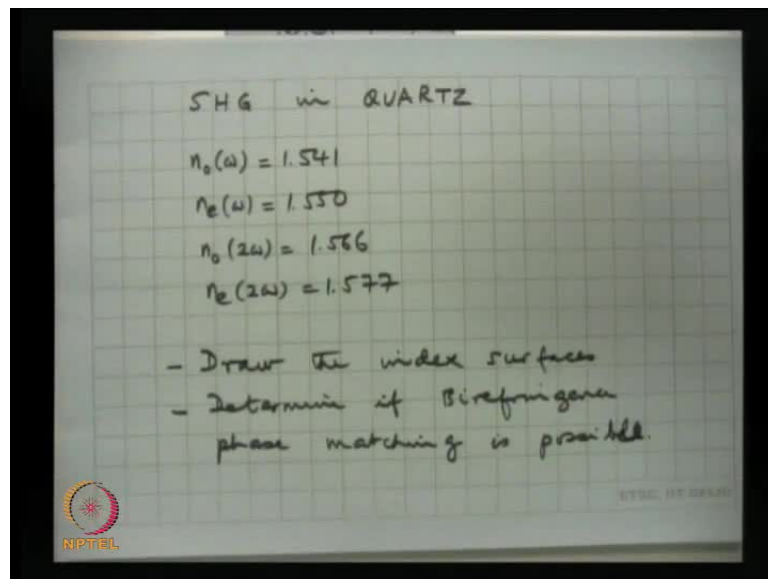


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**Module No. # 03**  
**Second Order Effects**  
**Lecture No. # 08**  
**Non - Linear Optics (Contd.)**

We will continue with discussions on non-linear optics. Do you have any questions? Yes. Okay, what I thought was, I will again leave you with another question, which is, for you to think about. It is a very short question, so this is not a quiz, but it is a question for you.

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So, let us look at second harmonic generation in QUARTZ. The, At one particular wave length, the refractive indices are given like this 1.541  $n_e$  of  $\omega$  is equal to 1.550  $n_o$  of  $2\omega$  is equal to 1.566 and  $n_e$  of  $2\omega$  is 1.577 So, draw the index surfaces and determine if Bi-refringence Phase Matching is possible; it is just a homework problem for you; please find out whether Bi-refringence is Phase Matching is possible, so, a very simple question.

So, what we will like to do, is to continue with our discussion on non-linear optics. So, we were primarily looking at second harmonic generation; and in second harmonic generation, we were, we found that as an example in KDP, we can achieve Bi-refrignence Phase Matching, provided, we propagate at an appropriate angle with the optic axis.

So, for the wavelength, we consider, ruby laser wavelength, this angle comes out to be 50.5 degrees. So, if this is the optic axis, I need the propagate at an angle of 50.5 degrees with the optic axis, in order to achieve phase matching. The omega frequency is an ordinary wave and the second harmonic will be an extraordinary wave.

So, the first question is, what orientation should I choose? Or, does it matter or does it not matter? That means, I can be 50.5 in the x y x z plane, I can be 50.5 in the y z plane, I can be 50.5 at some other plane, some other orientation - right? This is the optic axis.

So, there is the full cone of angles I can choose. Does it matter? Number one. And, what is this de-coeffecient that I will have to use in my efficiency calculation? Because, remember, we wrote a scalar equation  $2 \epsilon_0 d E^2$ , what is d? Because, for KDP, there is a dtensor. So, what we did was, we started looking at the expression for the non-linear polarization vector itself.

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$$P_i^{(2\omega)} = 2 \epsilon_0 d_{ijk} E_j E_k$$

$$P_i^{(2\omega)} = \frac{1}{2} \left[ \epsilon_0 d_{ijk} E_j^{(\omega)} E_k^{(\omega)} e^{2i(k_1 z - \omega t)} + c.c. \right]$$

KDP

$$d = \begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix}$$

NPTEL

So, let us recall, we started with  $P_i$  is equal to  $2 \epsilon_0 d_{ijk} E_j E_k$  where  $E_j$  and  $E_k$  are the total electric fields,  $P_i$  is the total non-linear polarization; then, for second harmonic generation, we wrote  $E_j$  as the sum of the electric field, of the fundamental and the second harmonic;  $E_k$  as the  $k$ th component of the sum of the electric field, of the fundamental and second harmonic; took a product and found out, that the non-linear polarization at  $2\omega$  is given by half of  $\epsilon_0 d_{ijk} E_j(\omega) E_k(\omega) \exp(2i k_1 z - i\omega t + \text{complex conjugate})$ .

So, we substitute for  $E_j$  at the total electric field, which is, the sum of the field at  $\omega$  and  $2\omega$ ; we substitute for  $E_k$  and calculate this non-linear polarization term at frequency  $2\omega$ .

Also, note that,  $d_{ijk}$  is also equal to  $d_{ikj}$  because, **there is no**, there is no change if I interchange the indices  $j$  and  $k$ . So, I can contract that last two indices of  $d_{ijk}$ ; and instead of the 3 by 3 by 3 matrix, I will have a 3 by 6 matrix.

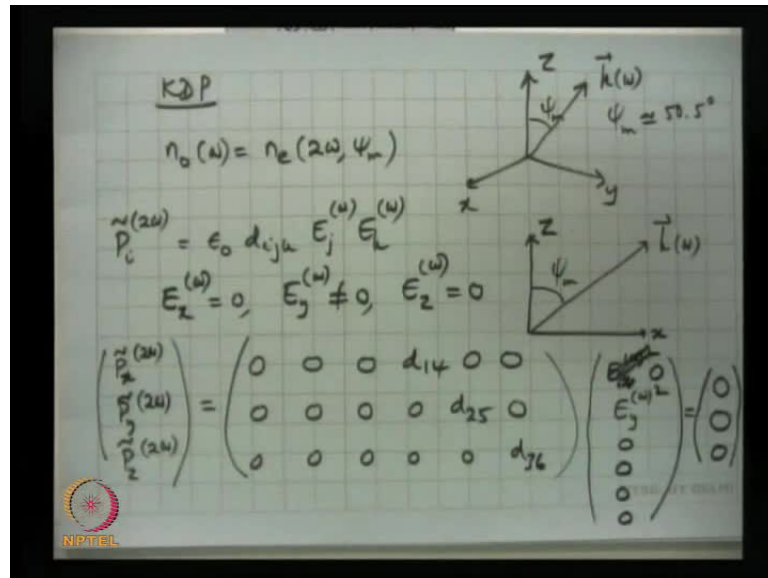
So, for example, for KDP, I wrote down the **d tensor**, **last**, yesterday, so, this was  $0 \ 0 \ 0 \ d_{14} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ d_{25} \ 0$  and all elements except the last one are 0. This form of the d tensor can be actually obtained by knowing the symmetry of the KDP crystal.

You can actually show that all these elements are 0 and these elements are the only ones which survived. All crystals belonging to the class of KDP will have the same d tensor

So  $d_{14}$  is actually  $d_{123}$ ,  $d_{41}$  is  $d_{23}$ ; and this is  $d_{213}$ ; and this is  $d_{312}$ ,  $d_{61}$  is  $d_{12}$ ; so, these are all contracted indices; **so, if you have to find...** This is also the d tensor in the principle axis system.

If I want to calculate  $d$  in another coordinate system, I would have to use the transformation properties of the tensor  $d_{ijk}$ , not the contracted one, but the d contracted d at d tensor; because, this is the tensor 3 by 3 by 3 by 3 matrix.

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So, any transformation property of  $d$  is related through the transformation of  $d_{ijk}$  and not this 3 by 3, 6 matrix. So, let us let us look at couple of examples to understand how do I use this  $d$  tensor? So, the first example we look at this KDP.

So, in KDP, what I have found out is, first thing is, for second harmonic generation, I need to achieve phase matching; to maximize the efficiency of conversion, I need to have phase matching, that means, the refractive index of the wave at  $2\omega$  must be equal to the refractive index at wave of the wave at frequency  $\omega$ .

Now, in KDP, because it is uniaxial, I found that it is possible to do this, provided, I - **so this is x, y and z** - provided I propagate at an angle making  $\psi_m$  by  $\psi_m$  is equal to approximately, 50.5 degrees. If I propagate like this, this is  $k$  vector of  $\omega$ , if I propagate in this direction making an angle of 50.5 degrees with the optic axis, I will be able to achieve  $n_o$  of  $\omega$  is equal to  $n_e$  of  $2\omega$   $\psi_m$ ; that means, the fundamental wave at frequency  $\omega$  must be an ordinary wave and the second harmonic will be an extraordinary wave. Now, let me try to calculate what will be the non-linear polarization term, and does it matter whether I choose this angle of propagation in any orientation, as long as that makes 50.5 degrees with the optic axis.

So, let me take, for example, **let me take**, I propagate in the  $xz$  plane at 50.5 degrees, this is  $\psi_m$ ; now, I will use this equation, remember, this non-linear polarization term is

given by here, this equation; so, remember, I wrote this equation  $P_i \tilde{\omega}^2 = \epsilon_0 d_{ijk} E_j \omega E_k \omega$  and we wrote this in matrix form, yesterday. This is written as  $P_x \tilde{\omega}^2$ ,  $P_y \tilde{\omega}^2$  and  $P_z \tilde{\omega}^2$ , is equal to the d tensor  $0, 0, 0, d_{14}, 0, 0$  - this is for KDP crystal.

And remember, I must have a 6 row column matrix here. What was the first element?  $E_x \omega^2$ . So, let me write just  $E_x \omega^2$ ; okay, now let me, **before I**, write this. Now, remember, to achieve phase matching, the electric field of the fundamental wave must be an ordinary wave. So, what is the orientation of electric vector of the ordinary wave propagating in this direction, in the x z plane? Along y axis. The ordinary polarization is perpendicular to the optic axis and to the propagation vector. So, to achieve phase matching, I will have to launch the light at frequency  $\omega$  as an ordinary wave; so, I will launch the  $\omega$  frequency as bi-polarized wave. So, the only element which survives is actually,  $E_x \omega$  is equal to 0,  $E_y \omega$  is not equal to 0 and  $E_z \omega$  is equal to 0.

Please note, **I am**, I have to choose the  $\omega$  frequency as an ordinary wave because, that is when I get phase matching; and I must propagate at a certain angle. So, because of this restriction, my incident electromagnetic wave at frequency  $\omega$ , is an ordinary wave polarized along y; so, this only element that survives, is  $E_y \omega$ . So, what happens to this column matrix? The first element is  $E_x$  is  $E_x \omega^2 = 0$ ; second element -  $E_y \omega^2$ , which is non-zero; third element - please look at the matrix -  $E_z \omega^2 = 0$ ; fourth  $2 E_y \omega E_z \omega$ , 0; then,  $2 E_x \omega E_z \omega$ , 0; and the last,  $2 E_x \omega E_y \omega$ , which is also 0.

For a given crystal, I know this tensor; this column matrix is obtained by the way I launch my  $\omega$  frequency wave, which components it has; these two matrices will then determine - what is the non-linear polarization generated in the crystal? What is the product of these two? All elements are 0. So, there is no non-linear polarization, the medium is non-linear; but the orientation of my electric field is such that, in this crystal, if I orient my electric field along the y axis and launch as an ordinary wave, there is no non-linearity generated in the medium. Because, the elements, some of the elements are 0; if this element was finite here, or one of these, the second column should have been finite, **we could have** picked up this  $E_y \omega^2$  from there.

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KDP

$$n_o(\omega) = n_e(2\omega, \psi_m)$$

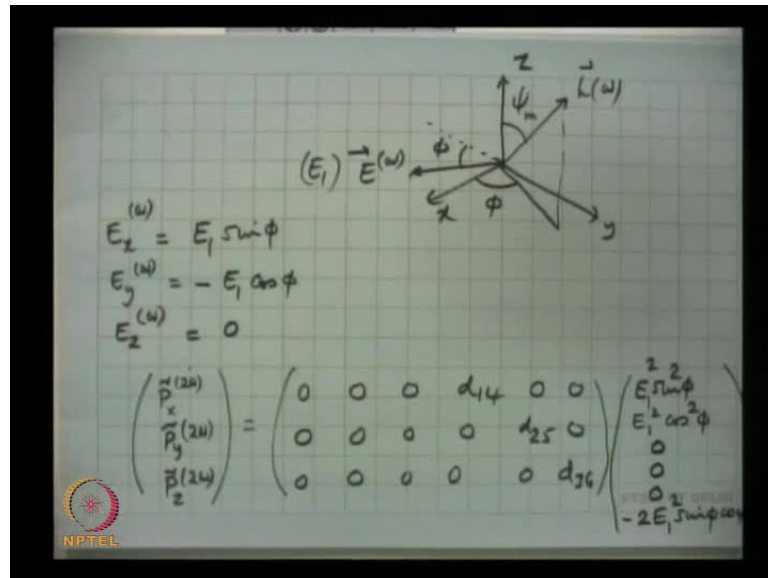
$$P_i(2\omega) = \epsilon_0 d_{ijk} E_j(\omega) E_k(\omega)$$

$$E_x(\omega) = 0, E_y(\omega) \neq 0, E_z(\omega) = 0$$

$$\begin{pmatrix} P_x(2\omega) \\ P_y(2\omega) \\ P_z(2\omega) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix} \begin{pmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

So, although I am phase matched, I do not have a non-linear polarization. So, here is the situation where there is no coupling but there is a resonance; the two have the same frequency; the two have the same velocity; the fundamental and second harmonic of the same velocity, but there is no generation of second harmonic, because there is no polarization generated at 2 omega frequency; all 3 components are 0. So, now, let me change the orientation of my electric field direction; please note here, by looking at this matrix, I am saying these elements of which are non-zero and my wave has to be ordinary, so let me take another direction of propagation.

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So x, y and z; so, this is k vector of omega; this is the perpendicular from here to this; see this is phi; so, the projection of this vector on the x y plane makes an angle phi with x axis; and this angle is till psi m, the phase matching angle, which means, what have done is, I am not choosing in the x z plane, but I have choosing in some other arbitrary plane. If phi is equal to 0, I get the x z plane; if phi is equal to 90 degrees, I get the y z plane; otherwise, I have some arbitrary plane.

Now, again, the omega frequency must be an ordinary wave, so, what will be the orientation? In which plane will the electric field of the ordinary wave propagating in this direction lie? x y plane, because ordinary polarization is always perpendicular to the optic axis and the propagation direction, which means, perpendicular optic axis, means, it must be x y plane. And in the x y plane, it must be at right angles of the k vector; so, it will be a vector like this, if I project back this, how much is this angle?

(O)

Yes, now, the crystal has certain symmetry orientations; x and y are not interchangeable for the d tensor; for the epsilon tensor, the anisotropic property, that is, linear property, epsilon matrix does not change if you rotate the x y in the x y plane, but d tensor can change; it is another tensor of a higher order, it can change, but the epsilon matrix does not change.

So, there I could chose in any arbitrary direction, but here, the x y z is the symmetry axis or the crystal axis as the principal axis. So, what is this angle phi? So, what is E x of omega? So, let me call this amplitude of this field as E 1; this is the field of E 1, E 1 is the amplitude of the field; so, what is E x of omega? E1 sin phi; E y of omega - minus E1cos phi; and E z of omega - 0

Now, so, what happens is, so, P x of 2 omega P y tilde of 2 omega P z tilde of 2 omega will be again the same matrix 0,d 14,0,0,0,0,0,d 25,0 into... Now, I get the matrix, the column matrix here. So, what is the first element? E x square. So, E1 square sin square phi; the second element is E by omega square, which is E1 square cos square phi; the third element is E z square; the fourth element is 2 E y E z, which is 0; the fifth element is 2 E x E z, 0; and the last one is 2 E y E z which is, minus, sorry, minus 2 E x E y, so, minus 2 E 1 square sin phi cos phi.

So, what is this lead to here? So, if you now calculate from here, what you will find is, P x tilde of 2 omega will be 0 because, the only element which survives here is the fourth one; and the fourth element column fourth element is 0 here; then, the next row, this is 1, that is 0, and the last one is finite.

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$$\begin{aligned}
 P_x(2\omega) &= 0 \\
 P_y(2\omega) &= 0 \\
 P_z(2\omega) &= -2 d_{36} E_1^2 \sin\phi \cos\phi \\
 \phi &= \frac{\pi}{4} \quad P_z(2\omega) = -2 d_{36} E_1^2
 \end{aligned}$$

So, what I will get is P x tilde of 2 omega is equal to 0, P y tilde of 2 omega is equal to 0 and P z tilde of 2 omega is equal to minus 2 d 36 E 1 square sin phi cos phi; and you see



phi is 0, this is 0, that means, if I propagate in the x z plane, phi is equal to 0, means, x z plane it is 0; if phi is  $\pi$  by 2, it is also 0; so, there is an optimum direction. What is a optimum value of phi?

45, actually  $2 \sin \phi \cos \phi$  is  $\cos$   $\sin 2 \phi$  and that is, phi is equal to  $\pi$  by 4, I must choose; then, P z of  $2 \omega$  becomes minus  $d_{36} E_1$  square. So, I cannot choose an arbitrary phi to maximize the non-linear polarization. I must choose phi is equal to 45 degree, that means, it is a plane making an angle of 45 with the x and y directions.

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$$E_z^{(\omega)} = E_1 \sin \phi$$

$$E_y^{(\omega)} = -E_1 \cos \phi$$

$$E_x^{(\omega)} = 0$$

$$\begin{pmatrix} P_x^{(2\omega)} \\ P_y^{(2\omega)} \\ P_z^{(2\omega)} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix} \begin{pmatrix} E_1^2 \sin^2 \phi \\ E_1^2 \cos^2 \phi \\ 0 \\ 0 \\ 0 \\ -2E_1^2 \sin \phi \cos \phi \end{pmatrix}$$

Now, so, the non-linear polarization, the wave is propagating like this, the non-linear polarization is generated in this direction. Now, this non-linear polarization has a component along the propagation direction and a component perpendicular propagation direction; the only component that will be responsible for the generation of the wave in this direction is the one which is perpendicular; because, dipole radiation, they will all add only along the perpendicular direction of the dipole oscillation. So, these polarization components along the axis, along the propagation direction, does not contribute of the generation of radiation in this direction.

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$$\begin{aligned}
 P_x^{(2\omega)} &= 0 \\
 P_y^{(2\omega)} &= 0 \\
 P_z^{(2\omega)} &= -2 d_{36} E_1^2 \sin\phi \cos\phi \\
 \phi &= \frac{\pi}{4} \quad P_z^{(2\omega)} = -2 d_{36} E_1^2 \\
 P_{NL}^{(2\omega)} &= -d_{36} E_1^2 \sin\psi_m \\
 &= d_{eff} E_1^2 \\
 d_{eff} &= -d_{36} \sin\psi_m = -d_{36} \sin 2\phi \sin\psi_m
 \end{aligned}$$

The only **the** perpendicular component will contribute, and that is, how much? So, I will get P,P let me write non-linear now, at 2 omega, which would be responsible for the generation of second harmonic is minus d 36 E 1 square into sin psi m. So, I can write this as d effective E 1 square, where d effective is effective non-linear coefficient minus d 36; actually, in general, this is minus d 36 sin 2 phi sin psi m.

This is for phi is equal to pi by 4; this is in general; if you choose an arbitrary direction then, this is the effective non-linear coefficient. So, what we have done? The problem which we did, the analysis which we did, use some d into E1 square; so, that d is now for this orientation in this crystal, it is minus d 36 sin 2 phi sin psi m; so, this is the effective non-linear coefficient that is responsible for the generation of the second harmonic. So, in a crystal, d tensor has 18 elements. Which element will contribute to my second harmonic generation will depend on the direction of polarization of the omega wave, the 2 omega wave, the direction of propagation and the crystal.

So, if I give you crystal, the first thing I need to check is, can I achieve Bi-refrindex Phase Matching? If I can achieve Bi-refrindex Phase Matching, what is the direction of propagation? I fixed the direction of propagation, I still have an uncertainty in the phi value; then, I write down the electric field, I take an arbitrary direction, making this angle of phase matching with the optic axis.

Then, write down the expression for  $P_x$ ,  $P_y$ ,  $P_z$  and find out finally, which is that element, which will be picked up in the final non-linear polarization? So, please note, if you happen to choose the wrong orientation value of  $\phi$ , you will get no second harmonic generation. There is non-linearity, there is no non-linearity generated because of the orientation of the electric field in the crystal, that you have chosen.

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$$E_x^{(\omega)} = E_1 \sin \phi$$

$$E_y^{(\omega)} = -E_1 \cos \phi$$

$$E_z^{(\omega)} = 0$$

$$\begin{pmatrix} P_x(2\omega) \\ P_y(2\omega) \\ P_z(2\omega) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix} \begin{pmatrix} E_1^2 \sin^2 \phi \\ E_1^2 \cos^2 \phi \\ 0 \\ 0 \\ 0 \\ -2E_1 \sin \phi \cos \phi \end{pmatrix}$$

So, what I have done is, I take some arbitrary direction of propagation, making an angle  $\psi$  with the optic axis. Why I have chosen  $\psi$ ? Because, that is the direction in which I get phase match.

I still have a freedom of **choose** choice as  $\phi$ , so, I choose some arbitrary  $\phi$ ; and I also know, that for phase matching, the  $\omega$  frequency must be an ordinary wave. So, for this propagation direction, I find out what is the electric field of the ordinary wave, that will give me the components of the  $E$   $\omega$ ,  $x$   $y$   $z$  components of  $E$   $\omega$ ; knowing the  $x$   $y$   $z$  components of  $E$   $\omega$ , **I calculate**, and the  $d$  tensor of the crystal, I calculate  $P_x$   $P_y$   $P_z$  of  $2\omega$ ; and then, from there, I get finally, that 4 component of non-linear polarization that will be responsible for the generation of the second harmonic, which I get in this equation like this.

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$$\begin{aligned}
 P_x^{(2\omega)} &= 0 \\
 P_y^{(2\omega)} &= 0 \\
 P_z^{(2\omega)} &= -2 d_{36} E_1^2 \sin\phi \cos\phi \\
 \phi &= \frac{\pi}{4} \quad P_z^{(2\omega)} = -2 d_{36} E_1^2 \\
 P_{NL}^{(2\omega)} &= -d_{36} E_1^2 \sin\psi_m \\
 &= d_{eff} E_1^2 \\
 d_{eff} &= -d_{36} \sin\psi_m = -d_{36} \sin 2\phi \sin\psi_m
 \end{aligned}$$

$P_z$ , I have got, but a component of this will be responsible finally, for the generation of the second harmonic. So, this component, I break up into one, which is along the propagation direction, and one which is perpendicular; the perpendicular component is, this times,  $\sin \psi_m$ . So, the non-linearity that is responsible is simply this equation; assuming  $\phi$  is equal to  $\pi/4$ , here; otherwise, I will get  $\sin 2\phi$  sitting here; so, I find that the non-linear polarization of  $2\omega$  is proportional to  $E_1^2$ , where  $E_1$  is the electric field strength of the fundamental; this is the total electric field strength because, the components are  $E_1 \cos \phi$  and  $E_1 \sin \phi$ .

So, this tells me, what is the effective non-linear coefficient that will be responsible for the generation of second harmonic? So, if I want to use that efficiency expression to calculate the power generated in second harmonic, I need to use this value for  $d$ , for the  $d$  expression in that equation. So, I know the  $d_{36}$  value, it is a standard value; I know the  $\phi$  and  $\psi_m$ , I have chosen; I substitute here and this is the effective  $d$  which I will have to use in the calculation efficiency of  $(\eta)$ , a proportional to  $d^2$ ; you see, that  $d^2$  is this now, square of this. So, in second harmonic, it is extremely important to not only do phase matching, but also, to choose the right orientation of the electric field, or the propagation direction, to ensure that you maximize the non-linear polarization.

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LITHIUM NIOBATE ( $\text{LiNbO}_3$ )

$$d = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

$d_{12} = -d_{22}$   
 $d_{13} = d_{31}$   
 $d_{14} = d_{15}$   
 $d_{16} = -d_{22}$

$d_{33} \approx 30 \times 10^{-12} \text{ m/V}$   
 $d_{15} = d_{113} = d_{xxx} = d_{xxz}$

LITHIUM TANTALATE ( $\text{LiTaO}_3$ )

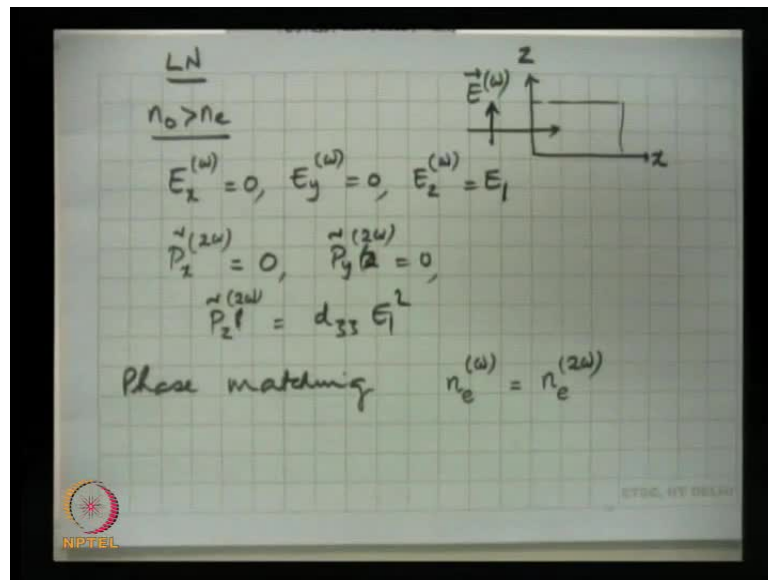
Let me look at another example: Lithium Niobate is a very important crystal and you will look at this in more detail as we proceed to the course. So, let me write the d tensor for this; **d 15**, there are only 3 plus 25 plus 3, 8 elements which survive; this belongs to another class of crystals. This is supposed to be d 12, this element is d 12; but d 12 happens to be equal to minus d 22, so, d 12 is equal to minus d 22; this element is supposed to be d 32, is equal to d 31. This element is supposed to be d 24, is actually equal to d 15; and finally, this is d 16, happens to be minus d 22.

So, please note d 33 means d 333 because the second 3 is 33, d 31 it d 311, that is, d xxx, 1 is x, 2 is y, 3 is z; this is, d yyy, this is supposed to be **5, is x z**; so, this d 15 will be d 113, which is actually d xxz or d xzx. So, **this element**, this matrix comes from again symmetry considerations of the crystal. All crystals belonging to the class of Lithium Niobate will have the same d matrix; Lithium Tantalate is another important crystal, LiT a O3; this is, LiN b O3; this is another crystal called Lithium Tantalate, LiT a O3, **which has** which is the same crystal class; it has the same matrix, but the values of the coefficients will be different.

Also, in this matrix, the largest element is d 33 and the value of d 33 is about 30 10 to the minus 12 meters per volt; it is one of the largest values of d available in inorganic crystals. So, because of this, in my non-linear interaction, I would like to use d 33 element. I want that d effective to become proportional to d 33, so that, I have the

maximum non-linear coefficient taking part in my non-linear process. So, to do that, I would have proper orientations of the electric field of the fundamental in secondary harmonic, but also, I need to make sure that I get phase matching; it **is not sufficient** I maximize the coupling, I also need phase matching.

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So, let us look at an example now, using Lithium Niobate; so, this is Lithium Niobate; my propagation direction is now, say x; this is z; this is my crystal now; and I launch a wave from here with z oriented, so this is the electric field of the omega wave; so, what polarization is this? **So, what is the...** Is it extraordinary, ordinary? This is uniaxial, so this is uniaxial with  $n_o$  greater than  $n_e$ , negative uniaxial. So, is it an ordinary wave or extraordinary wave?

Ordinary wave is always polarized perpendicular to the optic axis and k vector; this one will have, we have this polarization along the optic axis, so this is an extraordinary wave; this polarization, if I launch, that will be an ordinary wave. So, what I am assuming is, before looking at phase matching, I am trying to **say** see, whether if I launch a fundamental wave polarized along the z axis, what polarization will I generate? What non-linear polarization will I generate?

So,  $E_x$  omega is equal to 0,  $E_y$  omega is equal to 0 and  $E_z$  omega is equal to  $E_1$ ; there only one component, which is the electric field of the fundamental. Now, can you please

find out using this matrix, what is the non-linear polarization generated by this? I have to multiply this matrix by the 6 row column vector. So, what will be  $P_x$ ,  $P_x$  of  $2\omega$ ? Which element in this, will be effective? Which column? **Yes, this column**, because, it is in this 6 column vector here; only in third element survives;  $E_x^2$  is 0;  $E_y^2$  is 0;  $E_z^2$  is finite;  $E_y E_z$  is 0;  $E_x E_z$  is 0;  $E_x E_y$  is 0; so, only the third element survives and, that means, this column will be responsible; and so, you can show  $P_x$  of  $2\omega$  is 0,  $P_y$  of  $2\omega$  is equal to 0 and  $P_z$  of  $2\omega$  is equal to  $d_{33} E_1^2$ .

So, this gives me the  $d_{33}$  there; so, I will have a very strong non-linearity, non-linear polarization generated which will be oriented along the z direction because, **this polarization, this**, so, I take this Lithium Niobate crystal, I launch an electromagnetic wave at  $\omega$ , polarize along this z axis, which is the extraordinary wave; it enters the crystal, generates non-linearity at  $2\omega$ , which is also along the z direction; so, this polarization can generate which wave at second harmonic? **extraordinary**?

Because, this polarization will be only generating, this polarization oscillating like this, will generate an electromagnetic wave, polarize along z axis and that is an extraordinary wave. So, this polarization will generate an extraordinary wave at  $2\omega$  - right? Can I achieve phase matching? What will be the condition I need for phase matching now? Remember  $n_1$  is equal to  $n_2$ , so,  $n_1$  is  $n_e$  of  $\omega$ , **must be equal to...** that is not possible. So, I can choose this orientation, **to achieve**, to use the maximum non-linear coefficient, but I cannot use this because, I cannot have phase matching.

Now, the second technique which I have mentioned to you, Quasi Phase Matching, can help me; I will show you to use this coefficient in this direction because, I can use another principle to overcome the phase **mismatch** generated by the different velocities of these 2 waves. So, if I do not do Quasi Phase Matching, this interaction is not possible. I will not be able to generate the second harmonic at all; almost 0 efficiency, the efficiency is recalculated; remember, with these numbers only,  **$n_e$  value...** remember, and the phase **mismatch** was, such that, the coherence length was about 3 microns and efficiency was 10 to the minus 11 or so, with 1 watt of fundamental power; **that is not going to be any** generation at all.

(Refer Slide Time: 37:05)

$$E_x^{(\omega)} = 0$$

$$E_y^{(\omega)} = E_1 \sin \theta \quad (n_o^{(\omega)})$$

$$E_z^{(\omega)} = E_1 \cos \theta \quad (n_e^{(\omega)})$$

$$\begin{pmatrix} \vec{P}_x^{(2\omega)} \\ \vec{P}_y^{(2\omega)} \\ \vec{P}_z^{(2\omega)} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ E_1^2 \sin^2 \theta \\ E_1^2 \cos^2 \theta \\ 2E_1^2 \sin \theta \cos \theta \\ 0 \\ 0 \end{pmatrix}$$

So, I can even in fact generalize this little more and let me go to another orientation which you will come to later again and that is; so, this is x, y and z; this is my crystal again and I choose an electric vector propagation is like this; the electric vector make some angle phi or theta with the z axis. Is that a mode of propagation? If I propagate along x and if my electric vector is oriented at some angle to the optic axis in the y z plane, Will that propagate unchanged?

Will not propagate unchanged, because, I am propagating perpendicular to the optic axis so, it will have to broken up into two components; the y component will travel as an ordinary wave and the z component will travel as an extraordinary wave, but I can always launch my light in arbitrary polarization states. But, this light, now has two components y and z components; so, if I call this electric field as E 1, E x of omega is equal to 0, E y of omega is equal to E1 sin theta and E z of omega is equal to E 1 cos theta; please note, the component E1 sin theta remains E1 sin theta as they propagates; the component E 1 cos theta remains E 1 cos theta as they propagates.

This one will propagate with the refractive index n o of omega; this will propagate with the refractive index n e of omega; as they propagate, they will develop a phase difference that will change its polarization state, but these two components will always remain - these two components. I have it broken up into 2 modes, the modes are orthogonal; so, there is no coupling between the modes if I do not do anything; so, x y component



propagate, y component; z component propagates is z component, but these two can mixed together now, to produce non-linearity. So, let me calculate the non-linearity. So,  $\tilde{P}_x$ ,  $\tilde{P}_y$  and  $\tilde{P}_z$ ; so, let me write the d matrix - what is this matrix now?  $E_x \omega^2$ ,  $E_y \omega^2$ ,  $E_z \omega^2$ ,  $2 E_y E_z$ , so,  $2 E_x^2 \sin^2 \theta$ ,  $2 E_x E_z$  and  $2 E_x E_y$ .

(Refer Slide Time: 41:08)

$$\vec{\tilde{P}}^{(2\omega)} = \begin{pmatrix} 0 \\ d_{22} E_1^2 \sin^2 \theta + 2 d_{15} E_1^2 \sin \theta \cos \theta \\ d_{31} E_1^2 \sin^2 \theta + d_{33} E_1^2 \cos^2 \theta \\ 0 \end{pmatrix}$$

$$\tilde{P}_y^{(2\omega)} = d_{22} E_1^2 \sin^2 \theta + 2 d_{15} E_1^2 \sin \theta \cos \theta$$

$$\tilde{P}_z^{(2\omega)} = d_{31} E_1^2 \sin^2 \theta + d_{33} E_1^2 \cos^2 \theta$$

$$\tilde{P}_x^{(2\omega)} = 0$$

Now, let me **calculate**, multiply these two matrices and I will get the following matrix. So, the P vector, P tilde vector,  $2 \omega$ , the three components will give me, what is  $\tilde{P}_x$  of  $2 \omega$ ? 0,  $\tilde{P}_y$  of  $2 \omega$  -  $d_{22} E_1^2 \sin^2 \theta$  plus  $2 d_{15} E_1^2 \sin \theta \cos \theta$  and the last one is  $d_{31} E_1^2 \sin^2 \theta$  plus  $d_{33} E_1^2 \cos^2 \theta$ .

(Refer Slide Time: 43:05)

$$E_x^{(\omega)} = 0$$

$$E_y^{(\omega)} = E_1 \sin \theta \quad (n_o^{(\omega)})$$

$$E_z^{(\omega)} = E_1 \cos \theta \quad (n_e^{(\omega)})$$

$$\begin{pmatrix} \tilde{P}_x^{(2\omega)} \\ \tilde{P}_y^{(2\omega)} \\ \tilde{P}_z^{(2\omega)} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ E_1^2 \sin^2 \theta \\ E_1^2 \cos^2 \theta \\ 2E_1^2 \sin \theta \cos \theta \\ 0 \\ 0 \end{pmatrix}$$

So, it has now a y component and a z component;  $E_1^2 \cos^2 \theta$  and  $\tilde{P}_x^{(2\omega)}$  is equal to 0. What will  $\tilde{P}_y^{(2\omega)}$  or  $\tilde{P}_z^{(2\omega)}$  try to generate? There are two components now,  $\tilde{P}_y$  and  $\tilde{P}_z$ .  $\tilde{P}_y$  will try to generate the ordinary second harmonic and the  $\tilde{P}_z$  will try to generate the second harmonic of the extraordinary wave, extraordinary second harmonic wave.

(Refer Slide Time: 43:20)

$$\tilde{P}^{(2\omega)} = \begin{pmatrix} 0 \\ d_{22} E_1^2 \sin^2 \theta + 2d_{15} E_1^2 \sin \theta \cos \theta \\ d_{31} E_1^2 \sin^2 \theta + d_{33} E_1^2 \cos^2 \theta \end{pmatrix}$$

$$\tilde{P}_y^{(2\omega)} = d_{22} E_1^2 \sin^2 \theta + 2d_{15} E_1^2 \sin \theta \cos \theta$$

$$\tilde{P}_z^{(2\omega)} = d_{31} E_1^2 \sin^2 \theta + d_{33} E_1^2 \cos^2 \theta$$

$$\tilde{P}_x^{(2\omega)} = 0$$

$$k_e^{(\omega)} = \frac{\omega}{c} n_e^{(\omega)}$$

$$k_o^{(\omega)} = \frac{\omega}{c} n_o^{(\omega)}$$

$$k_o^{(2\omega)} = \frac{2\omega}{c} n_o^{(2\omega)}$$

$n_o > n_e$

Now, what is the phase matching condition? Remember, I told you, it is easy now to imagine in terms of photon picture and find out what will be the phase matching

condition. The input consist of this; what is  $E_1 \sin \theta \cos \theta$ ? It is coming from  $E_y E_z$ , this is simply  $E_y$  square, this is  $E_y$  square, this is  $E_z$  square, this is coming from  $E_y E_z$ , so, this second harmonic generation will be ordinary; if I use this component of the non-linear polarization to generate second harmonic, this will be because of  $E_y$  square, this will be because of  $E_y E_z$ . So, this will lead to the generation of **at** ordinary polarized second harmonic photon by merging an ordinarily polarized omega photon and an extraordinarily polarized omega photon.

Please note, this is the product of  $E_y$  and  $E_z$ , so, the electric field of the omega which corresponds to the ordinary and extraordinary, mixed together to produce an electric field of the second harmonic which is along the y direction and is ordinary. So, what will be my phase matching condition?

So, remember, now, that means, suppose I take Lithium Niobate with  $n_o$  more than  $n_e$ ; so, if I draw a vector diagram, I will have, this is  $k_o$  of omega,  $k_e$  of omega; this is  $k_o$  of  $2\omega$ ; let me explain -  $k_o$  of omega is bigger than  $k_e$  of omega because,  $n_o$  is more than  $n_e$ . Let me write here,  $k_o$  of omega is  $\omega$  by  $c n_o$  of omega,  $k_o$  of  $2\omega$  is  $2\omega$  by  $c$  into  $n_o$  at  $2\omega$ , and  $k_e$  at omega is equal to  $\omega$  by  $c$  into  $n_e$  of omega.

(Refer Slide Time: 46:26)

$$k_o^{(2\omega)} = k_o^{(\omega)} + k_e^{(\omega)}$$

Mathematically, when you look at the E y E z term, the E y will have an exponential minus i k 0 n o z, I will explain in the next class; and the E z will have an n e, so what is happening is, for phase matching, I need to satisfy this vector diagram; so, what is the condition I get on the refractive indices? **So, I must...** So, k o omega plus k e omega, the phase matching condition simply becomes k o of 2 omega, must be equal to k o of omega plus k e of omega.

(Refer Slide Time: 46:46)

$$\vec{P}^{(2\omega)} = \begin{pmatrix} 0 \\ d_{22} E_1^2 \sin^2 \theta + 2d_{15} E_1^2 \sin \theta \cos \theta \\ d_{31} E_1^2 \sin^2 \theta + d_{33} E_1^2 \cos^2 \theta \end{pmatrix}$$

$$\vec{P}_y^{(2\omega)} = d_{22} E_1^2 \sin^2 \theta + 2d_{15} E_1^2 \sin \theta \cos \theta$$

$$\vec{P}_z^{(2\omega)} = d_{31} E_1^2 \sin^2 \theta + d_{33} E_1^2 \cos^2 \theta$$

$$\vec{P}_x^{(2\omega)} = 0$$

$k_e^{(\omega)} = \frac{\omega}{c} n_e^{(\omega)}$ 
 $k_o^{(\omega)} = \frac{\omega}{c} n_o^{(\omega)}$ 
 $k_o^{(2\omega)} = \frac{2\omega}{c} n_o^{(2\omega)}$

$n_o > n_e$

This is E y E z term, E y component propagates with an ordinary velocity and is a propagation constant k o of omega; the E z component here, propagates with a velocity k e corresponded to k e of omega, so, this is a **situation where**, if you go back and look at the exponentials of E y and E z, the E y will have an exponential plus i k ordinary into z, the E z will have exponential i k extraordinary into z and the product of these two exponential must cancel off. I will do this in more detail in next time mathematically, but that should cancel off the exponential i k ordinary 2 omega on the left hand side.

So, it as if, if an ordinary photon and an extraordinary photon are merging to form a ordinary photon at frequency 2 omega.

(( ))Yes, that is the another process; if this process has to be effective, then I must have this process, for example, this is, I am writing for the second process, if I want to use this equation - this term, I must have this; if I want to use this term (( ))

Yes, exactly,  $2k$  naughted  $\omega$  will be  $k$  naughted  $2\omega$ , which is not possible because, that means,  $n_o$  of  $\omega$  is equal to  $n_o$  of  $2\omega$  which is not possible.

(Refer Slide Time: 48:15)

$$k_o^{(2\omega)} = k_o^{(\omega)} + k_e^{(\omega)}$$

$$\frac{2\omega}{c} n_o^{(2\omega)} = \frac{\omega}{c} n_o^{(\omega)} + \frac{\omega}{c} n_e^{(\omega)}$$

$$2 n_o^{(2\omega)} = n_o^{(\omega)} + n_e^{(\omega)}$$

So, this term actually does not create all unless, I do something else like Quasi Phase Matching. So, it is possible now; it looks as if this term now, gives me another equation, which is this equation -  $k_o$  of  $2\omega$  is equal to  $k_o$  of  $\omega$  plus  $k_e$  of  $\omega$ , so what is this mean?  $2\omega$  by  $c$   $n_o$  of  $2\omega$  is equal to  $\omega$  by  $c$   $n_o$  of  $\omega$  plus  $\omega$  by  $c$   $n_e$  of  $\omega$ .

Please always remember the propagation constant at  $2\omega$  is  $2\omega$  by  $c$ , so  $\omega$  by  $c$  cancels off and I get 2 times  $n_o$  of  $2\omega$  is equal to  $n_o$  of  $\omega$  plus  $n_e$  of  $\omega$ . Now, this may still not be possible, but this is a different equation now; if I can satisfy this equation corresponding to some pair of frequencies,  $\omega$  and  $2\omega$ , then, this term of the non-linear polarization will help me to generate **second harmonic at frequency  $2\omega$** , second harmonic frequency  $2\omega$ , which will be ordinarily polarized, which means, the light coming out will have bi-polarization state. So, what will happen is, if I take this crystal; here is my crystal, so, if I call this as  $z$   $x$  and  $y$ , I launch a light from here, oriented like this, at some angle, and out comes second harmonic like this, these along by direction; so, this is same angle for  $\theta$ .

(Refer Slide Time: 50:08)

$$\vec{P}^{(2\omega)} = \begin{pmatrix} 0 \\ d_{22} E_1^2 \sin^2 \theta + 2d_{15} E_1^2 \sin \theta \cos \theta \\ d_{31} E_1^2 \sin^2 \theta + d_{33} E_1^2 \cos^2 \theta \end{pmatrix}$$

$$\tilde{P}_y^{(2\omega)} = d_{22} E_1^2 \sin^2 \theta + 2d_{15} E_1^2 \sin \theta \cos \theta$$

$$\tilde{P}_z^{(2\omega)} = d_{31} E_1^2 \sin^2 \theta + d_{33} E_1^2 \cos^2 \theta$$

$$\tilde{P}_x^{(2\omega)} = 0$$

$k_e^{(\omega)} \rightarrow k_o^{(\omega)}$   
 $k_o^{(2\omega)}$

$n_o > n_e$   
 $k_e^{(\omega)} = \frac{\omega}{c} n_e^{(\omega)}$   
 $k_o^{(2\omega)} = \frac{2\omega}{c} n_o^{(2\omega)}$

$k_e^{(\omega)} = \frac{\omega}{c} n_e^{(\omega)}$   
 $k_o^{(2\omega)} = \frac{2\omega}{c} n_o^{(2\omega)}$

Again, as you can see, what is the angle theta I must choose to maximize this? 45. If I equally launch the ordinary and extraordinary components of polarization, then I can have maximum second harmonic generation of the ordinary wave, provided, I satisfy the phase matching condition. If I do not satisfy this phase matching condition, there is non-linear polarization generated, but there is no second harmonic generated; if I am unable to satisfy this condition, I will show, I can use Quasi Phase Matching to overcome this requirement of phase matching from here, I will, **sort**, be able to achieve phase matching for this process also.

So, there are two kinds of processes, one in which you would launch light of one polarization, one mode, ordinary and convert to extraordinary or, you launch extraordinary and convert to ordinary or, you launch a combination of ordinary and extraordinary and generate second harmonic of ordinary or extraordinary; one is called the type 1 process, the other is called type 2 process.

(Refer Slide Time: 51:32)

$$k_o^{(2\omega)} = k_o^{(\omega)} + k_e^{(\omega)}$$
$$\frac{2\omega}{c} n_o^{(2\omega)} = \frac{\omega}{c} n_o^{(\omega)} + \frac{\omega}{c} n_e^{(\omega)}$$
$$2 n_o^{(2\omega)} = n_o^{(\omega)} + n_e^{(\omega)}$$

There are different processes in non-linear interactions, so I am not restricted to only choosing one process; and this is very interesting because, we will come back to this problem later on because, as you will see, I can do a reverse process; if I can convert  $\omega$  to  $2\omega$ , can I convert  $2\omega$  to  $\omega$ ? It is the same non-linear process.

So, we will analyze this, I will show classically, you cannot; you need quantum mechanics. You cannot explain the process of, **the reverse process**, **some** harmonic generation; this is second harmonic, that sub-harmonic you launch  $2\omega$ , so I have a crystal, I launch  $\omega$ , get  $2\omega$ ; I launch back  $2\omega$  nothing comes out, classically, but I see like  $\omega$ ; it is a completely quantum mechanical process. So, in this process, I have 2 photons at  $2\omega$  merge to form 1 photon at  $2\omega$ ; the other process is 1 photon at  $2\omega$  comes and splits into 2  $\omega$  photons; and these 2 photons at  $2\omega$  at  $\omega$  frequencies, have some strange properties of entanglement.

So, this I will cover when we start to discuss more of quantum mechanics, but just mention, that this is a very interesting process, because, the 2 photons coming out will correspond to 2 orthogonal polarization states. One is an ordinary wave and the other is coming the extraordinary depolarization, and that is very interesting because, we would have some interesting properties, so we will stop here. Do you have any questions?

(( ))

It is a perturbative expansion, it is a perturbative expansion. I am considering non-linearity as the perturbation. I am actually substituting back into the Maxwell's equations; in principle, what I should have done? Which I have actually, I have assume divergence E is equal to 0, so, I have actually gone back to the isotropic calculations; but in principle, I cannot put divergence E is equal to 0; if there is anisotropy then, I must solve the entire problem little more carefully which I am not doing.

Because, in more situations, the predictions of this are very very accurate; so, if I need to do little more careful analysis, I must use the fact, that is, anisotropic in the crystal; so, when I propagate in directions other than, like this, along the principle axis, I cannot neglected divergence E is equal to 0; I cannot put that; so, I must go back and restart their analysis, but otherwise, this I am assuming the non-linearity is week enough here.

(( ))

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$$P_i^{(2\omega)} = 2\epsilon_0 d_{ijk} E_j E_k$$
$$P_i^{(2\omega)} = \frac{1}{2} \left[ \epsilon_0 d_{ijk} E_j^{(\omega)} E_k^{(\omega)} e^{2i(k_1 z - \omega t)} + cc \right]$$

KDP

$$d = \begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix}$$



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$$k_o^{(2\omega)} = k_o^{(\omega)} + k_e^{(\omega)}$$
$$\frac{2\omega}{c} n_o^{(2\omega)} = \frac{\omega}{c} n_o^{(\omega)} + \frac{\omega}{c} n_e^{(\omega)}$$
$$2 n_o^{(2\omega)} = n_o^{(\omega)} + n_e^{(\omega)}$$

Yes, this is, you see, even that equation P is equal to 2 epsilon 0; this is only the first term of the non-linear, this is only the first term; there are actually infinite series and this is also magnetic field coming in this is only electric field contribution; there also magnetic field contribution, so, there are more things in the equation which we can look at if I going to retell the non-linear optics. Anything else?

Thank you.