Basic Quantum Mechanics

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Module No.# 01

Introduction and Mathematical Preliminary

Lecture No. #02

Basic Quantum Mechanics: The Schrodinger Equation and The Dirac Delta Function

In this lecture, we will continue our discussions on wave particle duality and will give a very heuristic derivation of the Schrodinger equation. And then, we will discuss the Dirac delta function and Fourier transform that will be a bit of mathematics, butthat is necessaryto understand the solutions of the Schrodinger equation.

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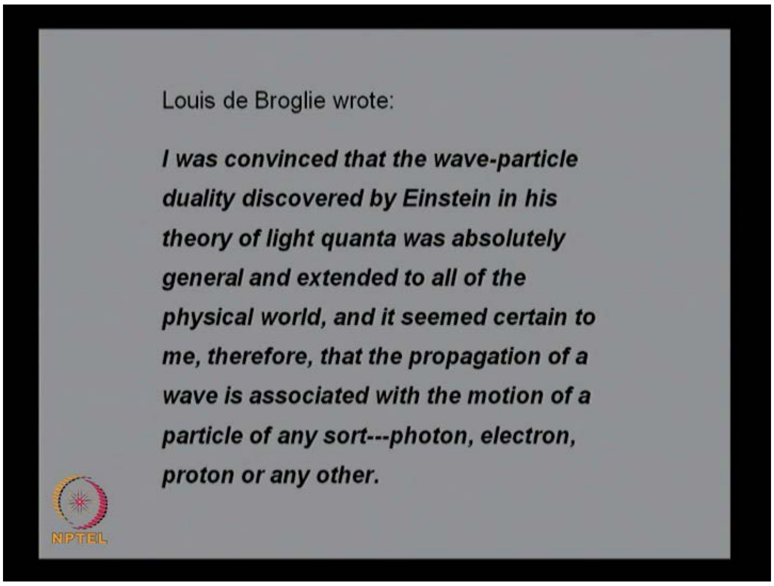

So, this is the second talk on quantum mechanics and as i mentioned in my previous talk de Broglie wrote that, Iwas convinced that the wave particle duality discovered by Einstein in his theory of light quanta was absolutely general and extended to all of the physical world, and it seemed certain to me that therefore, the propagation of a wave is associated with the motion of a particle of any sort photon, electron, proton or any other.

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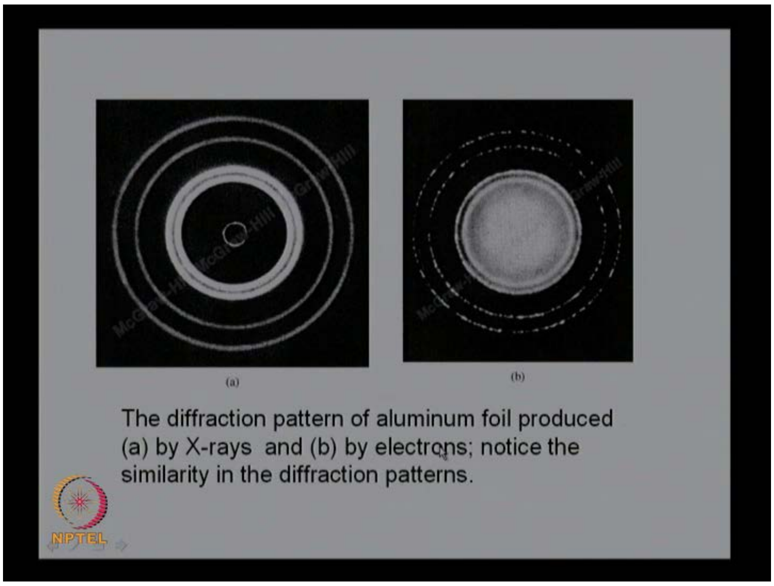

Now actually after de Broglie wrote this, the experiment, the experiments, the diffraction pattern by electrons were observed much later. So that is why de Broglie's contribution is considered to be outstanding. He predicted wave nature of electrons; he said that it could not just p for protons; it would be for electrons and protons and neutrons or whatever, you can think or alpha particles or anything that you can think of. And it was only later, he made this prediction around 1923 or 22 and the experiments were carried out only in 1926 or 27I think the famous diffraction experiments of electron.

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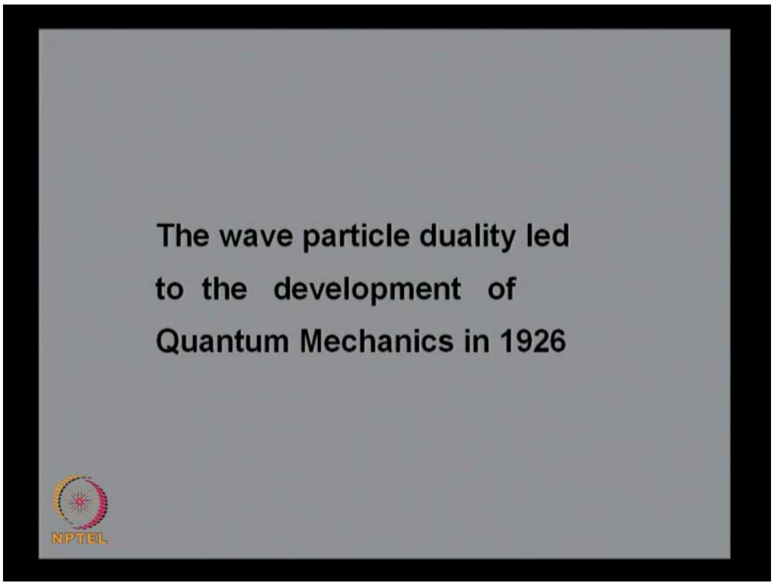

So, the wave particle duality led to the development of quantum mechanics in 1926.

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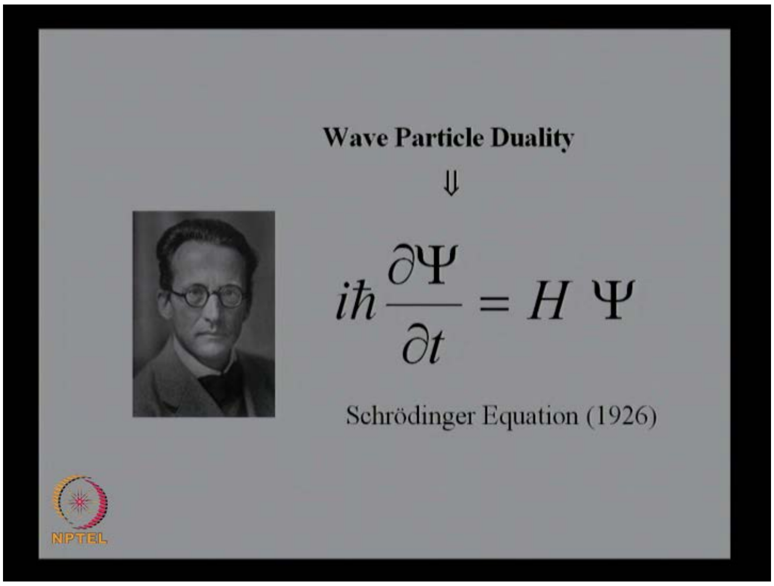

It lead to the famous Schrodinger equation andthe Schrodinger equation is given byi h cross delta psi by delta t is equal to H psi. So, let me tell you right in the beginning, if you ask me the question that, what is an electron? What is a proton? Is it a particle or a wave? Some people would answer that it is both; that answer is not quite correct. The correct answer is that it is neither. So the electron or the proton is neither a wave nor a particle; it is described by a wave function psi which is asolution of this equation.

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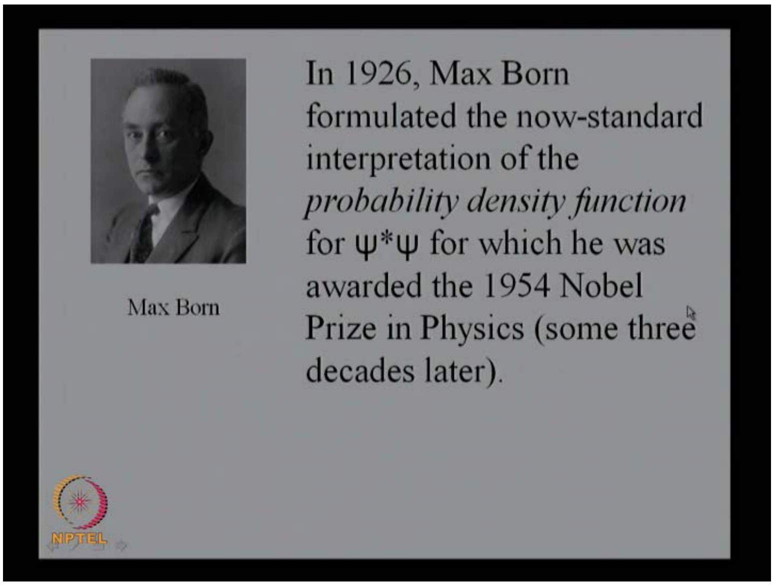

In 1926 itself, Max Born formulated the new standard interpretation of the probability density function for psi star psi for which, he was awarded the 1954Nobel Prize in Physics.

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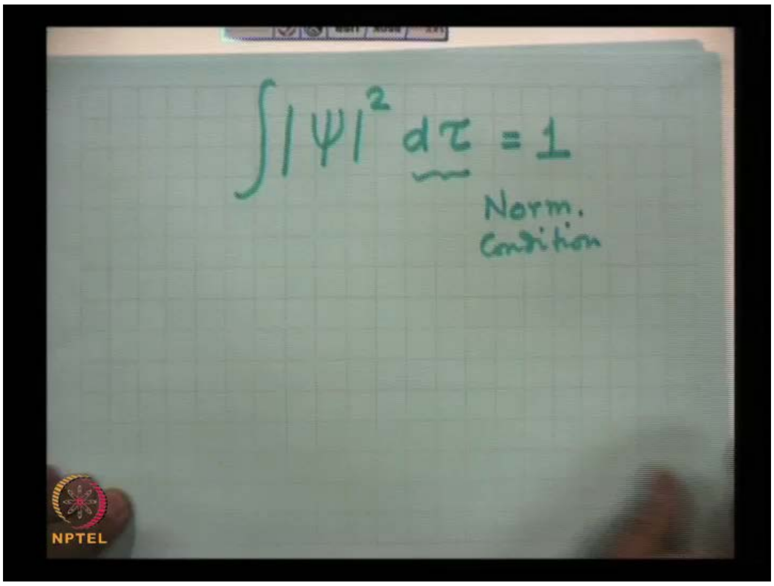

So mod psi square d tau mod psi square d tau represents the volume, the probability of finding the particle in the volume element d tau, and since the particle has to be found out somewhere, so the total integral, the total probability must be equal to 1. This condition is known as the normalization condition. Anyway we will discuss that, butfirst I would like to give you a heuristic derivation of the Schrodinger equation.

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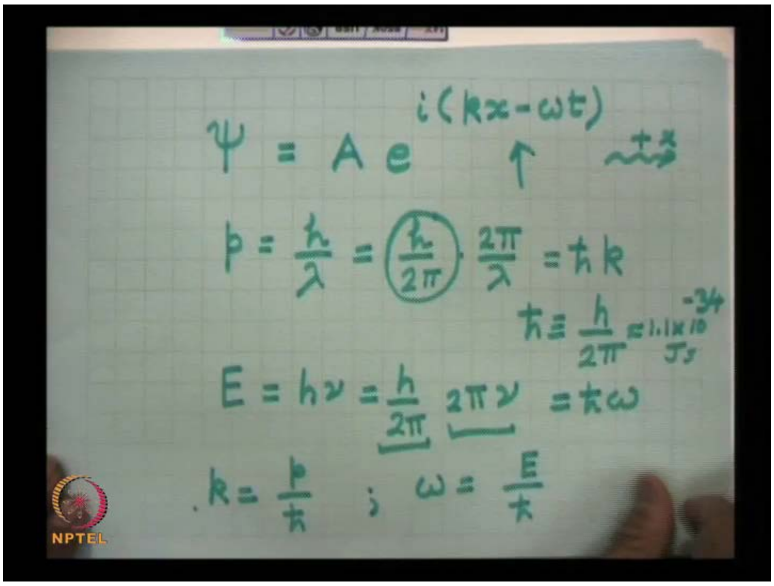

Now, let me take a plain wave; a plain wave is described by a wave function psi a one dimensional plain wave. So, you have A into e to the power of i kx minus omega t so this is a plain wave propagating in the plus x direction. Now in this equation, Isomehow inject the wave particle duality. So as was put forward by de Broglie that the momentum of a particle is related to the wave length by hby lambda. This equation is known as the de Broglie relation. So Idivide and multiply by 2 pi; so h by 2 pi into 2 pi by lambda; this quantity is defined as h cross. So this is h cross k, where h cross is defined to be equal to the plank’s constant divided by 2pi.

And it has a value about 1.1 something like that into 10 to the power of minus 34 joule second something like that. Then in this equation, we write the Einstein’s equation which is h; E is equal to h mu; once again Idivide and multiply by 2pi; so this is h by 2 pi into 2 pi mu. So this is h cross and this is omega; so this is h cross omega. So I get k is equal top by h cross and omega is equal to E by h cross and Isubstitute this in this expression.

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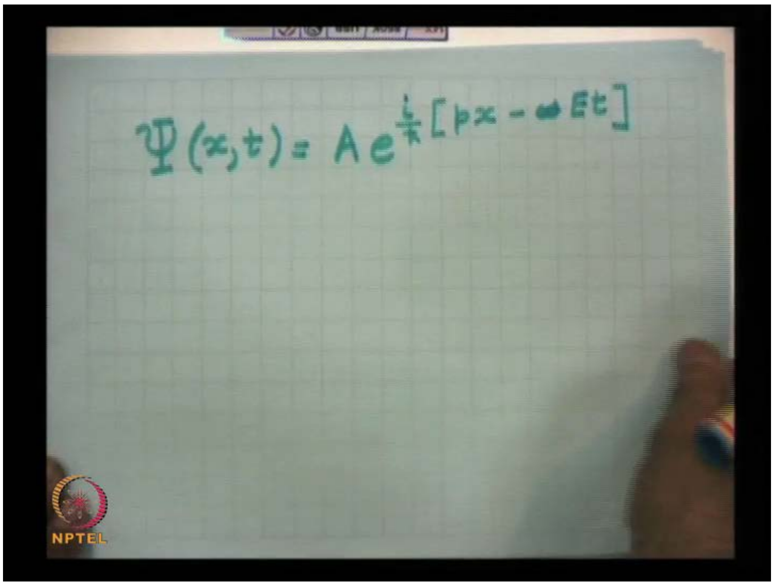

Therefore, I rewrite a one dimensional wave, psi x t is equal to A into E to the power of i k x; k will be p by h cross; so i by h cross px minus omega t sorry minus Et;let me do this again.

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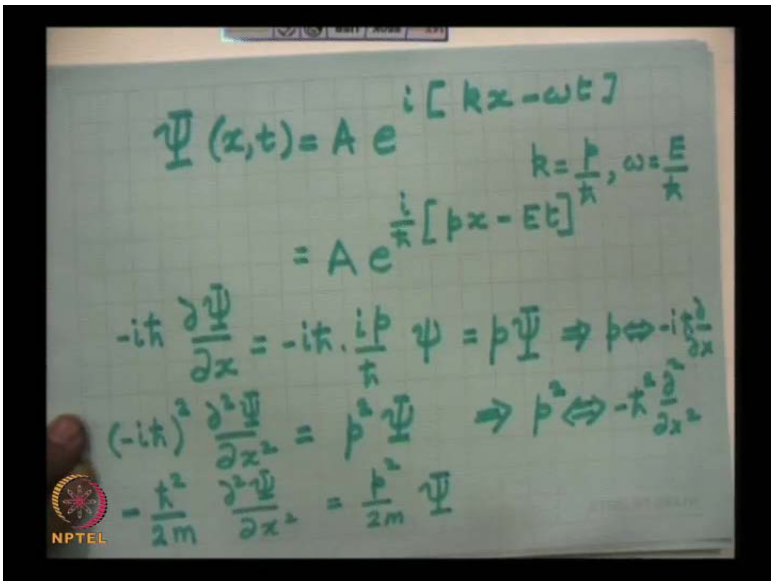

We have just now said that, psi of x t p one dimensional plain wave is equal toA e to the power of i kx minus omega t. So then we showed that, k is equal to p by h cross and omega is equal to E by h cross; so i substitute this here and I get, A e to the power of i by h cross px minus Et then, Idifferentiate partially differentiate with respect to x and multiplied by minusi h cross. So minusi h cross delta psi by delta x, this is equal to minusi h cross and if Idifferentiate this with respect to x I will get, i p by h cross; so multiplied by i p by h cross times the whole thing therefore, this is psi.

So h cross h cross cancel out; i times i is minus 1; so minus minus is plus 1; so this is p psi. In fact, this relation suggests if momentum operator for p because, p is associated therefore, p operating on psi is the same as p is associated with minusi h cross delta by delta x. This is not rigorous, butwe will write this down there, and we will discuss more little later. Idifferentiate this again, so I get minusi h cross times minusi h cross that is whole square; again differentiate so, delta 2 psi by delta x square.

So Imultiply by minush cross and if Idifferentiate this then again, I will get minusi h cross intoip by h cross and therefore, a little algebra will show that, this becomes p square psi. So this suggests that, p square is equivalent to the operator minus h crosses square delta 2 by delta x square. So this quantity minus minus is plus; i square is minus 1; so I get minus h crosses square and Idivide by say, the mass of the particle. Say mass of the particle is m let us suppose, minus h crosses square by 2m delta 2 psi by delta x squareis equal to p square by 2m into psi. Now let me write down the big function once again.

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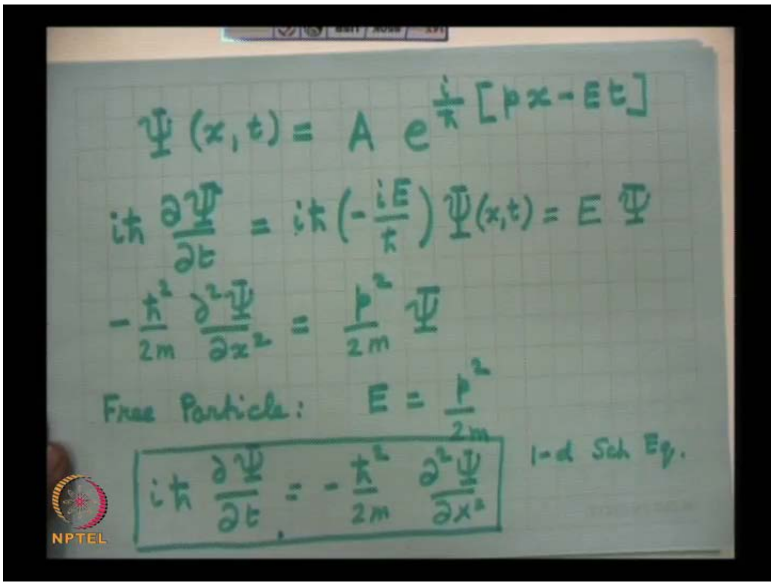

So I have psi of x t x comma tis equal to A to the power of i by h cross px minus Et. Now, Ipartially differentiate with respect to time and Iwritei h cross delta psi by delta t and Iobtaini h cross comes from here and then, if Idifferentiate with respect to time then, I will get minusi E by h cross and the whole thing again. So this will be times psi; psi as a function of x comma t so i times i is minus 1; minus 1 into minus is plus; h cross h cross cancels out so you get E psi.

Just now I have derived another equation which is minus h crosses square by 2m delta 2 psi by delta x square is equal to p square by 2m psi. Now for a free particle for a free particle, the total energy is thekinetic energy classically. So, you have E is equal to p square by 2m; so this side is equal to this side; so this must be equal to this; soi h cross delta psi by delta t is equal to minus h crosses square by 2m delta 2 psi by delta x square. This is known as the one dimensional Schrodinger equationSchrodinger equation for a free particle. Now the derivation that I have given is far from rigorous, butonce I have derived this equation, I will study its solution. So,let me do one more thing that I have derived two equations.

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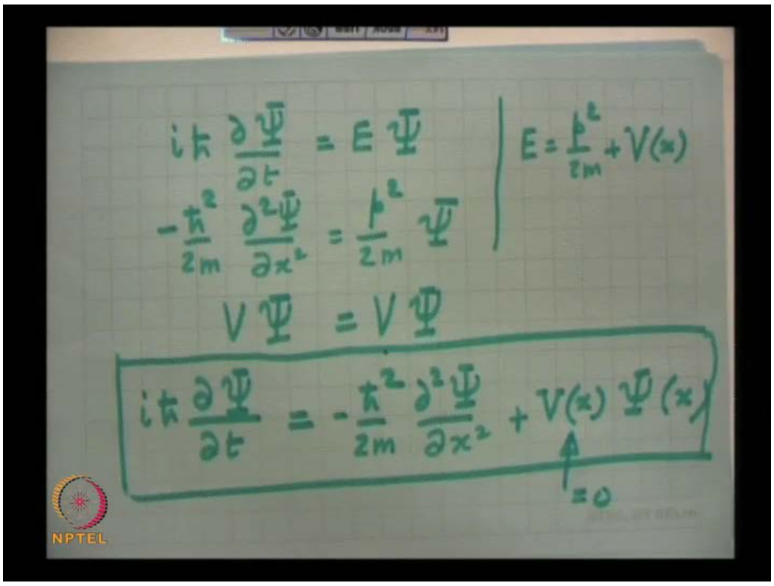

First Iderivedi h cross delta psi by delta tis equal to E psi, I have derived this then earlier, I have derived minus h cross square by 2m delta 2 psi by delta x square is equal to p square by 2m into psi. Now let us suppose, the particle is in the potential field;soyou have the total energy is the kinetic energy plus the potential energy. So, I write this equation V psi is equal to V psi; V is the potential energy. So, E is equal to p square by 2m plus V;so this must be equal to this plus this. So, this is the rigorously correctSchrodinger equation, one dimensional Schrodinger equation for a particle in a potential fielddescribed by V of x delta 2 psi by delta x square plus the potential is so much and psi of x.

So, this is the Schrodinger equation for a free particle for a particle in a potential field, when this is 0, when it is a free particle then this is 0 then, you have only these two terms for the Schrodinger equation.

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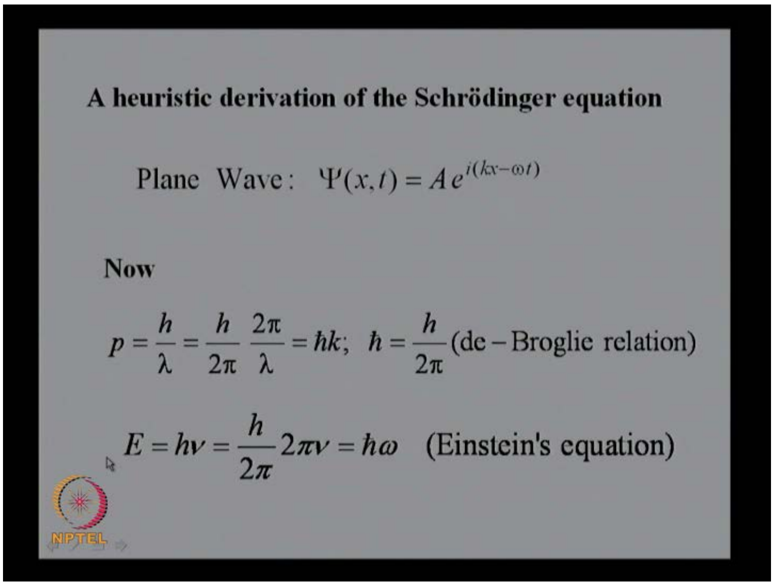

So let me go backto my slides andlet me give you, I will repeat the heuristic derivation of the Schrodinger equation. So Iwrite a plane wave propagating in the plus x direction as A into eto the power of i kx minus omega tNow, p is equal to this is the de Broglie equation h by lambda. So Imultiply by 2 pi and divide by 2 pi; so this quantity is the h cross; and this quantity is the width vector k; so this is h cross is known as the plank’s constant and E is equal to h nu; Idivide and multiply by 2 pi and I get h cross omega. So this is the known as the Einstein equation.

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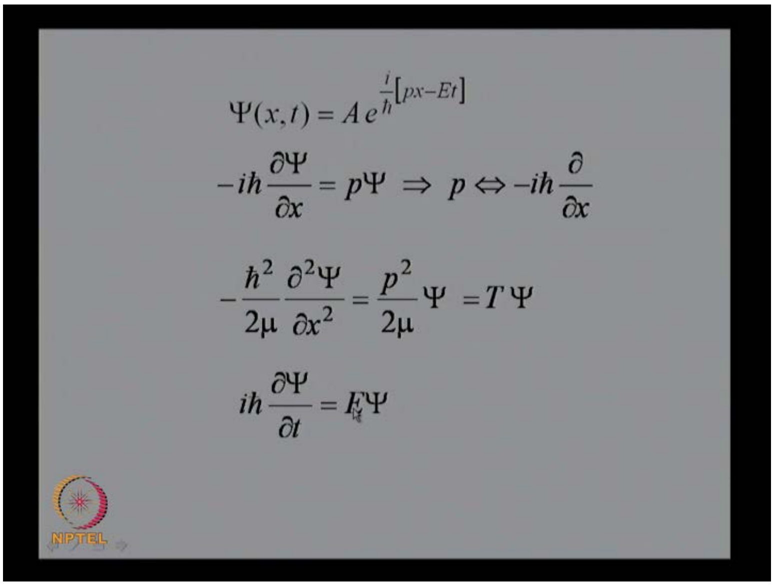

So Isubstitute p and E in this equation; that is for kyou will write p by h cross; for omega I will write E by h cross; so I get this. I differentiate with respect to partially respect to x multiply by minusi h cross and I will get minusi h cross times i by h cross p so that will become just p times i. So, p can be associated with the operator minus i h cross; p psi is equal to minusi h cross delta psi by delta x. Idifferentiate it again, here I amsorry, I have written the mass as mu minus h crosses square by 2 mu delta 2 psi by delta x square and Ip square when I when Idifferentiate this twice. So I will get i by h cross whole square p square; so h crosses square h crosses squarewill cancel out, p square psi which is the kinetic energy times; psi p square by 2mu is the kinetic energy and if Idifferentiate with respect to time, you will get E psi. So for a free particle E is equal to p square by 2mu; so this quantity should be equal to this quantity.

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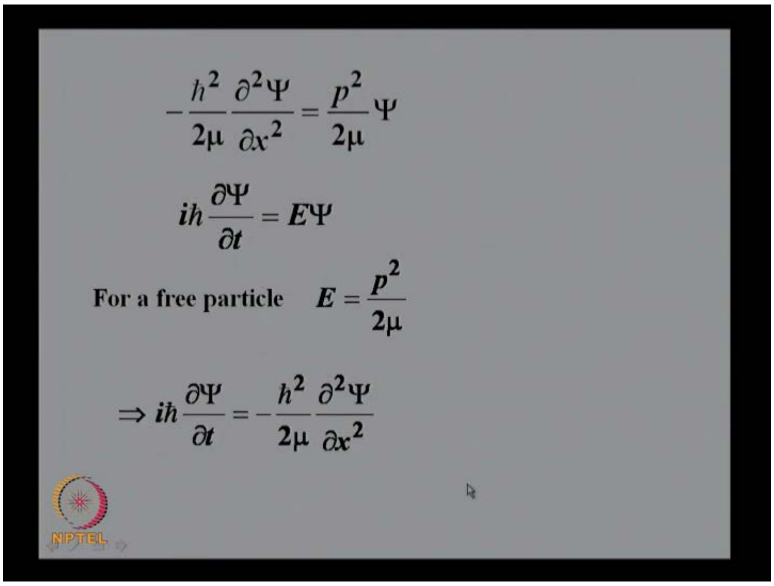

So for a free particle E is equal to p square by 2 mu; so I get the one dimensional Schrodinger equation for a free particle.

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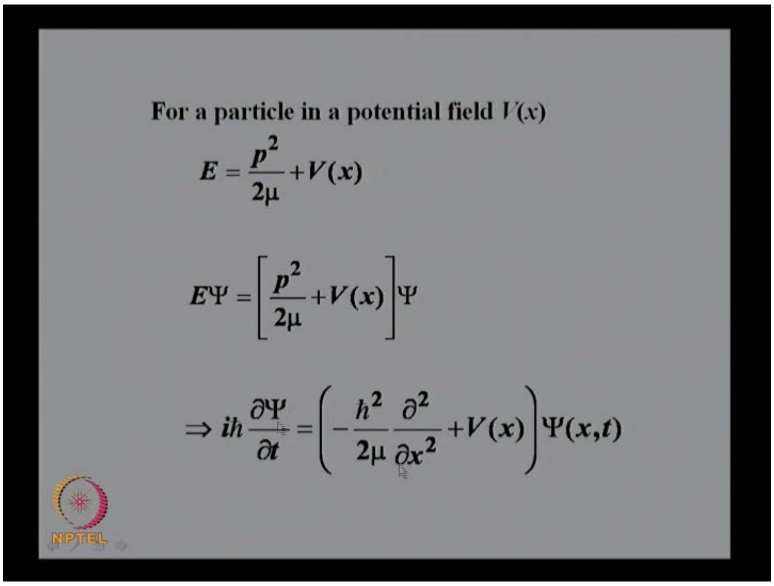

For a particle in a potential field V of x, E is equal to p square by 2 mu plus V of x; so psi is equal to p square by 2 mu plus V of x and then you get this. So this is rigorously correct one dimensional time dependent Schrodinger equation for a particle in a potential field V of x.

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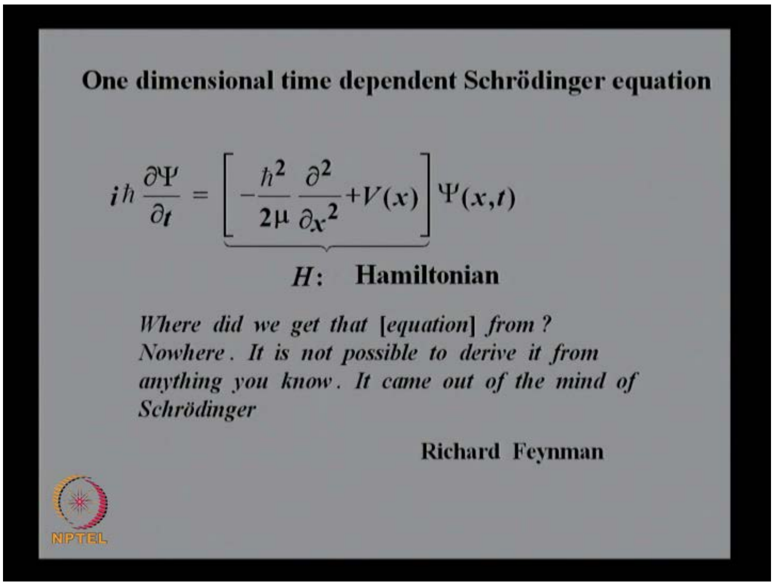

So one writes the thisquantity is the operated this is p square by 2 mu; so this is the operator corresponding to the total energy; so that is known as the Hamiltonian. Richard Feynman, as you all know his noble one of the outstanding physicist of the 20th century. He writes, where did we get that equation from and he says nowhere. It is not possible to derive it from anything you know; it came out of the mind of Schrodinger. So I have given a very heuristic derivation which lacks rigger. Somehow, I have been able to reach the Schrödinger equation and then, I will try to getresults by solving the Schrödinger equation.

This equation was first obtained by Erwin Schrödinger in 1926, and I will obtain the solutions and then we will find that this compares very well with experimental data. So that is the success of quantum mechanics. That is the success of the Schrödinger equations for which Schrödinger got the noble prize I think in 1933. So, we have obtained the Schrodinger equation. Now, we will use the Schrödinger equation to study its solution. However, we will digress here for a moment and we will do a little bit of mathematics and in this mathematics, we will try todefine what is Dirac delta function, and we will also discuss what we mean by the furrier transform of a function.

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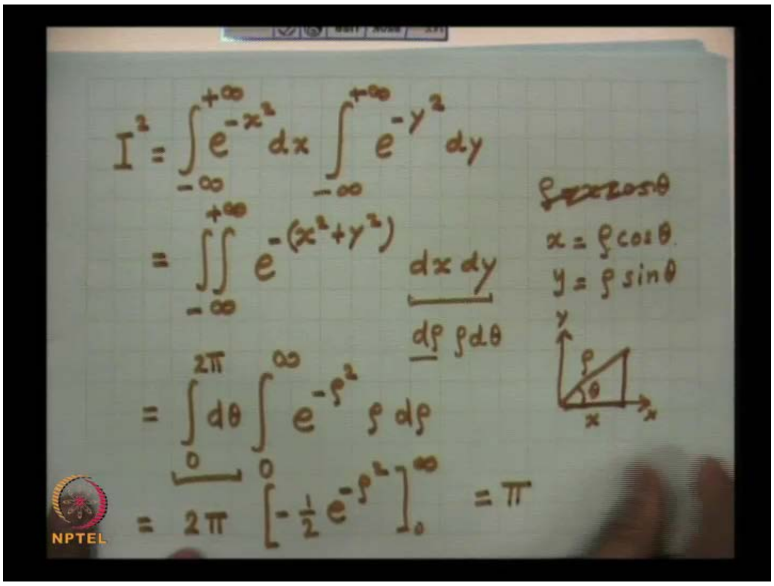

Even before that, let me mention thatthis integer eto the power of minus x square dx. Let me evaluate this integral; so from minus infinity to plus infinity. This integral let us suppose, Iwrite as I. So you will have Isquare will be this times this. So again the same integral since it is a definite integral, I will write this as eto the power of minus y square dy. So this is over the entire x y plane; so this will be minus infinity to plus infinity e to the power of minus x square plus y square dx dy over the entire x y plane from minus infinity to plus infinity. This integral is very easy to obtain. So you will have, you define the polar coordinates rho is equal to xcostheta and and I am sorry I am sorry x is equal to rhocostheta and y is equal to rho sin theta.

So you have here, this as the x and y coordinatesand these are the polar coordinates. So this is rho; so this angle is theta; so this is rhocostheta is x and rho sin theta is y. The area element in this plain as you all know will be d rho times rho d theta, and x square plus y square is equal to rho squarecossquare theta plus rho square sin square theta, that is just rho square. So I will have rho is of course positive; so d theta integral is from 0 to 2 pi and the rho integral is from 0 to infinity eto the power of minus rho square rho d rho. This integral is trivial;this is 2pi and this is half into eto the power of minus rho square with a minus sign with the limits 0 to infinity. At infinity this is 0; at 0 it is half; so this comes out to be pi.

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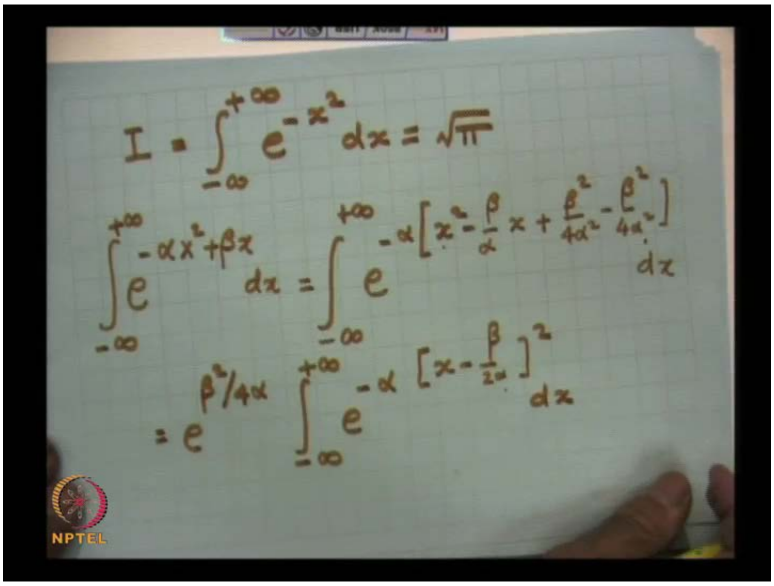

So, Iobtain the result that Iis equal to;this is a very important integral which Iwould like all of you to remember, and we will use this very often eto the power of minus x square dx is equal to under root of pi; Isquare was pi; so Isquare Iis under root of pi. Using this we can now evaluate this integral. So let us suppose, I have this integral eto the power ofminus alpha x square plus beta x dx;so this is from minus infinity to plus infinity. What you do is as most of you may be already be aware of you form a whole square of it; so you get minus infinity to plus infinity eto the power of minus alpha and then, you get x square minus beta by alpha into x and then, if you want to make a whole square, you have to add and subtract beta by 2 alpha whole square.

So beta square by 4alpha square and then minus beta square by 4 alpha square multiplied by dx. So this I cantake outside, we see beta square by 4 alpha square minus sign into alpha that is beta square by 4alpha. So this becomes eto the power of beta square by 4 alpha and this becomes eto the power of minus alpha x x minus beta by 2 alpha whole square into dx. So from minus infinity to plus infinity, Idefine this as a z variable.

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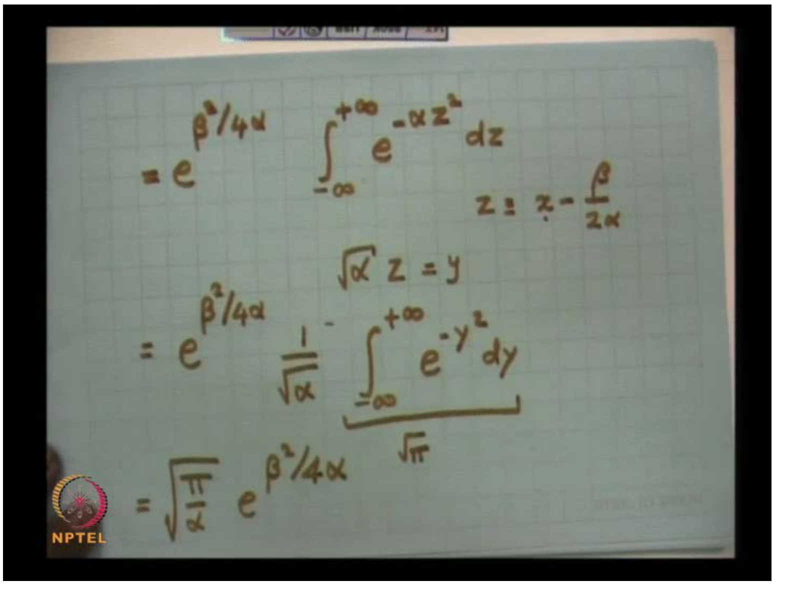

So Ifind that this integral becomes eto the power of beta square by 4 alpha4 alpha, and eto the power of minus infinity to plus infinity eto the power of alpha sayz square into dz where, z is define to be equal to x minus beta by 2alpha so that dx is equal to dz. And in order to evaluate that, you write down alpha under root of alpha times z is equal to y; so you get dz is equal to dy by under root of alpha; so you get eto the power f beta square by 4 alpha; so 1 over root alpha1 over root alpha and minus infinity to plus infinity eto the power of minus y square dy.

Now, this we are just evaluated; so this is square root of pi;so this quantity becomes equal to square root of pi by alpha eto the power of beta square by 4alpha. This is a very important integral; Iwould request all of you to remember this.

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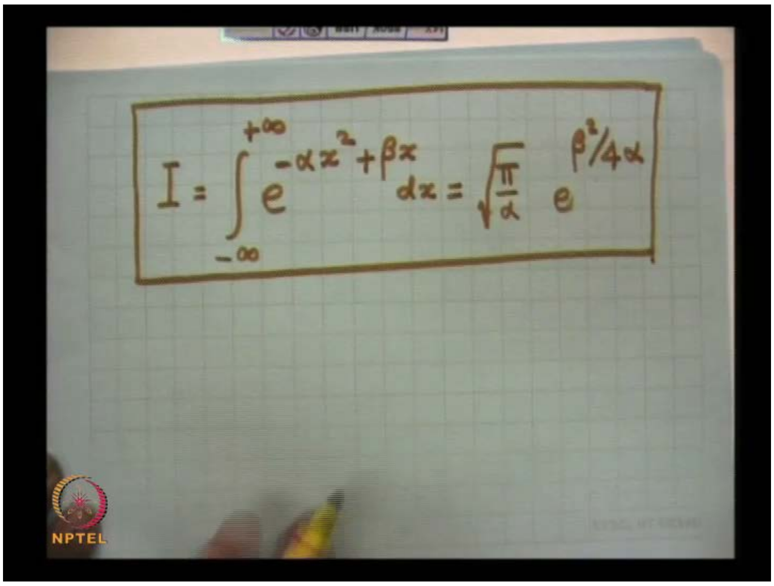

So let me write this down once again that, the integral Iis defined as integral from minus infinity to plus infinity eto the power of minus alpha x square plus beta x dx is equal to under root of pi by alpha into exponential of beta square by 4alpha. This Iwould request all of you to remember this. Now having done this let me now go to the definitionof the Dirac delta function.

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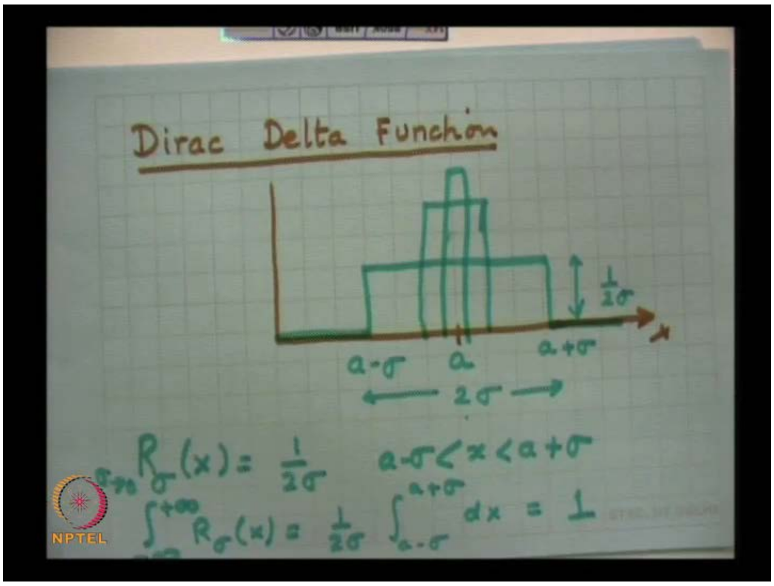

So, we will now discuss the Dirac delta function and the simplest representation of this Dirac delta function is like this. Let me consider a point a on the X axis; this is the X axis and Iconsider a rectangle; Iconsider a function please see this that which is which is 0 for like this and then, it becomes a rectangle function 1, 2, 3. So this is the point a; this is the point a minus sigma; and this is the pointa plus sigma. So that this line is 2 sigma and Iassume that the height of this function is 1over 2 sigma. So that the area under this rectangle is 1 over 2 sigma multiplied by 2 sigma; so the area is 1 independent of sigma.

So, Idefine a function R sigma of x which is equal to 1 over 2 sigma for x lying between a plus sigma and a minus sigma and 0 everywhere else so that the integral from minus infinity to plus infinity R sigma of x is equal to 1 over 2 sigma only from this limit. So this is a minus sigma to a plus sigma dx;so this is equal to 2 sigma; so this is 1. So let us reduce the value of sigma; so let me reduce this, butif I reduce the value of sigma, the height becomes larger and in the limit of sigma tending to 0, this becomes very sharp and very tall. So in the limit of sigma tending to 0, this is one of the representations of the Dirac delta function.

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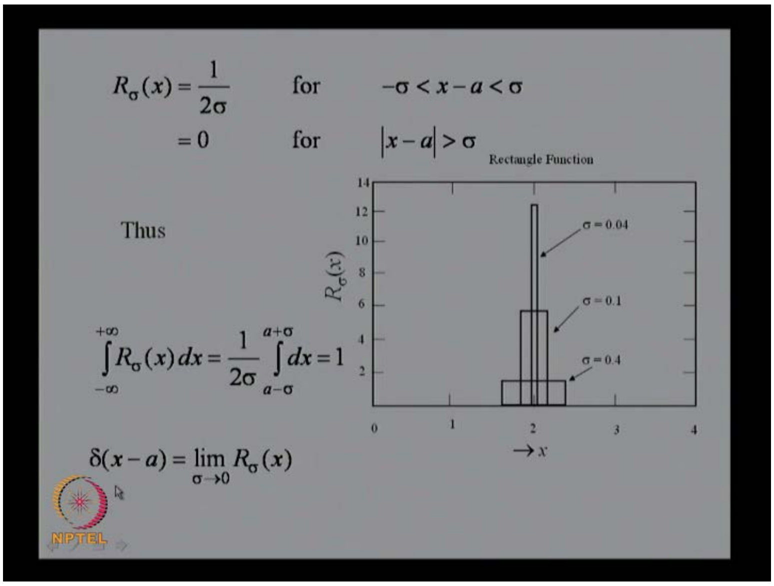

So let me show you; Iconsider a rectangle function which is 1 over 2 sigma for x lying between a minus sigma and a plus sigma and 0 everywhere else, sothat the integral of this function from minus infinity to plus infinity. So let us suppose, it is 0 everywhere and then it is so much and 0 everywhere else. So it goes from I have take a here to be equal to 2;so it is a minus sigma to a plus sigma; so the integral is 1. So now, Ireduce the value of sigma; so here sigma is equal to 0.4; so 1over 2 sigma is 1 over 0.8, it is slightly less than 1, and when you make it point sigma0.1, so this becomes larger. When you become sigma even smaller then, the width becomes smaller,the height becomes larger, butin each of the three cases, the area under the curve is unity.

If the limit of sigma tending to 0, we have a very narrow function and very tall function. The area under the curve is 1. That is one representation of the Dirac delta function. One of the many representations of the Dirac delta function.

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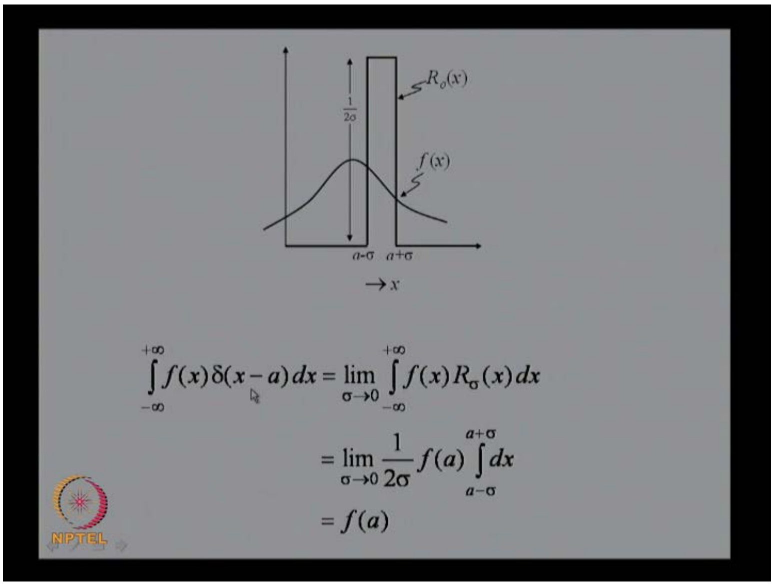

So, let me consider R sigma of x when sigma is small, so that the it is very tall and this width is very small. Imultiply this function by an arbitrary function of x. Now, the product of this functionand the rectangle function in this region 0; product of two numbers is 0 if one of the numbers is 0. So the product of f of x R sigma x is 0 accepting in the domain a minus sigma to a plus sigma. Why? Because R sigma of x is 0 here so therefore, f of x delta of x of minus a dx is equal to limit as sigma tends to 0 f of x R sigma of x. In this tiny interval f of x can be assumed to be constant because, this interval can be made as small as you like, and in this interval R sigma as a value of 1 over 2 sigma and when Iintegrate this, it becomes 2 sigma;so this becomes f of a; so this is theproperty. For any well behaved function f of x, if Imultiply by delta of x minus a and carry out the integration, I will get f of a.

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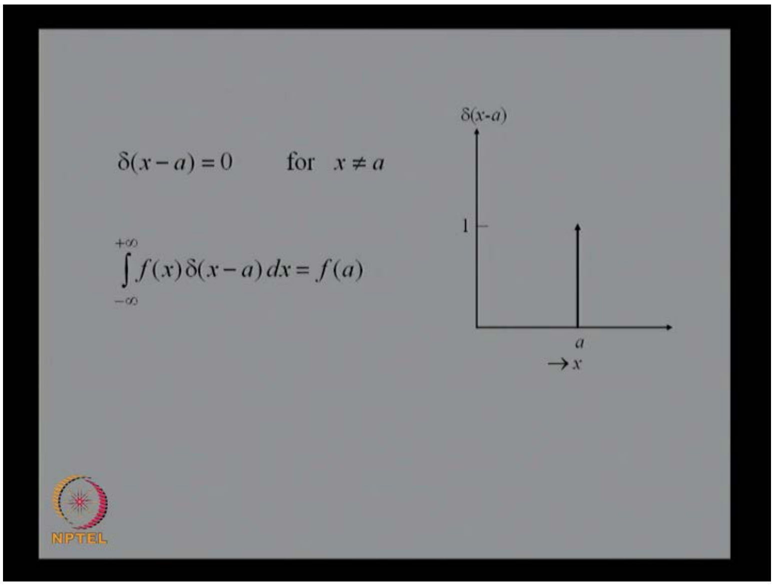

So this is the definition of the Dirac delta function. It is shown as a spike, an arrow of unit height at x is equal to a, butthe area under the curve is 1. Actually, it is infinitely thin and infinitely tall so that the area under the curve is 1, butit is shown by an arrow usually shown by an arrow; it is like an impulse and has the property that any function, any arbitrary function multiplied by delta of x minus a dx is equal to f of a.

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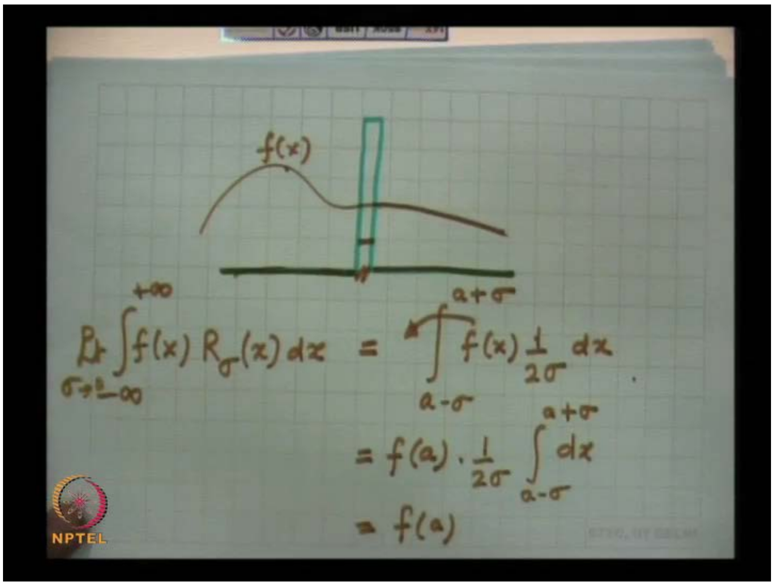

Let me do this once again; that you see what I havetry to tell you is that, I have a very tall function very tall function, and Ithis is 0 here this is 0 here. Now, Iconsider another function; this is the rectangle function. I have another function like this; this is f of x which is smoothly varying. Now the product of this function and this function in this region is 0because, this is 0 the product of f of x and this function is also 0. So the integral f of x R sigma of x dx from minus infinity to plus infinity is really between this limit and this limit because, the product is 0 everywhere else. So this isactually from a minus sigma to a plus sigma f of x R sigma of x is 1 over 2 sigma dx,butif this interval is very small then, f of x does not vary too much in that variable. So I can take it out of the integral and Iwrite this; this is no more approximate. If sigma is very small, I can choose as small as Ilike. So this becomes f of a into 2 sigma 1 over 2 sigma dx integral; so this is equal to a minus sigma to a plus sigma;so this is 2 sigma; so 2 sigma2 sigma cancel out and you get f of a. So this in the limit of sigma tending to 0

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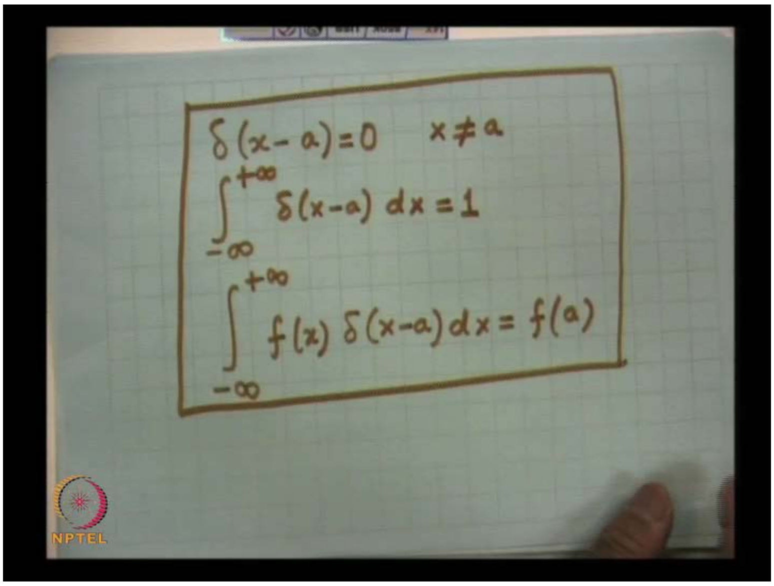

So I get the very importantresult thatI get the very important result that delta of x minus a is 0 everywhere else excepting at x is equal to awhere it is in infinite value such that, minus infinity to plus infinity delta of x minus a dx is equal to 1 and then, I have that minus infinity to plus infinity of a well defined function, well behave function f of x into delta of x minus a because, the product is 0 everywhere excepting at x equal to a dx. This is equal to f of x. These three equationsdefinethe Dirac delta function, butwhat I have given you is just one of the representations of the Dirac delta function.

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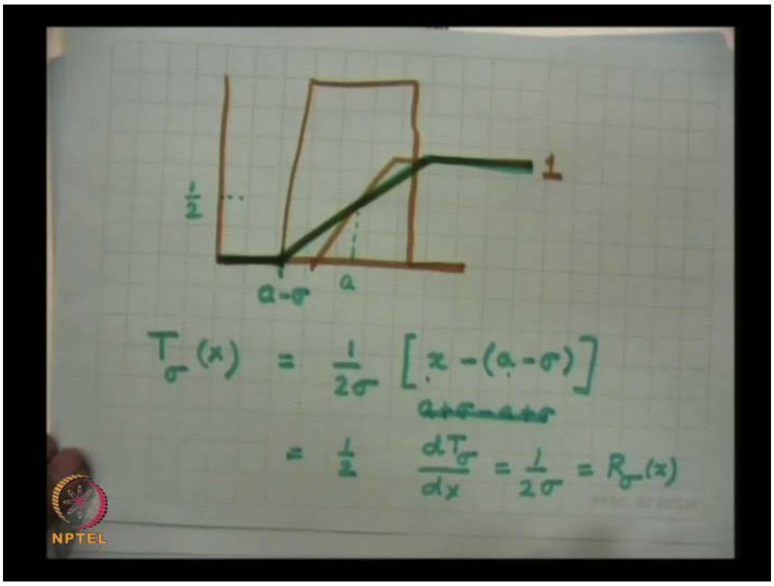

Let me give you one more definition. I have here let us suppose, a ramp function ramp function. Itake saylike this; it has a value 1 here and 0 here. So let me write this down with the blue color. So a ramp function is 0 here then, it increases linearly and then it becomes one here. So this is at x is equal to a, it has a value half. So if the ramp function is defined as T sigma of x then Iwrite that, this is equal to 1 over 2 sigma x minus a minus sigma. So at x is equal to a minus sigma, this is 0. So this is a minus sigma at x is equal to a plus sigma. So this becomes a plus sigma minus a plus sigma 2 sigma 2 sigma cancels out so that is 1, and that at x is equal to a, this is 0; this is sigma by 2 sigma. So at x is equal to a, it has the value equal to half.

So, in this region from between a minus sigma and a plus sigma, T sigma of x has this value. Now, Itake the derivative of this function; so I have this ramp function which is 0 here; it increases linearly here and it becomes one here, if Itake the derivative of this function. So dt sigma by dx,if you see this, if you just differentiate this; this is equal to 1 over 2 sigma. So this is my R sigma of x; this is my; this is the rectangle function. So the derivative of the function is if you see carefully that, in this here it is 0; here it is something like this and here beyond thisit is. So as Imake sigma smaller, it becomes sharper and sharper.

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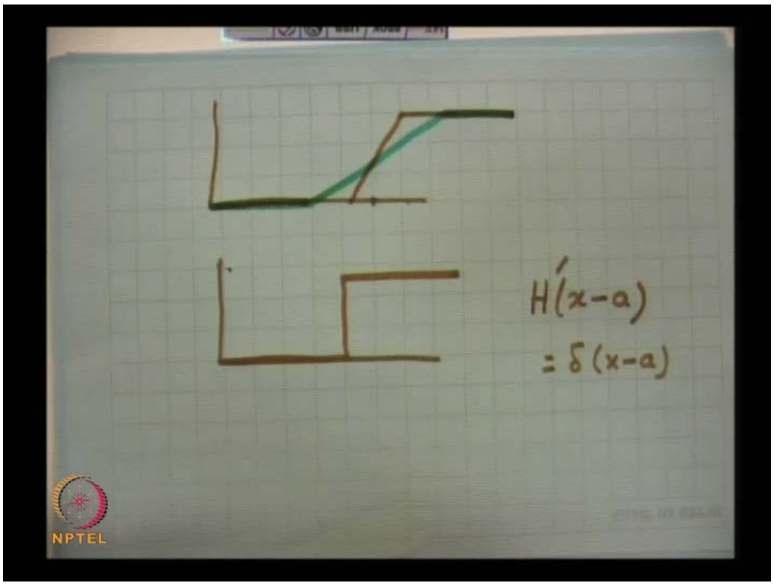

So therefore, if I have a ramp function like this if I have a ramp function like this; this goes like this and then, it goes like this and then it has a one value, then the derivative of this function because derivative is 0 here derivativeis 0here,derivative of here is unity; that this is the rectangle function, and as i make sigma smaller and smaller this becomes sharper and sharper. In the limit of sigma tending to 0, you will have the heavy side unit step function like this. The derivative of thisfunction, this is known as the heavy side unit step function and it is known as, it is described by H of x minus a. The derivative of the unit step function is a Dirac delta function. So H prime of x minus a is equal to delta of x minus a.

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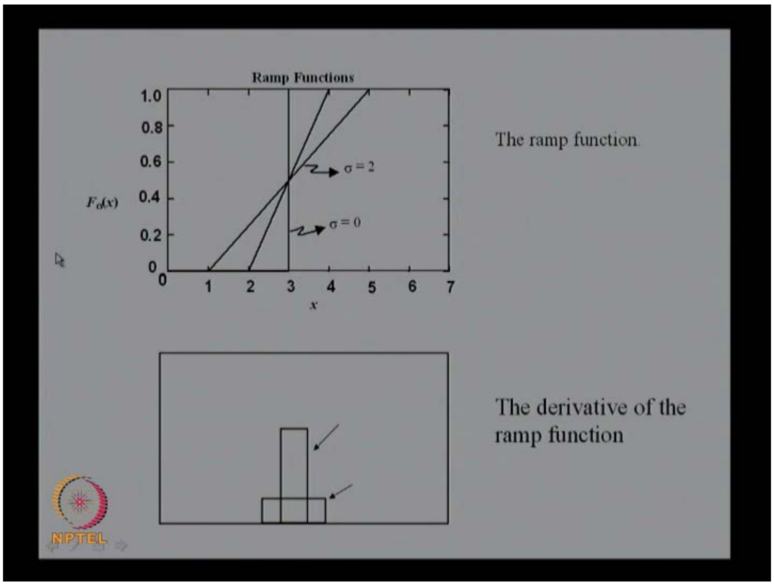

So let me show you in this slides thatthis is the ram function; soit is 0 here and then, it becomes when sigma is large. As you make sigma smaller and smaller this slope of this function becomes larger and larger. In the limit of sigma tending to 0, this becomes a unit stamp step function. I am sorry here I have written as F sigma of x. So this is the derivative of the ramp function and in the limit of sigma tending to 0, it will become a delta function.

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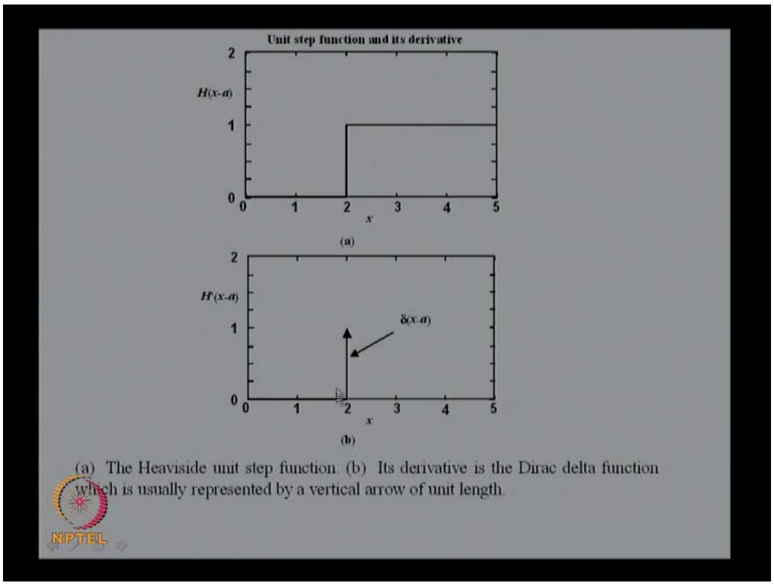

So this is the unit step function. This is known as the heavy side unit step function and its derivative is the Dirac delta function. So whenever you have a discontinuity in a function then, its derivative is a Dirac delta function. Now I will show you by a simple example, and you should be very careful in calculating the derivative.

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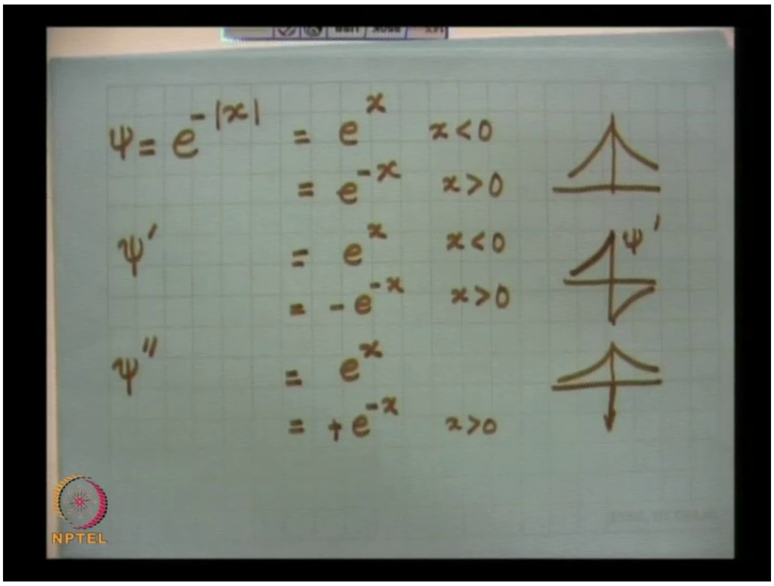

Say, let us consider a function like eto the power of minus mod x. Now eto their power of minus mod x; this means that this is equal to eto the power of plus xfor x less than 0 and eto the power of minus x; so psi is equal to eto the power of minus mod x; minus x for x greater than 0. If you take the derivative psi prime; this is eto the power of x for x less than 0 and minus eto the power of minus x x greater than 0, and if you keep on naivelydifferentiating this, it will become eto the power of x and eto the power of minus minus plus eto the power of minus x for x greater than 0.

So you will find thatthis is eto the power of minus mod x so psi double prime is equal to psi. Now this is incorrect because, this functionlooks like this. So its derivative has a discontinuity thederivative has a discontinuity. Discontinuity of minus 2 units; so the derivative if you approach from the left hand side this is psi prime has a value plus 1 and has a value minus 1, if you approach from the right hand side. So at this point at x is equal to 0, if you plot this, it will be something like this, butthere is a Dirac data function at the origin.

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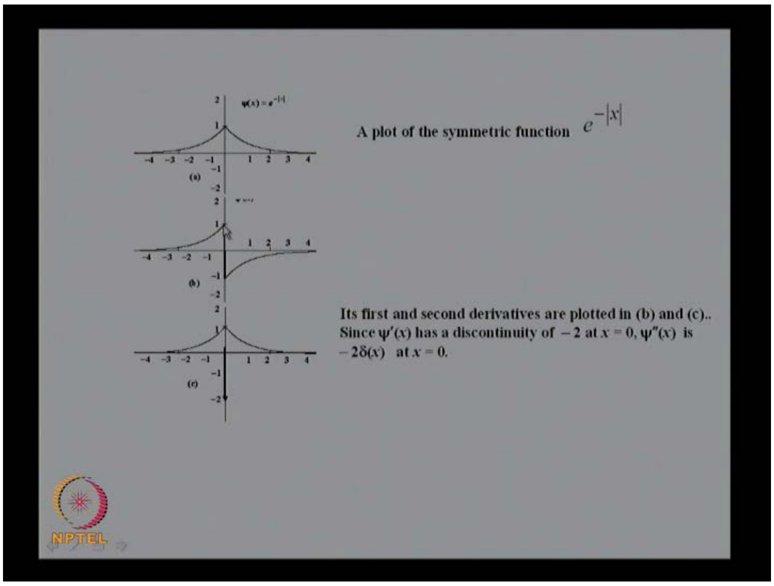

So let me illustrate this. So you have here, eto the power minus mod x, a plot of the symmetric function. Now its derivative has a discontinuity of minus 2 units. If the derivative has thediscontinuity of plus 1 unit then, the derivative then the derivative isif the function has a discontinuity of 1 unit plus then, it has its derivative is the delta function, butif it is minus 2 units so, psi double prime of x at x is equal to 0 is equal to minus 2 times delta x. So you must remember that, whenever a function has a discontinuity at a particular place then, its derivative is always a Dirac delta functionDirac delta functionmultiplied by the amount of discontinuity. If the amount of discontinuity is minus 5 then, derivative will be minus 5 times delta x. So in this particular case,the derivative was minus 2 units.

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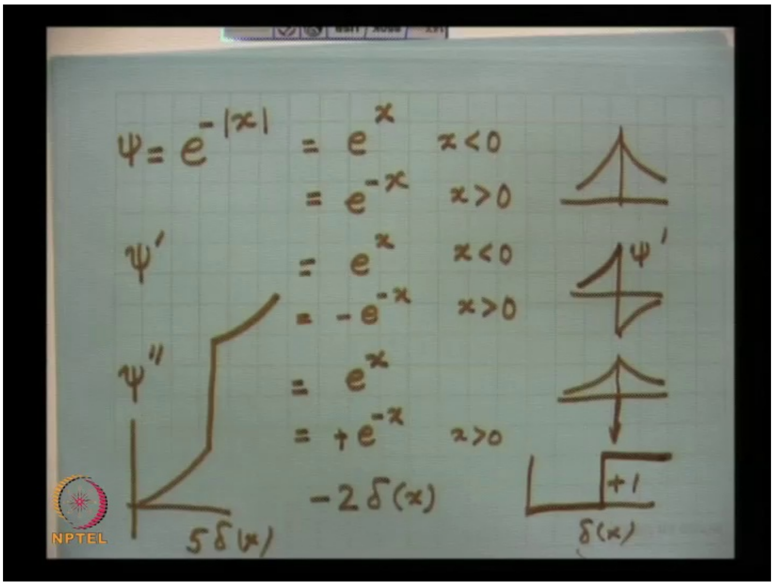

So the derivative at the origin is minus 2 delta x. In the unit step function, the derivative was plus 1 as Igo from, as i increase x the thethe discontinuity is plus 1. So the derivative here is just delta of x. If a function has behaves like this that, you have a function it goes like this and then, it makes a jump of 5 units. So at that point, the derivative will be 5 times delta x.

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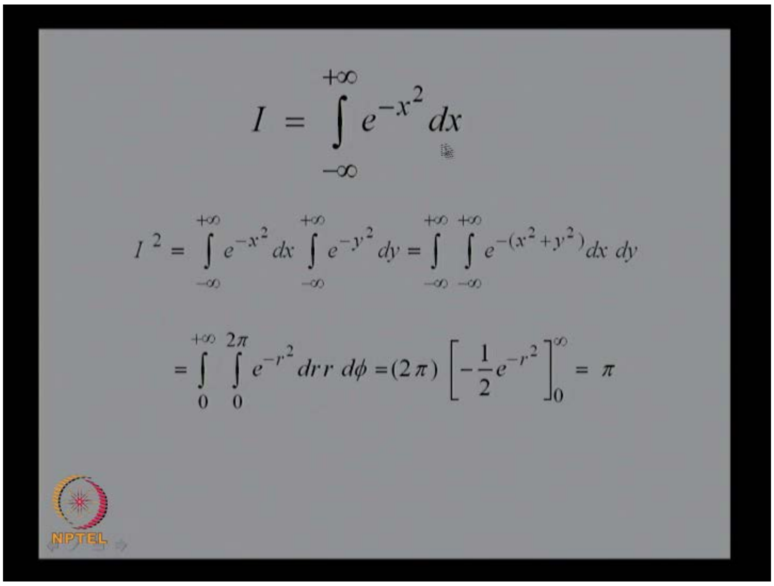

So this is here, I have derived the same equation that which Iderived sometime back at the integral I; I converted into polar coordinates and Iobtain that I square is pi. So I will be equal tounder root of pi.

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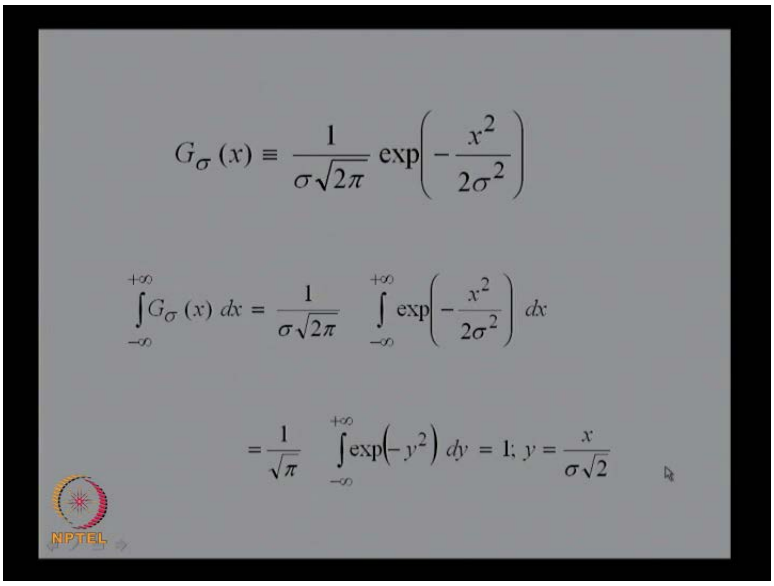

So let me consider the Gaussian function. The Gaussian function, Iwrite it as 1 over sigma under root 2 pi eto the power of minus x square by 2 sigma.

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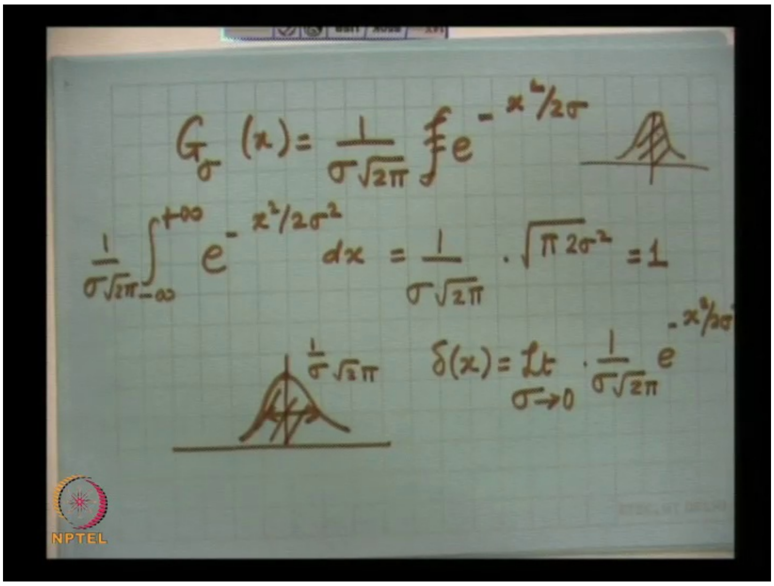

So let me write a Gaussian function like this. G sigma of x is equal to 1 over sigma under root of 2 pi eto the power of minus x square by 2 sigma square sorry sorry. There is no integral sign here. So it isait is a Dumbbell function like this; it is a Gaussian function peak at x is equal to 0. The integral of this function minus infinity to plus infinity, I can takeI cantake sigma under root of 2 pi outside. So the integral of this function is eto the power of minus x squareby 2 square dx.

So, if we use the formula; so this will be 1 over sigma under root of 2pi multiply by pi by alpha. So under root of pi alpha is equal to 1over 2 sigma square; so this is 2 sigma square; so this cancels out; so this will be 1. So, the areas under the curve is always 1 irrespective of the value of sigma. So let me plot this at x is equal to a, the width of the function at x is equal to say 0, the value is 1 over sigma. So the value of the Gaussian function at x is equal to 0 is about 1 over sigma so actually, 1 over sigma under root of to pi at the width of thefunction is about sigma.

So, as Imake the sigma smaller, the width becomes smaller, the peak becomes sharper, butthe area under the curve remains the same. So therefore, this is yet another representation of the Dirac delta function that delta of x is equal to limit of sigma tending to 0 1 over sigma under root of 2 pi eto the power of minus x square by 2 sigma square. This is known as the Gaussian representation of the Dirac delta function.

So Icome back to my slide and I have hereG sigma of x; Idefine as when I have three equal to signsthat means, one overthis is defined to be equaled to that. At this factor is such that, the integral under the curve is 1. So this is obvious, this is Itake the integral under the curve. So Iput y is equal to x pi sigma under root of 2. So this becomes very simple integration with the formula that Ihad given you earlier and this integral is one irrespective of the value of sigma.

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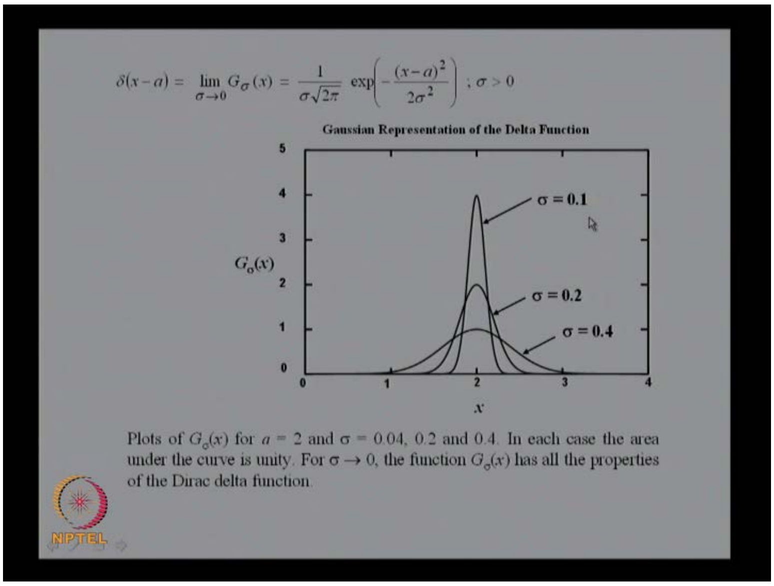

So Iplot this;so as the value of sigma become smaller and smaller, it becomes sharper and sharper, the width become smaller, butthe value at x is equal to 2 or if I have a displaced Gaussian, here I have taken a is equal to 2;this corresponds to sigma is equal to 0.04, 0.4; this corresponds to0.2,0.1. If you make even smaller 0.01, it will become taller and sharper. So for sigma tending to 0, the function G sigma of x has all the properties of the Dirac delta function. So this equation at the top is yet is the own as the Gaussian representation of the of the Dirac delta function.

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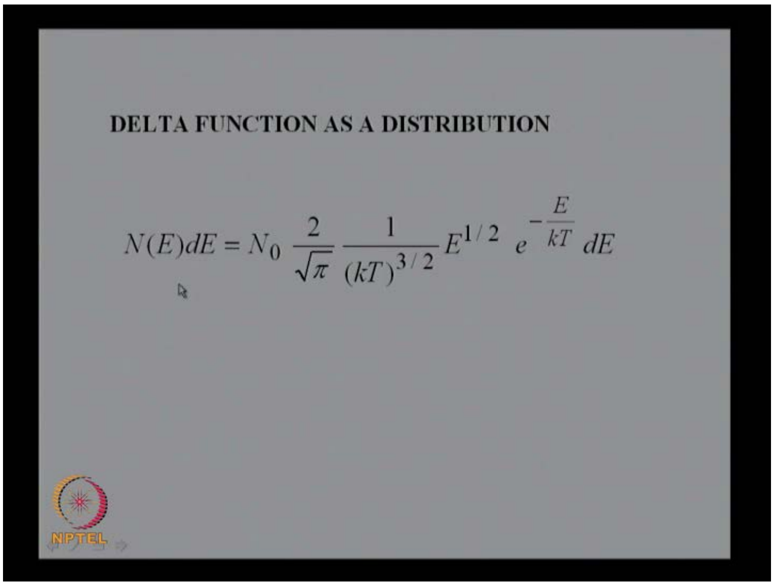

Now, in a room like this; let us suppose, the distribution of energy of the molecules is given by the Maxwellian distribution; so N of E dErepresents the number of molecules whose energy like between E and E plus dE. If Iask you the question how many molecules have exactly the energy E is equal to k T?The answer is 0because dEis 0. Iwant the number of molecules whose energy lies betweenkTandkTso dEis 0. So, if this distribution the total number of molecules which have exactly the energy E equal to k T is 0, butif we do have L number of molecules.

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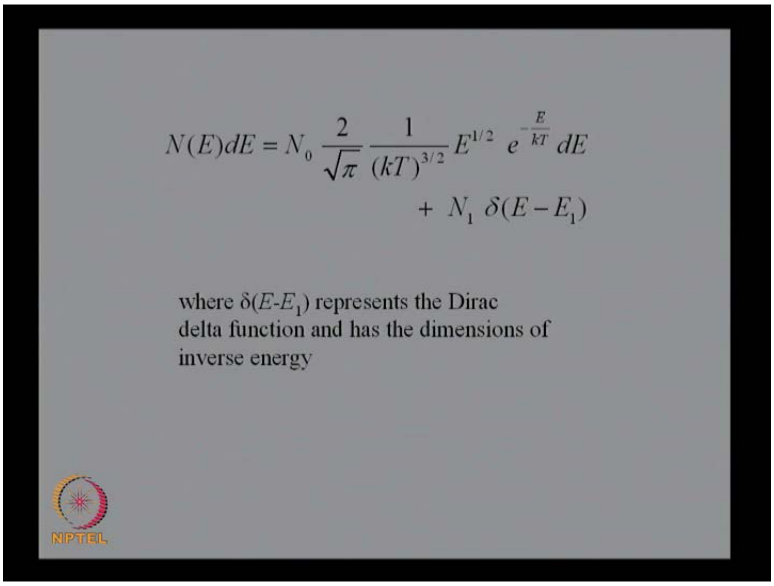

Let us suppose N 1 number of molecules, which has exactly the energy E 1 then to this, I must add this Dirac delta function. So delta function is a distribution and therefore, N of E dErepresents the number of moleculeN of E.

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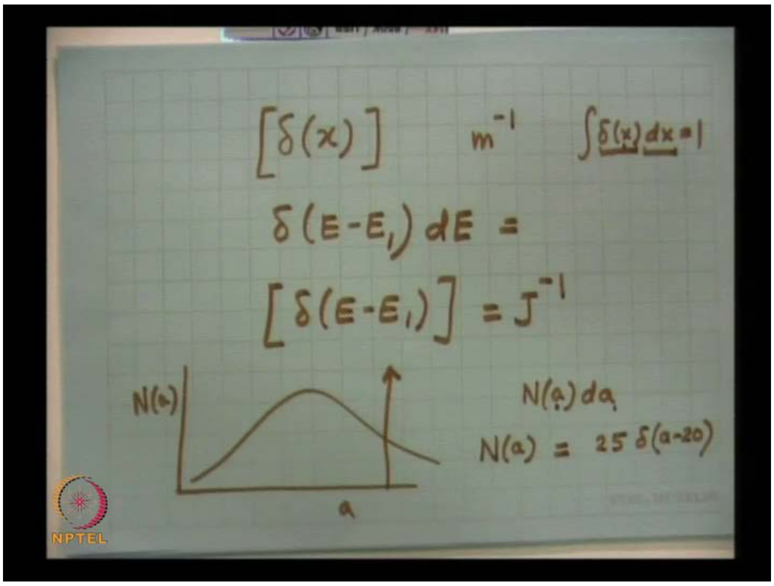

So let me summarize this by saying that, if I have delta of x and x is in meters then, the dimension of delta of x will be meter inverse. If delta of E minus E 1, if this is E is measured in joules then, this times dEwill give the number of molecules. So therefore, the dimensions of delta of E minus E 1 is joules inverse. Here, delta of x dx is equal to 1. So therefore, if x is in meters so dx has the dimension of meters, delta x will have a dimension ofmeter inverse. So it is a…It is not just an ordinary function,it is a distribution.

So, if you have if you have let us suppose, age distribution function of students in an university, if it is given by this N of a versus a therefore, N of a da represents the number of students whose age lies between a and a plus da. So if Iwant to ask that, what is the number of boys who have exactly the age 22 years? The answer is 0. But,if we do have exactly 20 students which have who have the exact energy of 20 years. So let us suppose so let us suppose, there are 25 students who have the energy who have the age exactly20 years so then, that is represented by this and that will be a represented by a Dirac delta function, and this dimensions of this will be age inverse.

So delta of x, if x is measured in meters then, it is delta of x is in meter inverse. If x is energy then, it isenergy inverse. If it is time then, it is time inverse. So we will continue next time onthe other representations of the Dirac delta function and the Fourier transform theory thank you.