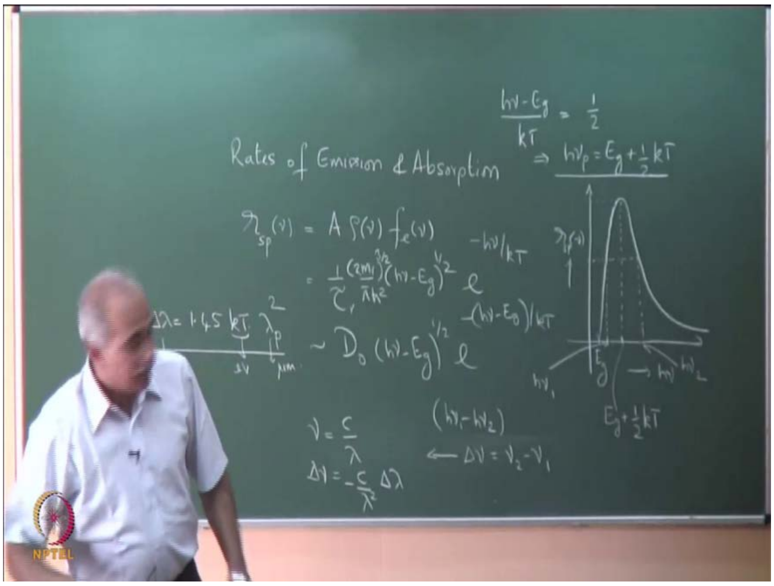

So, we start today’s topic is amplification by stimulated emission. So, recall that in the last class, we discussed about rates of emission and absorption and in particular, we discussed about the rate of spontaneous emission. We have got an expression for spontaneous emission that r s p of mu is equal to A into rho of mu density of states into f e of mu, where f e of mu is the probability of emission.

We substituted the various parameters, that is for rho of mu, we substituted the density. So, we have 1 over tau r. I am just recalling this is a recap. So, 1 over pi h cross square into h mu minus E g to the power half into we had shown that this is approximately f e is e to the power minus h mu by k T, and then we wrote this in the form of twice m r. There is an expression here, twice m r to the power 3 by 2 and we wrote this in the form of D 0 into h mu minus E g to the power half into e to the power minus h mu minus E g by k T and then, I had shown that express the variation looks something like this.

So, what I have plotted is r s p of mu spontaneous emission spectrum as a function of h mu and it starts at E g and then, we could find out the peak value. The peak value corresponds to E g is plus half k T. The peak of spontaneous emission occurs at energy value E g plus half k T. I had also to ask you to find out the full width that half maximum. So, you can find out the full width at half maximum. So, if this is the maximum, then half of it there you drop down and find out what are the corresponding values of h mu 1 and h mu 2.

Therefore, you can find out the bandwidth corresponding to this. So, corresponding to the frequency here, so we have let us say this is this point here is h mu 1 and this end here is h mu 2. Then, we have h mu 1 minus h mu 2, H mu 1 minus h mu 2 by k T. So, this is h mu. So, h mu 1 minus alright, let me I have written that just as h mu. So, h mu 1 minus h mu 2 and therefore, from this we can find out what is delta mu equal to mu 2 minus mu 1 and once you know delta mu, you can find out delta lambda that is the line width delta lambda is equal to lambda square divided by c.

Just a minute. So, mu is equal to c by lambda and therefore, delta mu is equal to c by lambda square into delta lambda with a negative sign. So, delta lambda is equal to lambda square into delta mu by c. So, I had given you an exercise to find the delta lambda. So, I had written an expression to show that delta lambda is approximately equal to 1.45 into k T into lambda p square approximately equal to. So, here k T is in electron volts and lambda p and delta lambda here are micron meters in this expression.

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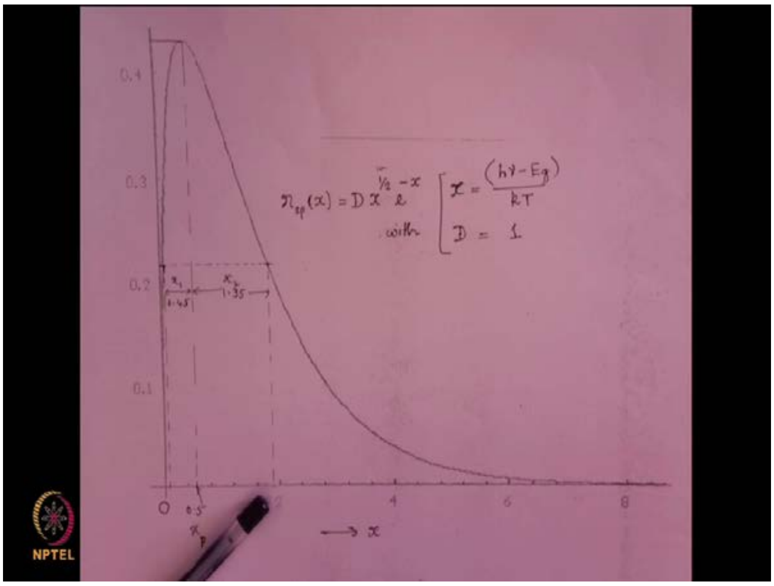

Here, I have a graph. I have actually plotted this function. I hope you can see this as you may not be able to read it. I have a slightly bigger one. Let me put as slightly bigger one as this is too small. Let us see whether this comes out, yes. So, what I would like to see is this is actual plot I have been qualitatively showing you here the variation like this. So, what I have plotted is r sp of mu. I have defined x as h mu minus E g by k T. Then, the expression for r sp becomes r sp equal to D into x power half. So, you can divide by k T to this and there is already a k T. So, e to the power of minus x and if I divide by square root of k T, this is x power half and k T to the power half has been absorbed in this D naught.

So, r sp can be written in this form with x equal to this. So, this is a plot of r sp versus x. So, you can see where the peak occurs. So, these are actual number calculate plotted by the computer. So, you can see that x p, if the peak occurs 0.5 x p is 0.5, so h mu minus E g by k T equal to half. This implies h mu is equal to E g plus half k T as I had written earlier it to you. The point is actually this shape, is really you get the shape like this and how you can find out at half the maximum value. You find out what the value of x is here and what the value of x is here and find out the difference between them, find out the delta lambda line width of a given source. The equation cannot be solved analytically. We have numerically solved or graphically you can obtain the solutions and get the expression.

So, today I will start with rates of stimulated emission and absorption. Please write to do this exercise and put some numbers and see what the line width you get is because I had plotted, I had drawn qualitatively, the spontaneous emission spectrum, but you can see that function you get lead in shape is like that.

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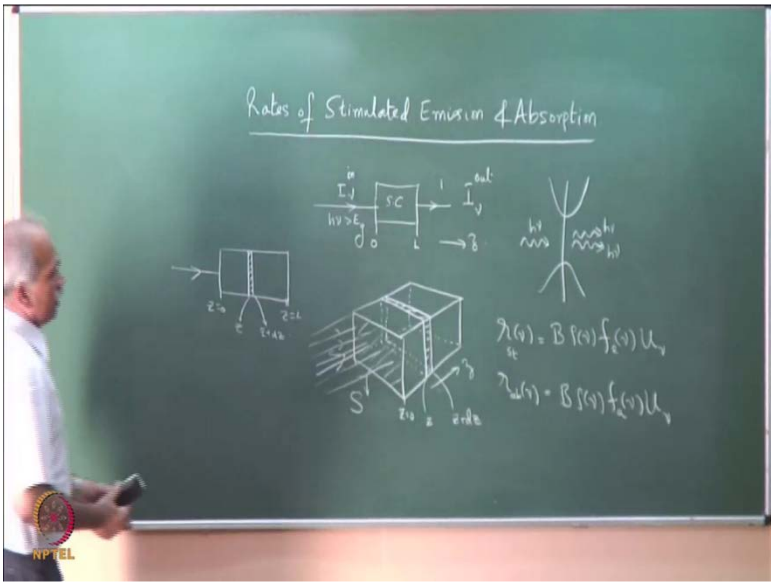

So, today we will discuss rates of stimulated emission and absorption. Our objective would be given a piece of semi conductor, a beam of radiance I mu of frequency h mu of energy. H nu is greater than E g. A beam of h mu here a light beam of radiance I mu entering this semiconductor here. What is the effect of the material property on the beam is the output here. If I call this as l equal to 0, let us say these are z direction of propagation, this is l is equal to 0 and l is a length of the semiconductor. What is this? This is the input, I mu of input and then, how is I out mu related to I in mu, whether that is attenuation amplification or more change. Under what conditions our specific objective will be to determine, under what conditions we will get amplification and what happens in thermal equilibrium? How does the semiconductor behave?

So, amplification, if you were looking at amplification, amplification can take place only by stimulated emission because that is the basic process of amplification, basic process of stimulated emission, where you have one photon which is incident here which leads to generation of two photon in simple terms. So, h mu here and gives out. So, this is a process of amplification. Amplification refers to coherent amplification. It is coherent amplification. So, stimulated emission is the one which is responsible for amplifications. You can have if the process of absorption, the photon can get absorbed and the output can get the minutes or attenuate. So, we want to find out what is the coefficient of gain or coefficient of loss in this semiconductor. That will be our objective.

So, let us consider, so let me draw this diagram again. This is a piece of semiconductor. So, a light is incident at z is equal to 0. So, this is the direction of the propagation is the z direction. So, z direction, so z is equal to 0. If I consider a small volume in a limit that is I consider a small slice of this here can infinite simile thickness. So, this is z here and this is z plus d z in the 2 d picture. Here let me draw in the 2 d picture. So, radiation is entering from here. So, I have considered a small slice here. So, this is z is equal to 0, and this z is equal to l this here and we have considered z at some position and z plus d z. Let S be the area of cross section. So, we want to get an analytical expression.

So, let S be the area of cross section of this surface and the beam is entering through the surface everywhere. The beam is present everywhere in the cross section. So, S is the area of cross section. If I consider this volume of small element and we know that the rate of stimulated emission at a frequency mu equal to B into rho of mu into f e of mu into the energy density u mu. Rate of stimulate emission is equal to the Einstein coefficient join density of states probability of emission and density at mu energy and rate of stimulated absorption or simply absorption is equal to B into rho mu into absorption. So, probability of absorption into u mu, both stimulated emission and absorption depend on the energy density of the radiation u mu. If I look at that small volume element, I am now focusing on the small volume element.

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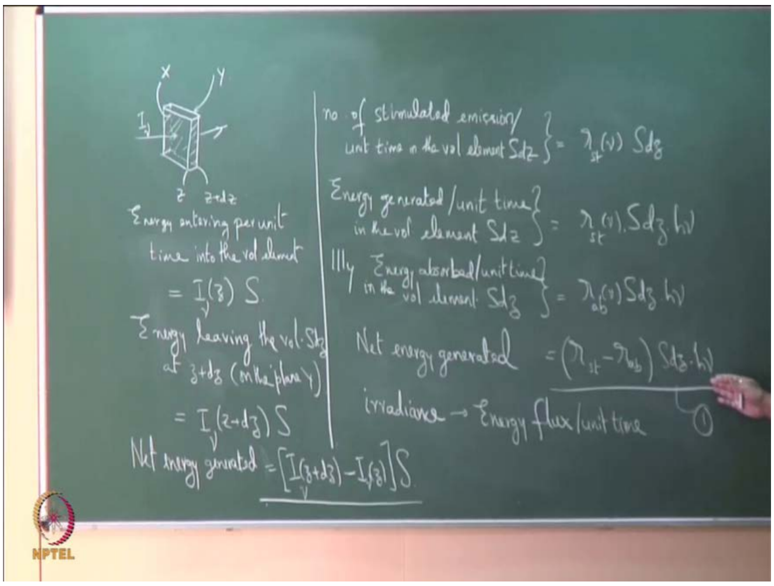

So, let me draw the volume element here that small slice. So, this is of the thickness dz. So, z and z plus dz, the number of stimulated transition emission. So, let me write it. Stimulated emission per unit time in the volume element S d z equal to rate of stimulated emission is this mean. What does this mean? This tells you the number of stimulated emission per unit time per unit volume. Why unit volume? Because density here rho mu is per unit volume. So, this is number of emissions per unit time per unit volume. Therefore, number of emissions per unit time in the volume as S dz will be equal to r st we are considering at a frequency mu into S dz.

So, every stimulated emission generates a photon of energy h mu and therefore, energy generated. We see the logic. Energy generated per unit time in the volume element S dz is equal to what will that be. This is the number of emissions. Therefore, energy will be equal to multiplied by h mu because every emission brings to a photon of energy h mu. Therefore, multiplied by h mu, this is energy generated in this volume element. There is a radiation of energy density u mu present and therefore, similarly energy is absorbed. I can find out number of absorptions per unit time in the volume element. S dz will be r ab into mu into S dz. S dz is the volume and therefore, energy generated here because of emission because of absorption energy will be absorbed of energy loosed energy absorbed per unit time in the volume element. S dz is equal to r ab into S dz into h mu. Therefore, net energy generated per unit time in the volume element S dz is equal to r st minus r ab. This is generated and this is absorbed. Therefore, the net generated is equal to S dz h mu net energy generated. I want to relate this energy generated to the intensity. So, I mu is the intensity or radiance or irradiance is energy flux per unit time. This is energy per unit area per unit time as the units of intensity because energy flux is energy per unit area per unit time.

Energy per unit time is power and power per unit area is intensity. That is why it is I energy flux energy per unit area per unit time energy. Per unit time is power and power per unit area is intensity. If I mu of z is the irradiance on this plane, so let me show this end as plane x and the other plane at the other end as y. So, energy entering per unit time into the volume element is equal to I mu of z, that is the irradiance of plane x I mu into the cross sectional area. The energy entering per unit time is intensity into cross sectional area, S is the cross sectional area of this surface, so I mu into z. What is the energy leaving from the other side? Energy leaving the volume element S dz at y, that is at the plane at x plus at z plus d z, on the plane y at z plus d z, on the plane y from the plane y at that plane is equal to I mu of z plus d z into S.

I is the intensity by irradiance and multiplied by the surface area, else the energy entering energy leaving. Therefore, the net energy generated in the volume element equal to what is leaving minus what is entering, so I mu of z plus d z minus I mu of z into S. Please see this net energy generated in the volume is this match r st minus r ab into S dz into h mu and net energy generated in terms of intensity is this much. So, they are the same.

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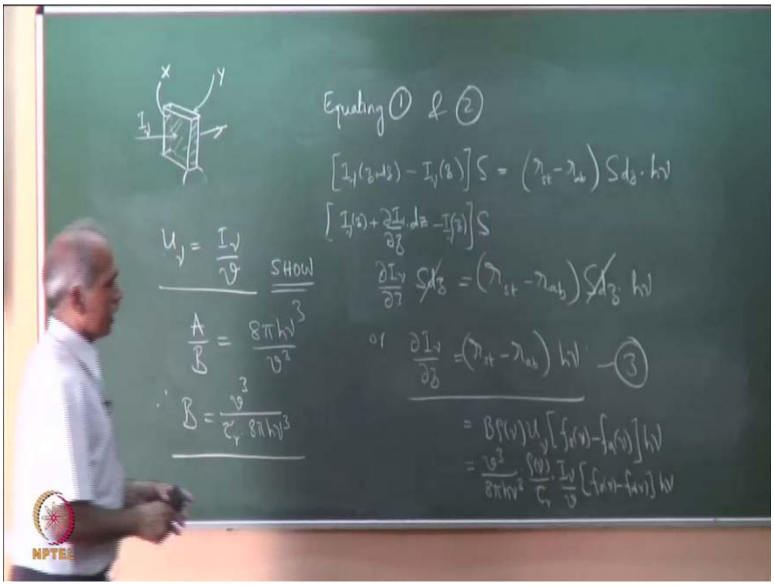

So, equating 1 and 2, I mu of z plus d z minus I mu of z into plus is equal to r st minus r ab into S dz into h mu. We have considered an infinite similar volume element. Therefore, I can write this I mu of z dz as I mu of z plus del I mu by del z into d z. The d z is the thickness of this minus I mu of z into S equal to this. So, this is equal to del I mu by dz into S dz equal to or. So, this goes half we have del I mu by dz is equal to r st minus r ab into h mu substituting for r st and r ab, and we should get the expression, so r st. So, this is equal to B into rho mu into u mu into f e of mu minus fa of mu, this portion into h mu into h mu.

How is I mu irradiance relative to energy density? U mu is equal to I mu divided by velocity. This is energy density. You can see energy per unit volume and what is this is energy per unit area per unit time. So, you can dimensionally, immediately you can see that they are the match dimension. If I mu is, you can show this let me write this to show that u mu is equal to I mu by v. Just use the (( )) for fundamental definitions of u mu and I mu and show that u mu is equal to I mu by v. So, this is equal to and what else do we have. We know that the relation between the Einstein coefficient is 8 pi h mu cube divided by h mu cube divided by v cube. See if it is in free space, but if it is in the medium, it is v that is c by n, where A is equal to therefore, B is equal to A is 1 over tau r. Therefore, you have v cube divided by tau r into 8 pi h mu cube. So, this is the coefficient B.

Einstein coefficient B is we have used A by B is equal to 8 pi h mu cube by v cube. So, B is equal to this. So, substitute them here in this expression. So, let me substitute here v cube divided by 8 pi h mu cube into rho of r rho of mu into tau r radioactive recombination time into u mu which is I mu divided by v. Velocity into f e of mu minus f a of mu is a very simple derivation, but for the sake of completeness that may not skip many steps.

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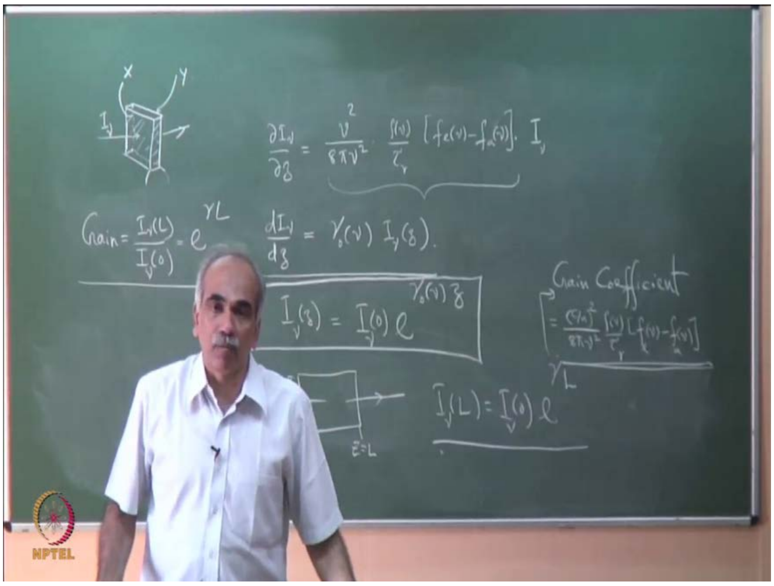

So, 1 h mu goes here with this h mu 1and becomes h mu square and v goes here. Therefore, we have del I mu by del z is equal to v square by 8 pi mu square. This v is I can write c by n into rho of mu by tau r into fe of mu minus fa of mu into intensity is a function of z. Please see this. Here does not depend on z independent of z intensity. I enters the medium and depending on the position z, it may be getting attenuated or amplified whatever, but I is a function of z and therefore, since I is a function of z alone, I can write this as d I mu by d z is equal to this whole coefficient here. It depends on the frequency. Therefore, I call this as gamma 0 of mu into I mu of z. So, what is the solution this gives? I mu of z is equal to I 0 of mu or I mu of 0 that is the entrance at the entrance point into E to the power gamma 0 of mu into z.

I have got an expression for the intensity at any value of z if we are looking at the output. So, this is z equal to 0. If I is 0, I mu of 0 is the intensity here at the output at z equal to l. This will be I mu of l is equal to I mu of 0 into E to the power gamma into l. So, I mu of l or I mu of z will be greater than I mu of 0. If gamma is positive, then I mu of z is greater than when will gamma be positive. So, this tells that there is amplification. So, what is output by input? Therefore, the gain is equal to I mu of l output by input I 0 I mu of 1 divided by I mu of 0 equal to E to the power gamma l. This is gain, this is not gain coefficient. This is gain and gamma is called the gain coefficient. So, gamma is the gain coefficient.

So, what is the gain coefficient? Gain coefficient gamma is equal to v square or c by n. Let me write it as c by n just because otherwise sometimes mu and v may get confused. So, 8 pi mu square into rho of mu density of states divided by tau r into f e of mu minus fa of mu. I have already called it as gain, but when will it be gain? It will be gain if gamma is positive and gamma is positive when everything here is constant, positive constant this is the density of states. So, it is the density of positive number. This is a number at a time positive number. Everything is possible. Only this can be positive or negative. When will this be positive? This will be positive when probability of emission is greater than probability of absorption. When will that happen? When will be the probability of emission will be greater than probability of absorption? This is the condition that we have already derived.

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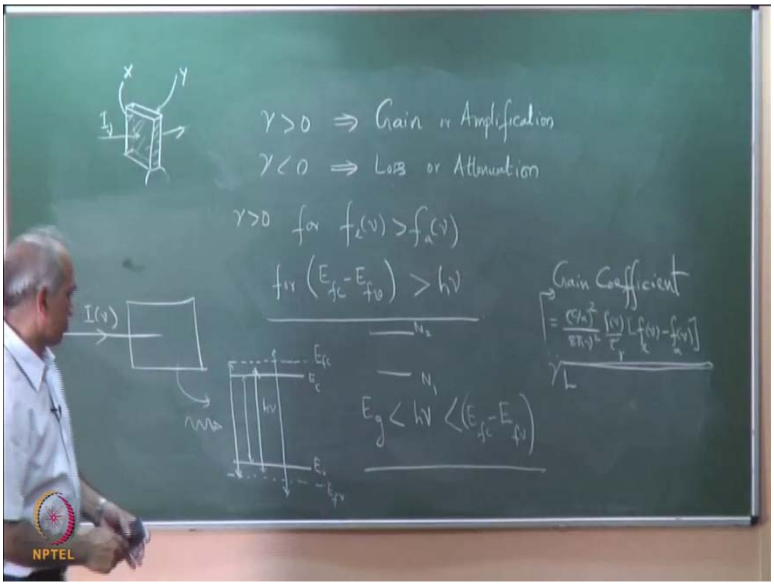

So, gamma is greater than 0 implies gain or amplification and gamma less than 0, that is negative implies lose net or attenuation and therefore, it is the sign of gamma which is important and which will determine that it will be determined by this factor f e of mu minus f a of mu. So, gamma is greater than 0 or f e of mu greater than fa of mu and this happens when E fc minus E fv, the difference between the queasy permi levels is greater than h mu. So, the separation between the queasy permi levels if this is greater than h mu, then for all those frequencies we have gain in the medium. So, if you have a semiconductor here and light of radiation I mu, now I put I of mu. If that radiation contains a spontaneous spectrum of frequencies I of mu, then if the material here is characterized by a band gap like this energy band diagram I am showing.

Now, let me show this is E c, this is E v and assume that in that semiconductor we have maintained E fc here and E fv here. So, this a valance conduction band and this is E g. For h mu greater than E g for all energy between all values of h mu between this, that is from E g here. So, E g less than h mu less than E fc minus E fv for all frequency mu, such that E g less than h mu less than E fc minus E fv. We have this condition satisfied. It means if I have radiation which is coming such that the energy difference corresponds to some value here, let us say h mu, then for this h mu which is greater than E g, but less than E fc minus E fv, we will have gain in the medium. So, if you pass a spectrum, then those frequencies for which energy corresponds to energy, satisfies this equation, we will have gain in the medium.

So, E f c minus E f v. So, this is the condition for gain. We have analytically showed that this is the condition for gain in a medium. This is equivalent. Therefore, this condition those of you have credit atomic laser physics, you know that there you have to have n 2 greater than none or a state of population inversion. So, a state of population inversion means n 2 is greater than none. This is the necessary condition for amplification by stimulated emission and in the case of semiconductors; the necessary condition for amplification by stimulated emission is this one. So, there is an equivalent. It is incorrect to say that number of electrons in the conduction band should be greater than number of electrons in the valence band. That is not the condition. The condition is this.

If you can maintain this condition, then all frequencies mu which satisfy this condition will get amplifier. What happens to other frequencies? So, if we go here for example, a radiation which corresponds to this frequency difference, so this condition is not satisfied immediately. That means, f e of mu will be less than fa of mu and therefore, this will be negative number which means gamma is a negative number which corresponds to loss or attenuation. This will determine the amplification bandwidth. We will discuss this in the next class, the amplification bandwidth of the amplifier.

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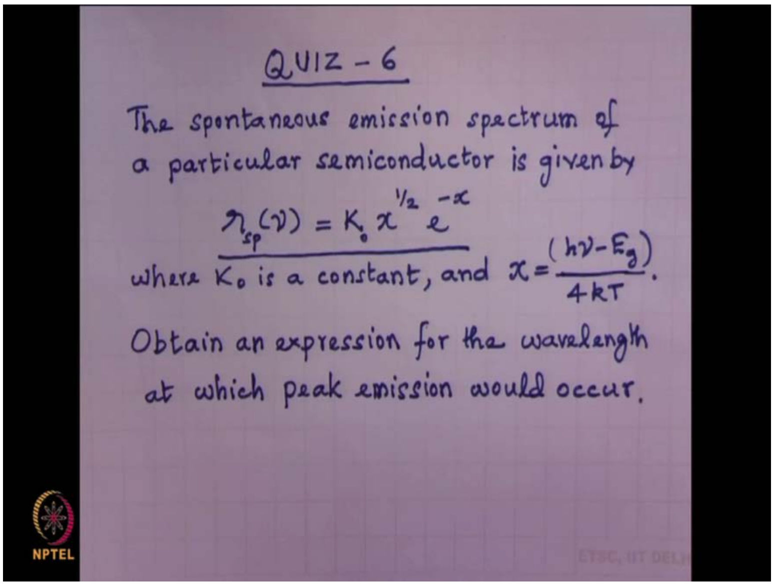

So, our next topic will be laser amplifier and an amplifier is characterized by gain and bandwidth and of course, noise also. So, we will take a quick quiz, a simple quiz .It is a very simple quiz. The spontaneous emission spectrum of a particular semiconductor is given by an expression given to you as r sp of mu equal to K 0 into x to the power half into e to the power minus x, where K 0 is constant and x is h mu minus E g divided by 4 k T. Obtain an expression for the wavelength at which peak emission would occur.