Quantum Electronics
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Module No. # 03
Second Order Effects
Lecture No. # 09
Non - Linear Optics (Contd.)

We continue our discussions on second harmonic generation. Do you have any questions?

Given an example of a crystal, in which k vector is along, at angle psi m to z and, it is in a plane, at some angle, and then E x comes out to be E i psi phi 1 and E y is equal to E 1 plus phi, and then later we used - Yeah, that is KDP, yes - So, I could not get the… So, from where do I get the psi m and p n and… after that?

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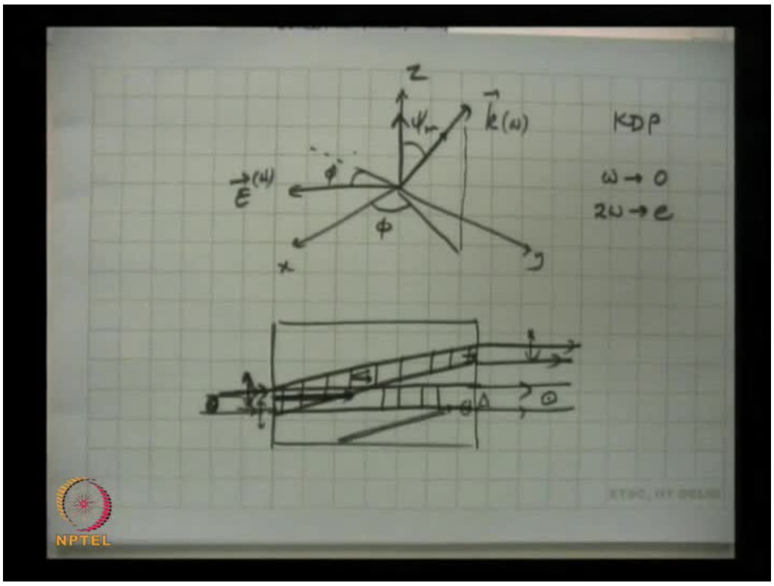

No, what we did was, it was an example of KDP; so, if you recall, we need to propagate at some direction to the optic axis, to achieve phase matching; and then, an angle was obtained as an angle psi m with the optic axis. So, this is the direction of propagation of the fundamental wave. We also found out that for KDP, the fundamental wave at frequency omega is an ordinary wave, and the second harmonic will be an extraordinary wave for phase matching. So, I consider a general direction of propagation making an angle psi with the optic axis, with the projection of the k vector on the x y plane making an angle phi.

Now, first of all, I know that omega is an ordinary wave; so, what will be the electric field direction of the ordinary wave going along this direction? Ordinary wave, if you will have in electric field, perpendicular to optic axis, and to the propagation directions; so, it must be in x y plane. And in the x y plane, if I project the y axis at the back, this E vector will be like this; this angle will also be phi; because this is a vector, perpendicular to this direction and this direction, then, I get the electric field components along y and x, calculate the non-linear polarization, and I find the non-linear polarization has only as that component.

This non-linear polarization has a component along the propagation direction and a component perpendicular to the propagation direction. So, the component perpendicular to the k vector will be responsible for generating the second harmonic wave, along this direction. So, I take this sin psi m component of the total non-linear polarization along the z component to calculate the non-linear polarization that finally leads to my second harmonic generation, so that is why, the sin psi m comes from there.

Yes, Mohit.

Why is it necessary, that we need the polarization to generate the further wave at 2 omega frequency, in the same direction of propagation?

That is the direction in which I have phase matching.

Okay.

The omega direction, the k vectors of the omega and 2 omega, if they point in the same direction psi m, then only, I have phase matching between the omega and the 2 omega wave; otherwise, I do not have phase matching.

What is the physical consequence of an extraordinary wave, when we see that the energy vector - the pointing vector - points at a slightly different angle, than the propagation vector? So, how can we physically, I mean, how can we observe the difference? This, I mean theoretical analysis, we can…

You mean the k direction and s direction?

Yeah.

How can we observe the difference between k vector and s vector?

Yes.

I showed you the other day, one picture for an acoustic wave propagating in a medium. So, if you take, for example, if you take anisotropic medium and you launch a beam like this; suppose my optic axis is, suppose my optic axis is, some direction here, like this optic axis. So, I am launching a wave in this plane, what will be the polarization state for it to be extraordinary?

In this plane.

In this plane, right? Because the ordinary wave will be perpendicular optic axis and the propagation direction which is in this plane, and so, it will be like this - ordinary; extraordinary is in this plane; so, this wave comes in with this polarization state. Now, what will happen is… Let me ask you question. What determines the direction of the k vector as the wave goes from one medium to another medium?

Sir, perpendicular to the plane.

What should be conserved?

Energy.

Energy is all right. But, for example, when a wave comes from one medium to another medium, the wave refracts, either towards normal or away from normal. What is conserved in the k vector, as it comes from one medium to another medium?

Parallel component.

Parallel component of the k vector, please remember; expected does not come into picture; it is k vector parallel component which is conserved, because, you need to satisfy the boundary condition for all values of the coordinate on the surface.

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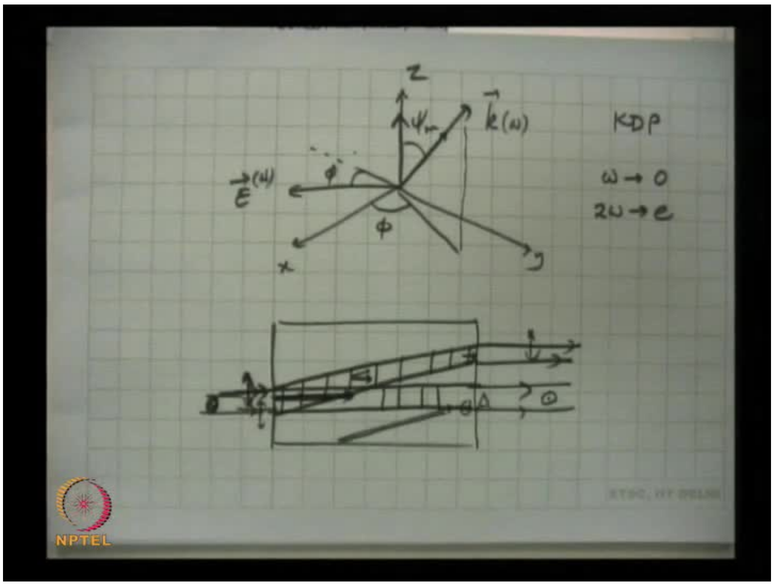

So, can you tell me, can you guess, what will be the direction of propagation to the extraordinary wave, k vector of the extraordinary wave, in this medium? Same direction? Because, the k vector, this is the surface, the component parallel or to, the k vector is like this, the component parallel to the interface is 0 of the incident wave; so, the component of the k vector of the refracted wave, parallel wave interface must be 0, that means, k vector must be like this for both the ordinary and extraordinary. The ordinary wave has also its s vector like this; so, it will go like this, the beam will go like this and come out.

The extraordinary wave also has its k vector like this, but because it is not propagating along the optic axis or perpendicular to the optic axis, its s vector is not parallel to the k vector, so this beam made, moves like this; this is the direction of propagation energy. Please note, if I draw the wave fronts of the extraordinary wave, it looks like this; the wave fronts of the ordinary wave is also like this; the k vector is always like this, but s vector is like this. And then, what happens here? Please note, k vector of this is like this.

And, it will come out like this; so, these two beams will be parallel. What will be the polarization state of this?

Perpendicular.

Perpendicular. This is ordinary wave; this will be perpendicular; so, this is a component which splits the incident components polarization components into 2 orthogonally polarized components. So, this beam, if you add an infinitely extended plane wave, you do not know where it is coming out from; you are not precisely defining a point on the beam. So, similarly, but to look at this, you need to look at a beam; and if you take a beam, the beam will propagate like this; and this is what I showed you in an image, two images - bi-refringence; so, you will see if you had a spot of light here, you will see 2 spots coming out.

And, if I rotate this crystal about, in this axis, in this plane, this beam will rotate; this image will not rotate; so, and the plane containing this beam and this beam is the plane containing the optic axis. So, this is, I will visualize that I take a beam of a finite cross section - this is the cross section - the beam, ordinary wave will move like this, the extraordinary wave will move like this, at an angle; and that, I showed you in a picture, one of this pattern, which actually, beam moves at an angle.

That is the incident beam at an angle into the incident surface, then, we have a splitting over there itself.

I need to look at… I need to make parallel component of k vector - continuous, and I will have a Snell's law, the same Snell's law.

Because, the refractive index is different for extraordinary and ordinary, we will have a splitting over there itself.

Over there itself, yes. The direction of k vector is itself will be different now, for the ordinary and extraordinary; the ordinary wave will have its k vector parallel to s vector; the extraordinary wave will have its s vector, even not parallel to its own k vector; so, there will be further refraction. So, I can actually… But Snell's law is simply the component of k vector parallel to the interface, should be continuous.

So, please note, that if I apply Snell's law to the direction of the wave front, I have no problem; but if I try to apply the Snell's law to the s vector, I do not satisfy; because, the incident ray is at a 0 angle and this refracted ray is a finite angle, it is not possible. But the Snell's law is for k vector, so, I can always use Snell's law to calculate the k vector direction of the refracted wave in the second medium; and, once I know the k vector direction and the optic axis direction, I can calculate the s vector direction, which will give me the direction of propagation of the energy or the ray in the medium. Yes, anything else?

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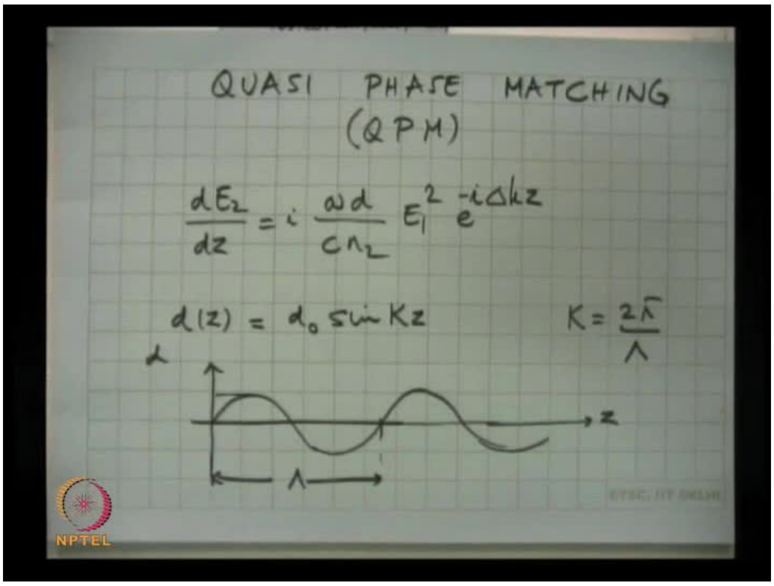

So, let us continue with the discussion on non-linear optics. So, what I want to bring in today is this concept of Quasi Phase Matching (QPM). So, one other techniques that is used to achieve phase matching is Birefringence phase-matching, in which, I use the anisotropy of the crystal; one of the waves is ordinary and the other wave is extraordinary; so, I use the extraordinary and ordinary refractive indices of medium to achieve phase matching between the 2 waves. But the problem is, there are many crystals like gallium arsenide, gallium nitride, zinc calоride; these are all cubic crystals which are isotropic but are non-linear.

So, I cannot use Birefringence phase-matching for this; not only that, it may be possible, that for a given set of wavelengths, I may not be able to satisfy this condition at all; as I showed you, there are some situations where the index surfaces do not intersect; then, I cannot use my Birefringence phase-matching. So, what do I do with this? There are other techniques and Quasi-phase-matching is a very important technique which actually was proposed way back in 1962 by Bloembergen and his group. And, it is… I will tell you, the, what is the basis for this.

Let us look at this equation again, d E 2 by d z is i omega d by c n 2 E 1 square e to the power minus i delta k z. So, if I solve this equation, if we solve this equation assuming no pump depletion, which means, E 1 is a constant and I got a solution for E 2, and that contained an sync factor, and that sync factor is coming from here. So, my objective is to eliminate this term from the equation; one way is to make delta k is equal to 0 which is phase-matching, Normal phase-matching; the other is…. Now, let me look at this following situation.

Suppose, I could generate a medium in which d was a function of z, a period function. So, let me take, for example, d 0 sin K z where K is equal to 2 pi by some period lambda, which means, I am trying to look a medium in which d goes like this; so, this is d as a function of z; d 0 is the maximum value here; and this distance is lambda periodic function of z, sinusoidal function of z. I will come to little later, how do I have a medium like this? So, let me assume… Then, I could, by some mechanism, produce a medium, in which d was periodically varying with z; d is equal to d 0 sin K z, so, let me substitute it to this equation.

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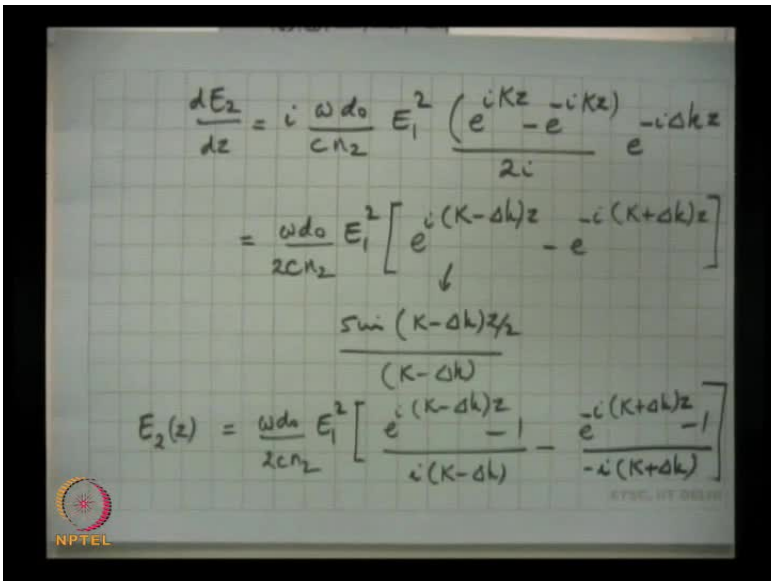

So, what happens to this equation? d E 2 by d z is i omega d 0 by c n 2 E 1 square, now, sin K z, I write as exponential i K z minus exponential minus i K z by 2 i into exponential minus i delta k z. So, this is equal to omega d 0 by, c n 2, 2 c n 2 E 1 square exponential i K minus delta k z minus exponential minus i K plus delta k z. Now, I can solve this equation again with no pump depletion, which means, assuming E 1 as a constant. So, what will this term give me? This will lead to another sync function; you will get something like sin K minus delta k z by 2 by K minus delta k…

Let me actually solve this equation. So, this is simply, if I integrate; so, if I integrate this equation what will I get? E 2 of z is equal to omega d 0 by 2 c n 2 E 1 square exponential i K minus delta k z minus 1 by i times K minus delta k minus exponential minus i K plus delta k z minus 1 divided by minus i times K plus delta k. Just the integral from 0 to z, and I am assuming E 2 at 0 is equal to 0; there is no second harmonic at the incident plane; there is only a fundamental; so, I can express these two in terms of sine functions.

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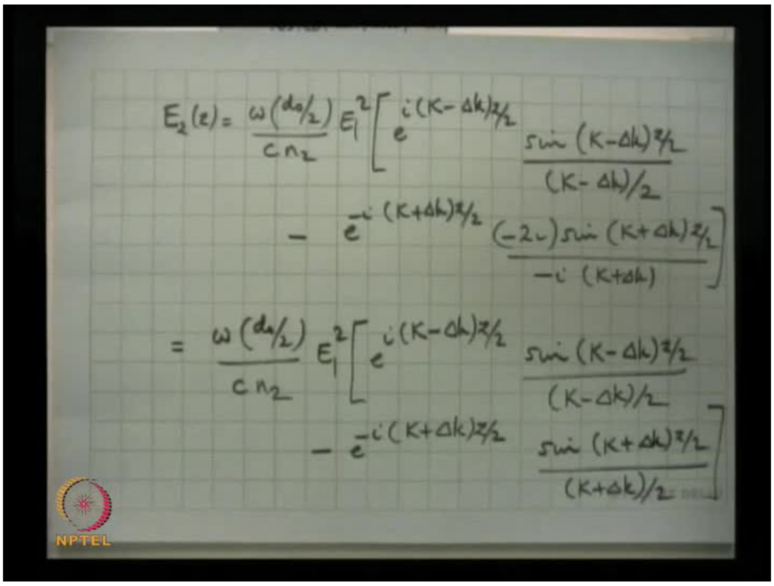

So, let me write this equation; so, I get E 2 of z is omega; let me write this, like this - d 0 by 2 by c n 2 E 1 square. Now, I will get 2 terms, the first term will be exponential, i K minus delta k z by 2 into sin K minus delta k z by 2 by K minus delta k by 2 because, this is 2 i; if I take this one of the factors out, this becomes 2 i sin K minus delta k z by 2; that 2, i take it with the denominator here and write it like this.

And then, the second term will be minus, so, this is actually, this is, exponential minus i K plus delta k z by 2 into minus 2 i sin K plus delta k z by 2 by minus i K plus delta k. So, this is actually omega into d 0 by 2 by c n 2 E 1 square into exponential i K minus delta k z by 2 sin K minus delta k z by 2 by K minus delta k by 2 minus minus i K plus delta k z by 2 sin K plus delta k z by 2 by K plus delta k by 2.

Is it all right? Yes, Mohit you are saying something? No, okay.

If I multiply by z in the denominator now; and the numerator, I get a sync function here, there is another sync function here.

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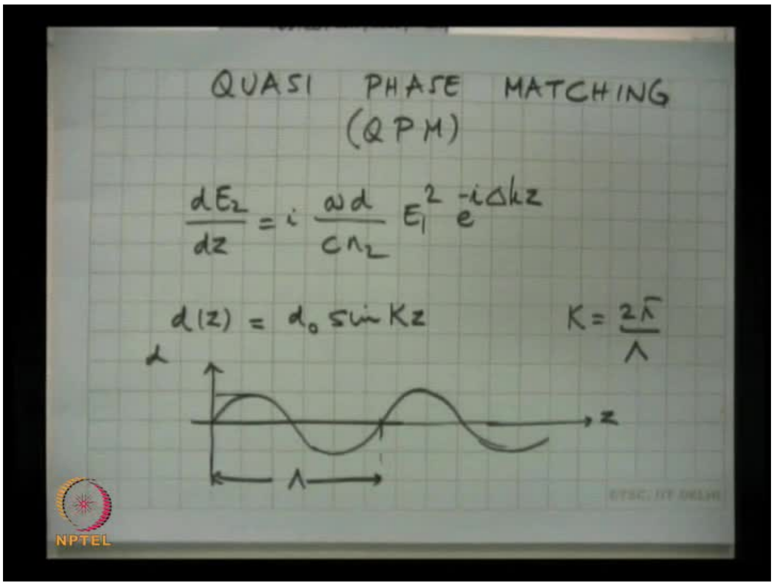

Now, look here; I have a sinc function or at K minus delta k - this capital K, remember, is the period, is the spatial frequency; lambda is the period; capital K, is called the spatial frequency, inverse of the period in space.

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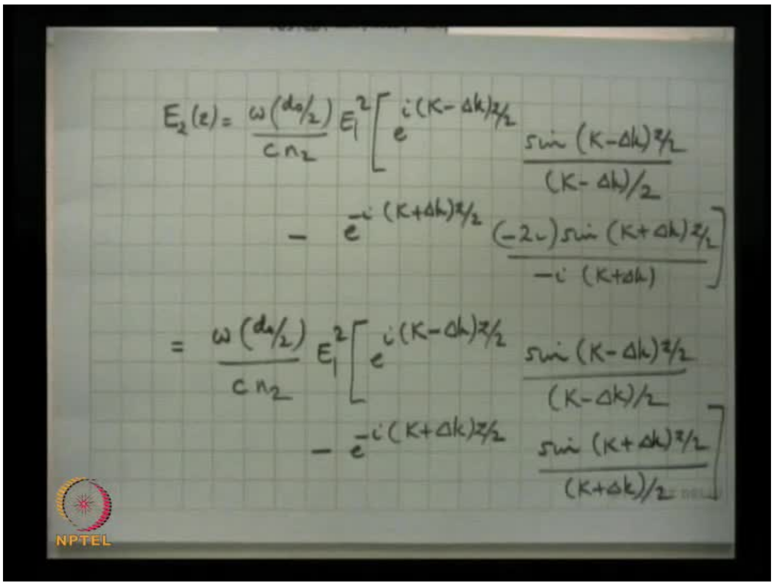

If I choose capital K is equal to delta k, this will give me z; this term will still be very small, because capital K plus delta k will not be close to 0; it will be close to 2 delta k; this term will become 0, this K minus delta k can be made 0 by choosing appropriately, capital K is equal to delta k. If I do that, this will become z; this will be some sync function; this will hardly contribute to my E 2; the main contribution will come from this term, and I will have some kind of a matching, phase-matching; I have gotten rid of one another terms exponential i delta k z.

What I have done is, I have, by taking a sinusoidal dependence of d, I have converted the exponential minus i delta k z to a sum of 2 exponentials; and one of them, I can make 1; if I choose capital K is equal to delta k, this is 1, this is not 1; and this 1 integrates to give me z; in fact, that is what is happening.

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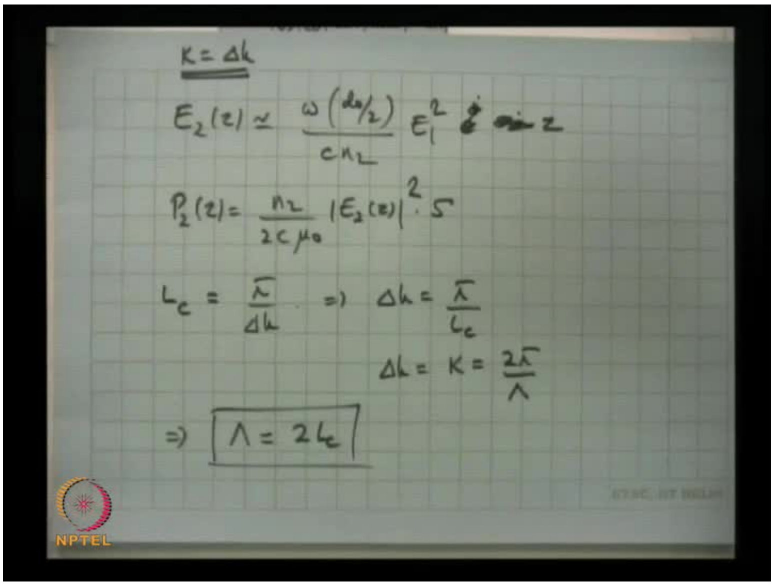

And, because capital K is in my control, if I can do this, if I can make a medium with this kind of a distribution of d, capital K is in my control; and, given a delta k, I can choose the capital K to make, capital K is equal to delta k; and this term will then give me maximum contribution, and so, what will be E 2 of z? Then, I can forget about the second term saying, that this will be very small; and I will have E 2 of z, will be approximately given by omega times d 0 by 2 by c times n 2 E 1 square exponential i, okay, k is equal to (()), so, that goes off; no, this is simply z.

So, I am assuming K is equal to delta k; so, this is exactly like phase-matching term. When the phase matching took place, I would have exactly got this, except that, now, I got d 0 by 2 instead of d 0 or d; the non-linear coefficient has now, actively become half of the original non-linear contribution, because it is a sinusoidal variation. But, it grows with z; it is not periodic, it is growing; so, the power in the second harmonic will now grow quadratically with z, at least for low conversion efficiencies; because, P 2 of z will be n 2 by 2 c mu 0 mod E 2 square into the area and it will be proportional to… All the factors you will get, exactly the same as before, except that, d will be replace by d 0 by 2, that is all in the phase match situation.

So, what I have done is, by having a periodic variation of the non-linear coefficient, I have been able to overcome the phase mismatch term, which is, content in exponential minus i delta k z. What is this capital K delta k? So, what is the coherence length? Pi by delta k. So, this implies delta k is equal to pi by coherence length; and for Quasi-phase-matching, this is called Quasi-phase-matching; for Quasi-phase-matching, delta k is equal to K is equal to 2 pi by lambda, so, this implies, the periodicity required is…

So, if I make, if I periodically vary the non-linear coefficient with a period, which is twice the coherence length, I seem to be adding of the second harmonic power rather than periodically oscillating. Now, why is this happening? Let us see, let us try to understand - what happens at the coherence length? I have told you.

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Maximum efficiency, but the phase difference between the non-linear polarization and the electromagnetic wave was becoming pi.

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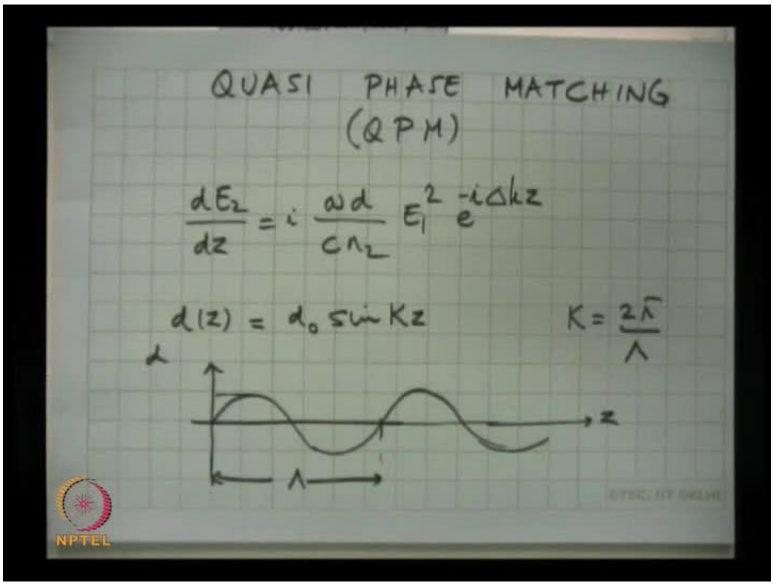

So, if I have to do Quasi-phase-matching, this must be the coherence length, because capital lambda is twice the coherence length; so, this must be the coherence length; twice the coherence length, 3 times the coherence length, 4 times the coherence length.

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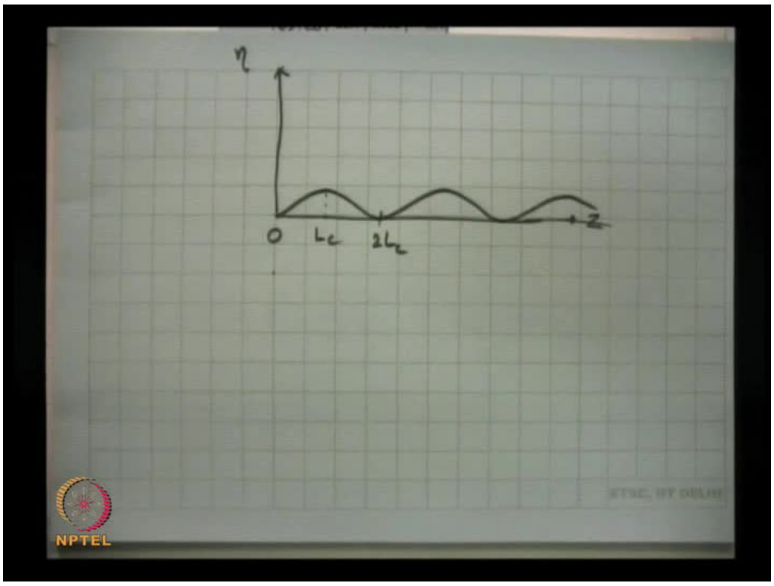

What is phase difference of pi minus 1? So, let me try to plot now. What is going to happen? If I look at efficiency versus z with delta k is equal to 0, with no Quasi-phase-matching, it was going like this; this was the coherence length.

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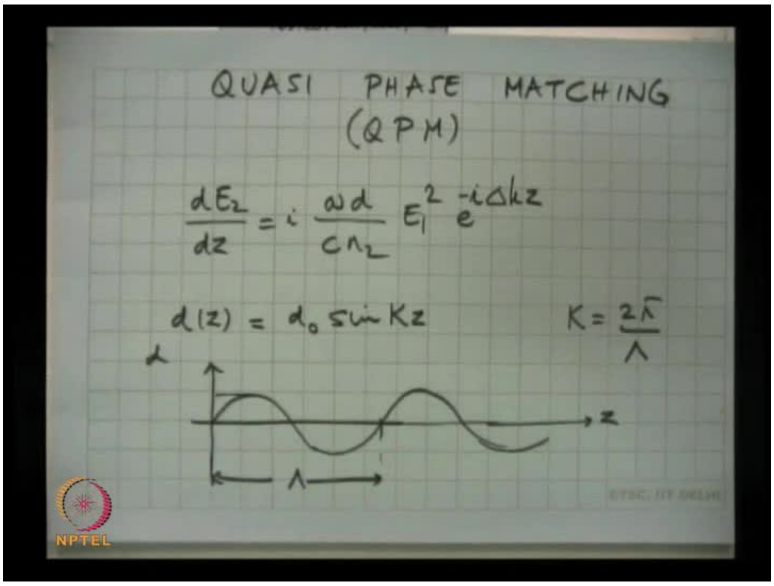

Now, what is happening with this d? This d is positive for one coherence length, half coherence length, sorry, twice coherence length, so, this is positive for 1 coherence length, negative for the next coherence length; next, again positive; so, it changes sign every time you go through 1 coherence length; magnitude is changing in between; but you change the sign, change of sign means bi-phase change. Non-linear polarization is proportional to d; so, when I change the sign of d, I change the sign of non-linear polarization. So, when the waves are propagating, at this distance, the non-linear polarization and the electromagnetic field got pi, out of phase; I force them back in phase by changing the sign of the non-linear polarization and bringing them back in phase.

Again, in propagating through a coherence length, they will generate another phase difference of pi and I re-bring them back in phase. Every time the non-linear polarization and the electromagnetic wave get out of phase by pi, I force them back in phase by reversing the sign of d, which essentially changes the polarization by a factor of minus 1; because, P non-linear is proportional to d. So, when I change the sign of d, I change the sign of non-linear polarization, which means, I add or subtract a phase difference of pi, which means, I bring them back in phase.

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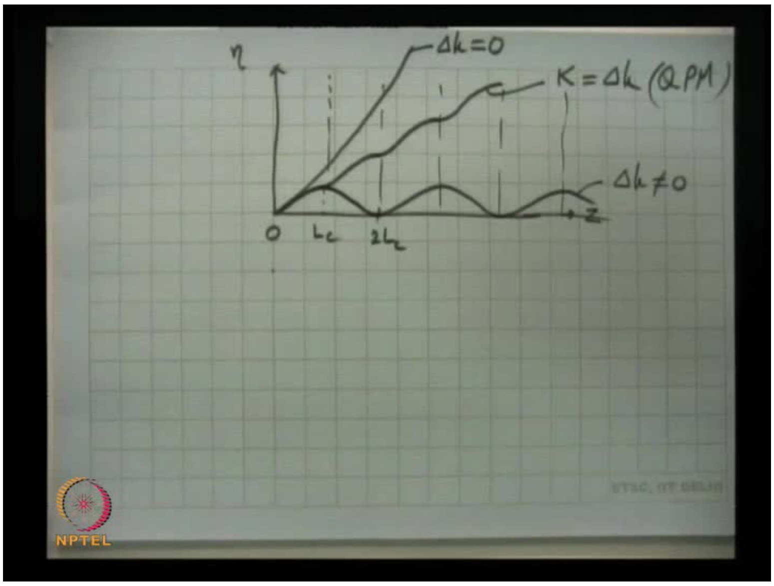

So, what will happen now is… So, in the Quasi-phase-matching case, the non-linear polarization is changing sign every one of this distance, so it comes here. At this point, if you did not do anything, these non-linear, the wave at second harmonic would have gone down but, it does not go down, it goes like this now because, you have brought them back in phase. At this point, if you did not do anything, it would have gone down like this, but you change the sign again, bring them back in phase.

So, every time the non-linear polarization and the electromagnetic field get out of phase, you force them back in phase; if you did not, if you had perfect phase-matching, it would have gone like this; this is delta k is equal to 0; this is K is equal to delta k; this is delta k not equal to 0; this is QPM; this is the perfect phase matching and this is not phase matched.

Note that the non-linear coefficient is now down in this, by a factor of 2; because, this is a similar equation to what we have written earlier, except that, instead of d, we have now d 0 by 2; and, what is d 0 by 2? d 0 by 2 is actually the coefficient of exponential plus i K z from here, except for i factor.

Because, d 0 sin K z is d 0 by 2 e to the power i K z minus d 0 by 2 exponential plus i K z, with the factor of 1 by i. So, d 0 by 2 - why is d 0 by 2 appearing? Because, when I wrote the sin in terms of exponentials, it actually, that two factors came from this term; the sin is being written as a sum of exponentials, this 2 is actually coming from here; and this 2 is what is effectively reducing the effective non-linear coefficient, that is, taking part; and that is the reason this is not raising as fast as here, it is much more slower, but the power is continuously adding. So, this is the principle of Quasi-phase-matching where, whenever there is a phase change of pi between the polarization and the, which is the source and the wave, I forcefully bring it back in phase; I do not need any bi-refringence for this.

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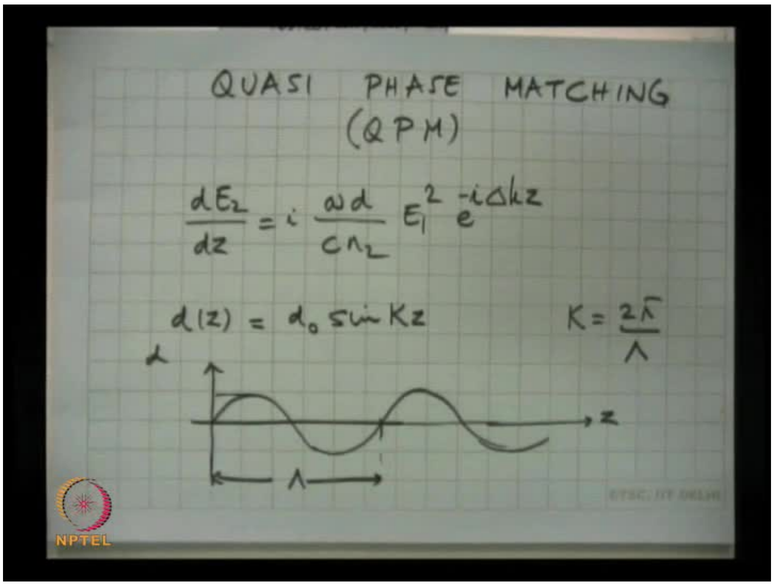

Now, remember, what was the period we had calculated coherence length? We have calculated for the lithium niobate crystal, about 3.3 microns; so, I need every 3.3 microns, the sign of d should change, so, this period is then supposed to be 6.6 microns; it is a very small period. Now, so, the first question is, of course, how do I achieve this? And, if I do not achieve exactly this, what is the result?

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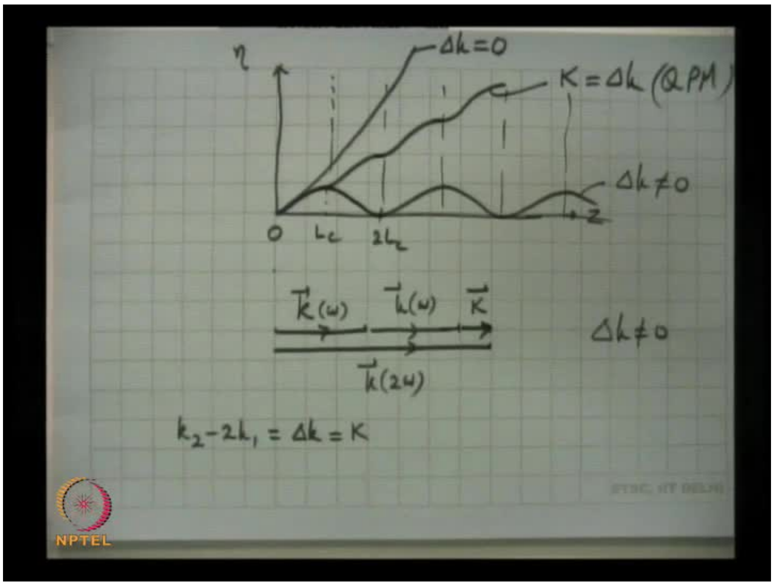

Now, I want to draw the vector diagram, and look at what is happening? Now, you see, here is situation where k at omega is here, k at omega; and k at 2 omega is not equal, it is bigger; there is no phase matching, delta k is not 0; this sum of k 1 plus k 1 2 k 1 is not equal to k 2; the length of this vector is k 2; the length of each vector is k 1; propagation vector at omega frequency, propagation vector at 2 omega frequency. So, how much is this length? This length is k 2 minus 2 k 1, which is capital K. A periodic modulation of the non-linearity gives me a small vector in the momentum space; this is a drawing of momentum, momentum of the photon; there is an extra small element, extra small vector capital K vector, which is coming because of this periodic variation; and that, I am using to close this triangle, to close this figure here and to make sure that sum of these three must be equal to this.

Bragg's diffraction of x-rays can be visualized like this; it is essentially, you have waves coming from here, you have a periodic lattice which has a spatial frequency vector that leads to coupling of this wave, electron waves or x-rays into another wave, in the another direction; you can draw vector diagrams for x-ray diffraction. Because, it is after all change in direction of the wave by giving an extra momentum, so, the vector diagram for this is essentially, that 2 k 1 is not equal to k 2, but you have a small element which is left there and that is what is being provided by the periodic variation of the non-linear coefficient.

So, Quasi-phase-matching is essentially a process in which you overcome the difference between k 2 and k 1, in this process, second harmonic, for achieving what is called as Quasi-phase-matching, and that is overcoming this difference by a spatial frequency vector capital K. So, instead of the second harmonic periodically growing and dying; now, the second harmonic grows, continuous to grow; it has an upward trend, but of course it is not as much as in the perfectly phase matches case, but at the same time, it is growing; and today, this is a very important technique that is commercially used to achieve very high non-linear effects, in, even in media, in which this was not normally possible.

Now, in gallium arsenide, to convert at 10.6 micron wavelength, what is 10.6 micron wavelength? Which laser? Carbon dioxide laser, CO 2 laser gives you 10.6 micron, so I can do a second harmonic of 10.6 micron to get 5.3 micron; this comes out to be, this period comes out to be of 110 microns or so. So, what we have done is, I make gallium arsenide; and by orienting the crystal appropriately, I can change the direction of the non-linear term, the value of the coefficient.

So, I take a set of gallium arsenide, sticks, slabs of 105 micron thick, I rotate alternately like this, stick them together and have a beautiful Quasi-phase-match crystal which works for second harmonic generation; there is no bi-refringence, but you do with 3 microns is not easy.

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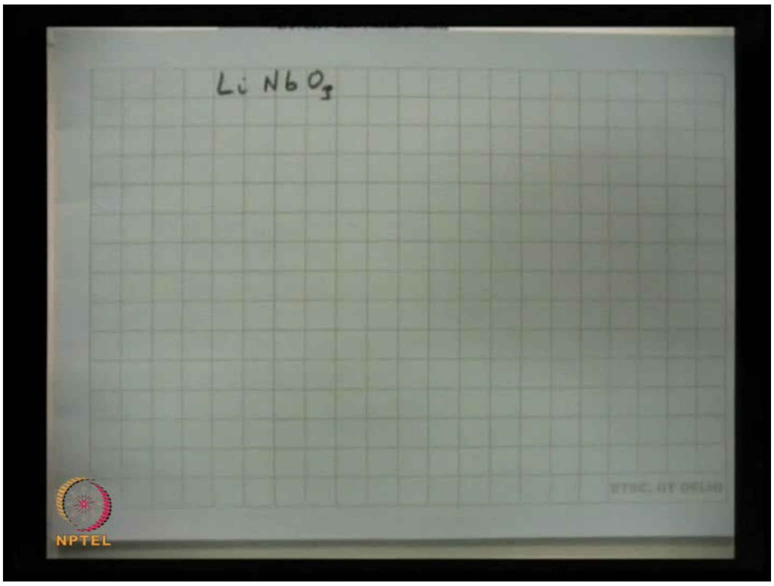

So, I will tell you little later, what are the techniques used to generate this kind of a thing in lithium niobate, but first thing is, how do I do this? So, now, in a crystal like lithium niobate, which is a very important crystal; this crystal is ferroelectric, that means, it has a spontaneous polarization at room temperature.

And there is temperature called curie temperature, if you heat the crystal above the curie temperature, it loses always spontaneous polarization. So, when you come down below, the curie temperature, you need to orient the crystal by in applied dielectric field, and you can orient all the domains, and you can come down polarized; the crystal gets polarized here; just like magnetized media, you have polarized media.

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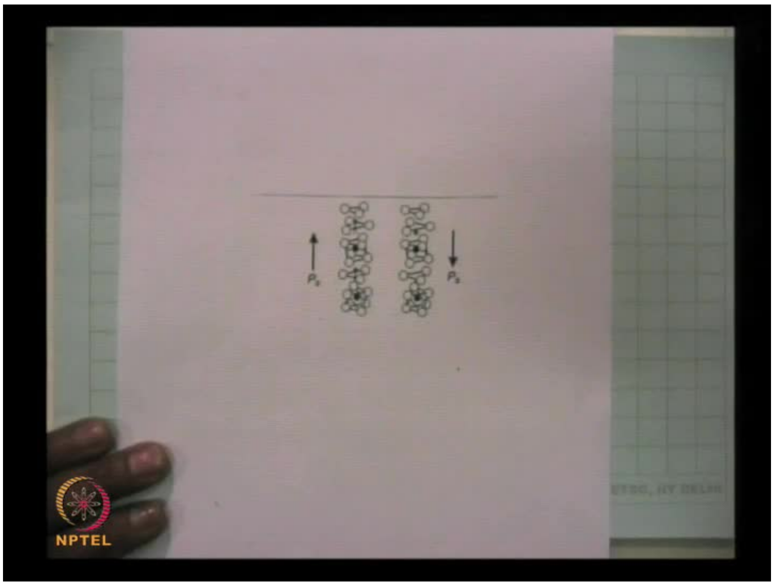

Now, the crystal looks like this. Now, let me show you a plot here. There is an asymmetry in the positioning of the lithium, which is the small grey circle here and the solid circles are the neobium atoms; and these three, three empty circles are all oxygen atoms, L i N b O 3. The lithium atoms can be above the oxygen plane or below the oxygen plane, both are stable situations; because of this asymmetry, if the oxygen atom are sitting in this position, the spontaneous polarization is pointing up; if you displace the lithium into the lower surface, below the oxygen plane, the spontaneous polarization reverses itself; actually, this is just this one rotated by 180 degrees, it is the same.

So, actually if you heated it to curie temperature, the position of the lithium comes on the plane of the oxygen atom, and so, there is no axis then, axis is just gone. Now, so, what I need to do is, this spontaneous polarization is connected to the coefficient the d tensor and the axis of a crystals.

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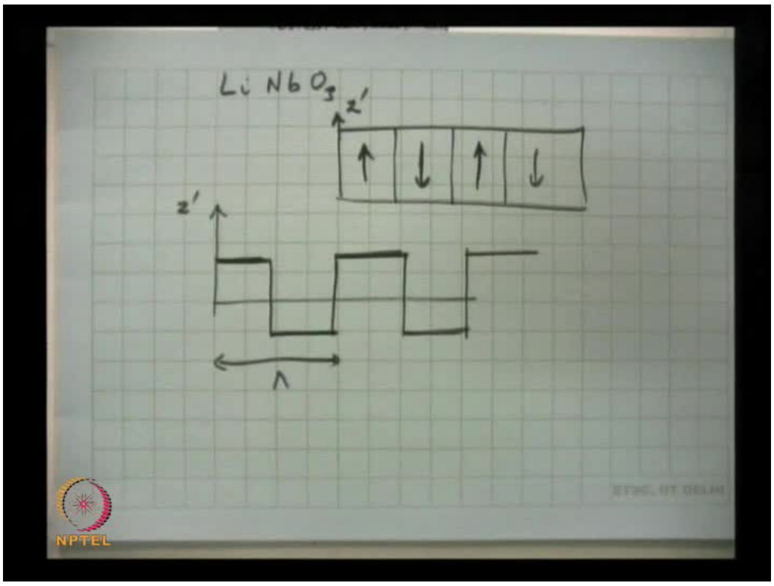

So, what I need to do is, to reverse the sign of the non-linear polarization, I need to push the lithium ion across the plane below; so, but what I can have is then, suppose, I have a crystal, I generate a crystal in which I have regions with the polarization pointing up down, up down; if I write this as my z prime axis, so, in the z prime axis, d will be positive, negative, positive, negative like this.

Because, in the z prime axis, this is pointing up, so this is positive; here, it is pointing down, so it is negative; its pointing up again, positive, negative. So, in my laboratory coordinate system, this portion of the crystal will have a positive, d vector, d component here, d tensor, will be negative. So, essentially, I have to understand what happens to the d tensor if I rotate my coordinate system.

So, what is actually happening is, the spontaneous polarization of the crystal is pointing up here, then down, so, this is… What I have done is, essentially, instead of taking the crystals and staking them together, if I can generate this, where the crystal has is made up of domains; here, the domains are pointing up; here, the ferroelectric domains are pointing down; up, down, up, down periodically. And, if I do this period to be lambda, I will have a periodic variation in d; I do not have a sinusoidal variation, I have a periodic variation in d with a period lambda. Now, what? Does it have a sign variation?

(( ))

I have to wave a Fourier series. If I take a Fourier series of this…

(( ))

At frequencies also will be there. So, let me look at a Fourier series. So, let me take a d, so, this is, which is, this z axis, this z prime axis is like this; this is a d value; d is positive, negative, positive, negative.

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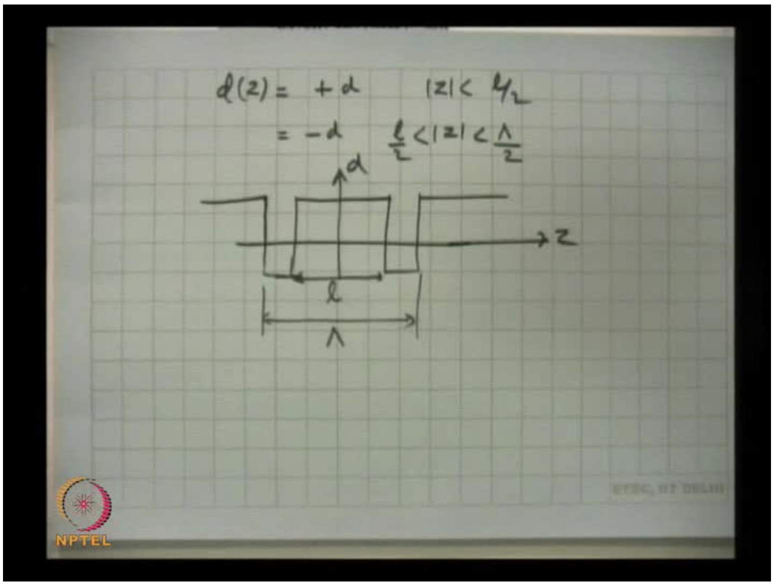

So, let me look at a d tensor like this. So, d of z, let me draw a figure here and then write the corresponding…; so, d of z is plus d for mod z less than l by 2, and is equal to minus d for mod z greater than l by 2, less than lambda by 2 (( )) yes, 1 second, 1 second; this is d z, yes, sorry.

Sir, what is the purpose of this analysis? To find out the efficiency in certain cases?

Yes, first of all my question is, I do not have a sine dependence; I shown you for a sine dependence; I cannot generate this normally with… I cannot vary the non-linear coefficient continuously in a sinusoidal equation, but what I can do is, do this - periodic plus minus plus minus or, in fact, I can also do plus 0 plus 0; I can kill the non-linearity at some region by doing some chemical operation by diffusing ions or whatever it is. So, periodically, I can kill the non-linearity; I still have a periodic non-linear variation, d varies periodically. What is important is the fourier component, spatial fourier component of that distribution. So, I want to just write an equation, giving you the spatial frequency components of this.

Sir, to find out the efficiency, could I just not find out it the addition 2 E 2 in 1 period?

Yes

That will be proportional to z

Yes

Then accordingly, I can find out… necessarily, do I have to do this analysis to find out this equation - in a case where E 1 is

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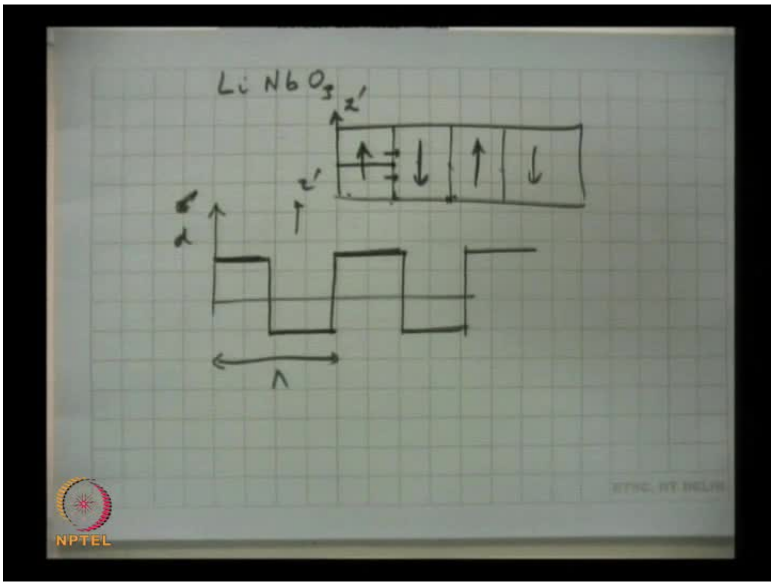

For example, I start from here; first I calculate over this distance, then I calculate over this distance; but please remember, at this point, there is both E 1 and E 2 incident; the equation which I have derived does not include this. So, I must calculate how E 1 will vary when E 2 is also present; also, E 1 remains constant.

We are taking the case with E 1 remains constant.

E 1 is remaining constant but E 2 is still present; here, the situation with E 2 z is equal to 0; E 2 of input is not equal to 0; so, the solution that you will get for E 2 of z with E 2 is equal to 0, is not the same as you will get for E 2 not equal to 0 and also depends on the phase difference.

So, this will be the same because d E 2 by d z depends only on E 1 square.

Yes.

And that I am taking as the constant.

Yes.

Only variable here is d.

Yes, but see, I have written E 1 E 2 of z minus E 2 of 0 is equal to this thing. So, the next boundary condition becomes E 2 of z not equal to 0; then, can I calculate from z is equal to this value, to this value? E 2 of z is increasing, so E 1 of z from here to here will not be the same as E 1 of z increase from here to here.

Approximately, we are taking that only, the rate of increase should be same.

So, what will be your expression for efficiency - for example.

Sir, the increase that have in the first half of the period, the increase in E 2 will be same in the second half of the period and so on, in every half of the period; I will have same increase in E 2 for as long as E 2 is very much smaller than E 1.

Yes, it is; even at the end, I will be assuming E 1 to be almost constant.

Sir, then we do not need that analysis.

Increasing E 2, you are saying.

Yes, sir the increase in E 2 in an every half period will be the same because, the slope is constant now.

Yeah.

It means, between the final and the initial. So, there is no really need for this analysis.

What will be the effective non-linear coefficient that is responsible for the increase? Suppose, I give you a length l, how would I calculate? Would I just divide by the length by the period.

I will calculate in - the effective?

Yeah

I can do it for the (( ))

I do not know; I have to check. I have to check whether if I divide l by the number of periods or the number of half periods, will I get the same. I have to check this, I have not checked this, but what I will do is the following, that…

Sir, over here, one more thing, we have assumed that, if physically, when I am shifting the polarization, shifting it; what I am doing is, rotating the crystal. So, when I do it some on the d tensor, I am assuming that the d tensor will become negative.

Not all elements, I am assuming that the element that I am using in my non-linear process is changing sign. Now, I will give this as problem later to you. What happens if I have a coordinate system which is rotating at about one of the principal axis, which elements of the d tensor will be change sign, which elements will not change sign? It is not necessary all elements, the d tensor change sign. So, I am assuming here now, that this helps me to change the sign of the non-linear coefficient that I am using in my non-linear process.

So, the elements of the d tensor actually contribute to the polarization, they should take change sign.

Yes, for example, in one of the example, we have looked that d 33 was coming into picture, so, I need to measure d 33 changes sign; if d 33 does not change sign in my process, then I have no use; so, similarly, I can use d phi 1 for some process. Does d phi 1 change sign?

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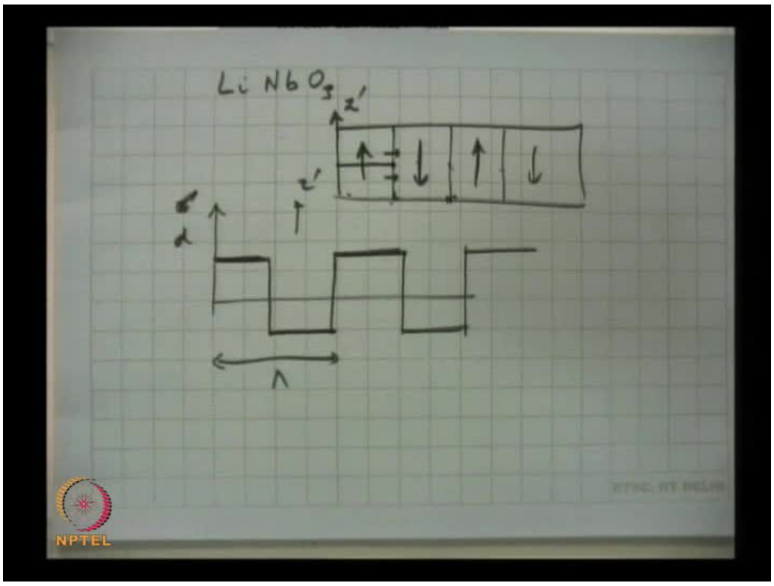

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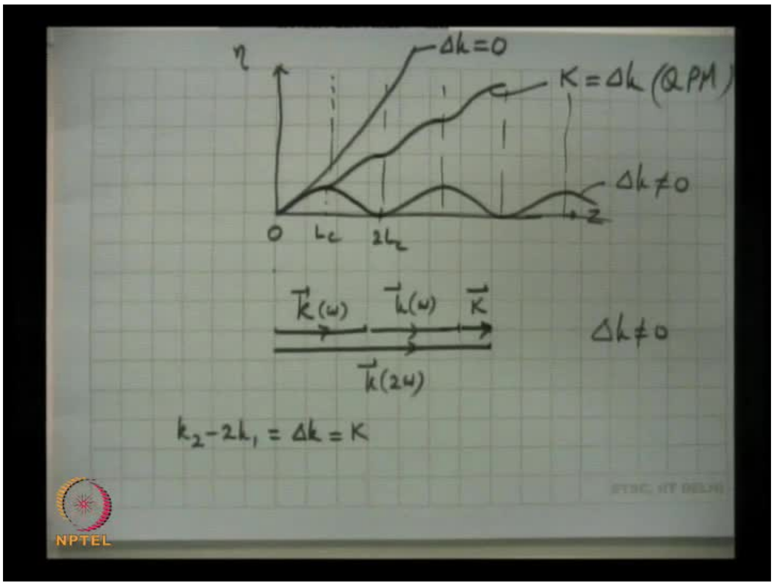

So, that I must check separately; I am assuming that d changes, and it is only that changed d that I am using in my interaction process - number 1. Number 2 is not necessary, the periods be equal, be the duty cycle be half, for example, I will show you that it is not necessary that I change every L c, I can change every 3 L c.

Because the fourier series will have that…

Yes, it will come here; and then, it does this; then, it increases now much slower, because the fourier series contains higher order of spatial frequency which I can use to phase-match. So, there is the first order Quasi-phase-matching, third order Quasi-phase-matching, fifth order Quasi-phase-matching. Because, the period required here is much larger, 3 times, ten microns, but what is the price I will pay? The fourier coefficient will be small, so, the effective non-linearity will be small; so, all these I will get from this analysis. All I need to do is a fourier series of this and because as I showed you, the term which is actually cancelling this, is exponential i K z.

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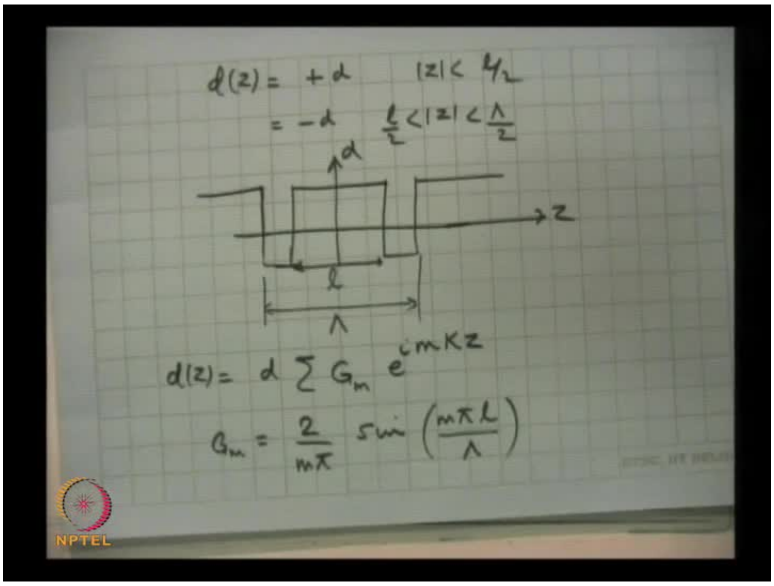

So, that is assuming a sign variation, but this has many spatial frequency components, so, m th m times k will be responsible for cancelling this term and I will get a single shot calculation of what is the effect in non-linearity which is helping me; and whether that fourier coefficient, if it happens to be 0 here, then, there is no effect, that is, in that non-linearity is absent in my system. So, let me leave this problem to you - simple fourier series; so, this is my function, as a function of z. So, assume d of z is equal d times sigma G m exponential i m K z, fourier series in terms of exponential, not sine cosine, calculate G m.

So, let me give you the expression 2 by m pi. So, G m is the m th fourier coefficient corresponding to the spatial frequency m K. So, the first term will be, if I use m is equal to 1, it will be 2 by pi; and, if I use l is equal to half lambda, which is 50 percent duty cycle - this figure which I have drawn is a 50 percent duty cycle - this is equal to this.

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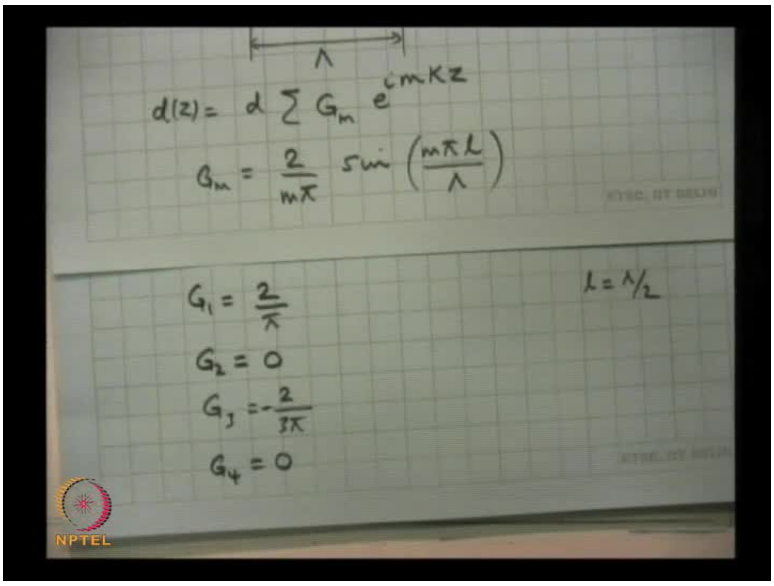

Here, I have drawn a figure in which l may be different from lambda, half a lambda. So, if I take l is equal to lambda by 2, if I take l is equal to lambda by 2, which is the duty cycle of… What is l is equal to lambda by 2? Yeah, l is equal to lambda by 2 - right? Then, G 1 will be 2 by pi. What will be G 2? What will be value of G 2? So, let me tell G 1 is equal to 2 by pi; G 2 will be… yes, G 3… All even coefficients will be 0, so, if I were looking for using…

(( )) Minus (( ))

So, this d becomes is sum of various exponential terms; the minus sign will finally not matter, in my, because it d square proportionality. But, otherwise, anyway, there is a minus sign. So, if I use this term, if I use this fourier coefficient for my Quasi-phase-matching, it is called first order Quasi-phase-matching. I can use this term G 3 for phase matching, it is called third order Quasi-phase-matching, fifth order Quasi-phase-matching, and so on.

If I want to do second order Quasi-phase-matching, what should I do? I must change the duty cycle, l by lambda, I must change. Now, I leave this problem to you. What is the best l by lambda that I will choose to make G 2 maximum? Not maximum among them, but G 2 should not be 0, but the maximum value, with that I can get, if there is a periodic variation and d. What should be the value of l by lambda, so that, G 2 is the maximum value? I leave this problem to you, please work it out.

So, I think we will stop for the quiz; so, what is interesting is, I can use a periodic variation in the non-linear coefficient to compensate for the phase difference that exist, that generates as they propagate and bring them back in phase; every time they get out of phase, I bring them back in phase. So, we will now close here and we will have the quiz.

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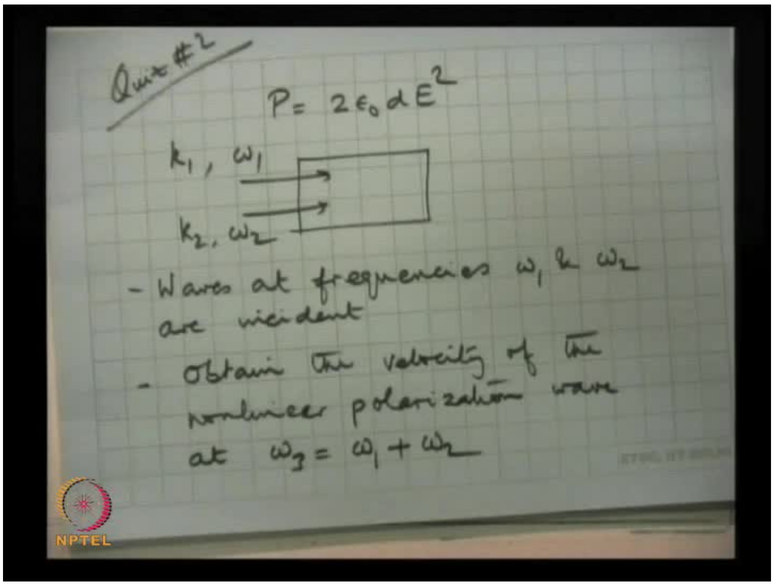

This is a crystal with a finite non-linear coefficient d; and waves at omega 1 and omega 2 are incident in the crystal; the propagation constants are k 1 and k 2. So, because of this non-linear effect, there will be non-linear polarization of the new frequency omega 3 which is the omega 1 plus omega 2. What is velocity of this polarization? Because I would need to know this to understand phase matching, so what is the velocity at which this non-linear polarization at omega 3 frequency is propagating?