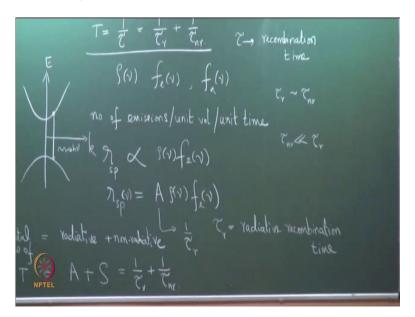
Semiconductor Optoelectronics Prof. M. R. Shenoy Department of Physics Indian Institute of Technology, Delhi

Lecture - 19 Rates of Emission and Absorption

(Refer Slide Time: 00:29)



In the last class, we discussed about optical join density of states rho of mu and the probability of emission f e of mu or p e of mu, and probability of absorption of f a of mu. We want it to find out the number of emission. Finally, our objective is today in this class, we will discuss about rates of emission and absorption. Our objective rates of emission and absorption refers to number of emissions or absorptions per unit volume of the material per unit time because once we know this number, then multiplied by h mu will give us the power energy per unit time is of the power output from this device or material.

So, the number of unit emissions per unit volume, this is the density of optical join density of states that is number of states available for photons of frequency mu to interact with. So, rho mu is the number of states, this is the emission probability and therefore number of states multiplied by the emission probability will give you number of emissions. So, rho of mu into f e of mu will give you number of emissions per unit volume, because this is per unit volume and therefore the rate of spontaneous emission, rho rate of emission or spontaneous emission r s p is proportional to rho of mu into f e of

nu or r s p, s p standing for spontaneous emission. Why it is spontaneous emission? I will come in a minute.

So, this I can write as equal to a rate constant A into rho of mu into f e of mu. A is the rate constant and you can show that this is nothing but one over tau r where tau r is the radiative recombination like that. So, this is the radiative recombination time. Radiative transition recombination time means recombination time for radiative transitions. Why I mean radiative transitions is the following.

If you see the E k diagram, as you can see I keep drawing this many times. So, E k diagram, a photon, an electron which is in the conduction band can make a downward transition in energy to the valence band giving out spontaneously on its own a photon of energy h nu. It is also possible that this recombination may not result in the emission of a photon at all. This recombination is also possible through phonon emissions. Vertical transitions are also possible with phonon emissions and therefore, in general or this may be sitting here, it makes an oblique transition, but the point is a transition from conduction band to valence band may take place with emission of a photon or without emission of a photon. Therefore, total number of spontaneous downward transitions comprise of radiative plus non-radiative transition.

So, radiative plus non-radiative transition. So, total number of spontaneous transitions because it is on its own. It is coming down. Spontaneous transitions would comprise of radioactive and non-radioactive components and therefore, correspondingly the total rate constant if I denote it as t, then this will comprise of a radioactive constant A plus non-radioactive constant which I denoted as S, total rate constant equal to radioactive constant plus non-radioactive constant here the rate constant. Therefore, this is inverse of time is rate constant. Therefore, it inverse of time. So, this is written as 1 over tau r plus 1 over tau nr. In other words, t is equal to 1 divided by tau is equal to 1 divided by tau r plus 1 divided by tau nr, where tau is the lifetime of the transition or spontaneous recombination time.

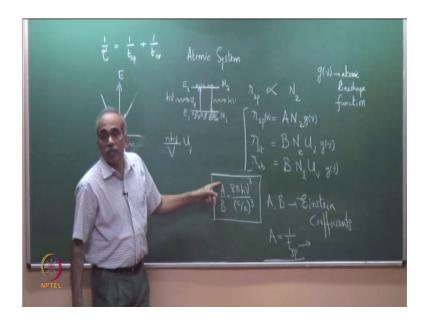
So, this is recombination time, this is radioactive recombination time, this is non-radioactive recombination time. The relation is 1 over tau equal to 1 over tau r plus 1 over tau nr. We will discuss about this a little bit more little later, but at this point I will just make a statement that in direct band gap semiconductor, tau r is of the same order

which means the rate constant A is of the same order at the rate constant S, which means the spontaneous emission. This is spontaneous emission rate of photons r s p of mu of frequency mu. If this is equal to this, it means out of the total recombination, 50 present of them result in emission of photon and 50 present of them result in emission of no photon that is no emission of photons.

If A equal to S, if tau r equal to tau nr, it means that A equal to S total rate constant is this which comprises of these two radioactive non-radioactive. Therefore, in a direct band gap semiconductor, the probability of radioactive transitions is equal to, nearly equal to the probability of non-radioactive transition. Almost every electron combining with a whole of a 50 present probability of a emission of a photon and 50 present probability of no emission of photon, but in the case of indirect band semiconductors, tau nr is much smaller than tau r. Therefore, S is much greater compared to A. We will put some numbers a little later when we go to materials.

Therefore, non-radioactive transitions dominate much more over radioactive transition. That is the important of tau r and tau nr. So, I want to bring at this stage an analogy with atomic systems because most of you know that the elementary theory of lasers, most of you are familiar to the elementary theory of lasers. You will see that is the complete one to one analogy and then I would like to proceed further.

(Refer Slide Time: 09:07)



See atomic system. If you consider the two level atomic systems with N 1 number of ground state and N 2 number of atoms in the excited state with energy E 1 and energy E 2 and energy E 1, then there are number of atoms, large number of atoms in the ground state which is given by the Boltzmann distribution. There are fewer numbers of atoms here. The three basic atomic processors are the same interaction processors, an atom sitting here and can make spontaneously downward direction giving out the emission of photon h mu. The number of transitions N 2 is the number of transitions per unit volume of the material, the number of transitions, downward transitions spontaneous transition is proportional to the number of atoms which are sitting here and therefore, the rate of transitions spontaneous transition is proportional to if I am looking at spontaneous emission, then it is proportional to N 2 or rate of spontaneous emission is equal to A times N 2.

There can be stimulated emission in the presence of photons of energy h mu if there is energy density u mu of photons of energy, h mu u mu is the energy density number of photons per unit volume of frequency mu that is if you have N number of photon incident when n h mu divided by volume of the material v is unit. So, this is equal to u mu energy density. So, is E mu u mu is the energy density, then rate of stimulated emission is equal to A constant B into N 2 into u mu and rate of absorption is proportional to the number of atoms in the ground state. Therefore, rate of absorption is equal to B into N 1 into u mu.

Please see when there is an energy density, when there are photons of energy h mu here, they can stimulate downward transition, but they can also cause absorption of upward transition. So, the absorption depends on the energy density u mu and the number of atoms in the ground state emissions depends on the number of state atoms N 2 here and also, on the energy density u mu. That is why we have this kind of expressions to make it to incorporate more features into this. Actually atom interacts with the radiation over a certain range of frequencies. So, there is a line shape condition, g mu atomic line shape function. Both you have studied in basic laser physics lined shape function. If you have not studied, that does not matter g mu and therefore, everywhere this r s p will be a function of mu and N 2 into g mu. G mu gives the strength as interaction at the frequency mu that is any atomic system interacts with radiation over a range of frequencies because

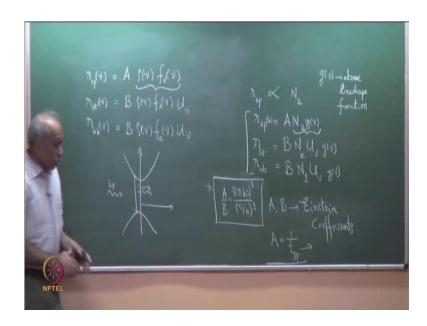
of various mechanisms which are called line broadening mechanisms. This g mu is the line shape function which determines the strength of interaction at a particular frequency.

So, I have carried traditional term genuine that is N 2 into g mu, N 1 into g mu. So, spontaneous emission at a frequency mu is proportional to N 2 into g mu and A is the proportional. So, these are the three expressions that we have for a long degeneration function. This for the absorption and emission B is the same. Otherwise, you have to write here B 2 1 and B 1 2 for a non-regenerated system become the same. These coefficients A and B are called, what they are known as? These are known as the Einstein coefficient.

So, A, B are Einstein coefficients. A is equal to over t s p where t s p is the spontaneous emission life time. You can write this as d N 2 by d t equal to minus A N 2 into g mu and simplify, and you can show that A is equal to 1 over t s p, where t s p is the spontaneous emission life time in atomic physics. Also, there is a relation that is life time tau is equal to 1 over t s p plus 1 over t n r non-radioactive transition. So, t s p here is first to spontaneous emission, but also their atom may come down without even giving a photon and they are called non-radioactive transitions. It is the same relation that we have in a semi-conductor. 1 over tau life time is equal to 1 over radioactive life time plus non-radioactive life time. So, A is given by 1 over t s p spontaneous emission life time and the relation between A and B, relation between the Einstein coefficients is given by A by B is equal to 8 pi h mu cube divided by c cube or a medium of refractive index n. It is c by n cube. This is the relation between Einstein coefficients A and B and A is equal to 1 over t s p.

Why I am writing this is because the coefficients that we have in semi-conductor A and B also have the same relation. Therefore, this have been written for the atomic system and now, I write for the semi-conductors and then you can see what is the comparison from here in an atomic system and in semiconductor here we are writing N 1 and N 2 are number of atoms. Here, we are dealing electrons and holes, but you see the similarity and the relations.

(Refer Slide Time: 17:10)

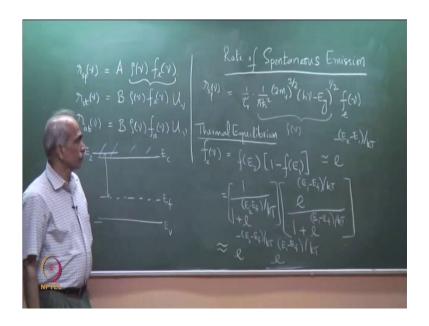


So, for the semi-conductors r s p of mu is equal to the constant A into rho of mu into f e of mu. This spontaneous stimulated emission r s t of mu is equal to constant B. Actually, this is A dash and B dash value is not the same. The relation is the same. So, B into rho of mu into f e of mu into energy density u of mu and r absorption of mu is equal to B into rho of mu and now, f a of mu into u mu, and A and B have the same relations here and this case A is the spontaneous emission life time. Here with A is equal to 1 over tau r radioactive recombination life time.

There are techniques, these are measurable parameters. You will see ultimately that everything will be in terms of life time because lifetime is a measurable parameter. If time permits, later on I will discuss that. The measurement techniques are one can measure the lifetime. So, A is equal to this is completely one to one corresponded and what do you see that if you replace n 2 into g mu. This tells you effective number of atoms available for the transition and this tells you the same thing. So, row mu into f e of mu is replaced by this, otherwise it is the same. So, these are the rate of spontaneous emission stimulated emission and absorption would be the difference between stimulated and spontaneous is the energy density. Spontaneous emission does not require any radiation to be present. It spontaneously makes the transition in stimulated emission. You need radiation to be present and therefore, u mu here refers to the density of states. So, u mu here refers to the energy density in the medium.

So, in stimulated emission if I want to illustrate stimulated emission, then I have photons coming here of energy density u mu and electron which is here making a downward direction and leads to, so there are more number of photons here can induce more number of downward transition. That is why stimulated emission rate is proportional to u mu. Larger the number of photons which are incident here, larger will be the possibility of stimulated emission and the coefficient here B. So, these are the Einstein coefficients. They are the same as this is not in magnitude, but in nature and the relation is the same in semi-conductors. So, let us find out first the rate of spontaneous emission. First, we will discuss spontaneous emission because I have already mentioned spontaneous emission is the basis of operations.

(Refer Slide Time: 21:08)



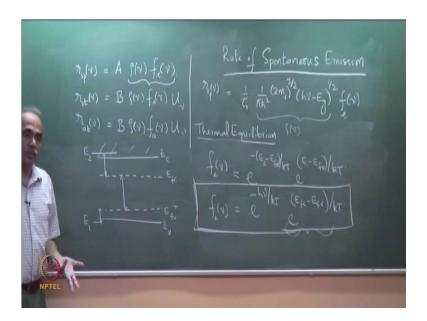
Now LED's. So, we will discuss the rate of spontaneous emission. We will find out what is this rate. We have determined the expression here, what is that rate. So, rate of spontaneous emission r s p at the frequency mu because the density optical join density can form the frequency mu. R s p of mu is equal to 1 divided by tau r into tau v or mu density optical join density of space, 1 divided by pi h cross square into twice m r to the power 3 by 2. Please check the expression into h mu minus E g to the power half. So, this is rho of mu. So, this is rho of mu multiplied by f e of mu probability of emission. Let us see the probability of emission f e of mu is equal to recall first thermal equilibrium.

Let us see the thermal equilibrium. A thermal equilibrium I want to find out f e of mu. So, we have a semi-conductor. The primary level is somewhere I do not know, B type or N type and just taking a semi-conductor not taking a p n junction and this is E C. The semi-conductor in thermal equilibrium f e of mu is equal to f of E 2 and probability of having an electron at state E 2 into probability of having the whole at state E 1 minus f of E 1. So, substitute at E 2 is 1 divided by 1 plus e to the power E minus E x to E 2 minus E x by k T into this one will give me e to the power E 1 minus E f by k T in the numerator divided by 1 plus e to the power E 1 minus E x by k T. Let us approximate a little bit here. Even remember that E 2 is an energy level here, the conduction in the valence band in the conduction band.

So, E 2 is an energy level in the conduction band, E 1 is an energy level in the valence band here. So, you can see that this gap is E 2 minus E f is much greater than k T at room temperature. This is the Boltzmann approximation E 2 minus E f is much greater than k T because k T is about 0.05 E B. Therefore, just like to we did Boltzmann approximation for carrier concentration under, then Boltzmann approximation E 2 minus E f is much greater than k T. Therefore, the exponent will be large number. I can neglect to one with respect to e to the power large exponent which means this second term here is a first term, here the first term this one is simply approximately, let me write approximately I want to see what kind of numbers are there. Approximately equal to e to the power. So, only this term minus goes to the numerator E 2 minus E f by k T, the second term here is approximately equal to e to the power E 2 minus E f by k T.

What about this term? This is a large number E 1 minus E f by k T. The large number you can neglect, but it is a large number, but even is less than E f. Please E 1 is here, E f is here, E 1 is minus E f is a negative number. So, it is a large, but a negative number. Therefore, this is closed to 0 and therefore, I can write this term is closed to 0 by neglect this. So, what we have left is this is only e to the power E 1 minus E f by k T. Then what is that? See plus E f and minus E f by k T that goes off. So, left to it e to the power of minus E 2 minus E 1. So, this is, therefore this is approximately equal to e to the power minus E 2 minus E 1 by k T, E 2 minus E 1 is what h mu. So, this is equal to let me erase this.

(Refer Slide Time: 27:29)



So, what we have got this f e of mu. F e of mu is equal to e to the power; I should not erase this e to the power minus. Let me write again. E 2 minus E f by k T into e to the power E 1 minus E f by k T which is equal to f e. This term is approximately equal to because you have made a broad man approximation here. This is equal to here we have e to the power of minus h mu by k T. So, f e of mu is e to the power of minus h mu by k T. You can put some number if you are talking of visible photons. Corresponding to visible light, h mu is about 1.25 Ev, and k T is extremely small compared to that which is also large number e to the power minus this. So, f e of mu is an extremely small number and probability of emission at thermal equilibrium is extremely small.

Suppose, I was in queasy equilibrium, we were in the state of queasy equilibrium. So, we will substitute in a minute, but just I have to proceed, suppose we were in queasy equilibrium which means let us say we have a pump. So, because of original is here may be exist this medium by may be by eliminations, may be by current injection whatever reason, so that the E f separate out 2 E f. So, now, we have one E f for the conduction band and E f c and one E f for the valence band, E f v. So, I erase this thermal equilibrium case. So, this was equilibrium and now, this is a queasy equilibrium.

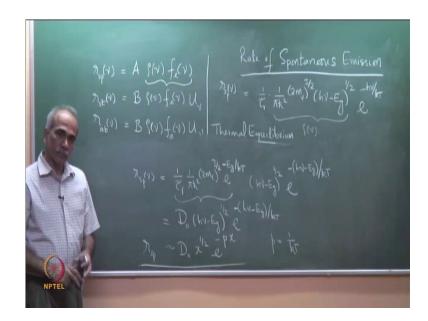
So, what difference would we see? We will see I have this is not enough of quality here. So, this gap is sufficiently large between E 1 and here, so that I can still apply Boltzmann approximation. Assume that I am still able to apply Boltzmann approximation. So, what

difference is in? This expression will come in this. We will have E f c and here E f v. There is the only difference that E f c E 2 minus E f c by k T and E 1 minus E f v by k T. Please write this expression again.

What do we then have earlier? E f positive gets cancel with each other. Now, we have this multiplied by e to the power minus E f c minus E f v by k T s p. The probability of emission in quasi equilibrium has changed to this expression, where this was in thermal equilibrium. That is now multiplied a term E f c minus E f v by k T. See in this diagram that I have drawn E f c minus E f v is this gap. It is much larger compared to k T 0.025 E v and therefore, this exponent here is very larger number. Now, this is very large no doubt, but this is also large. The point is by pushing the semi-conductor into quasi equilibrium, you have additional multiplication factor which will increase f e of mu by orders of magnitude. A semi-conductor in thermal equilibrium had f e of mu approximately equal to e. To the different means, you have an additional factor which is also a very large factor power minus h mu by k T which means the probability was very small by pushing the semi-conductor into quasi equilibrium may be by injection of carriers, may be by elimination. In other words, you can change the probability of emission by orders of magnitude.

We will see this simplification later that is how in an LED before bias that is nothing 0 or you just forward bias, it light starts coming out. What are you doing by forward biasing? You are simply raising the probability of emission. Alright, we will discuss more about the LED's little later. So, let me continue with thermal equilibrium in case, so I erase this.

(Refer Slide Time: 33:17)



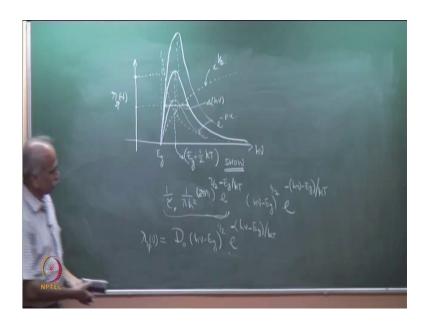
So, in thermal equilibrium, the rate of spontaneous emission is given by, so we have the expression here e of mu. Please write. You could write this again, e to the power minus h mu by k T, f v of mu is replaced by e to the power for minus h mu by k T. We want to simplify this dual form. If you multiply this expression by e to the power E g by k T and e to the power minus E g by k T, r s p is equal to 1 over tau r 1 over pi h cross square twice nr to the power 3 by 2 e to the power E g by k T into h mu minus E g to the power half. I have simply multiplied by e to the power E g by k T and e to the power of minus E g by k T e to the power h mu minus E g by k T, e to the power minus E g by k T is here and e to the power E g by k T.

Just to write it in the form of x to the power of, so this therefore, this constant here if we call it as some constant D or D 0, then this into h mu minus E g to the power half into e to the power h mu ho. This is minus e to the power minus h mu by k T in which case this minus minus plus. So, should have minus here. Please correct it. So, e to the power minus h mu minus E g because of h mu is greater than E g, therefore minus h mu minus E g. So, please correct this.

So, this is of the form some constant should be 0 and into x to the power half into e to the power minus A x, where A is some constant or p, let me write p because A has used for lattice constant. So, e to the power minus p x where p is equal to 1 by k T. This is of the form x to the power half that is h mu minus E g calling as x x to the power half into e to

the power minus p x, where p is 1 by k T. So, this is the rate of spontaneous emission. I want to see that what kind of variation that I will get that is I have written it that form.

(Refer Slide Time: 36:50)



So, how does it vary? I want to find out rate of spontaneous emission. So, this is h mu photon energy h mu and here, I plot in r s p of mu. So, where will it start? H mu equal to E g curves all density of everything is valid for h mu greater than E g. So, this is E g h mu equal to E g is x is equal to 0. So, some here has h mu increases, this goes as x to the power half. X to the power half goes the variation like this. X to the power half variation e to the power minus p x variation is exponential drop. So, e power minus p x at x is equal to 0. It has the value 1 at x is equal to 0 and then it draws some down rapidly is an exponential drop depending on the value of p. P is very large. As you have seen here is a large quantity 1 by k T, k T is 0.025. That means 1 by 0.025 40. So, p is about 40. So, is dropping like this. So, this is e power minus p x variation and this is x power half variation and D 0 is a constant.

So, D 0 does not involve mu. So, it is a constant. So, how is the net result look like? So, the net result looks like initially this function is very, so if the functions product of two functions. So, the product is 0 here, but suddenly this is a large value. So, the product jumps goes up to some maximum because latter on exponential takes over. Please remember that x power half ho. Why did I write x power minus half e power half? {lease correct this x power half. X power half is a much slower variation, then exponential

dependence is much faster. So, exponential takes over and then the net drops down. So, I plotted is rate of spontaneous emission as a function of h mu. It means, it is maximum some value here. So, this is r s p max corresponding to some value here. So, this is r s p max and it also has a certain line width. A certain line width in the sense where f w h m from here if you come down to half, its value is approximately half its value. So, this is delta lambda, it is delta h mu here in this case, but you can find out what is the corresponding in this case. It is delta h mu.

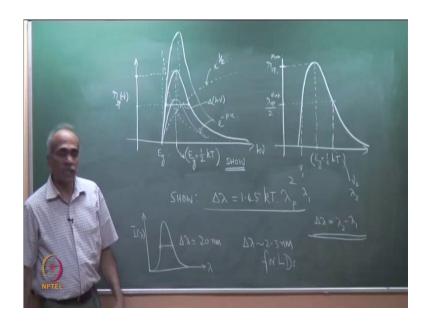
The full width at half maximum is the line width of this source full width at half maximum. How do you find out the maximum? Where does a maximum occur? We have an expression here for r s p mu is equal to D 0. It does not have mu h mu minus E g here to the power half into this. We want to find out the maximum. How to find out the maxima? So, how do you find the maxima? It differentiates with the function with respect to h mu and find out where is the maximum. So, please do this as an exercise and see what we get. This maxima corresponds to E g plus half k T. You show this that the maxima appear at E g plus half. Do this as an exercise and once you know that the maxima is set, E g plus half k T, simply substitute E g plus half k T here and you get what is the maximum value r s p of mu. If you know that occurs the maxima occurs that E g plus half k T substitute these value for h mu. So, E g goes away your left with half k T to the power half here again half k T. So, k T k T cancels. See what you get. So, you know what the maximum value will do here and we know every parameter in this for a given material, we know radioactive recombination life time constant, the reduced mass, all the parameters are known.

So, I take given temperature you know what we see and therefore, you can find out what is the maximum rate of spontaneous emission. Remember that rate of spontaneous emission is number of emission per unit time per unit volume. If you multiply this as a energy of the photon h mu, it will give you power output from that material power per unit volume. One other thing to note that the D 0, alright, let me write twice m r 1 divided by pi h cross square. What else we write? 1 over tau r, the D 0 value of D 0 tells you how far it goes. It is amplitude multiplied by time constant here. So, whether this variation goes like this, whether this equation thus like this it determined by D 0 and what is determined by, what determines D 0? D 0 is determined by temperature. All are constant for a given material. So, two things, one is tau r radioactive recombination time

and here T. So, if the temperature is larger than this exponential negative, then quantity become smaller and therefore, D 0 will be higher.

So, it is instructive to see that you will put different temperatures. For example, you put 250 k here, 300 k, 350 k. You see that it changes. Similarly, when you put T different values, the peak shift because this is E g plus half k T. Larger k T is the peak shift. So, deliberately I have shown the peak sitting likely this. So, this peak is because of higher temperature will have larger spontaneous emission coming out and also, the peak shifts by a small value. So, this tells us the rate of spontaneous emission from which we can calculate the spontaneously emitted power at any frequency greater then E g. So, please do this as an exercise to find out the h mu corresponding to the peak value and this one corresponding to the full width cut off maximum.

(Refer Slide Time: 45:40)



So, you can show in terms of lambda delta lambda, I think is nearly equal to 1.45 into k T. Delta lambda comes out to be I think it is 1.45 into k T multiplied by lambda p square. So, two exercises. Please work this out delta lambda is equal to, so delta lambda here refers to let me just explain this. How you would go about it and then I will stop. So, first for a given temperature, find out this. You know E g plus half k T. Half k T substituting this value, we know what is the maximum value r s p max. So, this is max where r s p max becomes half. So, this is a value where this is r s p max by 2. There are two values of h mu and hence, mu you will get two different values for which you will have the

value r s p max by 2 at two different h mu values. So, correspondingly we will get two frequencies. One, here for mu 1 and one, here mu 2 h mu 1 and h mu 2. So, you know mu 1 and mu 2. If you know mu 1 and mu 2, you can find out lambda 1 and lambda 2. Simply see by mu 1 and see by lambda by mu 2.

Therefore, delta lambda is equal to lambda 2 minus lambda 1. Why we want in delta lambda is normally the light, the line width of the force is expressed in terms of nanometers wavelength. Delta lambda may have are studied that an LED has a line width, delta lambda equal to 20 nanometers. What I have plotted? I have plotted intensity output, lambda as a function of lambda optical measurements are done in a wavelength. So, you change the wavelength and measure the spectrum of the banned LED output and you have intensity variation as a function of lambda. The full width of maximum is delta lambda which is typically 20 to 30 nanometer light emitting diode. If you measure the same thing for a laser diode normal laser diode, this delta lambda will be of the order of 2 to 3 nanometer for laser diodes for normal laser diodes febrile pare laser diodes.

We will study about special laser diodes which have extremely small delta lambda single frequency laser diodes. So, that is why please calculate delta lambda in terms of nanometers. Lambda p here corresponds to the peak wavelength, the peak wavelength that is lambda p, alright. So, I will see stop here and in the next class, we will discuss about rates of stimulated emission and absorption and see the condition for gain, and what condition we can gain from the semi-conductor spontaneous emission. There is no gain having an emission spectrum, but stimulated emission can lead to gain under certain circumference conditions, and we will see that the worth condition, we will get a gain.