Basic Quantum Mechanics

Prof. Ajoy Ghatak
Department of Physics
Indian Institute of Technology, Delhi

Module No. # 09
The JWKB Approximation and Applications
Lecture No. # 04
The JWKB Approximation: Tunneling Probability Calculations and Applications

In our last lecture, we had discussed, we had considered applications of the tunneling probability formula to problems of practical interest. We first had considered the parabolic potential and then we had started with a linear potential. So, we assume that I have a triangular barrier as it is written here.

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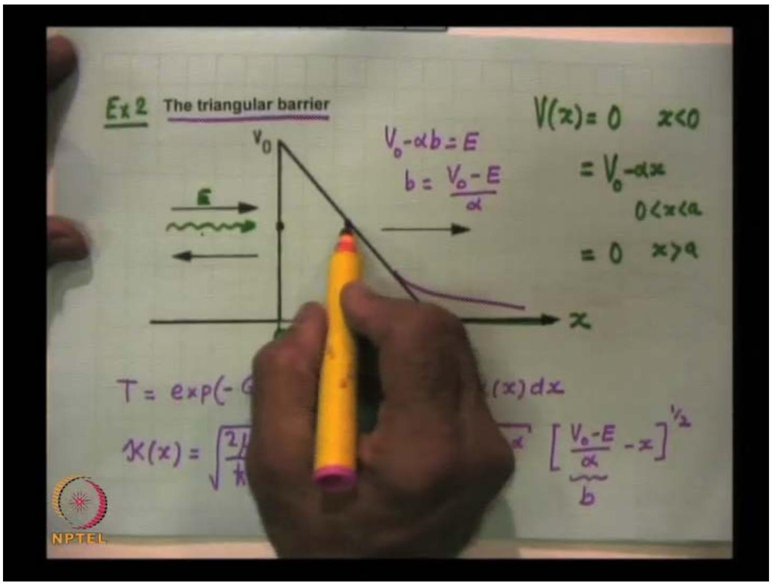

So, the potential energy function is 0 for x less than 0, and then it decreases linearly from a value V 0 by V 0 minus alpha x, and then beyond this point it is 0. Actually, it does not really matter, whether it tapers off like this or whatever happens here it does not really matter, because it is the variation of V of x between the two turning points that is important.

So, what is my kappa of x integral? So, the tunneling probability, as you know that exponential of minus G, where G is equal to two integral, from the turning points, a to b. So, here it is 0 to a, kappa of x d x and what is kappa of x? You have kappa of x is equal to 2 mu by h cross square under the root V of x in this region, between x is equal to 0 and a, V 0 minus alpha x minus E, raise to the power of half. If I take these, so what I will do is, that, I will write this down as, I will take alpha outside. So, this will be 2 mu by h cross square and alpha if I take outside, 2 mu alpha, then this will be V 0 minus E divided by alpha minus x, raise to the power of half.

So this is the value of the turning point (Refer Slide Time: 03:33). If this is the turning point b, then V 0 minus alpha b must be equal to E. Therefore, if I take b, so b is equal to V 0 minus E divided by alpha. So, this quantity is my turning point where the value of x at which the second turning point occurs.

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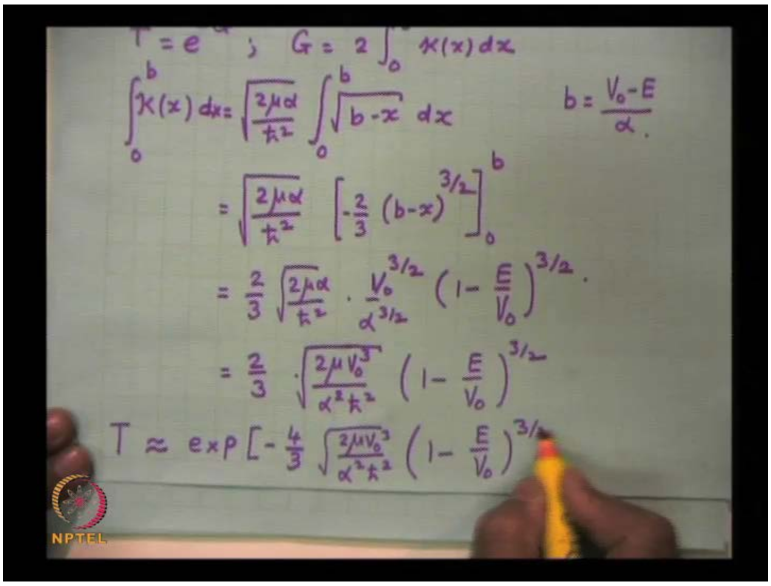

Let me write it down once again that T is equal to E to the power of minus G; G is equal to 2 integral 0 to b, actually it should not be a, it should be turning point b (Refer Slide Time: 04:33), 0 to b kappa of x d x and my kappa of x is equal to 2 mu alpha h cross square under root of b minus x. So, the integration is really very simple.

Therefore, the integral from 0 to b, so this will be integral, I hope this is clear, 0 to b d x. I must multiply this left hand side also by d x, and this will be equal to 2 mu alpha by h cross square will sit outside and the value of b as I had mentioned in my previous slide is V 0 minus E divided by alpha.

This is 2 by 3 with a minus sign, because and then b minus x rise to power of 3 by 2. This is the very simple straight forward integration from and then the upper limit, the integrand itself is 0 because x is equal to b, so that the upper limit it is 0. I get 2 by 3 b raise to the power of 3 by 2. So, I get this equal to 2 by 3 b raise to the power of 3 by 2. I will have 2 mu alpha by h cross square. So, 1 over alpha raise to the power of 3 by 2, because V rise to the power of 3 by 2 will have, and if I take V 0 also outside, so V 0 raise to the power of 3 by 2 and that is 1 minus E by V 0 raise to the power of 3 by 2.

If this alpha to the power of 3 by 2 so this becomes 2 by 3. So, 2 by 3 under root of 2 mu V 0 cubed, because if I take it inside the integral 2 mu V 0 cube and this is alpha to the power of half, this is alpha to the power of 3 by 2, so it is alpha cube will be under the integral, so this is alpha square h cross square, multiplied by 1 minus E by V 0 raise to the power of 3 by 2.

Therefore, the tunneling probability is WKB approximation, will be equal to E to the power of minus 2. So, exponential minus G and that is minus 4 by 3 times the whole quantity and that is a dimensionless parameter. So, this is dimensionless. It is always convenient to put parameters in terms of dimensionless parameters. It is multiplied 1 minus E by V 0 raised to the power of 3 by 2.

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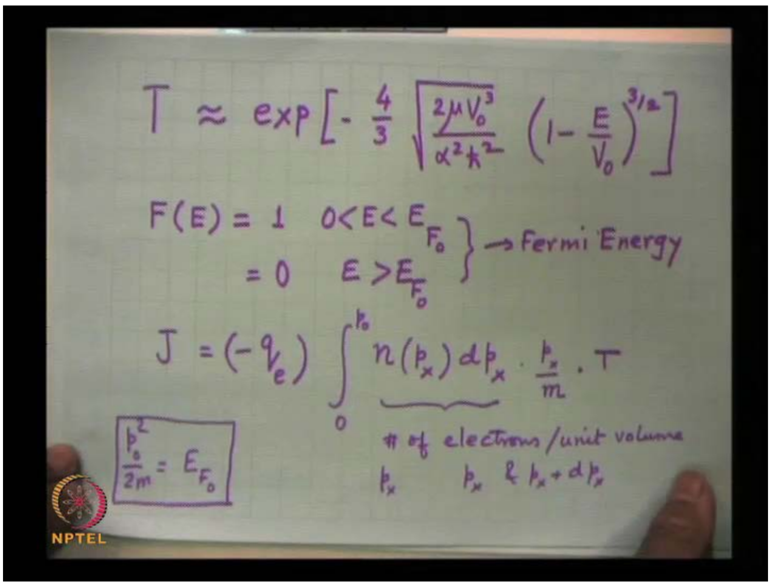

This is the tunneling probability. Let me write it down again, so that you can write it down. T is approximately in the JWKB approximation exponential minus 4 by 3 under the root 2 mu V 0 cubed divided by alpha square, h cross square, 1 minus E by V 0 raise to the power of 3 by 2.

This is the tunneling probability for a particle of energy E (Refer Slide Time: 08:47), incident from the left and seeing a triangular barrier and the maximum potential is V 0. So, what we said was before we discussed in our last lecture what we said was that the electrons in a metal like sodium or potassium, the valence electron is almost free.

In fact, quite sometimes back, we had developed the free electron theory of metals, and we assumed to be the electron to be completely free inside the metal. Now, we assume that this metal is at absolute 0. Now, electrons (( )) (Refer Slide Time: 09:34) statistics, so that we know that the probability of occupation of the electron is given by the Fermi Dirac distribution, which at absolute 0 is 1 for E lying between E less than E F 0 and is equal to 0 for E greater than E F 0.

This is the E F 0 is known as you probably know this as the Fermi energy and it depends on the temperature also. All states and all of the electrons inside the metal, so for x less than 0, you have the metal surface, and above the metal you apply an electric field, the electrons are free and there is a work function.

So all states up to this point (Refer Slide Time: 10:34) up to this level that is E F 0 are filled up. Now, the each electron depending on its energy seize this potential barrier and in front of it, so beyond the metal surface, there is an electric field that is applied, because of which it sees a potential of the type, a linear type of a potential, and there is a certain probability that it can tunnel through the barrier. This phenomenon is known as the cold emission of electrons.

One is the thermionic emission, if you heat the electrons, then the electrons get sufficient energy to get out of the metal. On the other hand, this is a purely, cold emission is a purely quantum mechanical phenomenon, and which we can understand through the simple analysis that we had carried out. It is a consequence of tunneling through the barrier of the electrons.

Let me calculate the current density that will be produced. What we will do is that the current density will be given by, say if the charge of the electron is let us suppose minus q e times (Refer Slide Time: 11:55), the number of electrons per unit volume, whose x component of the momentum lies between p x and p x plus d p x. So, let me write this down that this is number of electrons per unit volume, whose x component of the momentum, whose p x lies between p x and p x plus d p x, and then the velocity, this is the number per unit volume multiplied by the velocity that will be the current p x by the mass of the electron. Each time it hits the surface there is a small probability of it getting tunneling through the barrier.

So, the total number of electrons that will be coming out will be the integral from this and the corresponding say, p naught, p naught is such that p naught square by 2 m is equal to E F naught. That is the upper limit. Now, we will just outline you because this is not part of quantum mechanics so we will outline you the method for calculating this integral.

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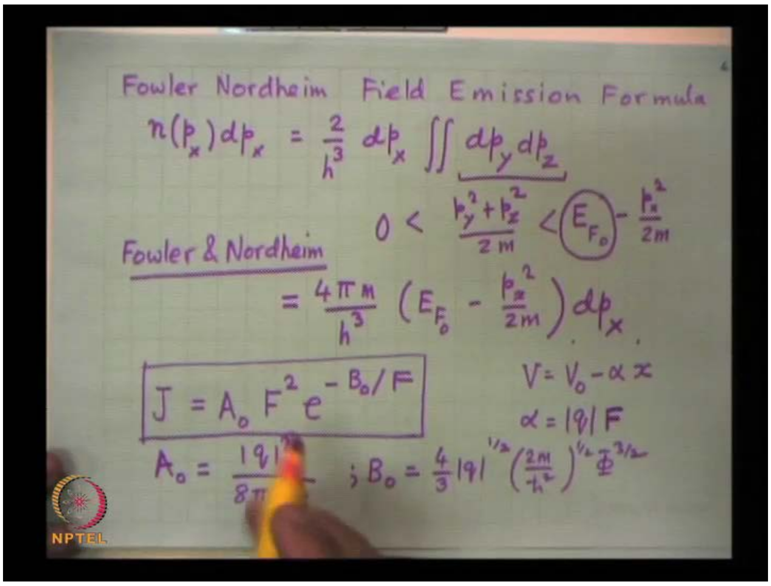

Now, n of p x, number of electrons, whose x component of the momentum, number of electrons per unit volume whose x component momentum lies between p x and d p x is equal to 2 by h cubed, it is just a simple phase space calculation, d p x double integral d p y d p z and this integral is calculated such that the 0, the p y square plus p z square by 2 m is less than E F naught minus p x square by 2 m.

E F naught is the maximum energy of the electron and p x square plus p y square plus p z square by 2 m is the total energy of the electron. So, the maximum value of p y square plus p z square by 2 m will be E F naught minus this. So, you can transform this to the polar space, carry out this integration, and then one finally, obtains this equal to 4 pi m by h cubed E F naught minus p x square by 2 m, d p x.

We have an expression for n of p x d p x, I substitute it here (Refer Slide Time: 15:39) multiplied by p x and then multiplied by this term and carry out the integration. One has to carry out the integration little approximately, the details are given in our book, but since it is not really quantum mechanics, so I will give you the final result. The final result is J is equal to A 0 F square; F is the electric field, e to the power of minus B 0 by F and the F is defined like this. Remember, that the potential energy function was equal to V 0 minus alpha x, so alpha is equal to modulus of the electron is the multiplied by the electric field.

The reason, why we have used the symbol F, rather than for the electric field E, so that it does not get confused with the energy that we have. This formula, so here A 0, if you do the calculation A 0 comes out to be mod q square by 8 pi h capital phi, where phi is the work function, that this is E F naught and this quantity is the work function phi (Refer Slide Time: 17:09). This quantity is the work function and then B naught is equal to 4 by 3 q raise to the power of half, 2 m by h cross square raise to the power of half, phi raise to the power of 3 by 2.

So, this formula for the current density was one of the early, the first of course, the example of the quantum mechanical tunneling phenomenon, was the alpha decay problem, which we will consider next and this formula was given by Fowler and Nordheim in early 1930s. So, this is known as the Fowler Nordheim field emission formula. Let me write it down. Fowler Nordheim Field Emission formula, it does explain the qualitatively, because of the many approximations that we have made in the experimental data in the case of some metals.

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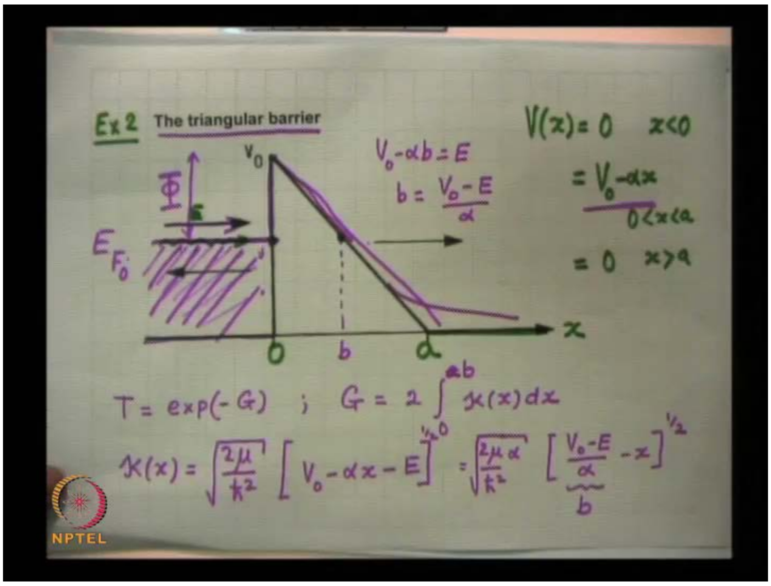

So, to conclude, we have here this on the left hand side is the metal surface where all the electrons have an energy less than or equal to E F 0. It has a certain work function, so that it can tunnel it through, so that classically it cannot go to the other side of the barrier. There is an electric field that is applied above the surface, because of which the potential energy function is a linear function of x.

In this linear potential energy variation, is due to the presence of the electric field. So, we can use the JWKB formula for the electron, which is near the Fermi level, which is the probability of it to tunnel through. Why do I say near Fermi level is because deep inside here the probability will become extremely small and it will contribute very little to the current, to the to the field emission current. So, this phenomenon is known as a Cold Emission or Field Emission of electrons from a cooled metal surface.

And if you therefore, apply an electric field to a cold metal, some electrons do come out that is because of the tunneling of the electrons from the barrier, and what we have tried to do is use the JWKB expression for the tunneling probability to calculate this. This can also model the alpha particle tunneling that we will be discussing next. So, let me now come to the alpha particle tunneling.

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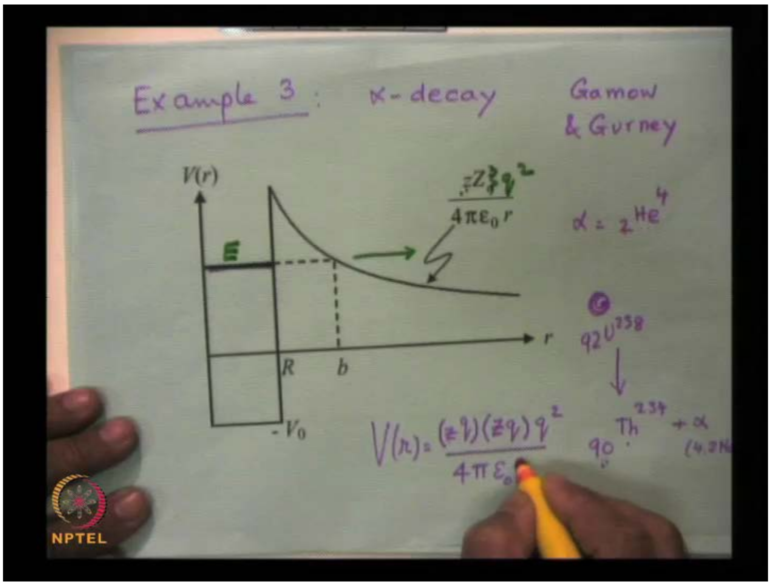

In example 3, tunneling of alpha, this is known as the alpha decay problem, and this theory was given Gamow and Gurney, in early 30s, I think around 1935 and so the alpha particle inside some of the heavy nuclei is inside a deep potential well. When it goes out of the, if it goes out of the nucleus then, as you know alpha particle is essentially the nucleus of helium particle. So, it has 2 protons and 2 neutrons. So, an alpha particle is the nucleus of the helium atom. So, it has 2 protons and 2 neutrons, so it is positively charged. I have a nucleus. Let us suppose, I consider the nucleus. Let me give you an example like 92 uranium 238.

Now, this is a radioactive substance. As you all know, this decays to 90 Thorium 234 plus an alpha particle. So, alpha particle has 2 protons and 2 neutrons, and it comes out with energy of about 4.2 m E V. So, this energy is 4.2 m E V and as soon as it gets out it experiences a repulsive force; it experiences a repulsive force.

So, the potential energy function is positive, so what is the potential energy function? V of r, we will write down as small z which is 2, because the charge of the alpha particle is z times q, 2 times q where q is the magnitude of the charged electron or of the proton. So z q is the charge of the alpha particle. Let us suppose z q is the charge of the nucleus, daughter nucleus. So, in this case capital Z is 90, not 92, multiplied by q square by 4 pi epsilon, I am working in the M.K.S system of units.

This is the repulsive potential energy function that it sees.

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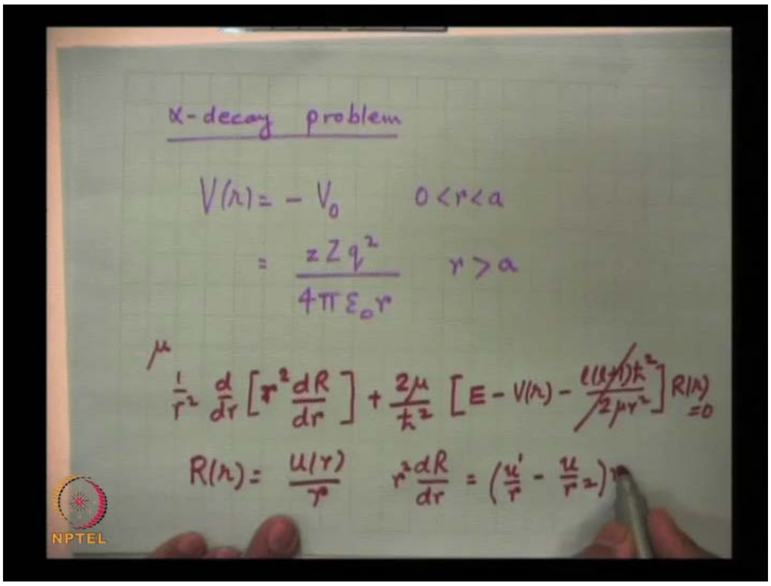

So, in the alpha decay problem, we will have V of r is equal to minus V 0 for 0 less than r less than a, and is equal to z Z q square, this small z is 2. Sorry, here I wrote q square twice, so this should not be there (Refer Slide Time: 24:24), 4 pi epsilon naught r. So, this is for r greater than a. Now, this is the potential energy that is experienced by the alpha particle. Let the alpha particle mass, I denote it by mu. So, as you all know that we have worked with the Schrodinger equation and the radial part of the Schrödinger equation was 1 over r square d by d r of r square d R by d r plus 2 mu by h cross square E minus V of r minus l into l plus 1 h cross square by 2 mu r square, R of r is equal to 0.

I assume l to be 0; zero angular momentum, and then as we have done before, I define a function small u of r through this relation, R of r is equal to u of r by r, and I do not have to, let me do the algebra. Therefore, d R by d r is equal to u prime by r minus u by r square. I can multiply this by r square, so this is r square, multiply this by r square. So, this becomes r u prime minus u, so this is straight forward. I think I had done this once before.

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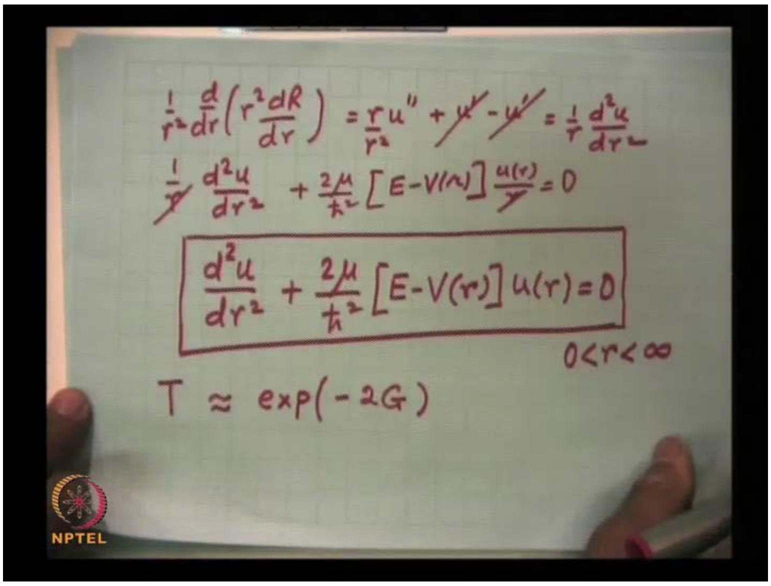

I can differentiate this, so d by d r of r square d R by d r. So, this will become r u double prime d 2 u by d r square plus u prime and differential of this is minus u prime, so these 2 term cancel out.

If I divide by r square, as you can see here, so I get 1 over r square divided by r square, so this becomes 1 over r d 2 u by d r square. So, this quantity becomes 1 over r d 2 u by d r square plus 2 mu by h cross square E minus V of r, R of r and it is u of r by r. So, why I am doing all this? I am trying to tell you that u of r, so r and r cancels out, so d u by d r square plus 2 mu by h cross square, E minus V of r, u of r is equal to 0.

This is once again the one dimensional Schrodinger equation but the only thing that we have to remember is that r does not take negative values, r goes from 0 to infinity and not minus infinity to plus infinity, r goes from 0 to infinity and that is all of you must remember. So, that now we can use the same formula for the alpha decay problem.

You have the energy, one of the turning points is R (Refer Slide Time: 28:41) and the other turning point is b, so where b is the value of r, where this coulomb potential becomes equal to E. I hope it is clear so, therefore, this thing is T; the tunneling probability is exponential minus 2 G and let me write down the formula for G.

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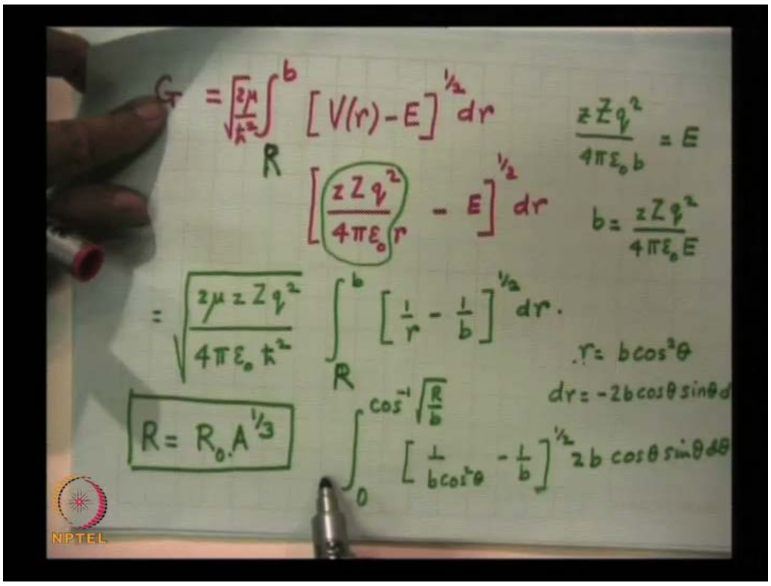

The formula for G is equal to, one of the turning point is 0, the other turning point is b, under root of 2 mu by h cross square, multiplied by V of r minus E, raise to the power of half d r. What is V of r? That is small z capital Z q square by 4 pi epsilon naught r minus E.

What I do is, I take this part outside, the square root of this part (Refer Slide Time: 30:32), this part outside. So, I will write down, this is equal to under root of 2 mu, just to patiently, in all quantum mechanical calculations, you just have to be patient, and this is 4 pi, epsilon naught, h cross square, 0 to b, 1 over r, minus 1 over b, raise to the power of half d r.

You see, because b is the point where this is 0, so z Z q square 4 pi epsilon naught b, is equal to E (Refer Slide Time: 31:32). So, this is the turning point, b is therefore, is equal to z Z q square by 4 pi epsilon naught E; E is the energy of the alpha particle. Now, let me integrate this and then we will come back to this, so we write down r is equal to, let me do this although it is a very simple integration, r is equal to b cos square theta. So, this becomes, I just consider the integral. So d r becomes 2 b cos theta sine theta d theta with a negative sign, so when r is 0 theta becomes pi, no i’m sorry i’m sorry i made a mistake

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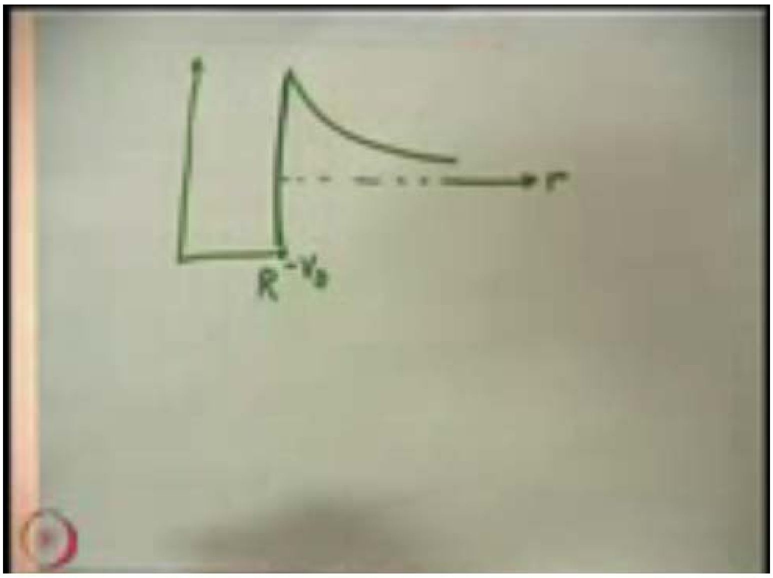

You see we had this potential energy function was something like this. This is the 0, this is r. So, this is minus V 0 and this distance is the radius of the nucleus.

So, the lower limit is not 0 is R (Refer Slide Time: 33:29). So, the lower limit, I hope you understand this that where did the figure go just 1 second let me yes

In this figure, one of the turning points is here. The particle is going from here, the alpha particle is incident on the boundary of the nucleus, and at that point r is equal to capital R and then it hits the point b, b is the other turning point.

So, the limits are limits are not from 0 to b but, from capital R which is the radius of the nucleus, and the radius of the nucleus is given by R 0 A to the power of 1 by 3. This is a empirical formula, where R 0 is the radius and this is about 1 Fermi, 10 to the power minus 15 meters. A (Refer Slide Time: 34:29) is the mass number of the nucleus. So, r is equal to b cos square theta and since there is a minus sign, then theta will be cos inverse under root of R by b when r is equal to R, and when r is equal to b, so cos square theta becomes 1, so theta becomes 0. Then the integrand will become 1 over b cos square theta minus 1 over b, raise to the power of half; d r is 2 b cos theta sin theta d theta.

I hope it is clear. So, I take the b outside and this b to the power of half comes outside, b to the power of half will be…

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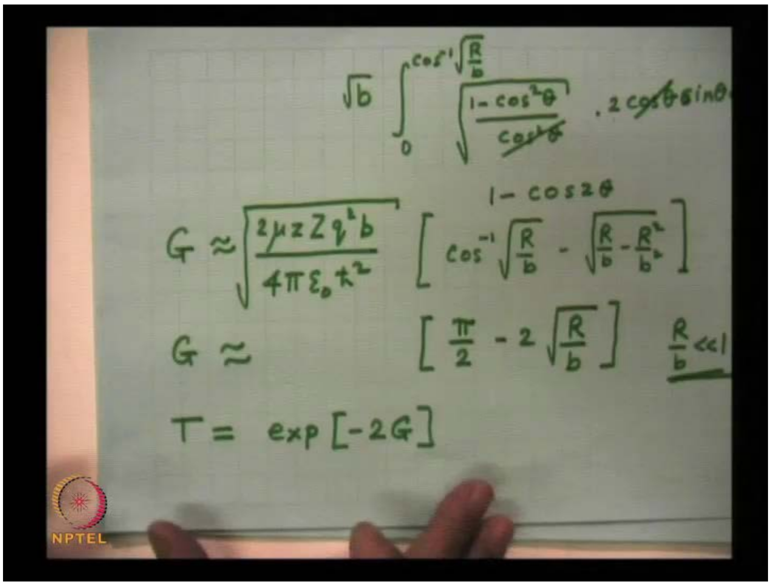

So, this will be integral, so let me put it below, and we will have under root of b and this will be integral from 0 to cos inverse under root of R by b. I had taken b outside.

So, 1 minus cos square theta divided by cos square theta, raise to the power of half, times 2 cos theta sine theta d theta. So, this is sine theta and this is cos theta, so this cos theta this cos theta cancels out. This becomes 2 sine square theta and 2 sin square theta is 1 minus cos 2 theta. Now, the integration is very simple.

The final result becomes, G becomes equal to 2 mu z Z q square, and b comes inside because of this factor divided by 4 pi epsilon h cross square under the root. This becomes equal to cos inverse under root of R by b minus under root of R by b minus R by b whole square. In fact, if approximation, so this can be written as pi by 2, when R is very small compared to b then minus 2 under root of R by b.

This is a fair approximation that R by b is much less than 1, is a about 0.1 usually, is usually a 0.1, so in that approximation this is the value. Now, therefore, an analytical expression for G and the tunneling probability is exponential as we all know minus 2 G.

Now, let me consider, so we have now an analytical expression for the tunneling probability. From this we will calculate the lifetime of the alpha particles, and we will get the result, which can be compared with experimental data and that is what we will do now.

You see inside the nucleus, let us suppose we have N such nuclei. So, we take what is known as a semi classical model in which the alpha particle is moving with a velocity V each, and it hits the boundary, and at each hitting there is a certain probability that it will tunnel to the other side. This tunneling probability, we have calculated by using the WKB formula.

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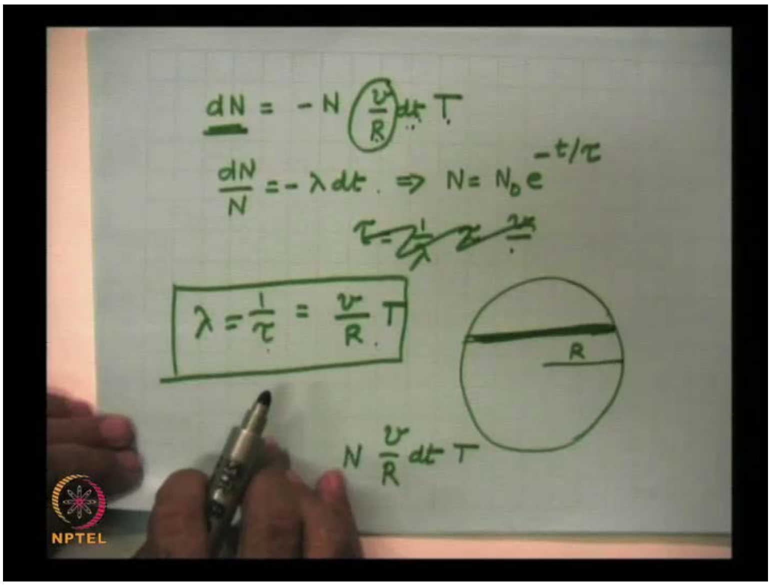

So the number of particles, which have number of alpha particles that have come out is the is proportional to the number of original nuclei or number of alpha particles, multiplied by the number of times the alpha particle hits the barrier. That will be about V by R times d t in time T. So, number of times the alpha particle hits the spherical surface of the nucleus per second is V by R, so in time d T it is so much.

And each time, the probability that it tunnels through is so much. So, you have d N by N is equal to minus lambda times d t, where lambda is the inverse of the mean free time; mean life time. So, this will give me N is equal to N 0 E to the power of minus t by tau, where tau is equal to 1 over lambda. This is the radioactive decay law and that is approximately equal to V by r, sorry sorry 1 over tau is equal to lambda and 1 over tau is equal to V by R into T (Refer Slide Time: 41:12).

So, once again, let us suppose, I exaggerate the nucleus like this; this is the nucleus whose radius is R, so approximately the alpha particle is assumed to be free and it is moving like in a straight line, in a semi classical model. So, number of times the alpha particle hits the surface trying to go out of the surface is V by R in times d t, the number of hits is V by r d t multiplied by the total number of nuclei that are present, and at each hit, there is a T probability of it getting tunnel through, tunneling outside. Therefore, the number of decays that take place and let us suppose that it is denoted by d N and because N decreases therefore, there is a minus sign.

So, minus N times V by R d t times T, if I write lambda is equal to so much then tau will become 1 over lambda or lambda is equal to 1 over tau is equal to V by R times T.

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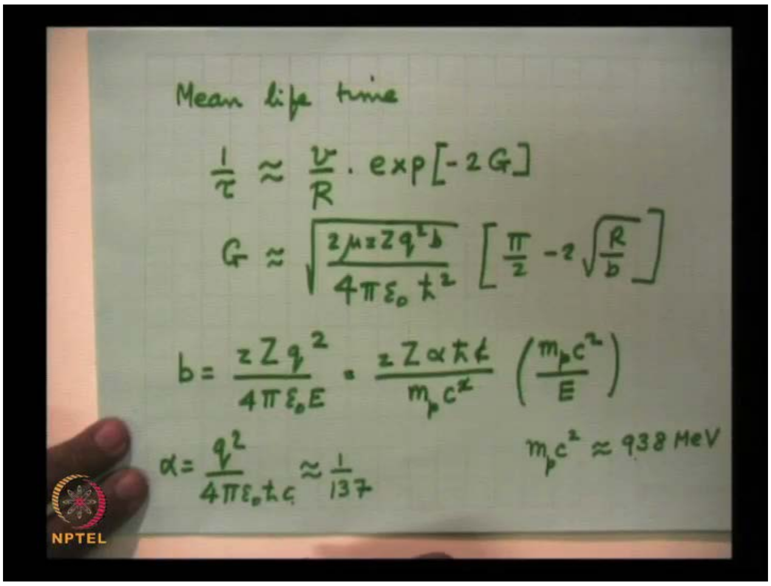

We have now an analytical expression for the lifetime, so for the mean life time is equal to, are given by 1 over tau, equal to V by R into the lifetime that is exponential minus 2 G. So, G as we all know is equal to under root of 2 mu z Z q square b, by 4 pi epsilon naught h cross square, approximately pi by 2 minus 2 under root of R by b. Now, one can do a little bit of algebra. Let me tell you a simple thing that b is equal to z Z q square by 4 pi epsilon naught E. As you know the fine structure constant is q square by 4 pi epsilon h cross c.

So, q square by 4 pi epsilon naught is alpha times h cross c. So, this I write as z Z alpha h cross c divided by E and what we do is I multiply this by m p c square and divide by m p c square divided by E. I tell you the moment why I did this, where m p is the mass of the proton, we know that m p c square; the rest mass of the proton is about 938 MeV. So, once it goes out from here, therefore, if I put m p c square is equal to 938 and then alpha is equal to as we know is equal to 1 over 137.

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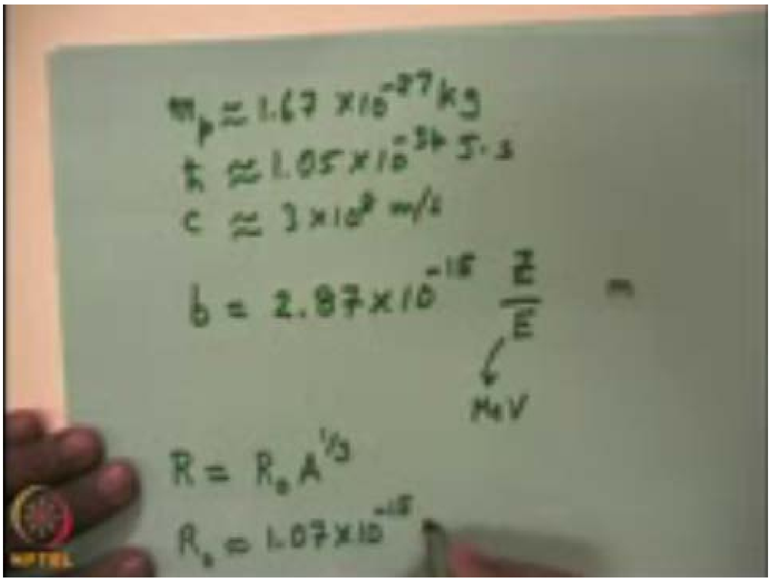

And then we use consistently, this M.K.S system of units, so in the M.K.S system of units m p is equal to 1.67 into 10 to the power of minus 27 kg, and h cross is equal to as you know 1.05 into 10 to the power of minus 34 joules second. I substitute for small z, Z remains like that and E is measured now in MeV.

Alpha is 1 over 137, h cross I know, m p I know and c is about 3 into 10 to the power of 8 meters per second. So, if I substitute that then in this expression, I get this following expression that b is equal to 2.87 into 10 to the power of minus 15 Z by E. This is measured in meters where E is measured in MeV. So, I would like you to fill out the intermediate steps.

And then the formula that we will be using for R is equal to, as I told you that this is equal to R 0 A to the power of 1 by 3. Then R 0 is approximately equal to 1.07 into 10 to the power of minus 15 meters.

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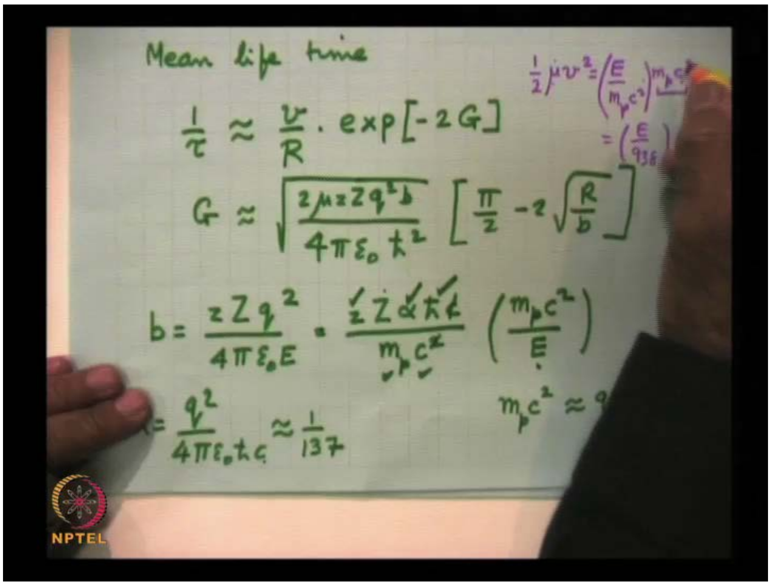

If you now use this and substitute it, in this expression, and also remember that the velocity of the alpha particle is related to the energy, by so much such as half mu V square is equal to E, and then you transform this also, where mu is the mass of the alpha particle, and what you do is you divide this by m p c square and multiply this by m p c square and so you get this part is E divided by 938 MeV and then m p c square, m p is the mass in kilogram, c is so much and then you obtain the mass of the alpha particle is 4 times mu p. Using that we can obtain an expression for V, and everywhere the energy of the alpha particle.

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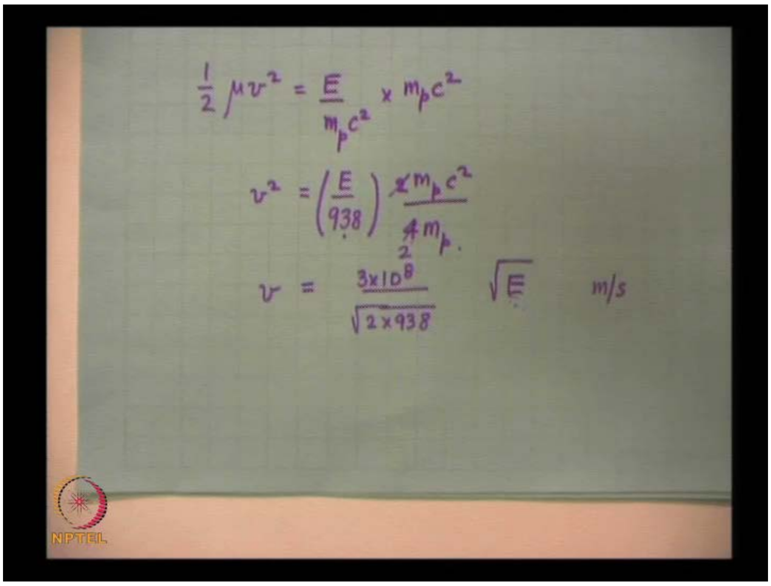

Let me do this again how to write the expression for V. So, you have half mu V square is E, the energy of the alpha particle. So, I divide by m p c square and multiply by m p c square. So, this is 938 m e V, so E by 938 m e V. Now, E is measured in MeV. So, V square is equal to m p c square, divided 2 m p multiplied by mu; mu is the mass of the alpha particle and approximately this is 4 times mass of the proton.

So, you have here, therefore, these 2 cancels out with 2 and m p, m p cancels out. V is equal to under the root of E and then this will be under the root of 2 into 938 multiplied by 3 into 10 to the power of 8. Now, V is measured in meters per second, but, E is written in terms of MeV. So, E has to be measured in m e v, because we have used m p c square.

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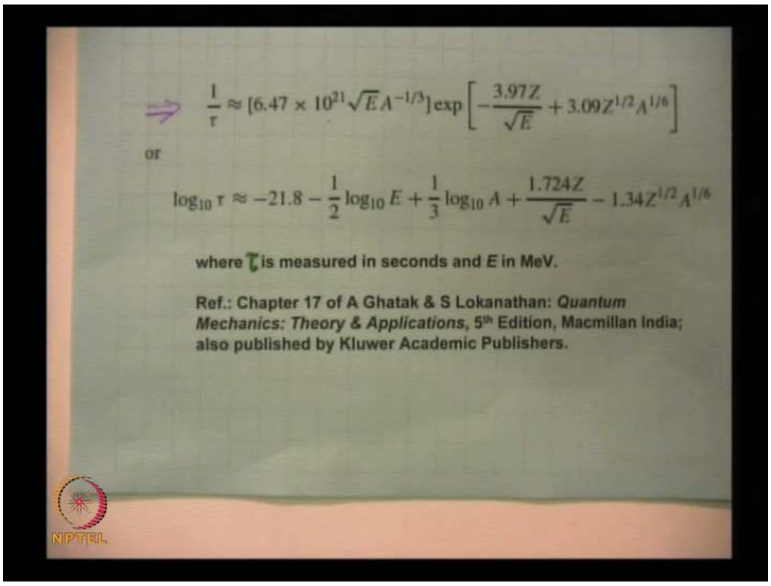

If you use this kind of algebra then the final formula, it is given in our book by myself and Professor Lokanathan on quantum mechanics theory and applications, fifth edition. So, you will get this expression for 1 over tau. So, here I read it out that 1 over tau is equal to 6.47 into 10 to the power of 21 square root of E; E is measured in MeV, A is the mass number of nucleus minus 1 by 3 exponential so much.

If you take the log to the base of 10, then you will get this approximate expression of minus 21. 8 minus half log 10, where tau is measured in seconds, and E is measured in MeV. So, E is the energy in MeV of the alpha particle coming out of the nucleus, A is the atomic number of the nucleus. Z is the atomic number the charge of the daughter nucleus; A, and all symbols are defined.

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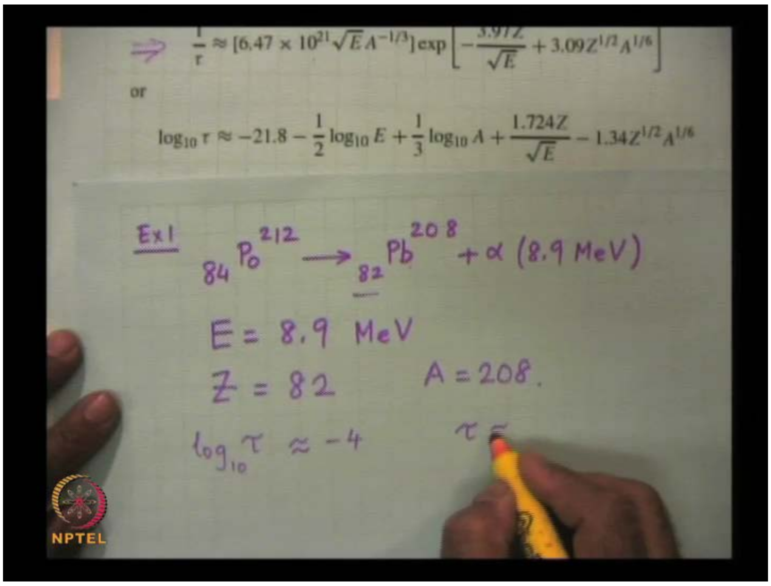

Let me now consider two simple examples. One is let me consider the decay of 84 polonium 12, example 1, going over to 82 lead 208, and an alpha particle of 8.9 MeV is emitted. So, you have here in energy E is equal to 8.9 MeV, million electron volts.

This Z is the 82 and A is 208. So, then what I do is that I substitute here E is equal to 8.9, and A is equal to 208, Z is equal to 82 and carry out this calculation. We will find that this comes out to be log tau to the base 10, it is a simple calculation comes out to be about minus four.

So this means, tau is about 10 to the power of minus 4 seconds. So, from first principles see the tremendous beauty of quantum mechanics, from first principles, we have been able to calculate without making any assumptions, the approximate lifetime of the alpha particle coming out of the nucleus.

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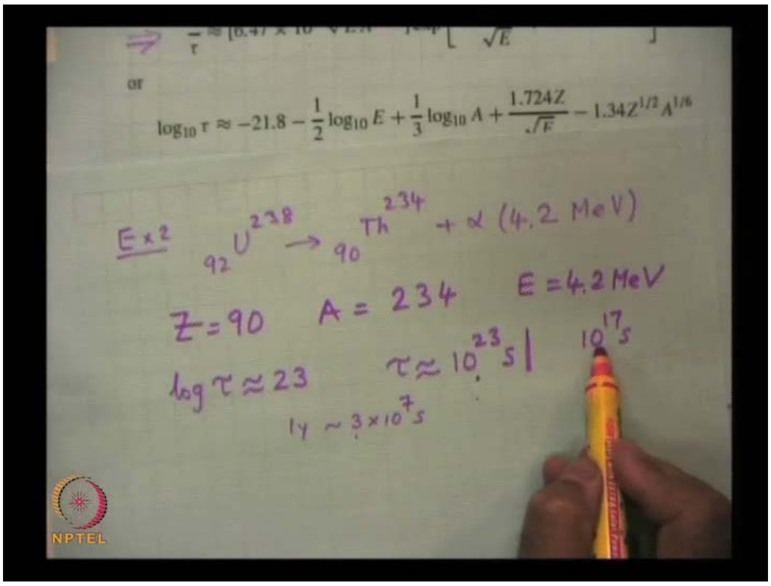

Let me take another example, and the example 2 is 92 uranium 238, giving you the 90 thorium 234 plus an alpha particle of 4.2 MeV, and here the Z is equal to 90, A is equal to 234 and E is equal to 4.2 MeV. Now, I once again substitute these numerical values in this equation and what I find is that tau comes out to be log tau now comes out to be 23. There log tau had come out to be minus 4, so tau comes out to be 10 to the power of 23 seconds.

You must all should know that 1 year is about 3 into 10 to the power of 7 seconds, the best way to remember that is 1 year is about pi times 10 to the power of 7 seconds, so this is 10 to the power of 16 years, so, this is a log tau comes out to be 10 to the power of 23 and we have this about a billion years, that is the lifetime that comes out. So, this is the value that comes out. Now, unfortunately the experimental value is 10 to the power of minus 7 seconds (Refer Slide Time: 55:18).

In this particular case, and here it is about 10 to the power of 17 seconds (Refer Slide Time: 55:28). So, one may say that the experiment really does not compare too well to the theory. But, our theory is very approximate; we have assumed that the alpha particle was moving classically, as a free particle inside the nucleus. This is not quite correct. But see that from the variation from 10 to the, that it has such a low value from 10 to the power of minus 4 seconds to 10 to the power of 23 seconds.

The experimental value is 10 to the power of minus 7 seconds to 10 to the power of 17 seconds, where 10 to the power of minus 7 seconds is less than a micro second, a tenth of a micro second, that is the lifetime of the alpha particle, measured lifetime of the alpha particle, and 10 to the power of 17 seconds is about 10 to the power of 10 years that is it is about a billion years.

And this huge range is explained and I had a slide, I am getting mixed up with all my slides.

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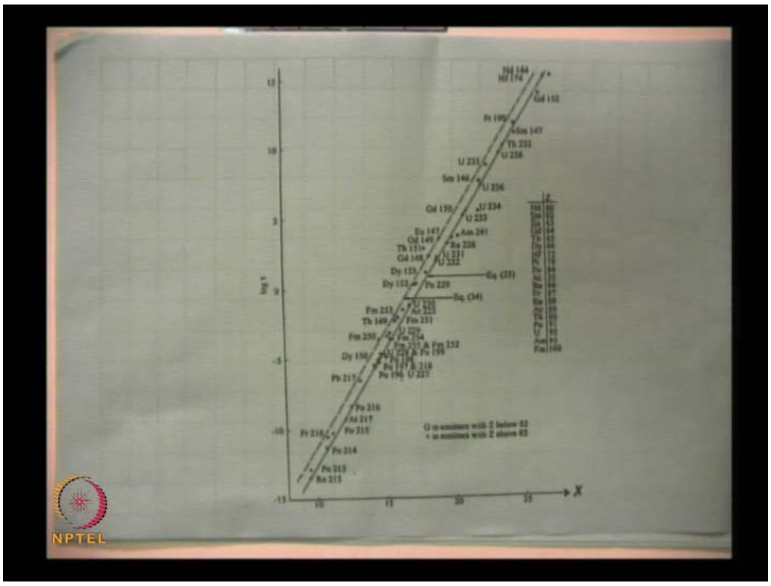

This is the experimental variation of the lifetime that tau is measured in years, the vertical scale. This is also given in my book that tau is measured in years and it goes from log tau is from minus 10 to plus 10, something like a fraction of a second to billions of years.

And this is what our approximate theory predicts, although it is off by a factor, but this huge variation in the lifetime from a few micro seconds to a billion years that is predicted by the WKB approximation. So, undoubtedly the JWKB approximation is a very crude model, a very approximate model that it does reflect the qualitative behavior that is experimentally observed the lifetime of the alpha (( )) alpha meter problems, varies from a micro second to millions of years. That kind of variation is also predicted by WKB theory.

However, it does give a slightly inaccurate result because of the fact that we have assumed the alpha particle to be free inside the nucleus and WKB formula itself, is not rigorously correct; is an approximate formula. Having said that we in the next lecture what we will do is we will start with a new another approximate method, mainly the Perturbation theory, but before that what we will do is that we will justify the connection formula that we have used in our WKB approximation.

Thank You.

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