Basic Quantum Mechanics
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Module No. # 03
Linear Harmonic Oscillator – I
Lecture No. # 01
Linear Harmonic Oscillator

Even the past 2, 3 lectures, we have been discussing the solution of the one dimensional Schrodinger equation for the particle in a box of dimension a. And we also considered before that, the free particle problem. Those were two important problems in quantum mechanics.

Today, we will be starting our discussion on the Linear Harmonic Oscillator problem; this is one of the most important problems in quantum mechanics and allows one to understand very clearly the relationship between classical mechanics and quantum mechanics. The plan is that, we will first give the solutions of the Schrodinger equation before doing the detailed mathematics associated with it. And then, we will discuss the physics of the solutions and obtain results, which correspond to the classical oscillator.

Once we have understood the consequences of the solution we will then, do a little bit of algebra and solve a differential equation; and obtain the exact solution of the Schrodinger equation of the one dimensional Schrodinger equation corresponding to the linear harmonic oscillator problem.

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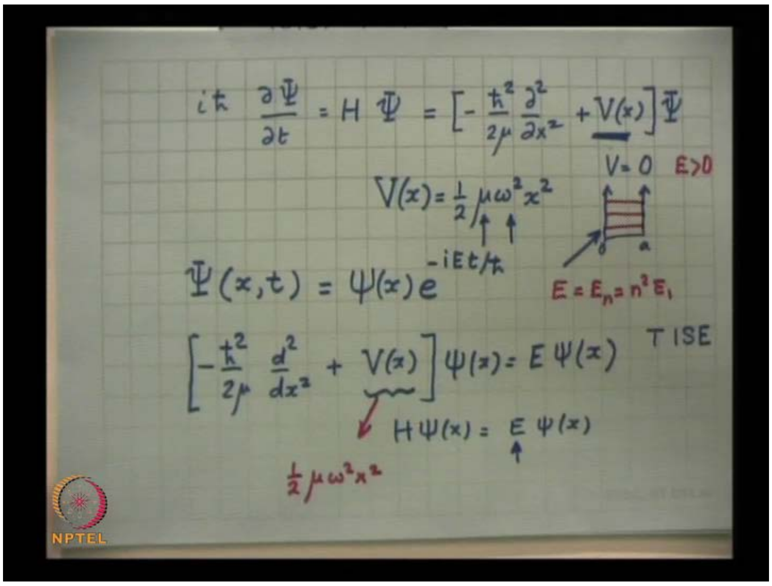

So, let us start with the with our familiar one dimensional Schrodinger equation, which corresponds which is given by i h cross as we all know delta psi by delta t is equal to H psi, where H is the Hamiltonian a operator corresponding to the total energy and this in the one dimensional case is given by minus h cross square by 2 mu delta 2 by delta x square plus V of x psi, which is a function of x and t. Now, we have solved this equation for the free particle, where V of x is 0. We have solved this equation for a particle in a box problem as I just now mentioned, confined in a one dimensional infinitely deep potential.

Today, we will give the solutions corresponding to the harmonic oscillator problem, which is equal which for which this is V of x is given by half mu omega square x square. We know the mu is the mass of the particle, omega is the classical frequency and x is of course, the x coordinate, this is the classical expression for the potential energy corresponding to the harmonic oscillator (Refer Slide Time: 03:46). And we will obtain we will write down first the solution of this equation corresponding to this potential.

Once again since psi since V is a function of x only does not depend on time. So, we can use the separation of variables to solve this equation and if we carry out this separation of variables; then, you will have psi of x of t is small psi of x time's a function of time, which can be shown to be equal to minus e i E t by h cross, where psi of x satisfies the the following eigen value equation minus h cross square by 2 mu d 2 by d x square.

Now, it is the total differential plus V of x psi of x, this we had done even in our last lecture, psi of x is equal to E times psi of x, this is the time independent Schrodinger equation (Refer Slide Time: 05:06), time independent Schrodinger equation.

Actually this is an Eigen value equation as I had mentioned last time that, H psi of x is equal to E psi of x, E is a number and H is the operator. And for a given potential we have certain discreet values of E, for the particle in a box problem we had seen that, that the energy Eigen values were something like this (Refer Slide Time: 05:42), E is equal to E n is equal to n square of E 1 we got discreet energies in a box. For the free particle problem all values of E greater than 0 were allowed.

So, we will solve this equation when V of x is equal to half mu omega square x square. So, we rewrite this equation for V of x is equal to half m omega square x square and if we do that, let me align this properly.

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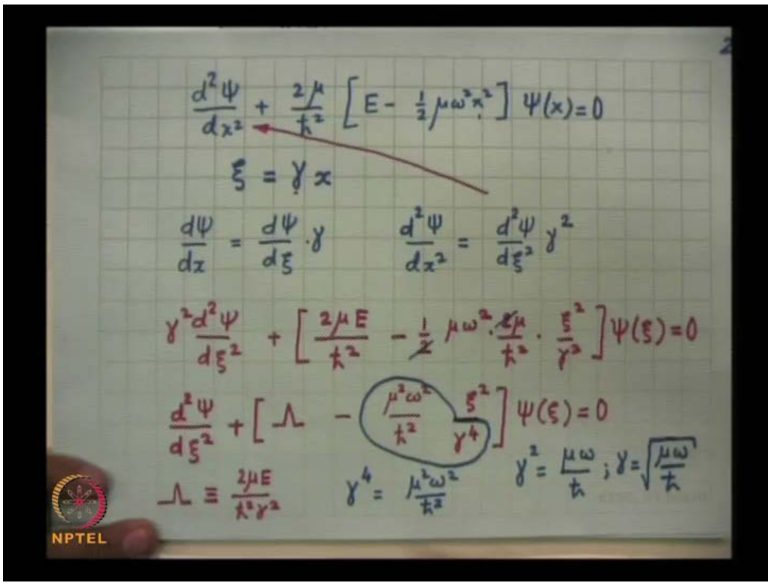

So, we will have d 2 psi by d x square plus 2 mu by h cross square E minus half mu omega square x square psi of x is equal to 0, this is the you just rearrange the top equation and you will get this particular equation. So, let me put it in a little convenient form, I introduce a dimensionless variable Xi which is proportional to x, gamma is the proportionality constant and I will decide on its value little later.

So, d psi by d x, if I introduce this independent variable, so d psi by d Xi into d Xi by d x, which is equal to gamma I differentiate it again I will get d 2 psi by d x square is equal to d of d Xi of this, multiplied by d Xi by d x; so, this will become d 2 psi by d Xi square multiplied by gamma square.

So, what I do is, I substitute this in this equation (Refer Slide Time: 08:10), in this equation and obtain please see this d 2 psi by d Xi square multiplied by gamma square I take 2 mu inside, so I will get 2 mu E by h cross square minus half mu omega square and then, I multiply by 2 mu by h cross square, so 2 mu by h cross square; and x here is Xi by gamma, so Xi square by gamma square. Now, psi is a function of Xi now.

So, I divide by gamma square, so I will get if I divide the whole equation by gamma square, so d 2 psi by d Xi square plus let us suppose I write this as lambda, where lambda is defined when I have 3 equal to sign here, it implies define to be equal to (Refer Slide Time: 09:30). So, this will be equal to 2 mu E by h cross square gamma square. Then, if you see this carefully then 2 2 cancelled out, mu square omega square by h cross square. So, this will be lambda minus mu square omega square by h cross square, then gamma square comes here, so it will be Xi square by gamma to the power of 4. So, there is already a gamma square here, so I multiply this by gamma square, so it becomes gamma to the power of 4. So, this multiplied by psi of Xi this is equal to 0.

I still do not know the value of gamma, I have not yet defined yet, I now choose gamma such that, this quantity is equal to 1 (Refer Slide Time: 10:40). So, that the coefficient of Xi square is unity. So, I choose gamma to the power of 4 is equal to mu square omega square by h cross square. So, if I take the square root then, I will get gamma square is equal to mu omega by h cross. And so, therefore, you will have gamma is equal to I define this equal to mu omega by h cross, this is the definition of gamma. So, if I choose gamma equal to under root of mu omega by h cross then, this quantity which I have encircled with blue becomes 1.

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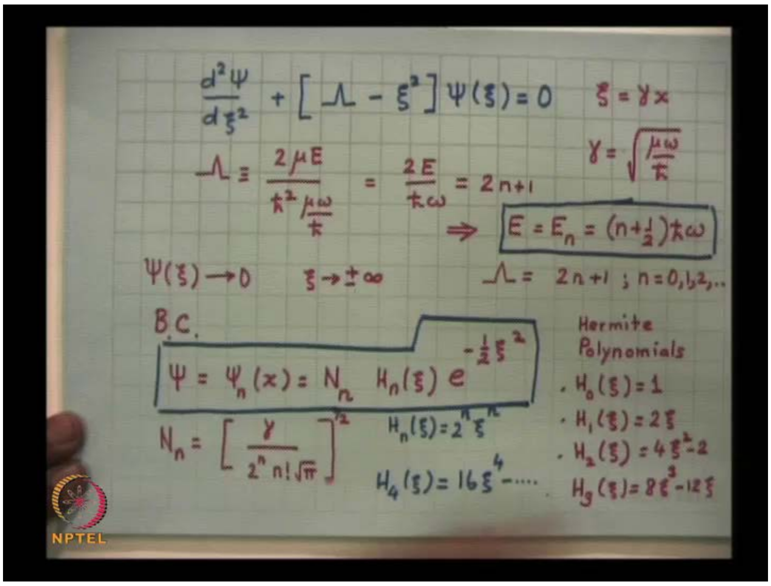

And so the Schrodinger equation becomes, if you see this carefully that, the Schrodinger equation then becomes d 2 psi by d Xi square plus lambda and this quantity is 1 (Refer Slide Time: 12:01), so this is Xi square psi of Xi is equal to 0.

So, I have the two definitions, the one definition is that of gamma, what Xi is equal to gamma times x, where gamma is equal to square root of mu omega by h cross, this is a dimensionless quantity (Refer Slide Time: 12:31), x has the dimensions of meters. So, gamma has the dimension of meter inverse and lambda is defined to be equal to 2 mu E by h cross square then gamma square; as you can see here (Refer Slide Time: 12:54), lambda is equal to 2 mu E divided by h cross square gamma square, so this becomes mu omega by h cross, so mu mu cancel out; so you will get 2 E by h cross omega.

So, therefore, we can reduce the the transform the Schrodinger equation we can transform the Schrodinger equation to this particular form (Refer Slide Time: 13:28). Now, we will later solve this equation we will find that, if I assume that psi of x or psi of Xi is made to go to 0 as x or Xi tends to plus infinity or minus infinity, this is known as the boundary condition these are known as boundary condition. So, if I make if I impose the condition that, the wave function has to go to 0 as x goes to plus infinity or minus infinity only then it will be a square integrable function, only then it will be a well behaved function.

So, if I impose this boundary condition we will find that, lambda must take only an odd integer 2 n plus 1, where n is equal to 0, 1, 2 etcetera, these are the Eigen values of this equation (Refer Slide Time: 14:58), this concept must be clear. These are the Eigen values of this problem that is only for lambda equal to 1, 3, 5, 7, 9 and so on, will the solution of the Schrodinger equation be well behaved at x equal to plus infinity and x is equal to minus infinity, for all other values of E there will exist a solution, but which will blow up at infinity.

So, therefore, these are the Eigen values of the problem and since lambda is equal to 2 E by h cross omega, so this means that, this quantity must be equal to 2 n plus 1 and therefore, this gives us the important result that, the energy Eigen values are quantized we will have E is equal to E n n plus half h cross omega. I must repeat I have not solved this equation yet, but I plan to do that later in one of the later lectures.

Today I will just give you the solution and try to make you understand the importance, the physical significance concept behind the solution. For these values of E, the Eigen functions are given by psi n of x is equal to some normalization constant N and these are H n of Xi e to the power of minus half Xi square; these are the normalized Eigen functions corresponding to the linear harmonic oscillator problem.

This is of course, a Gaussian function and H n of Xi are known as the Hermite polynomials after the French mathematician Hermite, these are known as Hermite polynomials. In fact, H 0 of Xi is equal to 1, H 1 of Xi is equal to 2 Xi, may be you have read this in your mathematics class H 2 of Xi is 4 Xi square minus 2 and H 3 Xi is 8 Xi cube minus 12 Xi I wrote four of them to show that, they are alternatively even and odd, this is even, this is even, this is odd and this is odd, these are known as Hermite polynomials (Refer Slide Time: 18:20).

And the normalization constant I think you must remember this, gamma divide divided by 2 to the power of n n factorial square root of pi. There is one important property that may help you to remember the Hermite polynomials that, H n of Xi the maximum power of Xi will be Xi to the power of n and the coefficient of that will be 2 to the power of n. So, please see this, for n equal to 0, 2 to the power of 0 is 1.

For n equal to 1, this is 2. For n equal to 2, this becomes 2 to the power of 2, so 4 Xi square; n equal to 3 this becomes 8. For H 4 of Xi for example, you can write down immediately that the first term will be 16 Xi to the power of 4 and so on. So, the coefficient of Xi to the power of n in H n of Xi is always 2 to the power of n. So, therefore, these are the discreet Eigen values of the problem (Refer Slide Time: 20:17), and these are the normalized wave functions of the problem.

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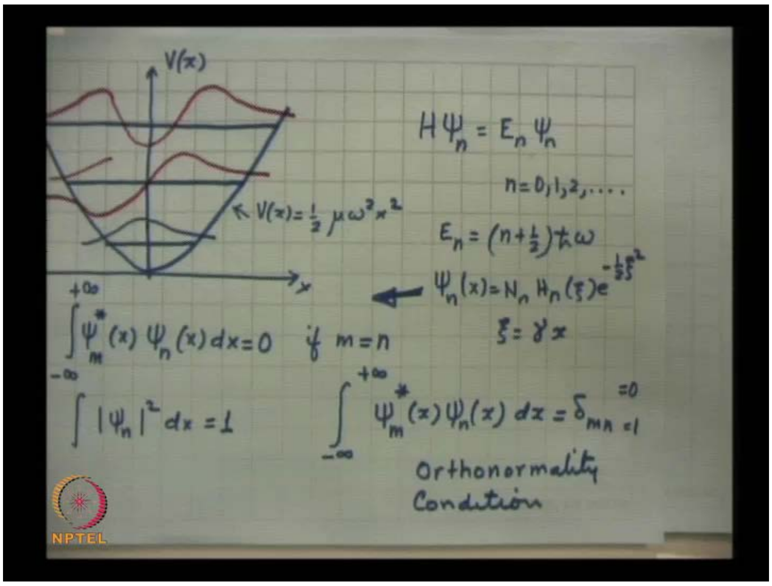

So, therefore, if I if I plot the plot the potential. So, it will be the parabolic potential, so this is my V of x and this is x, so the discreet energy states are half H cross omega, 3 halves H cross omega, 5 halves H cross omega and so on. And the first one is just a Gaussian function, the second one is a Gaussian function multiplied by Xi, so it will be sorry it will be something like this (Refer Slide Time: 21:15), and then this will be something like (Refer Slide Time: 21:24), these are alternately symmetric, anti symmetric, symmetric I will prove that theorem little later.

So, this is my V of x is equal to half mu omega square x square. So, we have we have now proved that, we have the expressions for H psi n is equal to E n psi n, n takes values 0 1 2 3 to infinity, so there are infinite number of wave functions; and E's of n is equal to n plus half H cross omega. And psi n of x as I told you, these are N's of n H n of Xi e to the power of minus half Xi square, where Xi is equal to gamma x.

As I had discussed in my last lecture, these wave functions form a set of orthonormal functions; and the coefficient N's of n that is orthonormal function means psi m star of x psi n of x d x is equal to 0, if m is not equal to n, all limits as before are from minus infinity to plus infinity. And when m is equal to n, mod psi n square d x I always I can always choose the value of N's of n such that, this integral is 1.

And in fact, the N's of n are chosen in such a way that, you have this condition that minus infinity to plus infinity psi m star of x psi n of x d x is equal to the Kronecker delta function. So, this is equal to 0, if m is not equal to n; and is equal to 1, if m is equal to n. So, this condition is known as the Orthonormality conditions this condition and these wave functions will satisfy satisfy this Orthonormality condition.

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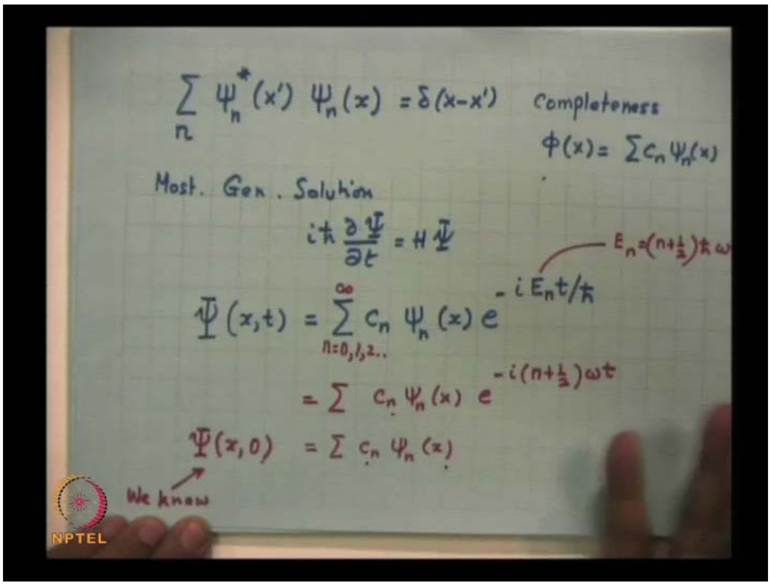

There was yet another condition that we had said and that was, that was the completeness condition that summation psi n star of x prime psi n x summed over n, this is equal to delta of x minus x prime, this is the completeness condition. This tells us that, any well behaved function of x any well behaved function of x like phi of x can always be expanded in terms of this complete set of something like the Fourier series, that in a particular interval any well behaved function can be expanded as you may have already read in terms of the Fourier series.

Similarly, in this space defined by from minus infinity to plus infinity, the Hermite gauss function that I had written down the Hermite gauss functions are, these these are known as the Hermite gauss functions (Refer Slide Time: 25:31). So, the Hermite gauss functions form a complete set of function that, any arbitrary function any arbitrary square integrable single valued function can be expanded in terms of this function. And therefore, the general solution the most general solution most general solution of the one dimensional Schrodinger equation that is i h cross delta psi by delta t is equal to H psi, where H corresponds to the linear harmonic oscillator problem will therefore, be given by psi when I write capital psi; that means, it is a function of x and time.

When I write small psi then, that is a function of x only. So, psi of x comma t is equal to a superposition that is C n just as we had done for the particle in a box problem e to the power of minus i E n t by h cross. But in this case and the summation is over n equal to 0, 1, 2, 3 to infinity in this case, I know the values of E n (Refer Slide Time: 26:59). E n is equal to n plus half h cross omega, so this becomes this becomes equal to summation C n psi n of x e to the power of minus i n plus half omega t, this is the most general solution for the harmonic oscillated problem. That is any state of the oscillator can be described by this sum, and this is how it will evolve with time.

So, let me give you an example. So, let us suppose I tell you and I will give you an example with which all of you will be familiar. Let us suppose I know the wave function at time t equal to 0, so this we know I know the state of the oscillator at time t equal to 0 and I want to find out that, how will the wave function sorry how will the wave function evolve with time, that is my question.

So, I substitute t equal to 0 here and I will get C n psi n of x I know this function (Refer Slide Time: 28:44). So, using this equation I will find out C of n, I know psi of x comma 0, I know psi n of x these are the Hermite gauss functions. So, I will now use the Orthonormality condition to determine C of n. And then, I will substitute it in this equation and carry out the summation. In most cases, it is not possible to obtain an analytical form, but in some situations, it is possible to sum it analytically and give a physical description of this system.

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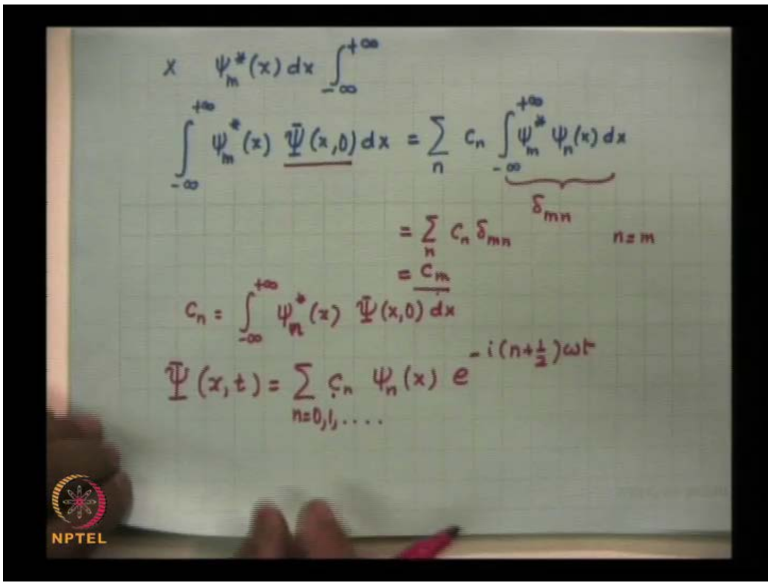

So, I come back to this equation and and I am sure all of you know this, let me let me write this psi of x comma 0 is so much. So, how do I find this, I am sure you know this we have done this before I multiply both sides by psi m star of x. So, I multiply both sides multiply both sides by psi m star of x d x and carry out the integration from minus infinity to plus infinity.

So, the left hand side will become all limits are from minus infinity to plus infinity, psi m star of x psi of x comma 0 d x is equal to summation (( )) integral psi m star of x psi n of x d x, this we know from the Orthonormality condition. This is equal to delta m n (Refer Slide Time: 31:06). Therefore, this equation becomes say, right hand side becomes C n delta m n, m is a fixed number, n is a running index, n goes from 0 to infinity.

So, in this series only the n equal to m term will survive and all the other terms will be 0. So, this will be equal to C's of m, so C of m will be given by this. And so, therefore, once I know C of m, so I can so I know that, C of n is equal to minus infinity to plus infinity psi m star sorry psi n star now of x psi of x comma 0 d x. If I know C of n then, in principle not in principle actually I can substitute it psi of x t is equal to C n psi n of x e to the power of minus i n plus half omega t. There is one more thing that, I I would like so, therefore, if I know psi of x comma 0 I can find out C of n. Once I know C of n I substitute it here and carry out the summation.

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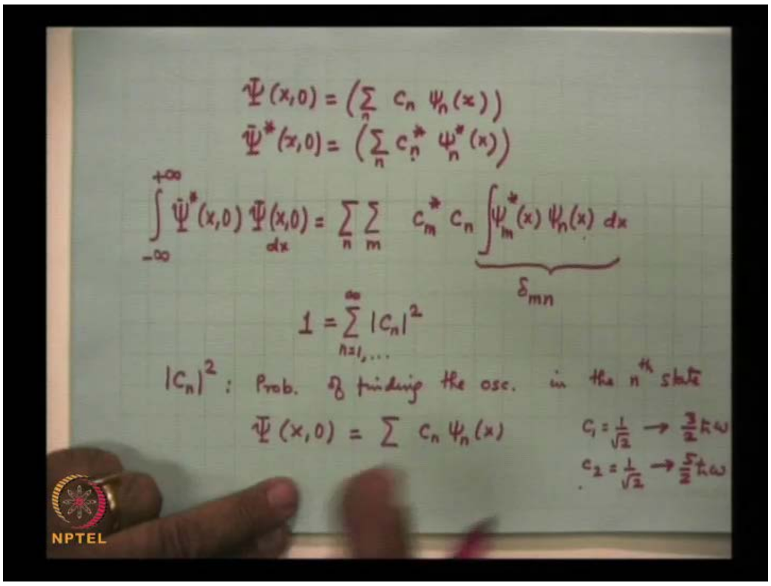

Before I proceed further let me mention one more thing that, psi of x comma 0 we wrote as C n psi n of x. The complex conjugate of that is, psi star x comma 0 is equal to C n star psi n star x. So, if I multiply this by this, when I have to multiply this sum with this sum, so that this end does not get both summations are over n, this does not get confused with this I replace n here by m.

So that, what I am trying to tell you is, psi star x comma 0 multiplied by psi of x comma 0, if I multiply the two equations and integrate. So, I will have two sums, one over n and one over this n, but this n should not get confused with this n. So, I will write here as m C m star C n psi m star x psi n of x, if I multiply by d x and carry out the integration then, these are the only two quantities, which depend on x, so I can carry this.

But the limits are all of course, from minus infinity to plus infinity, this from the Orthonormality relation we know that, this is equal to delta m n. So, only the m equal to n term will survive and therefore, this will be modulus of C n square. So, therefore, if initially I choose a wave function, which is normalized then mod C n square is 1, summation 1 2 3 and therefore, we can interpret C n square as the probability of finding the oscillator in the n-th state.

So, C n square will be the probability of finding the oscillator in the n-th state and this does not change with time (Refer Slide Time: 36:17). So, when I write psi of x, this is a characteristic of quantum mechanics, that is system can be in a superpose state. So, it is a superposition of various states. So, let us suppose C 1 is equal to 1 over root 2, C 2 is equal to 1 over root 2, so only two states are superposed. The probability of finding in the n equal to 1 state is half; probability of finding in the n equal to 2 state is half. So, the Eigen value corresponding to this is 1 plus half that is 3 by 2 h cross omega and the eigen value corresponding to 5 by 2 h cross omega.

Then you ask yourself the question that, does the oscillator have a given fixed energy? The answer is no. There is a half probability of finding 3 by 2 h cross omega, that is if you make a measurement of energy, then there is a half probability of finding 3 by 2 h cross omega; and there is a half probability of finding 5 by 2 h cross omega, this is the concept of superposition, which is one of the most important parts of quantum mechanics, one of the important aspects of quantum mechanics, it can be in a superposed state.

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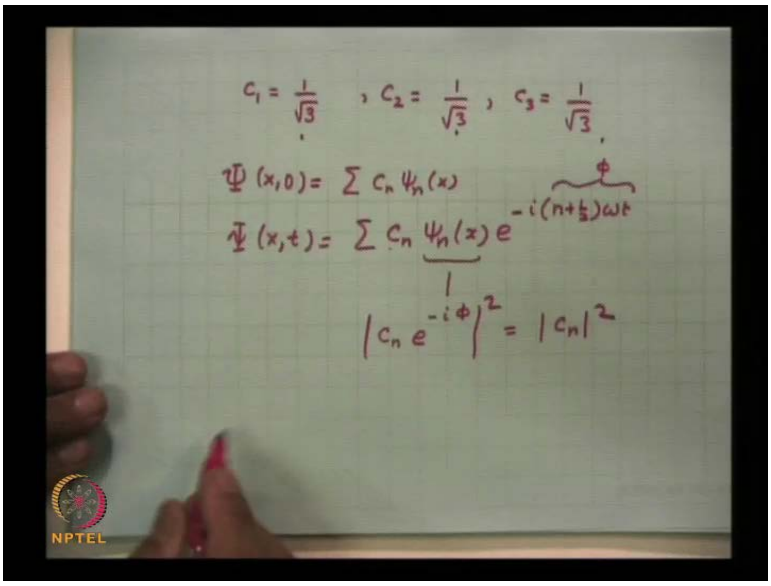

Let me take another very simple example that C 1 is equal to let us suppose 1 over root 3, C 2 is equal to say 1 over root 3 and C 3 is 1 over root 3, I have taken a very simple. So, there is a one-third probability of finding it in E 1, one-third probability of finding in E 2 and one-third probability of finding in E 3 and these probabilities will not change with time. And therefore, these states are known as stationary states, the probability of finding them in that state will not change with time.

However when I wrote down psi of x comma 0 is equal to C n psi n of x and psi of x comma t will be equal to how will it evolve with time, psi n of x e to the power of minus i n plus half omega t, the phase factor will change, the coefficient of psi C n the sorry the coefficient of C n the modulus square of that does not change with time. So, C n times this C n times the exponential factor let us suppose this I write as phi. So, minus i ph the modulus square of that is mod C n square. So, that does not phi depends on time that does not change with time, but different states will superpose with different phases at different times. And I will tell you a consequence of that.

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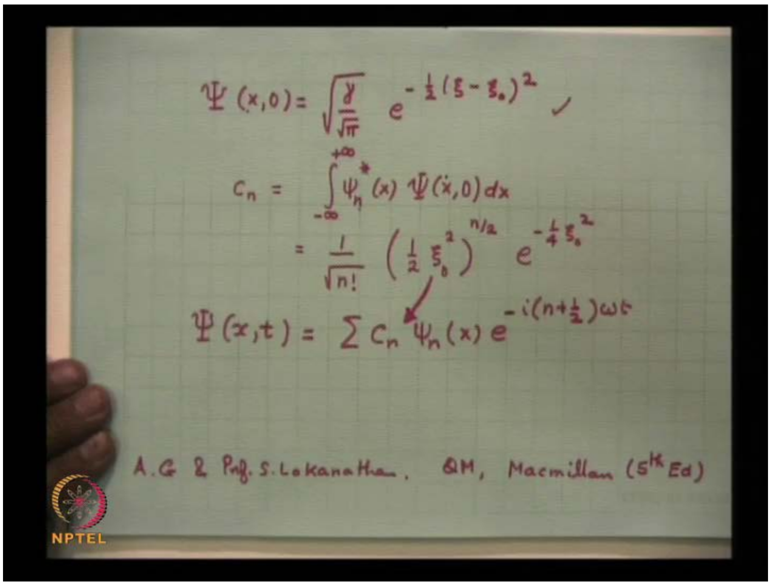

Now, let us suppose that, I choose a system I have my initial oscillator in this state that psi of x comma 0 is equal to say, this is a normalized function gamma under root of pi e to the power of minus half Xi minus Xi 0 whole square. If Xi 0 is 0, then this is just the ground state wave function. But, I assume that, Xi 0 is finite and this is the initial state of the system at time t equal to 0, the wave function describing the harmonic oscillator is given by this equation.

And my question is that if this is so, then how will this wave function evolve with time? And the answer is simple I first calculate C's of n that is psi n star of x psi x comma 0 d x, the algebra is bit complicated involved, it is given in many books including my book with professor Lokanathan; and the final result is that, slightly complicated square root of n factorial half Xi square raised to the power of n by 2 e to the power of minus quarter

Xi naught square. So, this is if psi of x comma 0 at time t equal to 0 is given by this expression then, the coefficient C of n if you substitute this expression in this and use for psi and f x the Hermite gauss function carry out the integration from minus infinity to plus infinity, it is little cumbersome, but very straightforward.

And it is given in many books including the the in my book that is myself and Professor Lokanathan S Lokanathan quantum mechanics and published by Macmillan, so all the details details steps are given there, this is the 5 th edition. Now, the next step will be to substitute this, so I want to now find out how it will evolve with time psi of x t. So, what I will do is, I will substitute in this equation, this has been the general recipe and sum it. So, I substitute this expression here (Refer Slide Time: 44:12), and carry out this summation. And it is one of those rare occasions that, you can actually sum the series and the final result is very straight very very beautiful.

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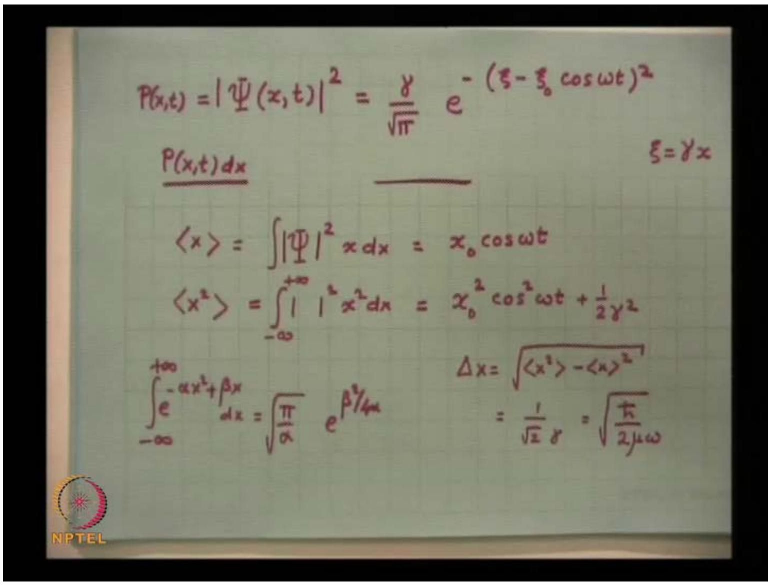

So, you will get if I sum that and take the complex conjugate of that and then, multiply that is mod psi x t square is equal to gamma by root pi e to the power of minus Xi comma Xi 0 cos omega t whole square. So, this is the probability distribution, p of x comma t, d x represents the probability of finding the particle between x and x plus d x. So, that probability distribution function is actually oscillating with time and I will show this to you in a moment graphically.

If I take if I calculate the expectation value of x, the average value of x then that as we all know was psi square x d x I multiply this by x you may remember that, Xi is equal to gamma x you can carry out this integration very easily just a few steps and you will find that, this will be equal to x naught cos omega t. And similarly, if you find calculate x square, so you will find that, mod psi square x square d x you will find that, this is equal to x naught square cos square omega t plus 1 over 2 gamma square.

I would request all of you to work out these integrals, this is very straightforward and you only have to use the integral that, e to the power of minus alpha x square plus beta x d x the same integral that I had mentioned couple of times before pi by alpha e to the power of beta square by 4 alpha, all limits are from minus infinity to plus infinity. So, then you can calculate delta x is equal to x square minus x average square under the root. So, this is this minus x 0 square cos square omega t, so this term will cancel out with this. So, I will get 1 over root 2 gamma and so this will be under root of h cross by 2 mu gamma. Let me show you the temporal evolution and then, we will come back to this slightly later.

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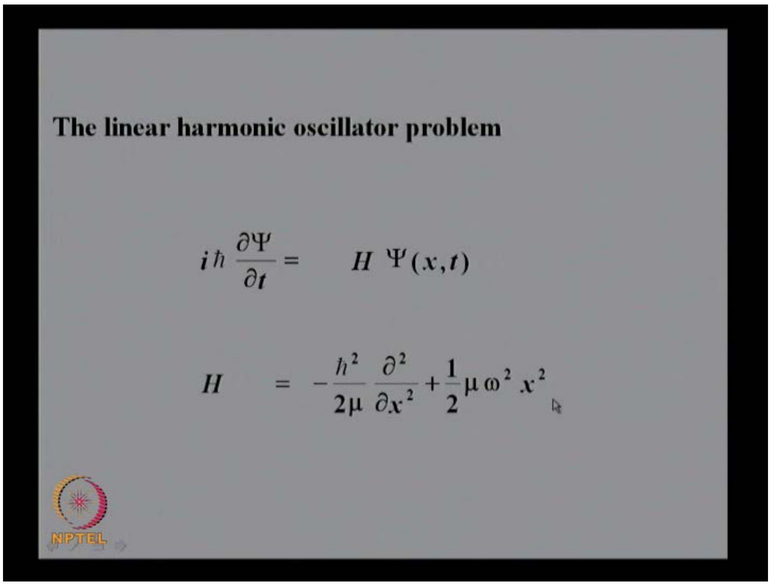

So, we discussed we were we have been discussing the harmonic oscillator problem. And as I told you this is just briefly going through what I have already done. So, this is my Hamiltonian (Refer Slide Time: 47:54), this is p square by 2 m and this is the potential energy function half mu omega square x square, this is the potential energy function corresponding to the oscillator.

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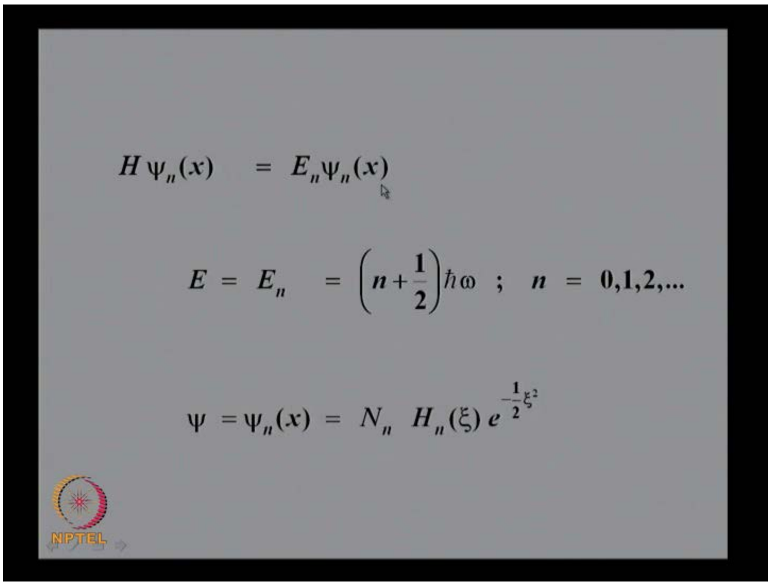

So, you have (( )), so you solve this Eigen value equation and you find that, for the wave function to be well behaved we will do this in detail that, the energy Eigen values are n plus half h cross omega; and the corresponding wave functions are the Hermite gauss functions, these are known as the Hermite gauss functions.

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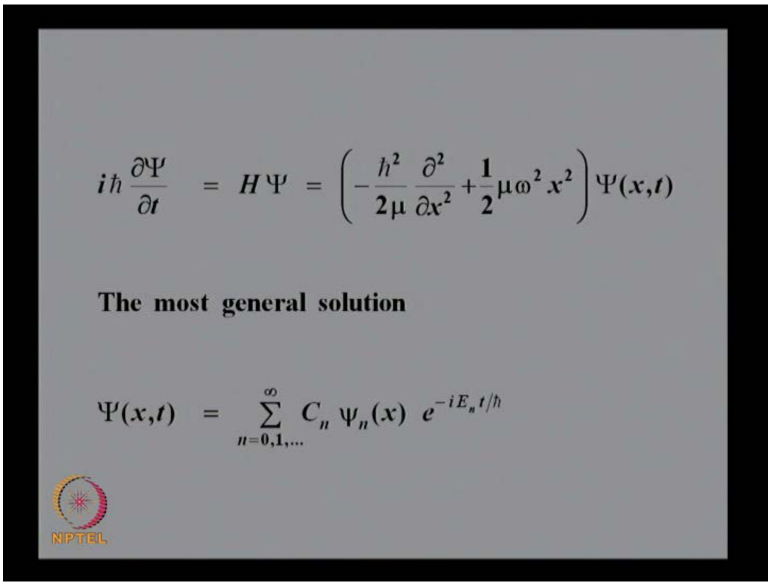

So, therefore, of the time dependent Schrodinger equation, this is the one dimensional time dependent Schrodinger equation (Refer Slide Time: 48:54), the most general solution will be given by this (Refer Slide Time: 49:03), these are psi n of x multiplied by this time or something like the modes of the system, the normal modes of the system. As you may have seen that, if you have a stretched string, it has certain normal modes of the system, these I have worked out in my book on optics and then, you displace the string in a particular way and then, you want to study it's time evolution.

So, you have to express this as a superposition of the normal modes of the system and therefore, these are the the psi n of x and these are the corresponding frequencies of normal modes, so this is how it will evolve with time. So, we this is the most general solution of the time dependent Schrodinger equation. Let me come back to this in a moment, but tell you we as I had told you earlier we have developed this software and these are the wave functions (Refer Slide Time: 50:23), so this is the ground state wave function and the energy eigen value is 1 times E 0, E 0 is half h cross omega.

Let me plot the first Eigen function, so this an anti symmetric function (Refer Slide Time: 50:41). So, the Eigen value is 1 plus half h cross omega 3 by 2 h cross omega and you can see, this is an anti symmetric function. The second state, which corresponds to n equal to 2, so the Eigen value is 5 by 2 h cross omega, so this is the second function second Eigen function (Refer Slide Time: 51:12). Then the third, this is an anti symmetric. So, alternately the wave functions are symmetric and anti symmetric, symmetric and anti symmetric.

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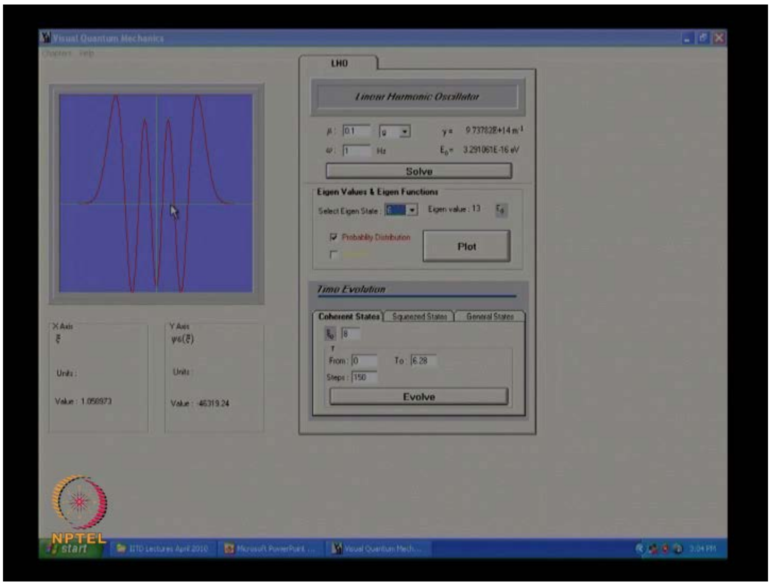

The fourth in fact, the fourth wave function has 4 zeros has 4 zeros and similarly, the fifth and the sixth, so these are the normalized Hermite gauss functions, which are the Eigen functions and these are the corresponding Eigen values n plus half h cross omega. So, these are this is for example, the psi psi 6 of x this is a this is a symmetric function of x, so it is 1 2 3 4 5 6 this. So, we will come back to this in a moment. So, these are my Eigen functions (Refer Slide Time: 52:21).

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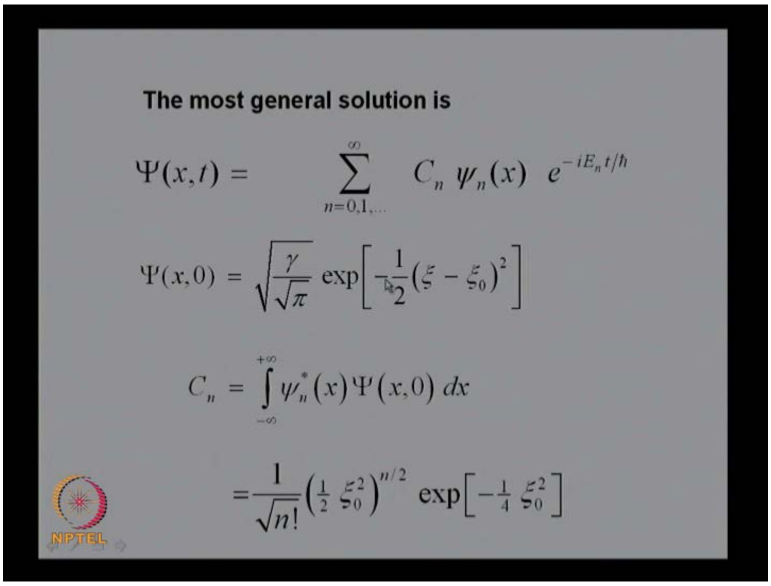

Now, as I mentioned before let us assume a particular case, that at time t equal to 0 the wave function is given by this expression (Refer Slide Time: 52:39), it is a displaced Gaussian it is a displaced Gaussian. Let me tell you in advance this is known as a coherent state of the oscillator. So, I want to, if this is my question is, if this is psi of x comma 0, what will be psi of x comma t? And the answer is simple, I first find out C's of n, hopefully I can find an analytical expression and then, substitute it back in this equation and carry out the summation.

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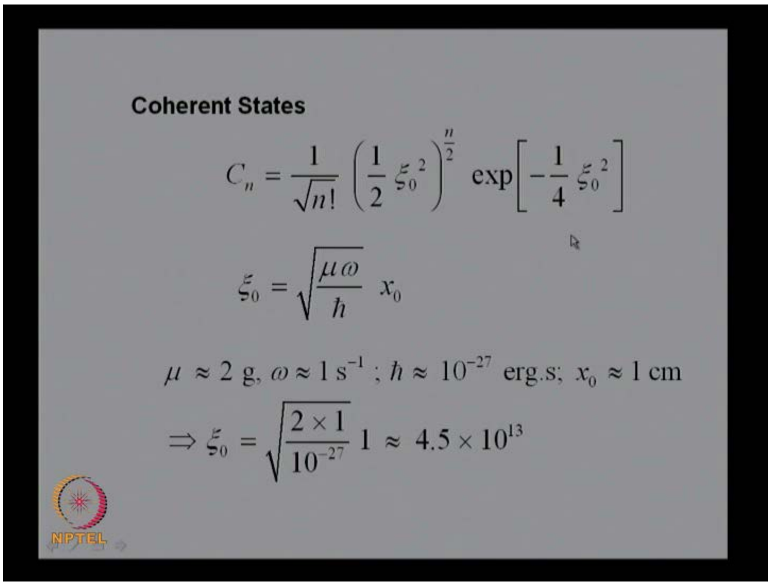

So, in this particular case in for this particular form of psi of x comma 0, I know I can determine analytical expression for C of n I substitute it here, carry out the summation and and I will obtain psi of x comma t mod square is equal to this (Refer Slide Time: 53:49), which represents the classical oscillator.

So, if I do this, this is known as a coherent state. Let me then let me then tell you that, if I superposed different states, so I am trying to superpose two states with the following coefficient. Let us suppose the first state 0.707 is 1 over root 2 and 0.707 is 1 over root 2, so there is a half probability of finding in the first state, half probability of finding in the second state. So, let me ok this and then evolve. So, this is how the probability distribution will dance (Refer Slide Time: 54:39).

So, let me make it slowly. So, you can see that, this is how the probability distribution will evolve (Refer Slide Time: 54:50). So, is the system has a particular energy? The answer is no, there is a half probability of finding E 1 and half probability of finding E 2. Let me do this once again, but let me put C 1 equal to 1 and C 2 equal to 0, so that it is in the first state, then what will happen is, if I evolve it will remain the same psi of x comma t whole square remains the same for all times, it does not change and that is because it is a stationary state, I hope you understand.

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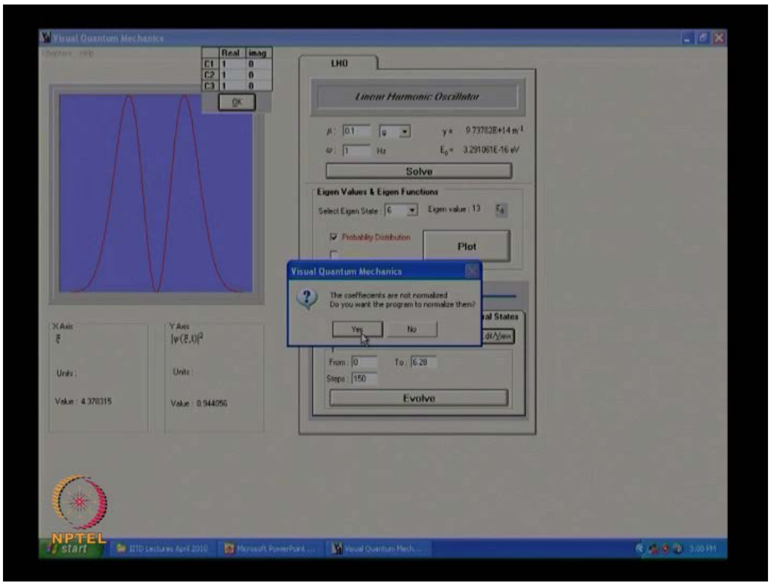

So, let me take say three states. So, let me have C 2 is equal to 1 and C 3 also equal to 1, equal probabilities of three states, so they are not normalized because, there should be actually 1 over root 3, 1 over root 3, 1 over root 3; so, the coefficients are not normalized. The software is asking me, do you want the program to normalize them? Let say yes and let us evolve, so this is like this (Refer Slide Time; 56:17), this is how the state of the oscillator will evolve with time.

Now, I go back to my coherent state and I say that, let C of n be given by this (Refer Slide Time: 56:40), only thing is I have to give a value of Xi 0 do you understand this, I then say let it be choose C n equal to so much, C 1 equal to so much, C 2 equal to so much, C 3 equal to so much, C 4, C 0 equal to so much, let C n be like this (Refer Slide Time: 57:04), it is a superposition of an infinite number of states.

But, I have to give the value of Xi 0 let me do that. So, that is known as a coherent state and I choose the value Xi 0 equal to 8 and I evolve with time, this is my classical oscillator (Refer Slide Time: 57:31). So, let me make Xi 0 large as I will tell you in a moment let me take it 100 and you will find that, this dances back and forth. So, let me make it in little more time steps, see this (Refer Slide Time: 57:48), so this is my classical harmonic oscillator.

So, we when you see in your first year laboratory, a long pendulum oscillating like this and you ask yourself the question, that in quantum mechanics we have solved this, to which Eigen value does it belong? The answer is, it does not belong to a particular value of E, it is a superposition of a very large number of states and these different states superposed so beautifully with time, with different phases that the wave packet oscillates back and forth just as you would see in a classical harmonic oscillator.

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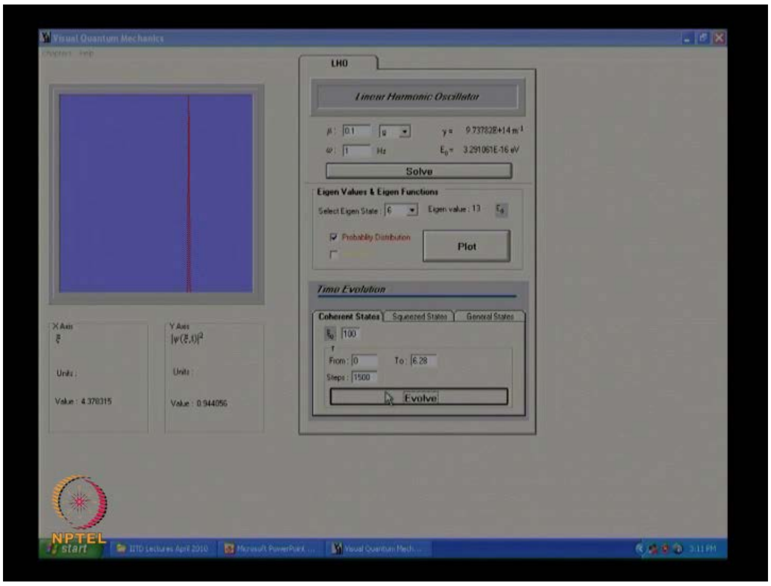

So, if I make it still these values of Xi 0, I will explain that in my next lecture, so then it will become even sharper. And let me make it in in larger number of time steps, this is a software which is available with my book on basic quantum mechanics, which is also been published by Macmillan. So, you will see that (Refer Slide Time: 59:14), the wave packet oscillates back and forth.

So, therefore, I end this lecture by mentioning that, my classical oscillator when it is moving back and forth to which energy does it correspond to and I will detail it in my next lecture that, it does not correspond to a particular energy, it is a superposition of billions of states, but but very closely packed energy, closely spaced energy levels. Does it have a definite energy? The answer is no, but the (( )) in the energy is extremely small. We will discuss this in more detail in my next lecture, thank you.