Basic Quantum Mechanics

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Module No. # 09

The JWKB Approximation & Applications

Lecture No. # 5

JWKB Approximation: Justification of the Connection Formulae

We have been discussing the JWKB approximation and I have given the solutions of the JWKB, in the JWKB approximation of the differential equation. Today, we will continue our discussions on that. We will consider one small problem in tunneling probability calculation. Actually, we will assign a problem and then we will discuss the generality of the WKB approximation. And, we will show that it can be used to solve a general class of differential equations.

Finally, we will give a heuristic treatment of the justification for the connection formulae. So, first we will start with the tunneling through the barrier problem, which we had discussed in considerable detail. And, we had considered a potential energy variation, which was something like this.

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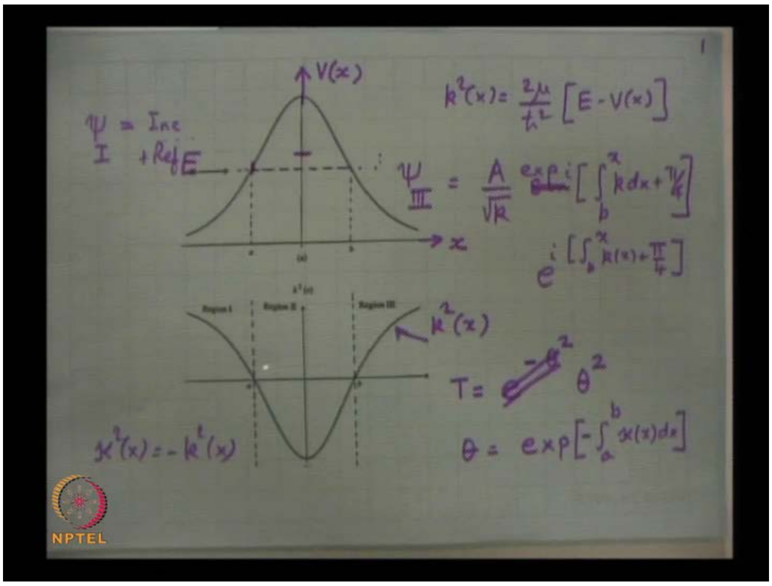

So, this is the V of x, a particle of energy E of mass mu is incident and this is the energy level and there were two turning points. Now, the k square of x; k square of x is equal to 2 mu by h cross square E minus V of x. So, since it is the inverse of minus sign of x, so k square of x, this is the variation of k square of x. And, x is equal to a and x is equal to b are the turning points.

So, here you will have oscillatory solutions, here you have exponential solutions and here you have again oscillatory solutions. So, what we did was in region three, we gave as, we wrote down a solution like this: A under root of top k sin of integral b to x k d x plus pi by 4.

Actually we wrote the solution. We can write down the solution as sin and cosine. So, instead, what we did was we consider an outgoing plain wave. So, we had exponential i times this. So, actually I should write it down as E to the power of i integral b to x k of x d x plus pi by 4 and wrote it as cosine plus sin. Then, used each of them to hop from this turning point here and then we looked at the turning point x is equal to A. And then, when we hopped through this turning point and obtained solutions here. and, the region, the solution and the region 1 was expressed as an incident wave plus as a reflected wave. And, we compared the coefficients and we found out that the transmission coefficient was equal to e to the power of minus theta square, where theta was equal to exponential.

So, I am sorry, the transmission coefficient was theta square and theta was equal to exponential minus a to b kappa of x d x because in the region of a to b, k square of x is negative. And so, therefore, we define kappa square of x which was equal to minus k square of x.

And, using that, and therefore using this relation we discuss the two, three problems; one, the tunneling through a parabolic barrier, then we discussed the tunneling through the triangular barrier and also to the alpha d k problem. But, I wanted to tell you today that, if we have a double potential like this and which has two peaks like the one that I have shown here and I assume the energy E and the V of x is variation like this. Then the corresponding k square of x, which is given by k square of x is once again 2 mu by h cross square E minus V of x. And, it will be something like this. So, there will be four turning points now.

Now, then what we have to do is, we have to first start with the solution in the region five. So, psi in region five will be an outgoing wave. So, we write down this as F by root k e to the power of i integral b to x k d x plus pi by 4. So, I write the solution here, which I write as F by root k, cosine of the same argument, cosine of this argument. So, let us suppose this is, say I write as capital phi plus i times sin phi.

And then, use connection formulae to go to the region four, then we look toward the turning point a and I go over to the region three and then we look to turning point minus a. And then, use the connection formula formulae to get to region two and then to region one. It is a very systematic process, very straight forward process. And, I would urge all of you, who want to learn the tricks of using the connection formulae, work this out yourself. It will help you to use the WKB JWKB approximation with tremendous efficiency and simplicity. It is very simple. You just have to get used to using these connection formulae.

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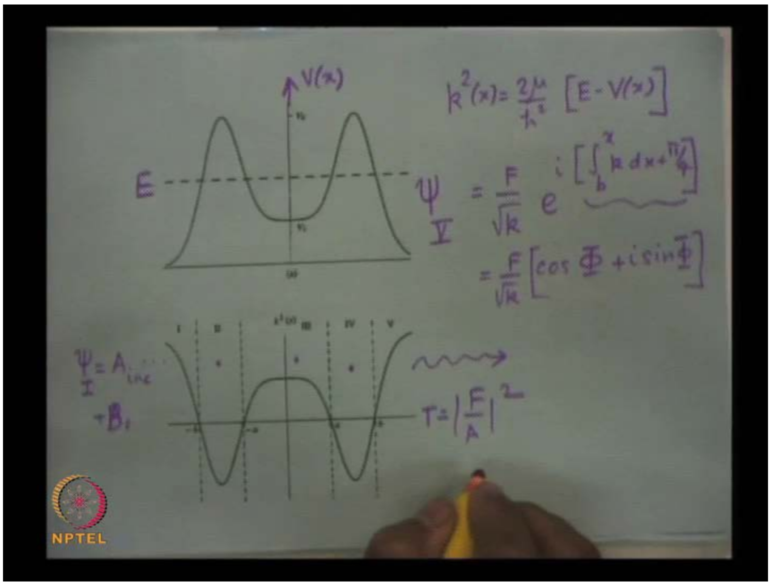

And, finally in region one, you express the solution as A times the coefficient of the incident wave plus E to the… and then you have the B, which is the coefficient to the reflected wave. And then, you will have F by A whole mod square will be tunneling probability. And, if you calculate that, you will find that the tunneling probability comes out to be…Let me write this expression. The, if you have a double humpty and these are the two turning points minus a minus b, plus a and plus b. This is the V of x variation, this is the V of x variation as a function of x.

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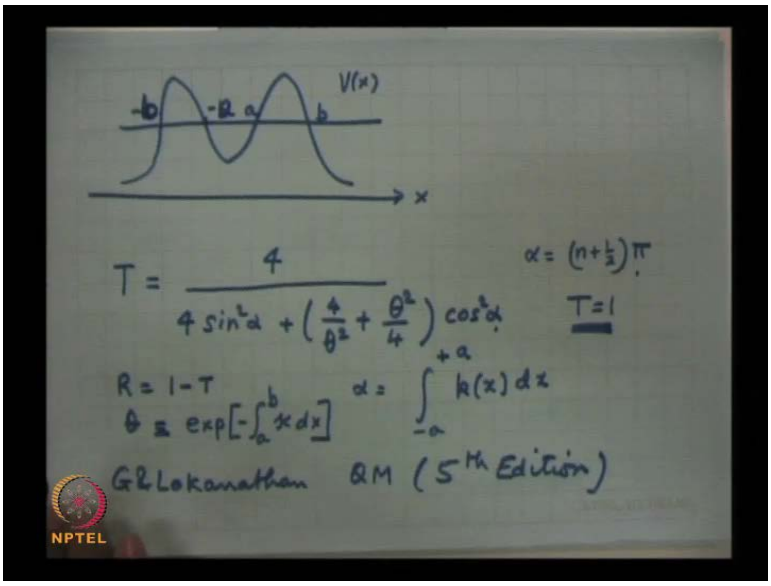

So, if you work this out and I would urge all of you to work this out. The tunneling probability comes out to be 4 by 4 sin square alpha plus 4 by theta square plus theta square by 4 cos square alpha. And then, similar expression for T; and, you will find that R plus T is equal to one, so that the reflection coefficient is 1 minus T.

What are the values of alpha and theta? The value of alpha is minus a to plus a. sorry. This is minus a and this is minus b. I am sorry. So, you have here; let me let me do this here. So, this is minus a and this is minus b. I hope all of you can see this. All of you can see this. So, the transmission coefficient becomes, so the alpha is equal to minus a to plus a. And, between minus a plus a, k square of x is positive. So, this is k of x d x. And then, theta is defined to be equal to exponential minus a to b kappa d x. As I mentioned to you, you first start with an outgoing plain wave here, outgoing wave here, then you hop to this region, then you hop to this region, then you hop to this region, then you hop to this region. And, in each hopping, you use the w JWKB connection formulae.

The remarkable feature of this simple formula is that, when alpha is equal to n plus half pi, when alpha is equal to n plus half pi, then cos square is equal to 0, sin square alpha is 1. So, T is equal to 1. These are known as the Fabry-Perot transmission resonances. Fabry-Perot transmission resonances. And, no matter what the shape of the potential variation be. The transmission coefficient is unity. These corresponds to, these corresponds, this is alpha is equal to n plus half pi. These are the Eigen values corresponding to the well.

So, those energy Eigen values which form for this particular well, you have transmission resonances. These are the modes, which resonate back and forth between the well. And, you have an unity transmission coefficient. So, to conclude that, for a double well, double hump potential well of the type shown here, you have, sorry, double. So, the… I would advice all of you to work out step by step, the tunneling probability.

It is the details, if you are unable to do that, the details are given in our book by myself and professor Lokanathan on Quantum Mechanics fifth edition. So, the details are there also, but I would like to, like you to work this out. And, obtain an analytical expression for the tunneling probability. And, this tunneling probability becomes equal to 1, irrespective of the shape of the potential structure when alpha becomes equal to n plus half pi. And, those are known as the Fabry-Perot transmission resonances.

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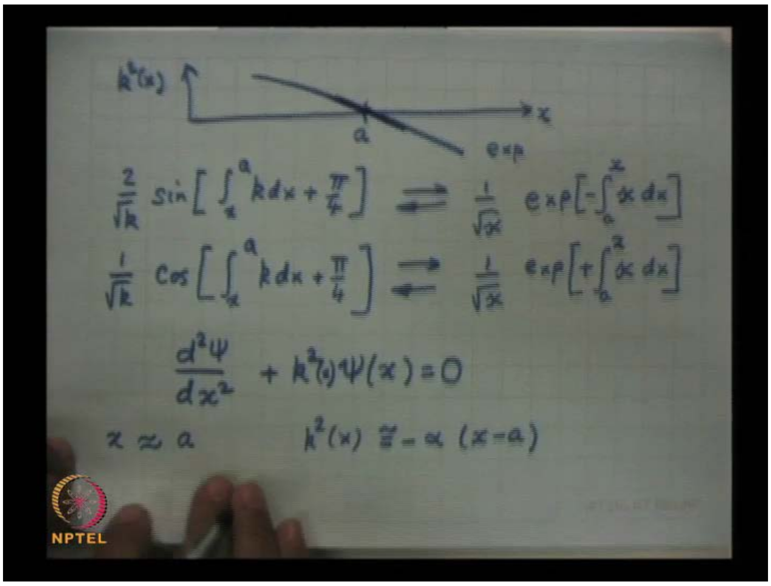

Now, we then, we then go over to calculating justifying the connection formulae. Justifying the connection formulae. For example, if you have a k square of x variation like this, this is the k square of x variation, k square of x variation. And, you have a 0, so that on the right side of the turning point, you have exponential solution. And, on the left side, k square of x is positive. So, we have sin and cosine solution. And, we had said without proof that 2 by root k, let us suppose the turning point is x is equal to a, then 2 by root k sin of x to a k d x plus pi by 4. It goes over to an exponentially decaying solution; minus exponential minus a to x kappa d x. And similarly, we had 1 over root k cos of x to a k d x k of x d x plus pi by 4. It goes over to the exponentially amplifying solution; so, 1 over root k exponential plus integral a to x kappa of x d x.

Now, I want to justify this. Now, near the turning point, near the turning point I can assume that the variation of k square of x is linear. So, the equation that we wanted, that we have been wanting to solve is d 2 psi by d x square plus k square times psi of x is equal to 0. k square of x, actually k square of x.

Now, we assume that in the vicinity of the turning point. So, at x, around x is equal to a, we assume that k square of x. In this case, it is decreasing function of x. So, we say that this is equal to minus alpha of x minus a approximately, so that in the vicinity of the turning point, we assume a linear variation for k square of x.

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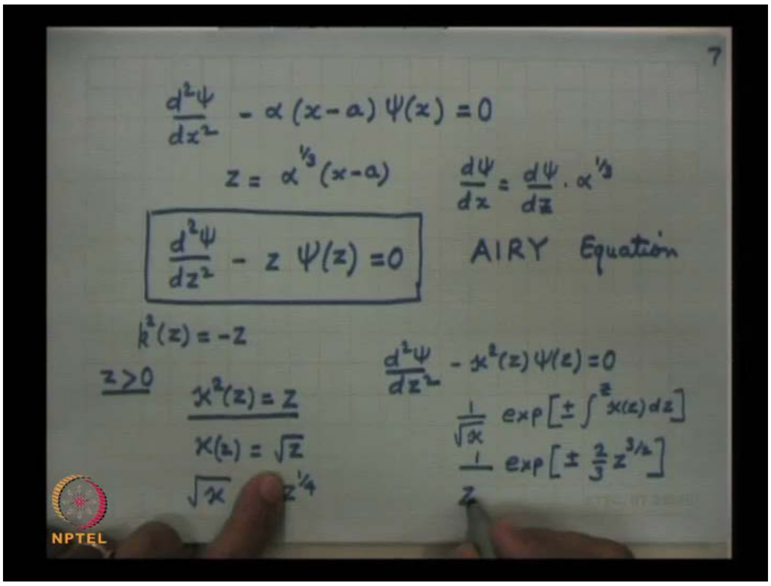

So therefore, my Schrodinger equation or the wave equation becomes d 2 psi by d x square minus alpha into x minus a psi of x is equal to 0. Now, I make a, I make a transformation I make a transformation that z is equal to alpha raised to the power of 1 by 3 into x minus a. Then d psi by d x, then d psi by d x will be d psi by d z into d z by d x. That is, alpha to the power of 1 by 3, then alpha to the power of 2 by 3 and then you will have x minus a is z by alpha to the power of 1 by 3 and so on. You do that and you will obtain d 2 psi by d z square minus z psi of z is equal to 0. This equation, this equation is known as the Airy equation after the name of the British astronomer G. B. Airy.

This is and the solution of this, the rigorously correct solution of these equations is known as the Airy functions. Now, first let me write down. So, here in this case k square of z is equal to minus z. So, for z greater than 0, I must write k square of z is equal to minus kappa square of z. And, kappa square of z will be equal to plus z. So that, for z greater than 0, this equation becomes d 2 psi by d z square minus kappa square of z psi of z is equal to 0. Now, since kappa square of z is equal to z, so kappa of z is square root of z and square root of kappa z is equal to z to the power of 1 by 4. Now, the WKB solutions of this equation are 1 over under root of kappa 1 over under root of kappa exponential plus minus integral z kappa of z d z. So, if I integrate this kappa of z d z, this will be exponential plus minus 2 by 3 z to the power of 3 by 2. And, outside it will be z to the power of 1 by 4.

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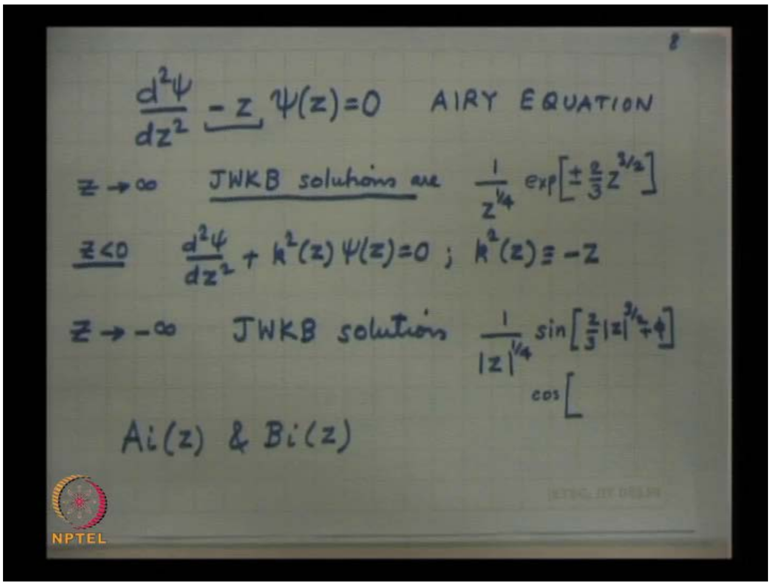

Therefore, therefore, the, of this equation d 2 psi by d z square minus z psi of z equal to 0. As z tends to infinity, the JWKB solutions are z tends to plus infinity. JWKB solutions are plus minus 1 over z to the power of 1 actually z to the power 1 by 4 exponential plus minus two-third z to the power of 3 by 2; small z everywhere.

Now, for z less than 0, this is positive. Minus z is positive. So, I must write it as d 2 psi by d z square, plus k square of z is equal to psi of z equal to 0, where k square of z is defined to be equal to minus of z. And then, as z tends to minus infinity, z tends to minus infinity, the JWKB solution are the JWKB solutions are 1 over mod z because now it becomes minus z mod z 1 to the power and then sin or cosine, sin and similarly cosine and two-third plus minus you can have, mod z to the power of 3 by 2 plus phi. And, similarly you can have the cos solutions.

So, the… So, of this equation, of this is the, as I mentioned this is the Airy equation. This is the Airy equation and obvious Airy equations. The WKB solutions are exponential this and that. Now, of the Airy equation, the rigorously correct solutions are denoted by A i of z and B i of z. These are known as the airy functions, which are related to Bessel functions. which are related to Bessel functions.

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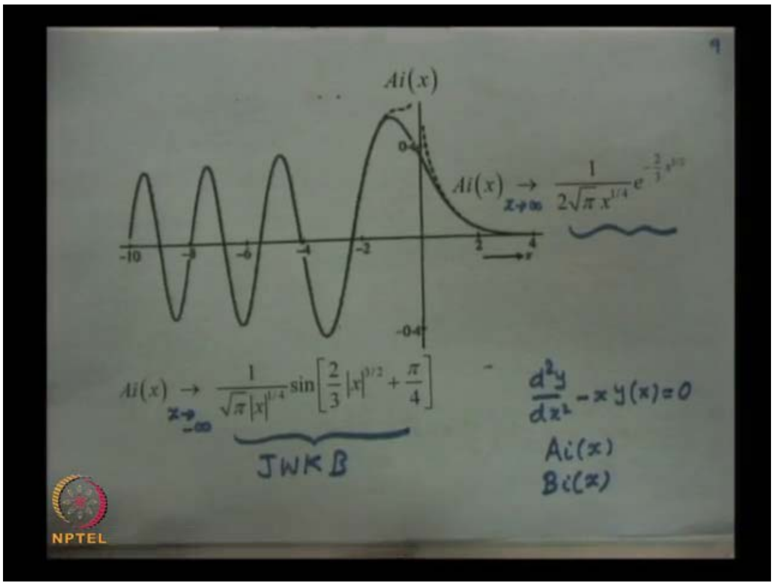

Now, the airy functions are the this solid line. You see, actually this is not in terms of z. This is the solution of the equation d 2 y by d x square minus x y of x. So, please replace z by x. So, of A i of x, the two independent solutions are A i of x and B i of x. And, A i of x has a asymptotic form as x tends to minus infinity, which is given by this. This I am not proving. But, it can be obtained from the asymptotic forms of the Bessel functions, which must be known to all of you.

Similarly, as x tends to plus infinity, this is the asymptotic form. If you can read properly, this is exponential minus 2 by 3 x to the power of 3 by 2. Now, these asymptotic forms are the JWKB solutions are the JWKB solutions. And so, therefore the sin term goes over to this term.

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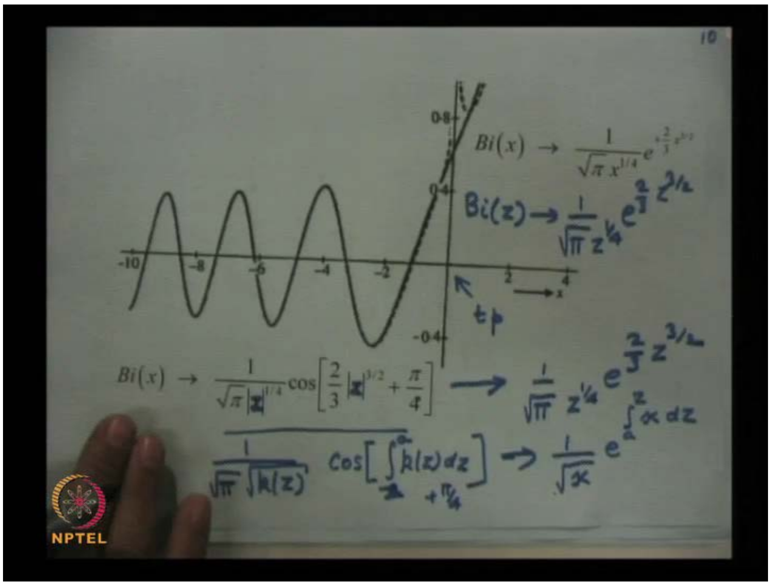

Let me first also show you the B i function. The other independent solution of this equation is the B i function, which are known as the Airy B i function. And, the asymptotic form is a cos plus pi by 4 as x tends to minus infinity. And, these asymptotic forms are plotted if you can see this as a dash line. You see the asymptotic solutions, which are just WKB solutions, they go to infinity at x is equal to 0; because this is the turning point; x is equal to 0 is the turning point.

So, here also the asymptotic solutions are shown by the dash line. And, they go to 0. And, the asymptotic solutions are the JWKB solutions. Here, the asymptotic solutions of B i of x is actually, I should have written B i of z goes over to 1 over root pi z to the power of 1 by 4 e to the power of 2 by 3 z to the power of 3 by 2. And, this also, I should have written z.

So, these are the same solutions that we had written down earlier. You see, exponential plus minus 2 by 3 z to the… So, one can say that cos plus pi by 4 goes over to 1 over root pi z to the power of 1 by 4 e to the power of 2 by 3 z to the power of 3 by 2. This suggests, this suggest that, since these are the WKB solutions, therefore 1 over root pi k of z, under root k of z cos of integral k of z d z. In this case, the turning point is at 0. So, from turning point say say z to a, that is 0. And, the turning point is, say a. This plus pi by 4 goes over to an exponentially amplifying solution. So, this is under root of kappa z e to the power of integral 0 to z.

In this case, therefore a to z kappa z d z; so, this is the, we have we have tried to justify the connection formulae because what we did was, we assumed the in the vicinity of the turning point, the solution to be the variation to be linear. When we assume the k square of x variation to be linear, then the solutions are rigorously A i of x and B i of actually not x, B i of z; where z is equal to alpha to the power of 1 by 3 into x minus a. Then, these are the WKB solutions. And, we showed by comparing the WKB solutions with the actual, with the exact asymptotic forms. We found that the sin from x to a plus pi by 4 will go over to an exponentially decaying solution and the cos term will go over to an exponentially amplifying solution.

So, this is the justification. We assume this to be linear, then the solutions become Airy functions. And, by looking at the asymptotic forms and comparing it with the WKB solutions, we write the connection formulae. And, as we have seen for to use these connections formulae, we will get exact results for the harmonic oscillator problems. And, even for the linear potential, if this gives very accurate results. We calculated the tunneling probability using the life time of alpha particle decay and showed the extreme variation. And, there are many comparisons that have been made to compare the validity of the JWKB approximation. And, it has been found as long as k square of x; that is, the potential energy variation is reasonably smooth, then JWKB approximation gives a fairly accurate solution.

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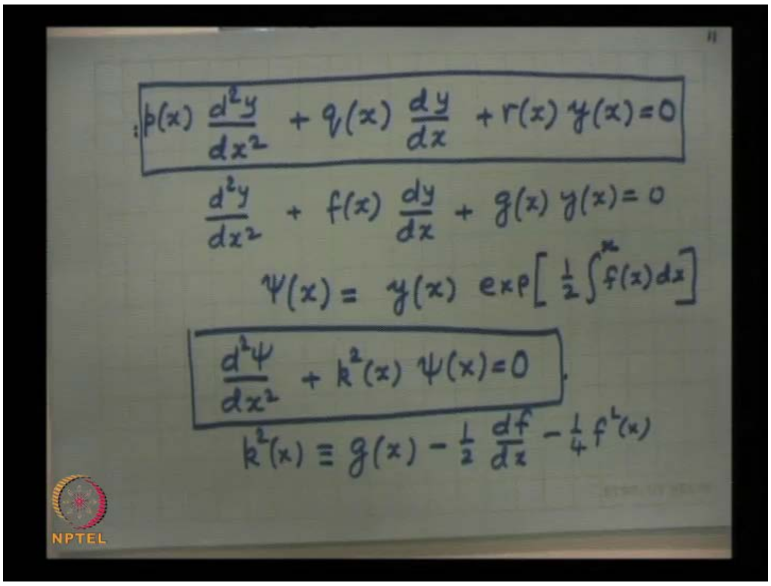

We finally, end up by saying that a general differential equation of the type p of x d 2 y by d x square plus q of x d y by d x plus r of x y of x d x equal to 0. I can always write this. I can always write this in the form of d 2 y by d x square, plus f of x d y by d x, plus g of x y of x. Obvious that, f of x is equal to q by p and g of x is r by p.

Then, we make two transformations. We make that, let us suppose that psi of x 1 psi of x is equal to y of x exponential half integral f of x d x. I leave this as an exercise for you. And, if you assume psi of x to be given by this and then you differentiate this and use this equation, you will find that d 2 psi by d x square satisfies this equation; k square of x psi of x is equal 0, where k square of x is now defined as g of x minus half d f by d x minus 1 by 4 f square of x.

So, what I was trying to tell you is that any second order differential equation of the type shown here, can always be transformed to any equation of this type. Any secondary second order differential equation of this form can always be transform to an equation, rigorously to an equation of this type. And then, we can use the WKB the w k b solutions to obtain approximate solutions of this equation. In fact, one can take the Bessel equation as an example and obtain solutions and they are, they often agree quite well with the exact solutions.

So, first you take, you look at any second order differential equation, try to transform it. Transformed airy differential equation of this type can be transformed to an equation of this type. And then, then one can obtain the WKB solution of that. So, in principle, we have given a recipe for obtaining a JWKB solution of a second order general; second order differential equation of this type. And hence, therefore w k JWKB approximation forms of very powerful method for solving the second order differential equations.

That concludes our discussion on the WKB approximation, in which to summarize this is a very powerful method for solving a second order differential equation. In which, k square of x is assume to be a smoothly varying function. It should not be very rapidly varying. It should not have too many zeroes because if it has too many zeroes, then you have to hop from one turning point to the other. But that, in principle, is possible as long as the variation of k square of x is fairly smooth. And therefore, it has been extensively used not only in Quantum Mechanics, but in Wave Guide theory in Plasma Physics and in in many other areas; diverse area of Physics and Engineering.

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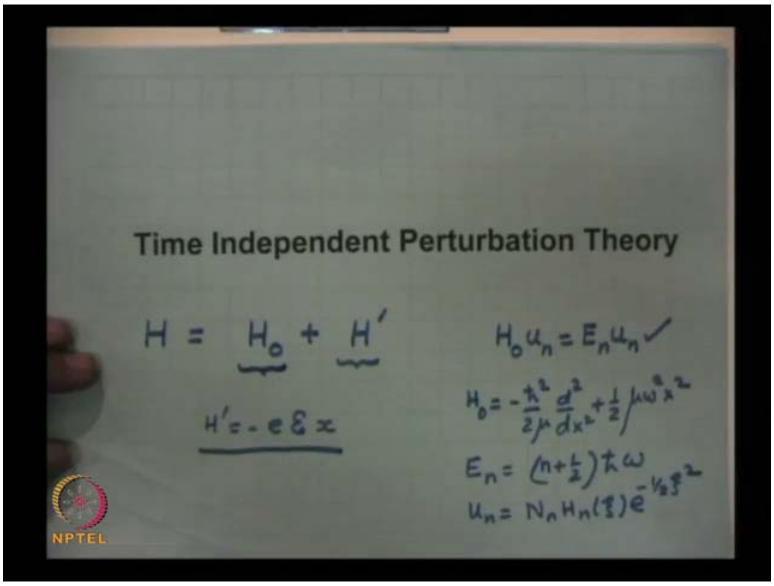

Our next lecture, in this lecture itself we will continue our discussions on developing and approximate method and that will be the Perturbation theory. We will continue our discussions on yet another very powerful approximate method. And, the method is known as the Time Independent Perturbation Theory. So, we have here in the Time Independent Perturbation Theory. We write the Hamiltonian as a sum of two parts; H is equal to H naught plus H prime.

Now, H naught is the Hamiltonian for which I know the solutions. So, H naught u n is equal to E n u n. For example, H naught may be the harmonic oscillator problem, like minus h cross square by 2 mu d 2 by d x square plus half mu omega square x square. Now, … we know the solutions. So, what are E n and u n? The E n, we know that is equal to n plus half h cross omega and u s of n are the normalized Hermite gauss functions the Hermite gauss function or H naught maybe the Hamiltonian, corresponding to the Hydrogen atom problems.

Now, therefore H naught is the is the portion of the Hamiltonian for which the solutions are known. Then, we add to it another term. And then, we try to calculate the effect of this term. So, for example, we have a charged oscillator. We put it in an electric field. And then, there is a Hamiltonian, which is something like minus, say e times the electric field times x.

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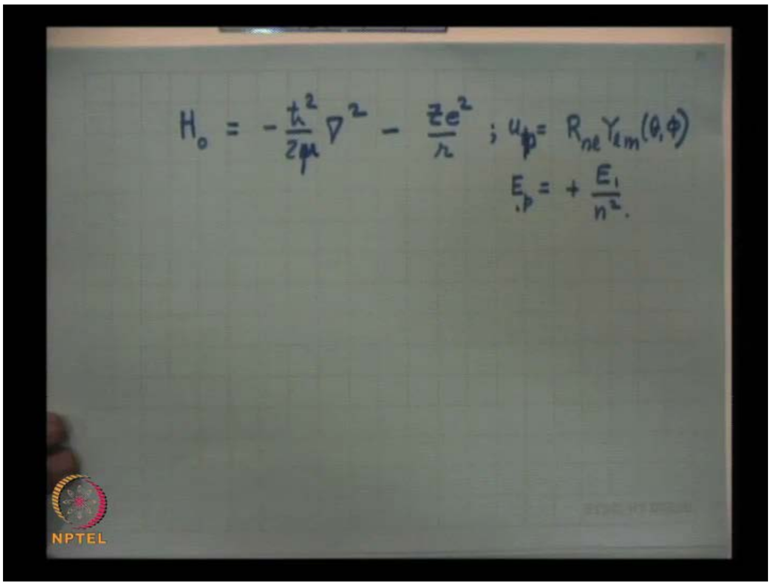

So, this is the perturbation or, you may have something like the H naught may represent the hydrogen atom problem, in which it is given by h h cross square by 2 m del square or mu actually mu, del del square minus z e square by R. This is v of R. So, for this problem, we know that u s of n is equal to R n l Y l m. Actually, these are three subscripts here. These are these are the wave functions. These are the wave functions.

This n and this n are different. So, this maybe something like say, we may write as u m or u p. u p u p is a combined form of these three. And, the E p, the energy Eigen values corresponding to be real state will be something like, the energy for the first state divided by n square. And, E 1 is about minus 13.6 electron mu. So, I know the Eigen values Eigen values and Eigen functions corresponding to H naught.

Now, I put the hydrogen atom in an electric field. And, they will be a perturbation or in a magnetic field. There will be a they will be an additional term, which will represent the interaction with the electric field and the magnetic field.

So, what will be the effect on the energy Eigen values? This is an extremely important problem in Quantum Mechanics as well as in Atomic and Molecular Spectroscopy. So, and, in any any, in most problems of interest, you cannot obtain a direct solution of the, a closed form solution of the Schrodinger equation. And, you have to apply an approximate method. One approximate method, we have already discussed. And, that was the JWKB method in which k square of x was assumed to be as slowly varying function.

Here, what we assume is that we have a solution, which is known to us, which is closed to the electric or magnetic field is weak enough, so that it makes a small change in the Eigen value structures of H dot. So, what we do is the method the method involves in writing the... Let me write it on a fresh page.

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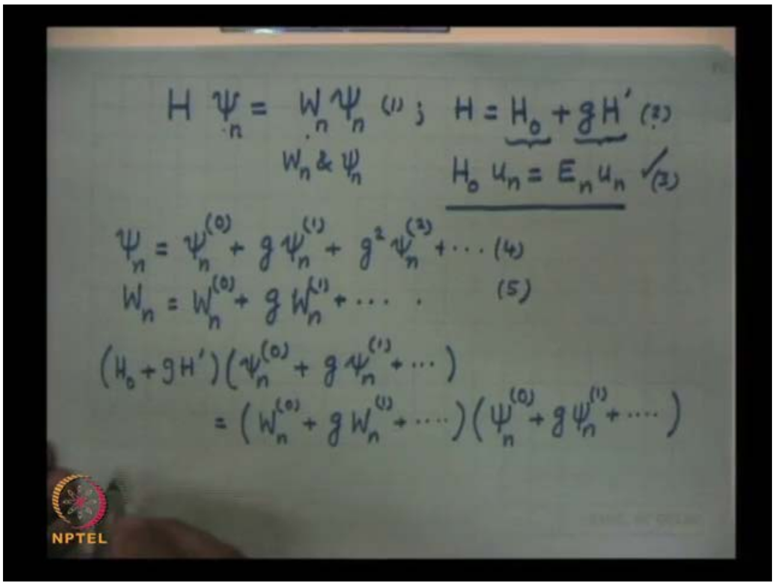

So, our objective is to solve this equation; H psi is equal to W psi. Where H, I write as H naught and I introduce a parameter g, if for example, H prime is due to the perturbation, due to the electric field, so it maybe the parameter is something like the strength of the electric field. So that, when the electric field goes to 0, you have the original Hamiltonian. So, g is a parameter, which is assumed to be less than one.

So, what we do is, we make a parametric expansion of psi and W. So, our objective is to obtain a specific state. Let us suppose, the n th state. So, our objective is to find W n and psi n. However, we know the Eigen values and Eigen functions of the operator H naught. So, the solutions of this are known. So, H naught u n is equal to E n E u n. So, this we know.

So, therefore let me state the problem clearly. The hydrogen atom is put in an electric field or in a magnetic field. Because of the presence of the magnetic field, it causes an additional term; it results in an additional term in Hamiltonian. If I exclude this term if I exclude this term, then I know the solution of the Eigen value equation corresponding to H naught.

This may be something like the hydrogen atom problem or the harmonic oscillator problem or a particle in a box problem. So, these solutions, the solution of the Eigen value equation corresponding to H naught are known to us. Our objective is that when we apply this perturbation, what will be the value of W n and what will be the value, what will be the corresponding Eigen function?

So, what we do is, we introduced the parameter g as I mentioned. And then, make a parametric expansion; that is, write psi n is equal to psi n 0 plus g psi n 1 plus g square psi n 2 etcetera. And similarly, we write W n is equal to W n 0 plus g W n 1 plus g square and so on. And, I substitute this. So, this let be equation 1, this be equation 2, this be equation 3, this be equation 4 and this be equation 5.

So, we substitute 2, 4 and 5 in equation 1. So, we get, please see this. H naught plus g H prime multiplied by psi n 0 plus g psi n 1 plus g square. Let me neglect the higher order terms right now. It is equal to bracket W n is W n 0; the zeroeth order term plus the first order term multiplied by psi n 0 plus g psi n 1 plus this.

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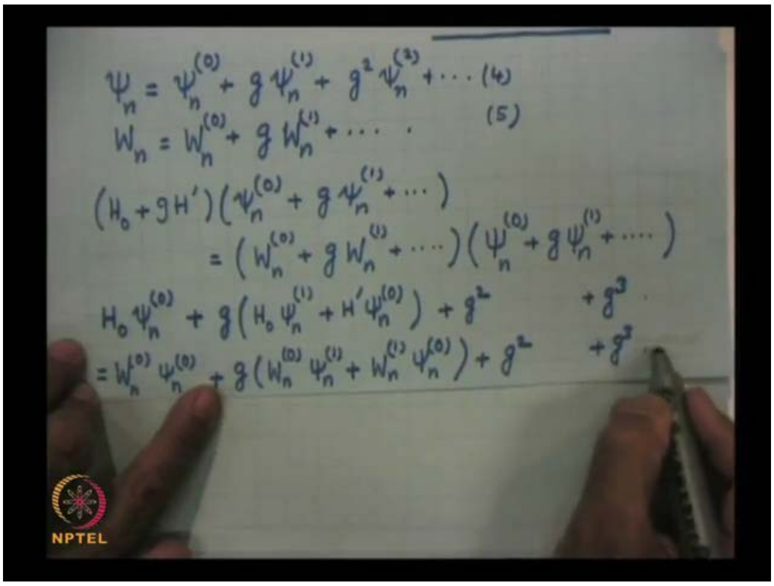

Now, I multiply this out and collect power, collect terms of different powers of g. So, you have you have the term which is independent of g is, H naught psi n 0 plus g will be g will be H naught psi n 1 H naught psi n 1 plus H prime psi n 0, plus terms which are proportional to g square, plus terms which are proportional to g cubed and so on. This will be equal to the first term will be is equal to W n 0 psi n 0 plus the term which is proportional to g, W n 0 psi n 1 plus W n 1 psi n 0, plus term which are proportional to g square, plus terms which are proportional to g cube.

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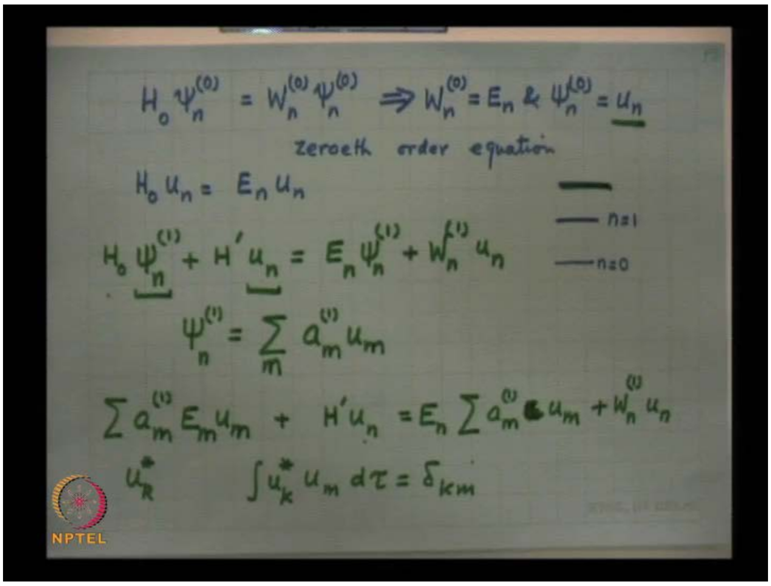

Now,… g tends to 0. And, there is always possible because we let the, let us suppose the external electric thing goes to 0. So, then all the terms cancels out. And, we had the zeroeth order equation, H naught psi n 0 is equal to W n 0 psi n 0. This is the zeroeth order equation. Zeroeth order equation. And, this is the same Eigen value equation, as for the H naught because for H naught, we wrote H naught u n is equal to E n u n. So, therefore we first consider non-degenerate states. And, I will tell you the difficulty that we encounter, when we consider degenerate states.

So, n is equal to 1 is this state or n is equal to 0, then n is equal to 1 or n is equal to 1 and so on. Each state is a non-degenerate state. Then, I can write this down immediately that W n 0 is equal to E n and psi n 0 is equal to u n. This is the zeroeth order solution; which is obvious because there is no electric field or magnetic field or something, there is no perturbation.

Now, then we have, so we said that this term is equal to this term. So, then these two terms cancel out. Then, we divide the whole equation by g. So, this goes out, this goes out, this goes out, this becomes g square, g square and g.

So, if I now make g tends to 0, then this is the first order term. So, the first order term becomes, you see becomes, H naught psi n 1 plus H prime psi n 0; which we have written as u n W n 0 is E n psi n 1 plus W n 1 u n. This is my first order equation. This is my first order equation.

Similarly, I can write down my second order equation also. But, we will restrict ourselves to only first order perturbation theory; because in most analysis, one uses first order perturbation theory. Then, what I do is that, we have we have already assumed that the, we are considering the perturbation to the n th state whose Eigen function is known. Psi n 1, I do not know. But, I know that u n form a complete set of functions. Therefore, let us expand it. Psi n 1, we can always expand psi n 1 as a n 1; superscript present that, we are considering first order of perturbation, times u n.

So, multiply this H naught, a n is a constant. So, H naught u n is equal to E n u n. So, H naught operating on this will give me summation a n 1 E s of n, u s of n, plus H prime u n is equal to E n summation a n 1. Actually, I am expanding. So, I must put it here as m because this is the dummy variable. So, this is A m 1 E m u m. I am considering the perturbation to the n th state. So, m n is fixed. So, this is a m 1 E n a m n 1 and u s of m plus W n 1 u s of n.

Now, I multiply by u k star and integrate. And, I know that these Eigen function for the non- orthonormal set; that is, u k star u m d tau is equal to the chronicle delta function. Remember that the harmonic oscillator functions also satisfy this orthonormality relation, where the integration is over the dash space.

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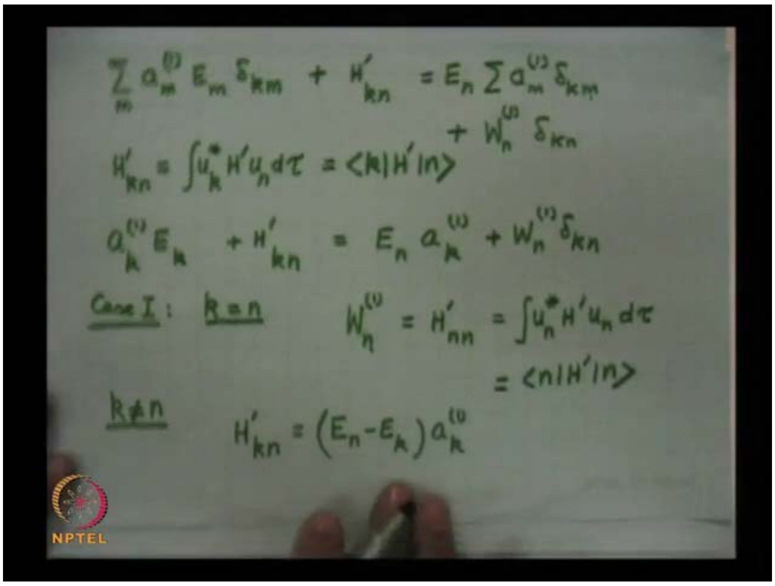

So, therefore if I multiply the entire equation by u k star, so we multiply the entire equation by u k star on the left and then integrate. So, the first term will become summation a m 1 E m delta k m plus H prime k n is equal to E n summation a m 1 delta k m plus W n 1 delta k n; where, H prime k n is known as the k n th matrix element; is the u k star H prime u n d tau. Symbolically, this is more convenient to write this as k H prime ket n. This is the k nth matrix element of this.

So, therefore if I sum the series, only the m equal to k term survives. And, so we have a k 1 E k plus H prime k n is equal to a n a k 1; because only the m equal to k term will survive and W n 1 delta k n. So, case 1, if I assume that k is equal to n. That, I multiply by u n star only. So, this term and this term cancel out. This becomes 1.

And then, W n 1 becomes H prime n n. So, this is the first order perturbation to the energy. So, this is u n star H prime u n d tau. And symbolically, it is n H prime n in the bracket notation. When, k is not equal to n, then you will have this term goes to zero. And, if I take, if I take the others, so H prime k n is equal to E n minus E k into a k 1.

So, you can use this, you can use this to calculate a k 1, which will be equal to H prime k n divided by E n minus E k. So, we have the coefficients because psi n 1 was equal to a k 1 u k. Actually, we had written a m 1 u m. This is a dummy variable. So, it does not really matter. But, here, and this I will discuss it greater detail in my next lecture. One see is that if, k is not equal to n, but E n is equal to E k; that is, if we have, let us suppose if 2 4 degenerate state, that is about, this is the ground state, this is a 2 4 degenerate state. So, this is u 0.

This is u 1 and u 2. u 1 and u 2 belong to the same energy state. So, that is a degenerate state. Then, this tells us that H prime k n must be 0. And, this I will explain it in greater detail in my next lecture.

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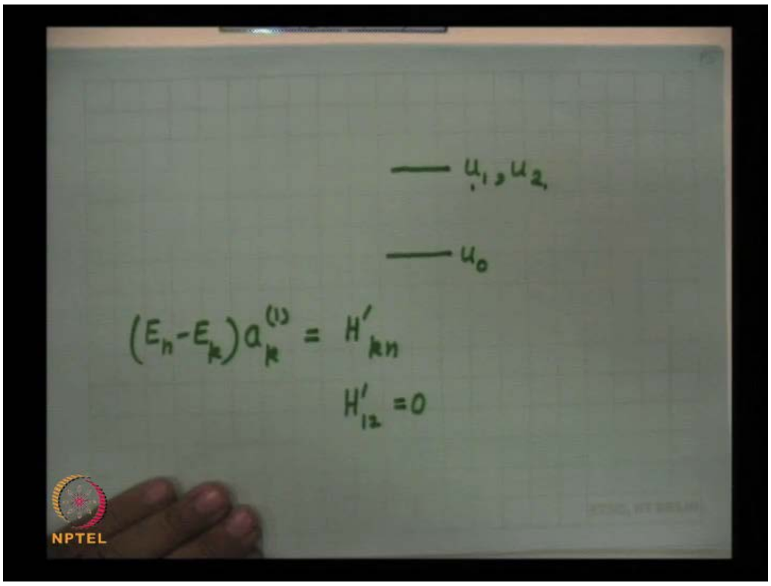

That, you see in degenerate state, if let us suppose u 0 is a ground state and the second state is 2 4 degenerate, then u 1 is a possible wave function, u 2 is a possible wave function; any linear combination is also a possible wave function. So, if that is so, then this relation E n minus E k times a k 1 is equal to H prime k n, says that I must choose such linear combination for which H prime k n, that is H prime 1 2 must be 0. That is the representation should be such that, the H prime in the subspace generated by the degenerate state vectors. …the H prime must be diagonal. And then, only the diagonal elements will give the energy Eigen values.

So, with that, if you have not followed this, we will give more illustrations. We will have, so the basic formulation is completed; that the first zeroeth order term is equal to E n, of course. So, these are the E 0 and E 1 and things like that. This we know. Then, the first order correction W n 1 is equal to; you just have to calculate the matrix element. First, we will do for non-degenerate states. And then, we will say, what precaution do we have to take for degenerate state perturbation theory. Thank you.