Basic Quantum Mechanics

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Module No.# 04

Simple Applications of Schrodinger Equation

Lecture No. # 01

Tunneling through a Barrier

In our last lecture, we had discussed the case when electron approach a potential step and we were calculating the reflection and transmission of that electron wave.

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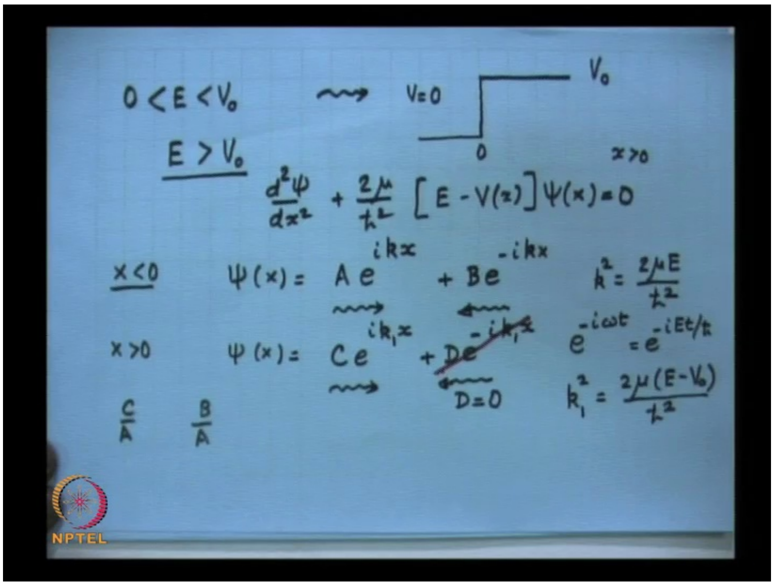


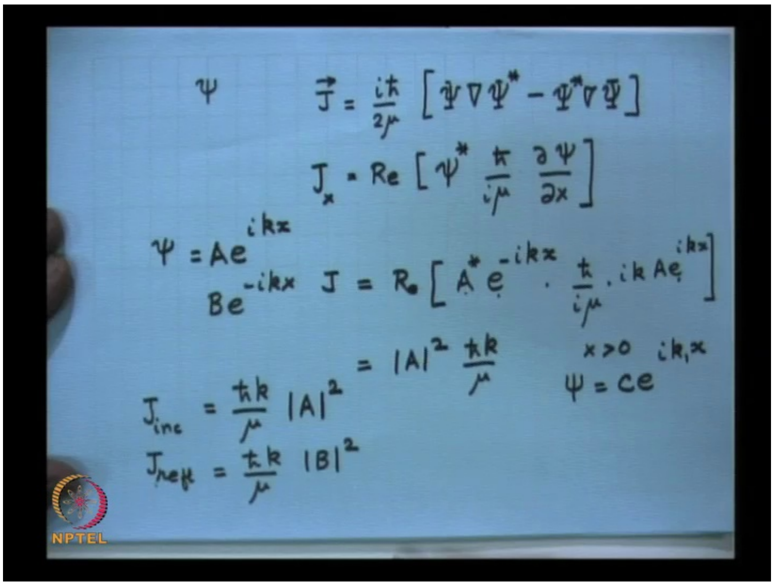
So, we considered a potential step, such that the potential is zero for X less than zero and it is equal to V 0 forx greater than zero. So, there are two cases, one in which E is less than V 0 and in the other we will have the energy of the incident particle is greater than V 0.

So, in our last lecture we had considered E greater than V 0 and the solution of the Schrödinger equation d 2 psi by dx square plus 2mu by h cross square E minus V of x psi of x is equal to 0. So, in the region x less than 0 this is V is 0. So, that in the region X less than 0 the solutions will be psi of x is equal to E to the power of i k x plus B into e to the power of minus i k x where k square is equal to 2 mu E by h cross square.

The first term represents a wave propagating in the plus x direction and the second term represents the reflected wave which propagates in the minus x direction and that is because, as I had mentioned earlier the time dependence is of the form of e to the power of minus I omega t. So, this is equal to E to the power of minus i E t by h cross and. So, when I multiply the space dependent part with e to the power of minus i omega t, this represents the forward propagating waveor the incident wave and this term represents the backward propagating wave or the reflected wave.

In the region I am assuming E greater than V 0, as we have done in our last lecture. So, in the region x greater than zero we will have the solutions psi of x is equal to c into e to the power of i k 1 x plus D into E to the power of minus i k one x where k one square k 1 square is equal to 2 mu E minus V 0 by h cross's square. So, once again this represents a forward propagating wave and this term represents a backward propagating wave.

Now, in the region two that is in the region X greater than 0, there cannot be any backwardpropagating wave because there is no reflection that can take place and. So, therefore, D is equal to zero this is my boundary condition. So, this term can be this term vanishes and. So, we match the boundary conditions and found out the relation between we had found out the C by Aand B by A before we interpret that.



Let me mention thatfew lectures back, we had associated with the wave function psi is the current density which was given by J vector this is the current density, which is equal to i h cross by 2 mu multiplied by psi grade, psi star,grade psi star minus psi star, grade psi. Now, this can also be written as the real part of psi star h cross by I mu delta psi in for the one dimensional case delta psi by delta X. This is the X component of the of the current density if the wave function depends only on the X coordinate which it is true in this particular example then it will become like this.

We if I have a wave function the space dependent part is e to the power of i k x then the associated current density will be J will be the real part of psi star that is suppose this is a into e to the power of i k x. So, A stare to the power of minus i k x multiplied by h cross by i mu delta psi by delta x will be i k into a into e to the power of i k x. So, therefore, AA star is mod a square and this term cancels with this term. So, this will be multiplied by h cross k by mu. So, my incident wave associated with the incident wave is the current density h cross k by mu multiplied by A square this should be also physically obvious because h cross k represents the momentum.

So, momentum divided by mass is the velocity, sothat is the current associated with the position probability density associated as mod A square. So, similarly the reflected wave, the reflected wave is described by B into e to the power of minus i k x and if you substitute it in this expression, you will get h cross k by mu into B square, this is my reflection coefficient, reflected current density associated with the reflected wave. Similarly, the transmitted wave, for the transmitted wave the wave function in the region x greater than 0 we had written down as c into e to the power of i k one x and if I substitute it here. So, the transmitted wave will be mod c squareh cross k 1 by mu.

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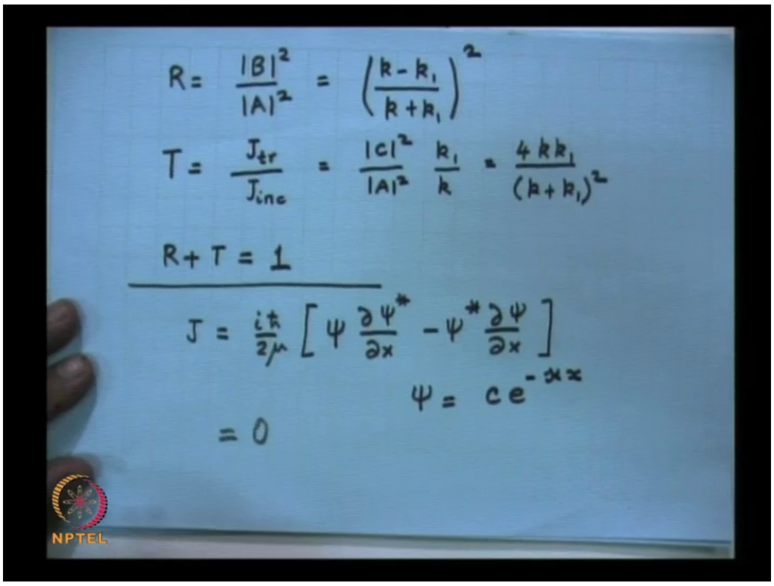

So, therefore, from these two expressions one can find out,one can find out that the reflection coefficient,the reflection coefficient is the reflected current divided by the incident current. So, that will be equal to mod B square by mod A square and this as we had found out in our last lecture this was equal to K minus K one divided by K plus K one whole square. The transmission coefficient will be equal to the transmitted current divided by the incident current and this will be equal to mod Csquare divided by mod A square and then this will be h cross k 1 by mu, divided by h cross k by mu. So, this will be k 1 by k and if you substitute the value of C by A whole square then this will come out to be 4 kk 1 divided by k plus k one whole square.

So, we get the result that this is the this is thetransmission probability and this is the reflection probability and if you add this two then you will get R plus T this is equal to k minus k 1 whole square plus 4 k k 1 will be k plus k 1 whole square and this will be 1, but one has to be little careful in calculating the current density and that is and therefore, the factors k1 and k will appearin the expression it is not just c by a mod whole square, but it is multiplied by k 1 divided by k.

Now, there is one more thing that I would like to mention that if psi is real, if the wave function is real then psi will be equal to psi star and if psi is real this quantity will be real, this quantity will be real and. So, this quantity will be pure imaginary sothe current density will be zero. So, if I have a wave function which looks like this,so the expression for the current density is that J is equal to i h cross by two mu psi if the wave function depend only on the x coordinate del psi star by del x minis psi star del a del psi by del x.

So, let us supposemy wave function psi is something like c into e to the power of minus kappa Xit is a real function then you can immediately substitute this here and you will find that the current density will be zero and this also follows from the fact that if psi is real, if psi is real then this quantity will be all this quantity will be real and this is pure imaginary, sothat the current density is zero. So, I leave it as an exercise for you to show that, if I use for the wave functions psi equal to somuch then the associated current density will be zero.

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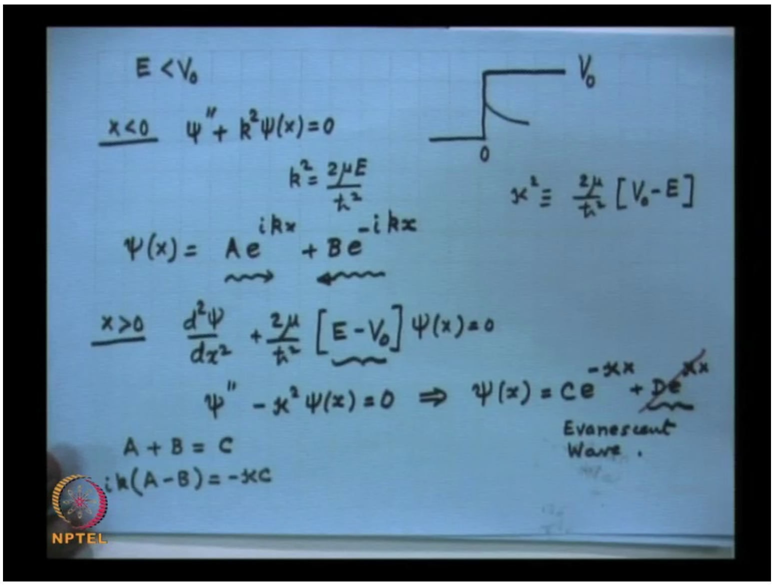

So, therefore, when we considered the case, then we next considered the case then E is less than V 0. If E is less than V 0, sothis is X is equal to 0 for Xless than 0 the Schrödinger equation is psi double prime d 2 psi by d x’s square plus 2 mu E by h cross’s square. So, that is k square psi of X is equal to 0. So, I am assuming now the energy is less than V 0 where k square is again the same quantity which is equal to 2 mu E by h cross’s square. So, the solution of this equation is again psi of x is equal to e to the power of i k x plus B into E to the power of minus i k x this represents the incident wave and this represents the reflected wave.

Now, we consider the region x greater than zero where the Schrödinger equation will be d two psi by d x’s square, plus 2 mu by h cross’s squareE minus V 0 E minus V 0 psi of x is equal to 0.

So, since E is less than v zero, sothis quantity is negative. So, that I write this as minus kappa square psi of X 0 where kappa square is equal to is defined to be equal to two mu by h cross’s square V zero minus a and the solution of this equation will be. So, this is psi double prime and the solution will be as I had discussed last time the evanescent wave, one term will be C e to the power of minus kappa x plus d into e to the power of plus kappa x this term willblow up at infinity will go to infinity at x is equal to infinity. So, Ikind of said this d equal to0. So, this is known as the exponentially decaying solution or this is also known as the evanescent wave, evanescent wave. I associated with this evanescent wave the current density is zero, the current density is zero. So, there is a certain probability of finding the particle in the classically forbidden region.

You see if I have E less than V 0 the total energy is less than V 0. So, the kinetic energy is negative classically speaking. So, this is a region where classically a particle will not be found; however, quantum mechanically there exists a probability of finding it in the classically forbidden region, but it is an exponential decaying solution and such a wave is known as an evanescent wave for an exponentially decayingwave. So, my solution will be e to c into e to the power of minus k kappa x and then we canapply the continuity conditions. So, at x is equal to zero we will have A plus B is equal to C this is one condition and then i k even if i have the derivative of the wave function equal to continuous then i k A minus B is equal to minus kappa C.

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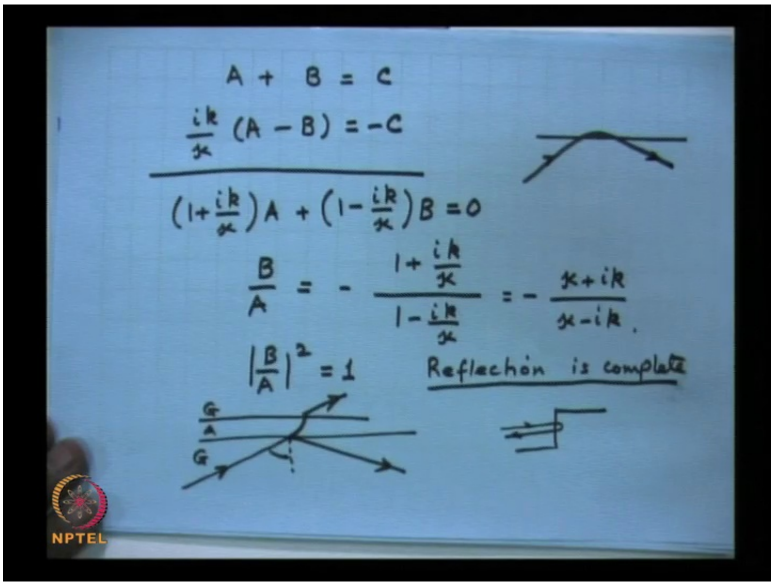

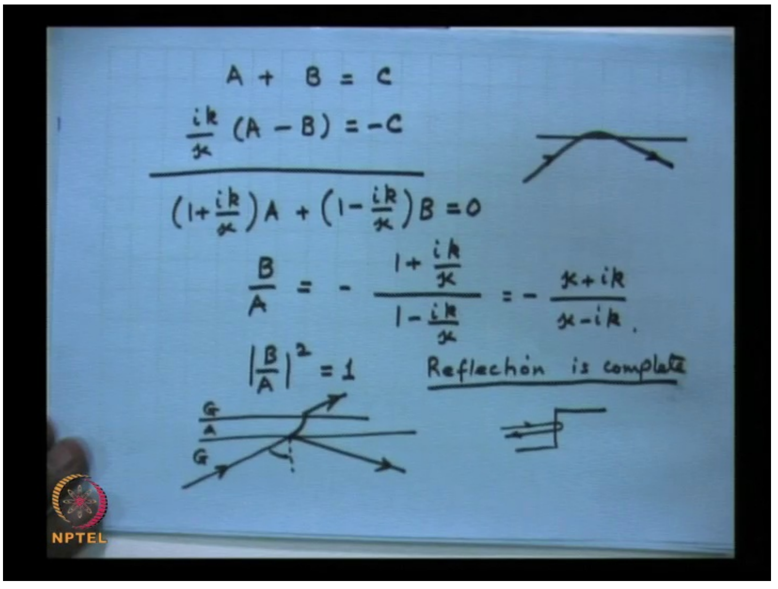

From this equation, we can find out what are the so for example, I have these two equations that A plus B is equal to C and i k by kappa A minus B is equal to minus C. So, if I add themif i add the two equations then I get one plus i k by kappa times A plus one minus i k by kappa into B this is equal to 0.

So, we will have we will haveB by A is equal to B by A, I can calculate and that will be that will be minus one plus i k by kappa divided one minus i k by kappa, sothis will be minus kappa plus i k divided by kappa minus i k and if I take the modules square B by A whole square then, this will be kappa square plus k square this will be kappa square plusk square. So, this will be one indicating that the reflection is complete reflection iscomplete.

Now, this is what I had try to tell you last time, that when I have an electromagnetic wave which is incident at a rarer medium at an angle greater than the critical angle then you have the phenomenon of total internal reflection the energy gets completely reflected; however, the reason evanescent wave here, the reason evanescent wave in the rarer medium and which can be used to tunnel. So, if you have a glass your glass surface then it will undergo internal reflection here and there is a certain probability that it can tunnel through the barrier. So, if you have if you have a potential step and if the energy is less than V 0 then physically one may understand that as if the particles sought of penetrates into the classically forbidden region and comes back the reflection coefficient is unity.

In this case also, in the case of in the case of reflection by a rarer medium what one understand that the wave enters the classically the rarer medium sought of enters and it comes out it gets slightly shifted to the right, sothe same thing happens even in quantum mechanics. You have a wave which is present in the classically forbidden region and the wave gets totally reflected, the reflection coefficient is one and, but there is an evanescent wave in theclassically forbidden region. So, that was the complete analysis for the for a wave electron wave incident on a potential step.

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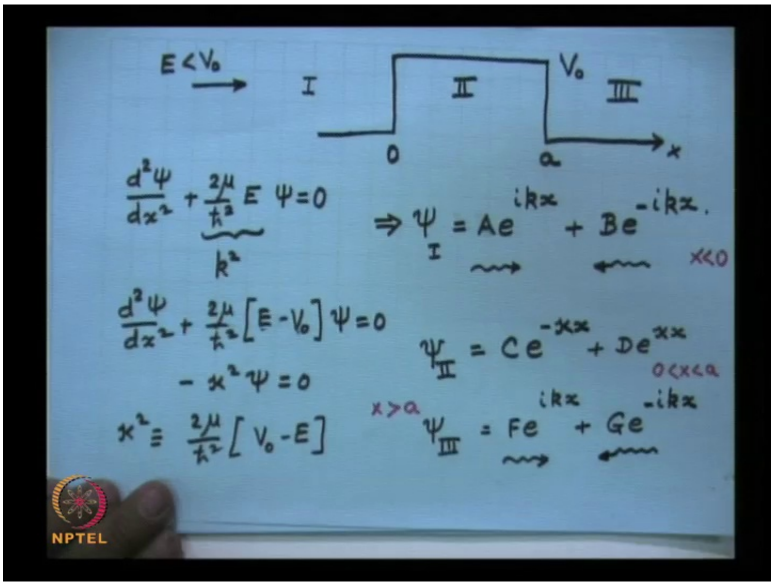

The next thing, that we will be discussing is reflection by a potential barrier that is you have here, you have a barrier of finite height, finite height and, sosomething like this. So, let us suppose this is my x axis and this is V 0 andthis is X is equal to 0 and this is X is equal to a. I have a particle which is incident, which is coming from the left whose energy is less than v zero whose energy is less than V 0 it is something like a tennis ball which in front of it there is amountain. So, we know that the tennis ball will roll up to a certain distance and will come back.

But here, there is a certain possibility that you will be tunnel through the barrier and will go through the other side, this is purely a quantum mechanical phenomenon this is purelya quantum mechanical phenomenon. And there have been experiments which are proved that indeed the particles will tunnel do tunnel through the barrier, do tunnel through a through a region which is classically forbidden.

So, let us do the mathematics once again we have to solve the Schrödinger equation in the three regions the three regions are X less than zero,X lying between 0 and a and greater than a. So, in region one in region one the Schrödinger equation is the potentialis zero. So, that d 2 psi by d x's square plus two mu by h cross's square E minus b. So, V 0 psi is equal to 0. So, this we again denote by k square. So, the solution in region one, in region one which is X less than 0 that is A into e to the power of i k x plus B into e to the power of minus i k x. So, this represents the incident wave and this represents the reflected wave in the second region, we will have d 2 psi by d x's square plus, two mu by h cross's squaree minus V 0, E minus V 0 into psi is equal to zero, but e is less than v zero.

So, I write this as minus kappa square psi is equal to 0 where kappa square is defined as we had done before two mu by h cross's square V 0 minus a and the solution of this equation is psi in region two, this is the region two; will be c into e to the power of minus kappa X this is the exponentially decaying solution and since there is a boundary we cannot reject the exponentially amplifying solution because it will become large, but it will not becoming infinite. So, there is noreason why we have to neglect this solution. In fact, it has to be taken. So, this is the solution in the classically forbidden region, then in the third region which corresponds to sothis region, this is the first solution corresponds to X less than 0 this is in the region 0 less than X less than a and then for x greater than a,for x greater than athe wave function will satisfy the same Schrödinger equation because the potential energy is 0.

So, thereforeif the third region,if the third region we will have f e to the power of i k x plus G into e to the power of minus i k x, thisrepresents a wave propagating in the plus x direction, this represents this term represents a wave propagating in the minus x direction and since there is no barrier, no further barrier or a potential change beyond this place. So, therefore, you cannot have a reflected wave because there is nothing to reflect it. So, G will be zero, sothis term we will have to neglect. So, will have five unknowns one A,B,C,D,F and we will have four continuity conditions two at X is equal to 0 and two at x is equal to a and using these four continuity conditions, one can find the ratio between any two coefficients. So, let us try to do that.

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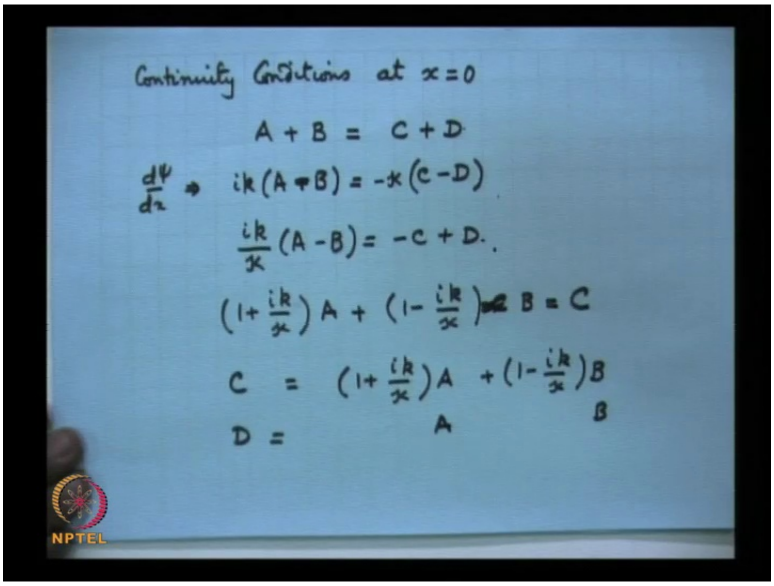

So, continuity of the wave function, the continuitycondition at conditions at x is equal to zero. So, if I apply at x is equal to 0, this will be A plus B and this will be equal to C plus D. So, we will have one condition as A plus B is equal to C plus D and similarly, if I take the derivative, if I take the derivative then this will be i k A times one because at x is equal to zero this will be one minus and here also minus kappa plus kappa D. So, we will have the continuity, this is the continuity of the wave function and continuity of d psi by d x, continuity of d psi by d xwill lead to i k A minus B is equal to minus kappa C minus

D. So, using these two equations I can write C and D in terms of A and B. So, I can write down as i k over kappa A minus B is equal to minus C plus D this is correct.

So, I add them and I will get one plus i k by kappa into A,this will be B plus one minus i k over kappa this is equal to C. So, this multiplied by B is equal to C. So, I can write C as one plus i k by kappa into A plus one minus i k upon kappa into B similarly,D also I can write it as a linear combination of Aand B, I just have to subtract this equation, from this equation soI will get an expression for D once we have obtained that. So, we have expressions for C and expressions for D.

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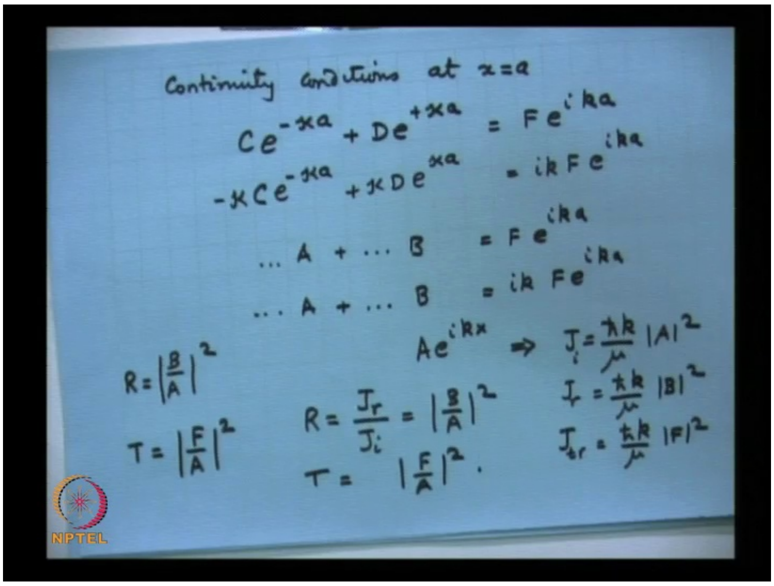

The we next apply the continuity conditions at x is equal to a, continuity conditions at x is equal to a. So, you will have the wave function was if you recollect the wave function was C into e to the power of minus i k x. So, you will have C into e to the power of minus kappa a plus D into e to the power of plus kappa a, this is equal to F into e to the power of i k a, this is the continuity of the wave function at the point x is equal to a. Similarly, if I take the derivative soI will get minus kappa C into e to the power of minus kappa a, plus kappa D e to the power of plus kappa a and if I take the derivative of this will be i k F into e to the power of i.

Now, this is slightly cumbersome algebra, but very straight forward we have just now obtained expressions for C and D in terms of A and B, I leave it as an exercise for you to substitute those expressions here for C and D and. So, thereforeyou will have something like this A some coefficientplus, some coefficient B is equal to F into to e to the power of i k a and again A plus, some coefficient B will be equal to i k F into e to the power of i k.

From these two equations, I can find out what is B by A whole square. So, this will be my reflection coefficient and then you can find out what is F by A whole square, which will be my transmission coefficient. So, actually the reflection coefficient will be you see associated with A into e to the power i k x the current density,the current density is given by h cross k by mu multiplied by A square. This is the incident the reflected will be h cross even the same k mu b square and the transmitted will be again h cross k by mu mod f square. So, since the same k appears in all the equations. So, the reflection coefficients which will be actually the reflected current divided by the incident current will just be equal to mod B by A whole square and the transmission coefficient will be mod F by A whole square. And if you, if you carry out this algebra I leave this is as a simple exercise.

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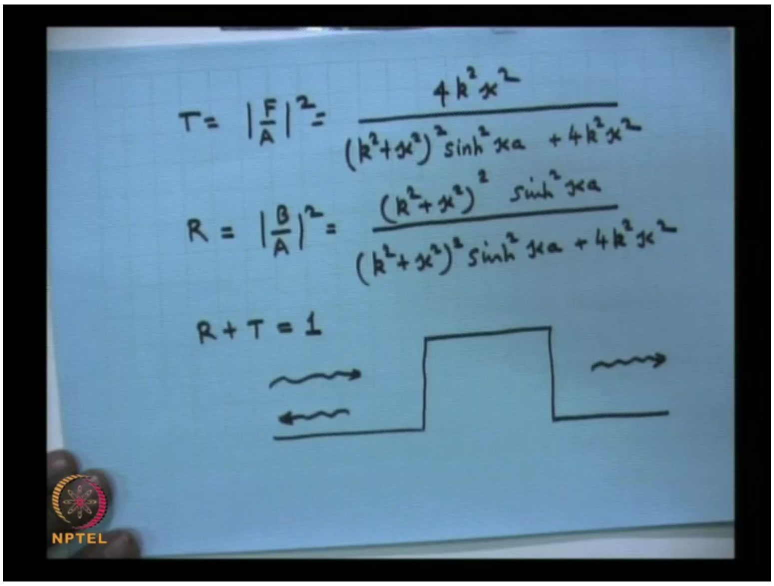

You will find that the transmission coefficient will be T is equal to a Fby a whole square this will be equal to four k square kappa square divided by, k square plus kappa square its fairly straight forward whole square, sin hyperbolic square kappa a plus four k square kappa square and the reflection coefficient will be equal to mod B by A whole square will be equal to k square plus kappa square whole square, sin hyperbolic square kappa a divided by the same thing that is k square plus kappa square whole square, sin hyperbolic square kappa a plus four k square kappa square.

As you can see if I add these two up the reflection and the transmission will be equal to one. So, thereforewhat we have shown above is a very important application of the solution of the Schrödinger equation, that you have an incident wave on a potential barrier like this, you have a reflected wave, you have a you have a incident wave there is a reflected wave and there is also a certain probability for the particle to tunnel through the barrier.

So, if an individual electron approaches a barrier if there is a certain probability for it getting reflected, there is a certain probability for it getting transmitted what will happen to an individualelectron no one can predict, one can only predict the odds the probabilities of the events and so therefore, if one is not making a measurement then it is both in the reflected beam as well as in the transmitted beam. Only when one makes a measurement then this then the electrons certainly collapses to a state when it is found either in the reflected beam or in the transmitted beam.

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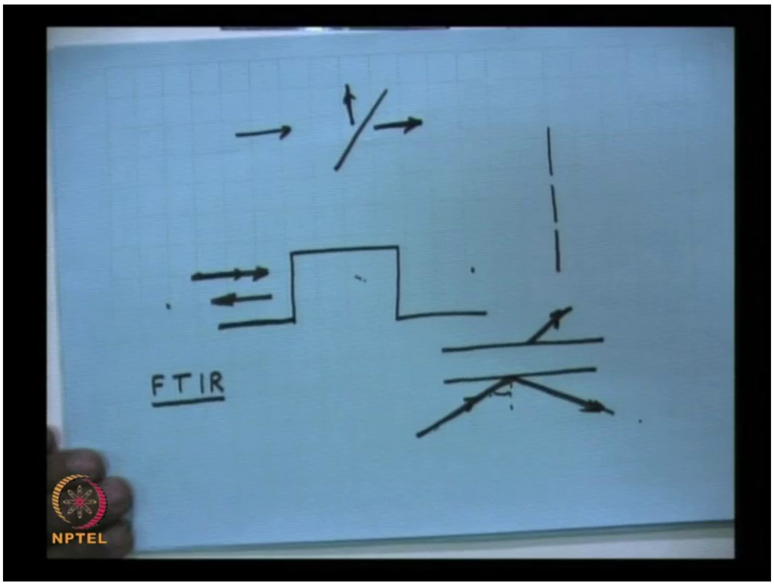

So, this is something similar to the famous Michelson interferometer experiment. I have aI have a photon this is very nicely discussed in Dirac's book that it can get reflected as well as transmitted.

So, there is a certain probability and say half probability of it getting reflected and half probability of it get transmitted. So, unless you make a measurement it is describe by a wave function which is present here as well as here. So, it is in both the beams and it is because of that these two beams can be further reflected and made to interfere, sothis is the same thing that we had discussed quite some time back that the electron passes through both slit both the wholes simultaneously.

So, similarly if you have a potential barrier if you have a potential barrier then an electron is incident from the left then after it interacts with the potential barrier, it is both in the transmitted beam and in the reflected beam there is a certain probability of it being found there in this region as well as in this region. So, this is the conceptthe underline concept in quantum mechanics thatthere is a certain probability of it being reflected certain probability of it being transmitted.

What will happen in a particular event? No one can predict, one can only predict the odds of happening in a particular measurement. So, thereforewe have here a tunneling through barrier and this the equations are almost identical to the to the phenomenon that I had mentioned very briefly in my last lecture, that is the phenomenon of frustrated total internal reflection now that is a entirely a classical phenomenon that in which you have in which you have a rarer medium and it is becausethe light wave is incident if there is the evanescent waves are created in this rarer medium and there is a the light wave is incident at an angle, which is greater than the critical angle there is a probability that it will be reflected back, but there is a small probability that it will tunnel through this rarer medium and appear on the other side, but this comes out from by classically by solving the classical Maxwell's equation.

So, this phenomenon is known as the frustrated total internal reflection andthe is quite analogous to the phenomenon of the one solves the same type of equation, even in considering thetunneling through a potential barrier. In fact, according to geometrical optics one may have always hundred percent reflection and one says that the relationship between geometrical optics and wave optics is the same as that between classical mechanicsand quantum mechanics.

In geometrical optics there will be always the ray will be reflected back and no ray will be found in this particular region, in classical mechanics a particle which is incident here will always be reflected there'll be no transmission because it cannot enter this region, this is a classically forbidden region in which the total energy is less than the potential energy and classically speaking therefore, it leads to a negative kinetic energy.

So, the particle can never enter inside the barrier and. So, therefore, it is always reflected back on the other hand, when you solve the Schrödinger equation on the other hand when you solve the Schrödinger equation you do find that there is a small possibility of tunneling through the barrier.

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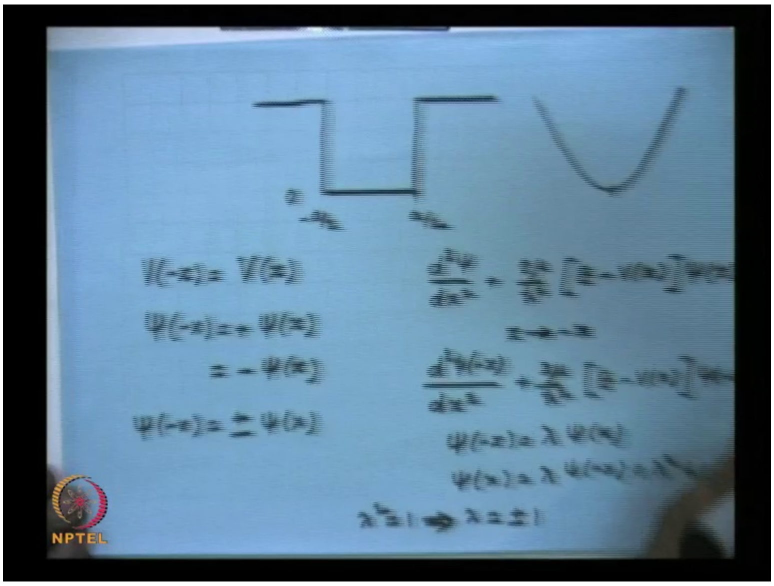

Now, I will do one more problem in one dimensionalone more problemof evolving the solution of the one dimensional Schrödinger equation and that is the potentialwell problem. That you have a potential well, in which say this isV equal to 0 for x less than minus a by 2 to x less than.

I have discussed sometime back, that if the potential energy function is a symmetric function that is beam of minus x is e V of x then the solutions can always be written down which are either symmetric or ant symmetric that is, the Eigen functions can be written down as psi of minus x will be either minus plus psi of x which is the symmetric functions or it will be minus psi of xwhich is the ant symmetric functions. So, this follows from the fact that I write down the Schrödinger equation d two psi by d x’s square plus two mu by h cross’s square e minus v of x e minus v of x, psi of x is equal to 0.

Now, if I make a transformation x to minus x and since v of minus x is equal to v of x. So, we find that d two psi of minus x satisfies the same Schrödinger equation two mu by h cross’s square e minus v of x psi of minus x. So, therefore, psi of minus x must be a multiple of psi of xand therefore, if I make the transformation again psi of minus, minus x. So, therefore, psi of x will be lambda psi of minus x. So, this will be equal to lambda square psi of x leading to lambda is equal to plus minus one, lambda square is equal to one imply lambda is equal to plus minus one. So, therefore, psi of minus x must be either plus psi of x or minus psi of x.

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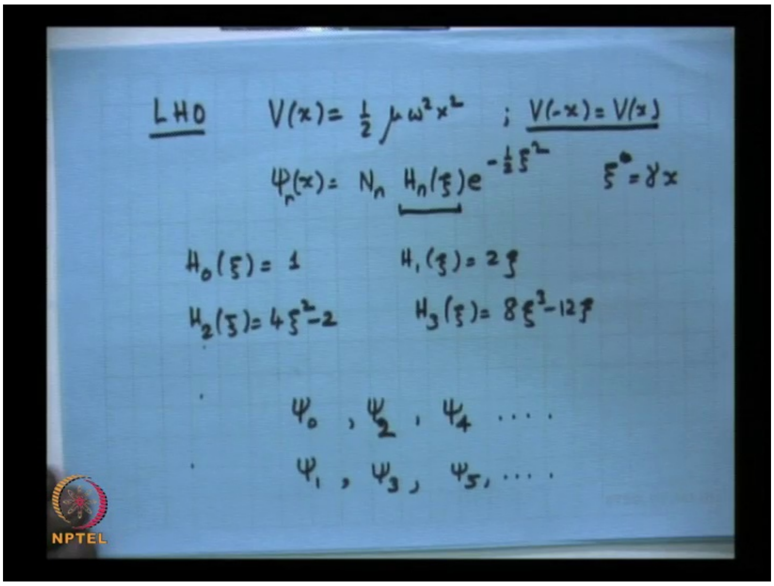

If you recollect that when we did the harmonic oscillator potential. It was something like this and then this potential function is symmetric with respect to x and the wave functions are wave function be, the wave function that we had calculated for the for the linear harmonic oscillator problem in which V of x is equal to half mu omega square x's squarein thiscase you have V of minus x is equal to V of x. So, we found we had found out the psi of x, psi n of x is equal to n of n this is the normalization constant H n of psi e to the power of minus half psi square, where xi is a multiple of x is xi is equal to gamma x.

Now, h n xi are alternately thehermit polynomials which are even and odd. So, for example, H 0 of xi is equal to 1, h 1 of xi is equal to 2 xi H 2 of xi is equal to 4 xi square by minus two something like that H 3 of xi will be equal to 8 xi cube minus twelve xi. So, H 0 of xi, H 2 of xi, H 4 of xi, H 6 of xiare even polynomials involving even powers of xi H 1, H 3, H 5, H 7 will involve only odd powers. So, thereforepsi 0, psi 1 of x,psi 0, psi 2, psi 4 they may be all even functions of x and psi 1, psi 3, psi 5, will be all odd functions of x. So, whenever the potential energy function is a symmetric function of x, whenever v of minus x is equal to v of x then the Eigen functions are either symmetric functions or ant symmetric functions.

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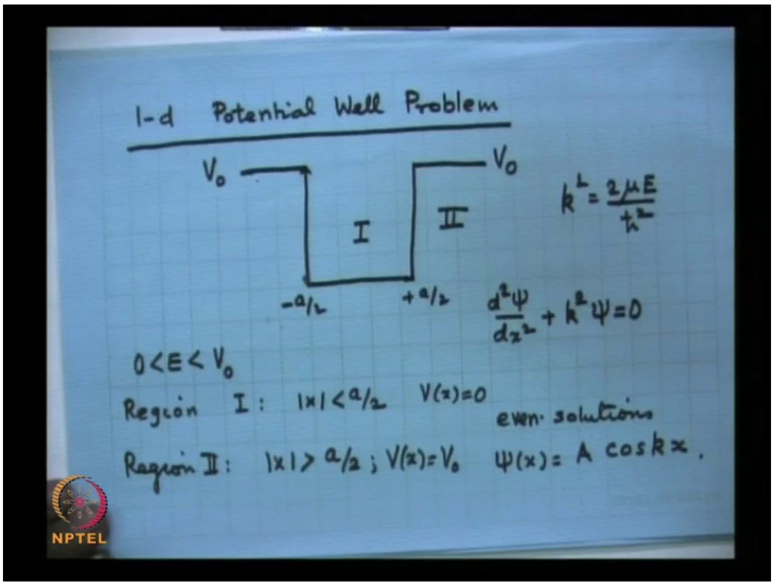

So, let us use this tosolve the one dimensional potential well problem, the one dimensional this is a very important problem one dimensional potential well problem in which the potential function as I had mentioned earlier, is that you have this is x is equal to minus a by 2 this is plus a by 2 and this is V 0, this is V 0. So, let me first consider the case where zero lies between, where the energy lies between 0 and V 0. So, let me write down the solution the Schrödinger equationin region one region one is region one corresponds to mod x less than a by two where the potential is 0. So, where V of x is equal to 0.

So, therefore, the Schrödinger equation is d 2 psi by d x's square plus 2 mu e by h cross's square which I write as k square psi is equal to 0, where k square is equal to 2 mu E by h cross's square we write the solution in terms of sin ad cosine functions. So, sin functions as we know is an odd function of x and cos function is an even function of xSo, we first consider the even solutions, sothe even solutions Idid this down as psi of x in region one. I can write this down as A cos k x now in region two, this is region two for x region two x greater than a by two and my V of x is equal to V 0, but E is less than V 0. So, that the Schrödinger equation.

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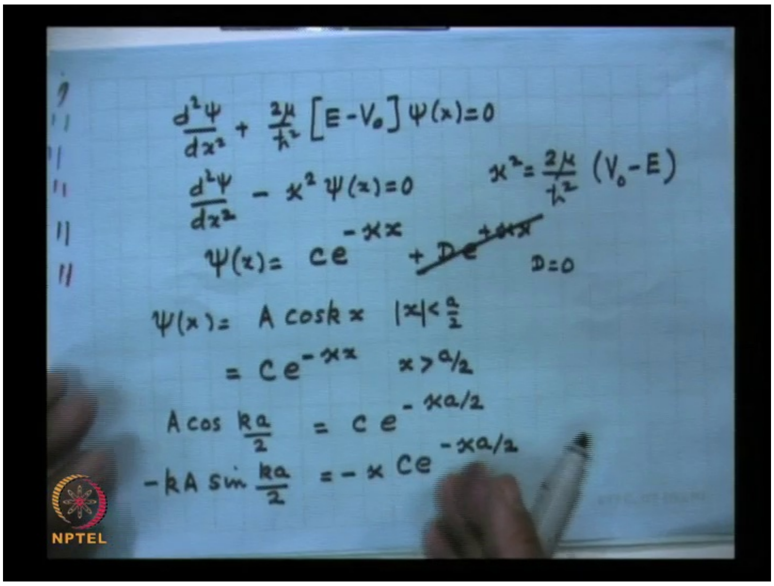

So, the Schrödinger equation becomes d 2 psi by d x’s square plus 2mu by x cross’s square E minus V 0 psi of X is equal to 0. Since E is less than V 0 as in the previous case I write this is a negative quantity and I write this as minus kappa square psi of x is equal to 0, where kappa square is defined to be equal to 2 mu by h cross’s square, V 0 minus E is less than V 0. So, this is d two psi by d x’s square is equal to soand the solutions are once again psi of x psi of x you will have c into e to the power of minus kappa x, which is the exponentially decaying solution plus D into e to the power plus kappa x.

So, in this region, soin region two you will have one which exponentially decreasesand one which exponentially amplifies and since this extends to infinity. So, this will lead to the exponentially amplifying solution, will lead to a wave function which blows up at infinity,which blows up at infinity and we cannot allow that and. So, thereforethe wave function has to move to zero at infinity and therefore, D must be equal to zero, sowe must neglect this term.

So, I will have two solutions, so psi of x is equal to A cos k x, for x less than a by two actually is mod x less than a by two and C e to the power of minus kappa x for xgreater than a by two. So, I match the boundary conditions, soI have A cos we see this cos k a by two will be equal to C into e to the power of minus kappa a by 2 that is the continuity of the wave function at x is equal to a by two and then I differentiate itand get minus k a sin k A by 2 I differentiate that and put x is equal to a by2, this is equal to minus kappa C into e to the power of minus kappa a by 2. This is a set of homogeneous equationsA and C. So, for a non trivial solution the determinant must be zero. So, I divide one with respect to the other and you will get if I divide this, soyou will get you will get minus, minus cancels out.

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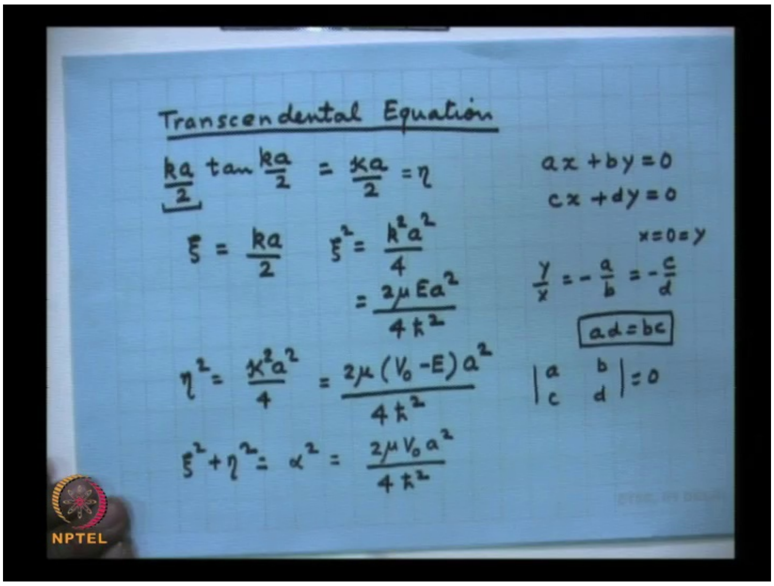

So, you get K tangent of k a by two this is equal to kappa, I multiplied both sides by a by two so, I get kappa a by 2.

I hope you understand, what I am trying to say that the determinant must be zero because let us consider it a set of 2 equations a x plus b y is equal to 0 andc x plus d y is equal to 0. Now one solution is of course, the trivial solution that x is equal to 0 is equal to y is equal to 0, but if I neglect this trivial solution then you can see from here that y by x is equal to minus a by b and in the second case y by x is equal to minus c by d. So, you must have a d is equal to b c. So, the this is said that if a set of homogeneous equation, if you have a set of homogeneous equations then for non trivial solutions this determinant a, b, c, d must be equal to 0; this is for non trivial solutions. So, here for example, we have a c here a c here. So, for non trivial solutions one trivial solution is A is 0C is 0 that corresponds to the wave functions the wave function vanishing everywhere, but otherwise if I do not include those trivial solutions then this must be equal to this is said to be a transcendental equation,now this is very important and I want to spend some time on this. So, that you understand the meaning of the word transcendental equation.

Now, I put this equal to xi, soyou have I define this as xi is equal to k a by two, somysomy xi square is equal to k square, a square by four or this is equal to two mu E a square by four h cross's square and kappa a by two,let us suppose I write it as eta. So, you will have eta square is equal to kappa square a square by four, sothis is equal to twomu kappa square is V 0 minus E, a square by four h cross's square. So, if I add them up, soI get xi square plus eta square soEE cancels out,you get you write this as alpha square which is equal to two mu V 0 a square by four h cross's square for a given potential, for a given value of V 0, for a given value of a and of course, h cross is a constant this is just a number. So, I get this therefore, eta is equal to under root of alpha square minus xi square.

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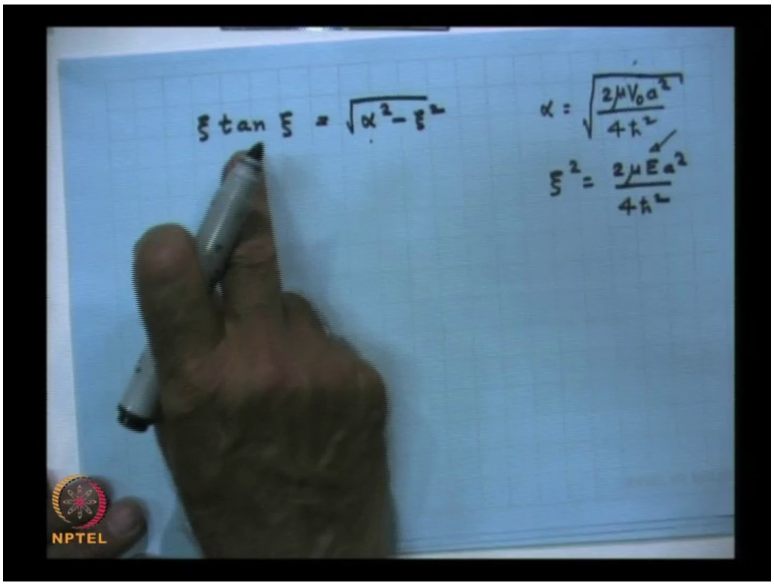

So, the equation that we obtain is xi tan xi is equal to eta that is under root of alpha square minus xi square. We saythat this is a transcendental equation; that means, for a given value of alpha, for a given potential as I had told you alpha square is equal to or alpha is equal to under root of two mu V 0 a square by four h cross's square.

For a given particle the mass is known, for a given potential v zero is known a is known h cross is a constant. So, this is a number now it is only for certain discrete values of xi that the left hand side will be equal to right hand side and what is psi square? So, what isxi square is equal to two muE a square by four h cross's square. So, there are only there will be only certain discrete values of xi and therefore, certain discrete values of E for which the left hand side will be equal to right hand side those are the Eigen values of the problem. So, we stop here in of my next class, we will try to understand the this we will discuss in greater detail the solution of the transcendental equation that I have described here.   .