Quantum Electronics

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Module No. # 03

Second Order Effects

Lecture No. # 06

Non - Linear Optics (Contd.)

We continue with our discussions on nonlinear optics. Do you have any questions from the last class.

Sir, how is it possible that the electromagnetic wave of frequency omega naught, they are making dipoles to produce frequency of 2 omega?

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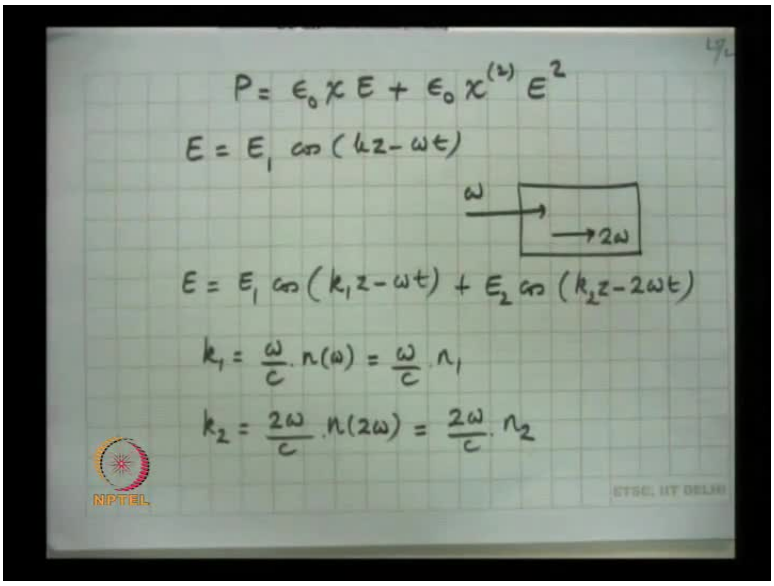

How is it that electronic magnetic wave at frequency omega is creating dipoles, which are radiating at 2 omega frequency? This is coming through the equation, which represent the nonlinear polarization term. So, if you go back and look at this equation, if this term is absent, then the polarization has the same frequency as the electric field of the incident light wave. So, polarization represents dipoles; so, if you have a linear medium, if the electromagnetic wave had frequency omega, the polarization also have frequency omega. So, the dipoles will oscillate at frequency omega and radiate omega frequency.

Now, because of this term - E square, if E has a frequency - omega, E square - there is a term at frequency 2 omega, so that means there is a component of polarization oscillating at frequency 2 omega also, apart from omega from here. So, that 2 omega frequency represents a component of the nonlinear polarization and polarization is dipoles. So essentially, the polarization at 2 omega frequency is generating electromagnetic wave at frequency 2 omega. So, the presence of this term induces nonlinear polarization. And if E has a frequency omega, this E square term will contribute a frequency 2 omega to P.

So, the polarization not only has a frequency omega, but also has a frequency 2 omega; that means, it has a combination; it is not simple harmonic any more, it has omega, it has a d c term, and also a 2 omega term. So, that 2 omega contribution to the polarization leads to the generation of electromagnetic wave with that frequency 2 omega; that means, there are individual dipoles radiating at 2 omega frequency, but please remember that even if I have a large number of dipoles radiating electromagnetic waves at a certain frequency, it is not necessary that they will all constructively interfere and add.

It is possible that they cancel each other off. So, there is no way and that is the phase matching condition, that I will explain; that if you do not satisfy the phase matching condition, what is happening is, the radiation from individual dipoles is not adding constructively; it is trying to sort of cancel off - partly cancel off or completely cancel off.

Does that answer your question? Anything else?

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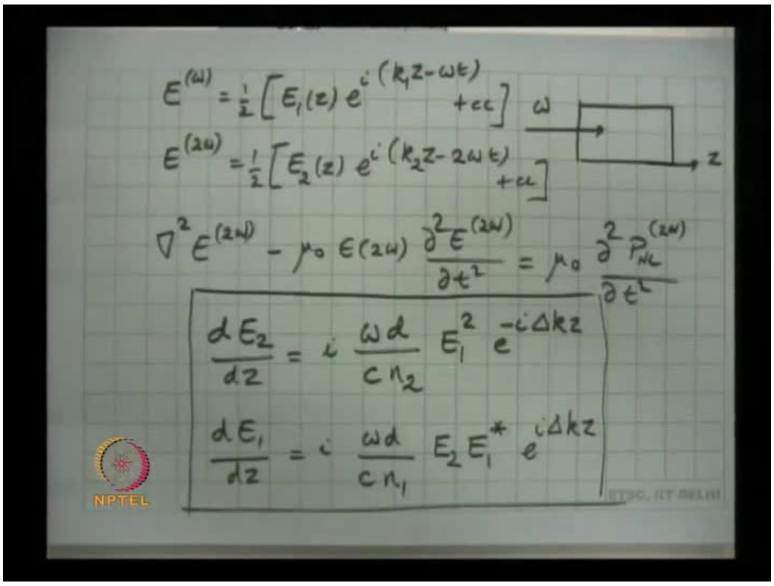

So, we will continue the discussion on second harmonic generation. Let me recall the problem we are looking at is, I have a crystal with a finite second order nonlinear coefficient, I launch waves at frequency omega, and I want to calculate what is the amplitude of electromagnetic wave at frequency 2 omega, as it gets generated along the crystal; so, this is the z direction. Now, let me recall this z direction is some laboratory coordinate z direction, this could represent optic axis, this could be at some angle to the optic axis; and this z direction is not the principle z direction of the crystal.

So, let me recall, so we had E omega given by half of E 1 of z e to the power i k 1 z minus omega t plus complex conjugate. Then, we wrote E 2 omega as half E 2 of z e to the power i k 2 z minus 2 omega t plus complex conjugate.

A plane wave at frequency omega propagating in the z direction. And inside the crystal, at any point you also have electromagnetic wave at frequency 2 omega, which we expect we generated by the nonlinear polarization at frequency 2 omega. So, what we did was, we substituted this equation into the nonlinear polarization term, calculated the nonlinear polarization at frequency 2 omega and at omega, substituted in the wave equation corresponding to 2 omega. Let us recall this equation was, E of 2 omega minus mu 0 epsilon of 2 omega del square E of 2 omega by del t square is equal to mu naught del square P nonlinear at 2 omega by del t square.

So, we substitute at expressions for E 2 omega and P nonlinear at 2 omega into this equation, neglected the second derivative of E 2 with respective to z, and use the fact that k 2 square is 4 omega square mu naught epsilon at 2 omega.

And after some simplification, remember we obtained this equation, d E 2 by dz is equal to i times omega d by c n 2 E 1 square e to the power minus i delta k z; this represents a differential equation corresponding to the rate of change of the amplitude - complex amplitude was second harmonic wave - as it propagates on to z direction. Remember, d is the nonlinear coefficient.

We will have more detail discussions on the tensile properties of d, but right now this d is some scalar quantity, setting here in effective nonlinear coefficient. And the fact that E 2 is proportional to E 1 square is because of nonlinearity; E 1 square is coming in this equation starts a nonlinear equation. And as I mentioned this equation cannot be solved just like this, because E 1 is also a function of z. So, if second harmonic waves get generated, the energy must be coming from the fundamental omega and so if E 2 increases with z, E 1 must decrease with z.

Now, as I told you in most situations, the efficiency of generating 2 omega from omega is usually very low - may be a percent or few percent. And in such a situation, I can approximately assume that, E 1 is independent of z and then I can integrate this equation.

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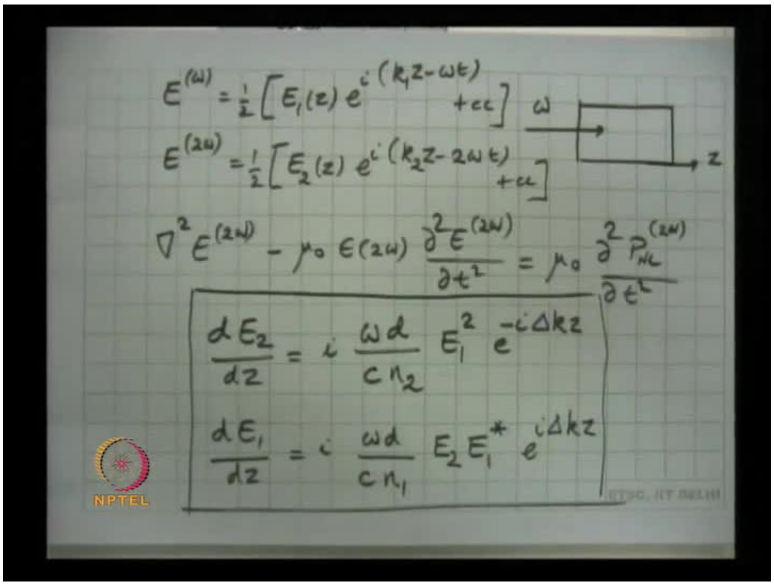

Now, before we integrate this, let me write down the equation for E 1 as a function of z. And I leave it as a problem you to obtain this equation exactly in the same procedure, that we adopted to obtain this equation. So, d E 1 by dz is i omega d by c n 1. Now, tell me, instead of E 1 square, what should I get? Last time, I told you that E 1 square, E 1 is a coefficient of exponential minus i omega t, so E 1 square will be minus 2 i omega t, which is E 2.

So, how will I get omega? Nonlinearity at omega will depend on what product? E 2 into E 1 star; E 2 corresponds to E to the power minus 2 i omega t, E 1 corresponds to exponential plus i omega t and the product will be the coefficient of E to the power minus i omega t, which is what E 1 represents. In fact, this E 2 E 1 star we had obtained as the nonlinear polarization term of that the frequency omega.

If you look back, we had shown that P nonlinear at omega is half of 2 epsilon 0 d E 2 E 1 star into exponential i k 2 minus k 1 z minus omega t plus complex conjugate; so, this equation we have derived. So, you can show that this equation, you get with exponential i delta k times z. So, these are the two coupled nonlinear differential equations, which describe the evolution of the fundamental and the second harmonic as a function of z.

So, I leave this derivation of the second equation to you. you use the same procedure, neglect d 2 E 1 by dz square, use an expression relating k 1 and epsilon at omega, and you will end up with this equation.

So, normally, we have to solve these two equations simultaneously to obtain the solution, but first what you will do is, we will look at the approximation, that the generation of second harmonic is very weak; efficiency is very low.

So, as I told you, suppose I launch 1 of watt power at omega and if I generate 1 milliwatt of power at 2 omega, I shall have 999 milliwatts at omega frequency. So, the electric field of 1 watt and 999 milliwatts is almost equal; they are not very different. So, I can assume E 1 does not change appreciably as I propagate, while E 2 has gone from 0 to 1 milliwatt. So, change in E 2 is significant, but the change in E 1 could be very small. So, this is under low efficiencies; for high efficiencies, I must solve these two equations simultaneously.

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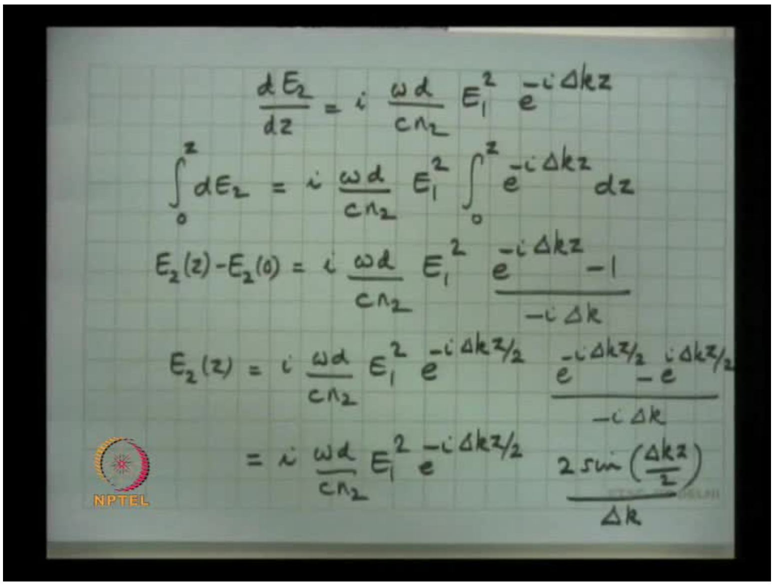

So, we look at the solution later, but right now let me look at the solution of the first equation under low efficiencies. So, let me rewrite this equation, d E 2 by dz is i omega d by c n 2 E 1 square exponential minus i delta k z

Now, if E 1 is a constant, I can integrate this equation; so, integral d E 2 from 0 to z will be equal to i omega d by c n 2, E 1 square is a constant, integral 0 to z minus i delta k z dz. Remember, delta k is k 2 minus 2 k 1. So, this will give me E 2 of z minus E 2 of 0 is equal to i omega d by c n 2 E 1 square exponential minus i delta k z minus 1 by minus i delta k.

Now, in most situations, there is no incident field at omega frequency. In fact, normally, in second harmonic generation, I have omega frequency instance. So, E 2 of 0 is 0, because that is the input field at second harmonic; so, I can write this as, E 2 of z is i omega d by c n 2 E 1 square, and let me take this factor common, minus i delta k z by 2. So, what will be left in the numerator? I will have minus i delta k z by 2 minus e to the power i delta k z by 2 by minus i delta k.

So, omega d by c n 2 E 1 square exponential minus i delta k z by 2, this is minus 2 i sign delta k z by 2, so minus i minus i cancels off, and I get 2 sin delta k z by 2 by delta k. So, this is the electromagnetic field at the second harmonic; remember E 2 is the amplitude - complex amplitude of the electric field - corresponding to the frequency 2 omega; E 1 is assumed to be constant and is the electric field of the incident wave at frequency omega.

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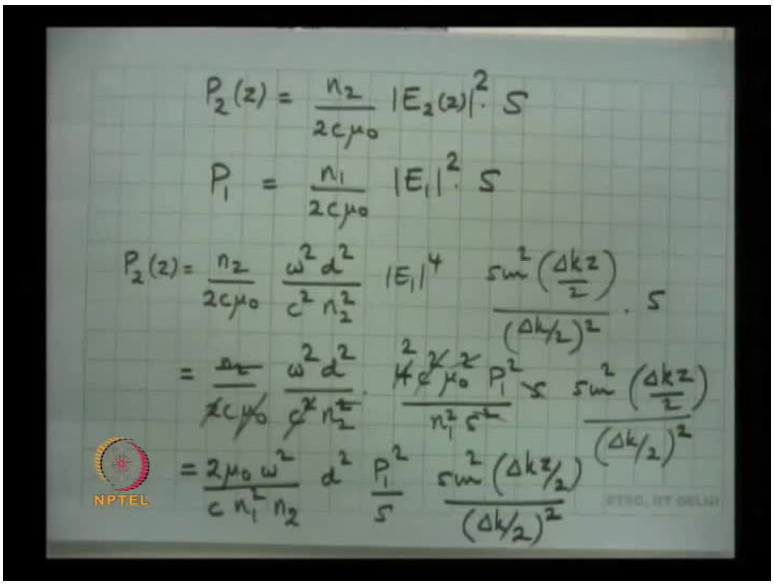

Now, what I am in interested in is, what is the power generated in the second harmonic, how much of energy is generated in the second harmonic. So, how do I calculate the power? I know that power is related to electric field through this equation. So, power at second harmonic is n 2 by 2 c mu 0 mod E 2 square into area of the beam.

So, if the beam has an area a, I am not taking any diffraction into account, there is area of cross section S; in that S area of cross section, this is the intensity; remember, intensity is related to electric field of a plane wave through this equation, the refractive index divided by 2 c mu 0 mod E 2 of z square into area; that is the power at the second harmonic. Similarly, I can define power at the fundamental as, n 1 by 2 c mu 0 mod E 1 square into S; E 1 is assumed to be a constant - almost a constant, so P 1 is the power incident at the fundamental frequency omega.

I have an expression for E 2 of z, so I substitute from here. So, let me do this mathematics here; so, P 2 of z is n 2 by 2 c mu 0; now, mod E 2 z square will give me omega square d square by c square n 2 square into mod E 1 raise power 4 into sin squared delta k z by 2 by delta k by 2 whole square.

I have substituted E 2 of z, that we have obtained as a solution into this equation. Now, I can replace mod E 1 raise power 4 in terms of P 1. So, this is n 2 by 2 c mu 0 omega square d square by c square n 2 square. Now, mod E 1 4 is 4 c square mu 0 square P 1 square by n 1 square into S square sorry there is an S here, into S, this is into S area into S into sin square delta k z by 2 by delta k by 2 whole square, simple algebra.

So, let me strike off common terms here, one n 2 goes off here, c square goes off, 1 mu 0 goes off, see you will have mu 0; so there is factor of 2, this factor of 2 cuts off here, so 2 mu 0 omega square by c n 1 square n 2 d square P 1 square by S, one of the S cancels off, into sin square delta k z by 2 by delta k by 2 whole square. P 2 of z is the power in the second harmonic wave at any value of z; P 1 is the power incident at the fundamental frequency omega and that is assumed to remain constant with propagation; d is the nonlinear co-efficient; S is the area of cross section of the beam; and delta k is k 2 minus 2 k mod. The first thing you notice is, when will, under what conditions of delta k will P 2 be maximum.

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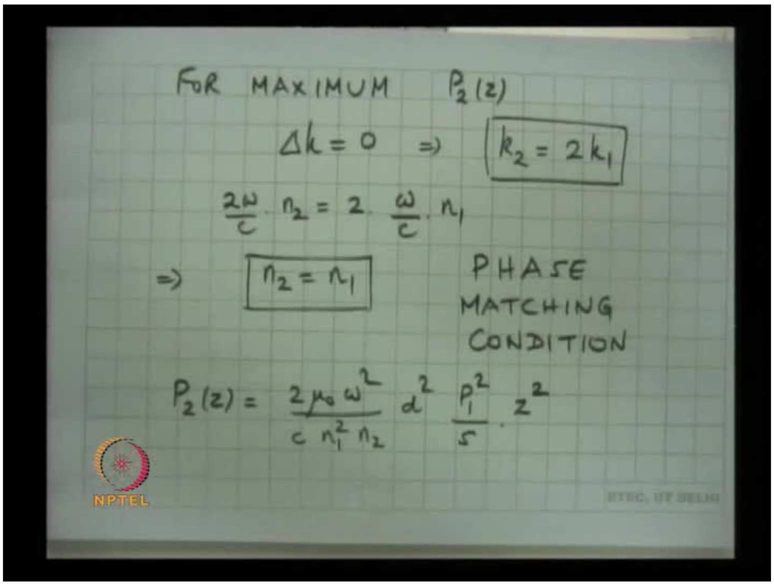

When delta k is equal to 0, what happens to this function - sin square delta k z by 2 by delta k by 2 whole square one or something else? z square; sin x by x is 1, but this is sin alpha x by x whole square; so, please remember, this is z. So, actually the maximum efficiency will happen. For maximum efficiency, for maximum P 2 of z, delta k is equal to 0, this implies k 2 is equal to 2k 1. And because k 2 is 2 omega by c into n 2 must be 2 times omega by c into n 1 this implies n 2 is equal to n 1; this is called the Phase Matching Condition.

So, maximum conversion will take place, if you satisfy the phase matching condition. And as I interpreted before, this implies that the velocity of the nonlinear polarization at frequency 2 omega is equal to the velocity of electromagnetic wave at frequency 2 omega.

Nonlinear polarization is the source of the electromagnetic wave. If the source and the wave that it generates travel at the same speed, then you can continue to add energy into the second harmonic. So, when you satisfy the phase matching condition, you find that the efficiency is maximum, the generation of second harmonic is maximum. And this is nothing but saying that the nonlinear polarization and the velocity and the wave that it is trying to generate the travel at the same speed.

This you will see in many other interactions as we go through and this is a very very important condition for efficient nonlinear interaction. We will actually calculate the power generated if I do not satisfy this condition and you will see the efficiency is to be million times smaller than if you were to satisfy the condition. So, the efficiency very critically depends on your satisfying the phase matching condition.

So, if I satisfy the phase matching condition, then P 2 of z will be given by 2 mu 0 omega square by c n 1 square n 2 d square P 1 square by S into z square; the sin square delta k z by 2 by delta k by 2 whole square is simply z square. So, this is the maximum efficiency and the efficiency grows as z square. If you double the length of the crystal, the efficiency becomes 4 times.

Please note that I cannot use this equation, if the efficiency becomes very large, because according to this equation, it looks as if I keep on increasing z and my efficiency can be larger and larger more than 100 percent; that is not correct, because this equation is valid only for low conversion efficiencies. The moment P 2 and P 1 become comparable, I have to use the second equation also simultaneously to solve.

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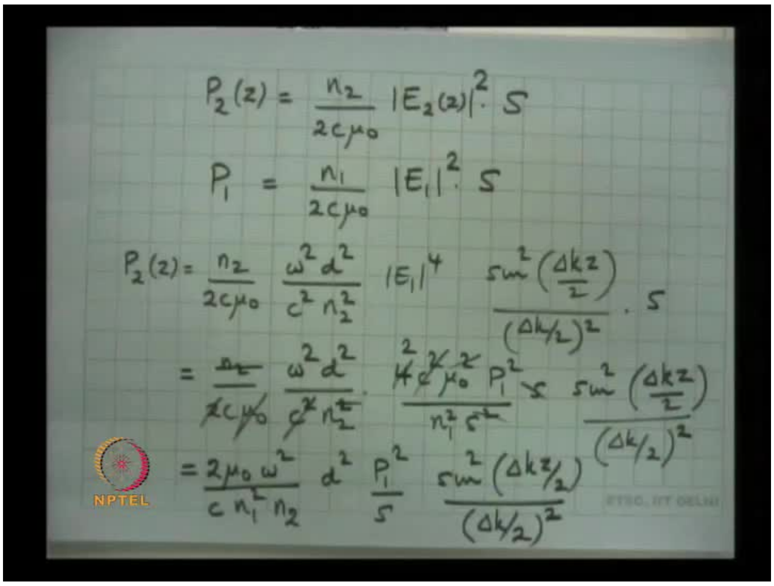

Now, with this equation, from this equation actually, we can define what is called as the efficiency of second harmonic generation, as the ratio of the power emit coming out at second harmonic divided the power incident at the frequency omega.

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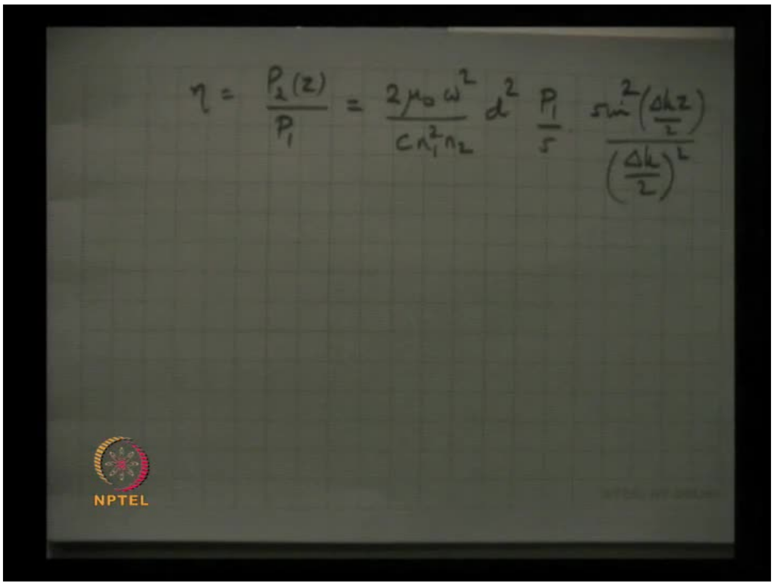

The power that is generated at frequency 2 omega divided by the power incident at omega is the efficiency is, P 2 of z by P 1 and this is nothing but 2 mu 0 omega square by c n 1 square n 2 d square P 1 by S sin delta k z by 2 by delta k by 2 whole square.

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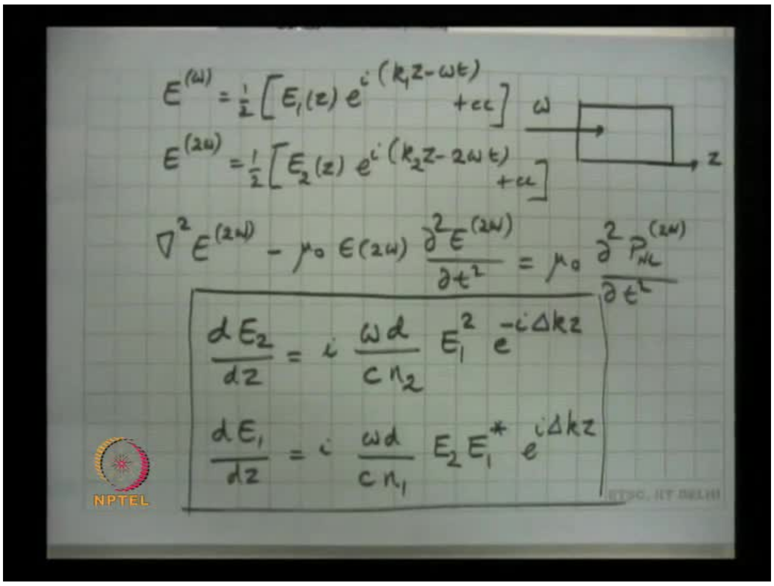

Consider both the equation simultaneously, after the efficiency is sufficiently high, we expect a saturation or do we expect that it will fall again?

No, I will solve these two equations later. And we will solve this, assuming delta k is equal to 0 and I will show you it saturates to 100 percent, as you keep on increasing the length; it will never reach 100 percent, unless you have an infinite length of meter large. So, the energy keeps on adding up, adding up, and finally…

But if delta k is not 0 as we now show that, this is a periodic exchange, I will plot that. So, first thing to notice here is the efficiency; this is the equation for the efficiency of the generation of second harmonic through the nonlinear interaction process; and first notice that the efficiency depends on the ratio of P 1 by S.

Sir, if we say that, if we consider high efficiency generation.

Yes.

Then, do we not have to consider the second derivative that indicated in the(())

No, second derivative neglect implies that the rate of change of E 1 and E 2 is small; it does not mean the total E 1 and E 2 are small. The rate at which E 1 is changing, because if you go back to that equation and see what we are neglecting with respect to what, then you will see that, it only means that the rate at which E 1 and E 2 are changing with z, over a wave length is very very small; that is, the change in E 1 and E 2 over a wave length is very small.

So, this dependent on P 1 by S is a typical nonlinear interaction, because efficiency depends on the input power. Normally, in linear processes, deficiency does not depend on the input amplitudes; this is a nonlinear process. So, if you take a certain power of light and if you decrease the area of cross section, the efficiency will increase for the same power.

So, if you focus a laser beam into a crystal to decrease the area of cross section, you will increase the efficiency. Of course, as I told you before while discussing the fraction, we have to be conscious that if I try to focus too much, I will defocus very quickly. So, I cannot have arbitrary z, if I try to focus too much.

So, the length over which I can interact over a small area of cross section are not independent of each other, but the fact that for the same power, I can increase efficiency by decreasing the area of cross section is interesting. And that is, which means that, I need to reduce the area of cross section of the beam, if I want more efficiency. It depends on d square - the square of the nonlinear coefficient and so the larger the nonlinear coefficient of the crystal, the higher deficiency. So, there is a constant effort to make, to look for materials with higher and higher values of nonlinear coefficients.

So, lithium niobate is one of the very high nonlinear coefficient crystal; and d, you can get of the order of 30 10 to the minus 12 meters per volt. So, the order of magnitude of d is above 101 to the minus 12 meters per volt and lithium niobate is one of the crystals which has one very strong nonlinear optical coefficient.

So, let me plot eta verses z; if I do phase matching, that means if delta k is equal to 0, this coefficient becomes z square. So, this increases quadratically with z; I am assuming this numbers are still a few percent only, otherwise this parabola will go on, keeping on going up. So, the actual efficiency if I were to calculate, we will be going along this, but then start to deviate from here for larger deficiencies.

What happens if delta k is not equal to 0? Its periodic; so, it goes up, come down. Now, what is this value? This is z is equal to 0. What is the value at which deficiency becomes 0 again? When the sin function becomes 0, so this length must be 2 phi by delta k, this is 4 phi by delta k, and this point is phi by delta.

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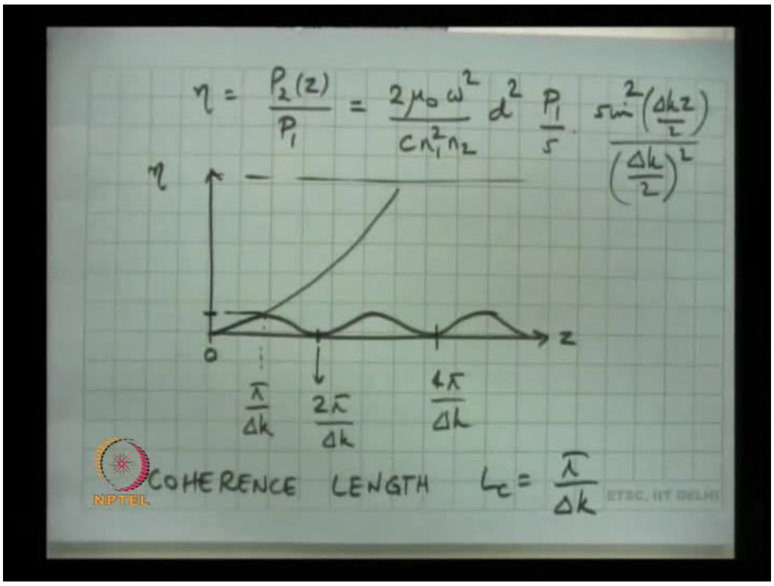

So, if you do not have phase matching, the power in the second harmonic increases up to a length phi by delta k becomes maximum at this distance and then starts to decreasing again. This length is called the Coherence Length, L c is equal to phi by delta k. This is not the Coherence Length of a light source having a spectral with delta lambda, this is another Coherence Length; that means, if you, if you have a finite delta k, the maximum efficiency you can get is, when you take a crystal of length - Coherence Length equal to Coherence Length - you cannot do anything better than that. If you have phase matching, the longer the crystal, the better it is; but if you do not have phase matching, then the best you can do is to have a crystal of length phi by delta k - Coherence Length.

So, now, what is happening? Let me try to interpret this figure as, so this is the plane at which the… so this is the crystal is here. So, the frequency omega enters from here; at the input, there is no second harmonic field. So, the omega field generates nonlinear polarization at 2 omega and omega also inside; the 2 omega field starts to build up. At this distance, it keeps on building up until the distance phi by delta k; beyond phi by delta k, the amplitude of two of second harmonic is decreasing. Now, so, what is happening? Where is this power going? It must be going back to omega frequency.

So, what is this process? Difference frequency generation. So, till this point, omega plus omega equal to 2 omega - second harmonic; beyond this point, 2 omega minus omega is equal to omega till this point, till the second harmonic disappears. This plane is exactly same as this plane; again, the second harmonic starts to grow. So, this keeps on repeating itself periodically and this periodic exchange is from omega to omega, and back to omega and then to omega, back to omega; it keeps on exchanging back and forth. And the maximum efficiency you can get here is, when you have, what is the value of this? When this function becomes…

No, it cannot be 1.

When sin becomes 1; so, this coefficient becomes… So, this maximum efficiency is this, entire quantity multiplied by 4 by delta k square. When this sin becomes 1, so that is the value; so, this is, the maximum efficiency you can get is, this multiplied by 4 by delta k square.

So, larger the delta k, smaller is the maximum value. So, the more you are phase mismatched, the smaller is the maximum efficiency you can generate from this process. So, why is it at this point, the energy is flowing back from 2 omega to omega? So, let me calculate what is the phase shift suffered by the nonlinear polarization in travelling from here to this point, and what is the phase shift suffered by the second harmonic electromagnetic field in going from here to here.

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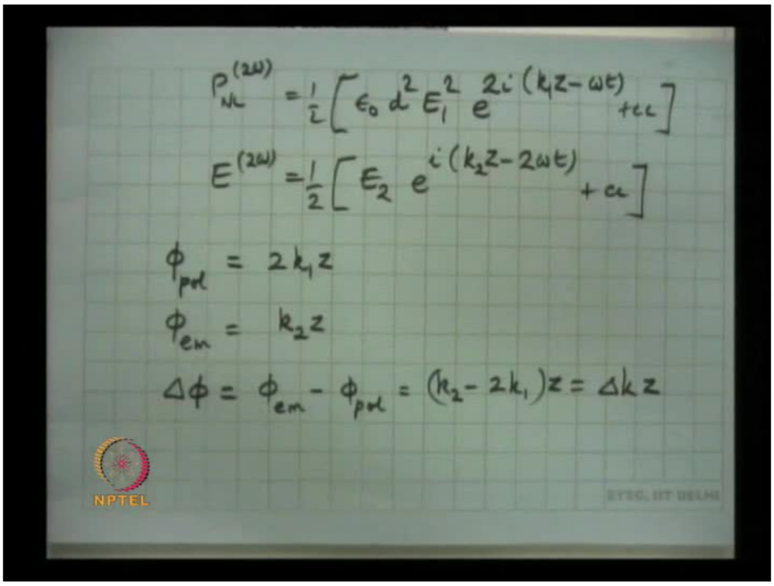

Now, if you remember, if you recall the nonlinear polarization wave is also a wave, so the nonlinear polarization; if you remember this is nonlinear polarization, 2 omega is half of epsilon 0 d E 1 square e to the power 2i k 1 z minus omega t plus complex conjugate; and the electromagnetic wave at 2 omega is half E 2 e to the power i times k 2 z minus 2 omega t plus complex conjugate.

The nonlinear polarization is a travelling wave; the electromagnetic wave at omega 2 omega is also a travelling wave. So, what is the phase difference - phase shift suffered by the nonlinear polarization in travelling a distance z? 2 k 1 z, because the phase is changing at the rate of 2 i k 1 z. So, if I propagate at distance z, the phase will change by 2 k 1 z.

What about the electromagnetic wave? k 2 z and they may not be equal; they are not equal if you phase matched. And so the phase difference between the polarization and the electromagnetic wave is, phi electromagnetic minus phi polarization, which is k 2 minus 2 k 1 into z, which is delta k z.

So, if you are not phase matched, the nonlinear polarization and the electromagnetic wave are actually getting dephased as you propagate. And what happen at the Coherence Length? z is phi by delta k; the phase difference becomes phi - there exactly out of phase.

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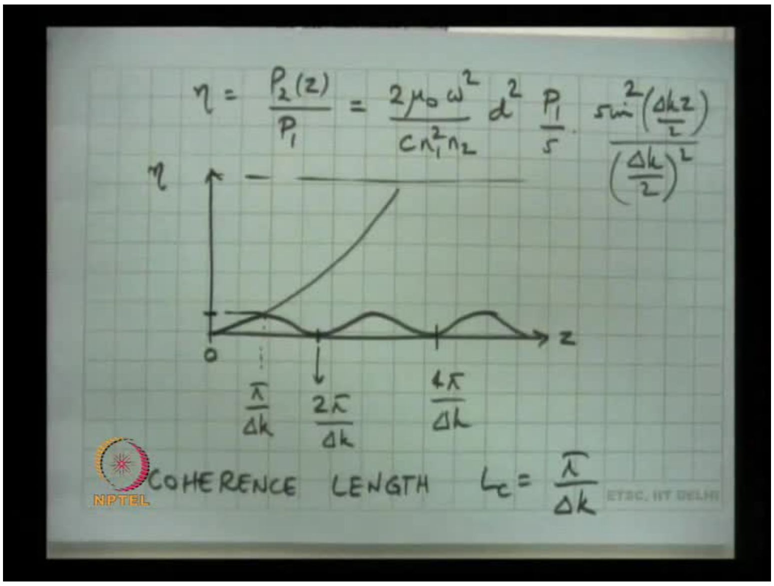

So, the second harmonic is travelling at a different speed as the nonlinear polarization, at a distance correspondent to Coherence Length. The field and the polarization get out of phase by phi and beyond that point, the polarization instead of feeding energy into the second harmonic, constructively adding; it is actually feeding, but out of phase and out of phase means destructive interference. And so, the amplitude of the second harmonic starts to fall, until you reach completely 0; at this point, there is no second harmonic and the second harmonic starts to build up again.

Remember, if you are, if you are, let me take the example of a swing, child sitting on a swing and the child has to sit and stand at a certain frequency. What is the frequency at which the child has to sit and stand? If the frequency of the swing is omega - 2 omega, right; it has to sit, then stand up, then sit and then stand up. So, the child sits and stands at twice the frequency of the pendulum of the swing.

And if the child is not at the right frequency - a slightly different frequency, initially the swing will increase amplitude and then start to decrease amplitude, because the child now starts to sits and stand at the wrong points. So, the swing will start like this and then go back like this, and then again start like this, it will keep on oscillating back and forth. So, the child is trying to pump the swing and the maximum pumping is possible, continuous pumping is possible provided, the frequency at which the child is sitting and standing is lightly matched to the frequency at which the swing is swinging.

If there is a small difference in frequency, it will be adding for some time, and then subtracting again, adding some time, subtracting; in fact, this example is a very interesting example. This is called a parametric amplifier, because all that the child is doing is to increase the length of the pendulum and decrease the length of the pendulum. It is modulating the time period of the pendulum by sitting and standing; and this modulation feeds energy into the system. This is called parametric amplifier and we have optical parametric amplifier, which we will discuss later.

So, what is exactly happening is, at the Coherence Length, the electromagnetic wave and the polarization are exactly out of phase by phi; and beyond that point, the second harmonic feeds energy back into the omega frequency. So, at this point, all the energy is back in the omega, again 2 omega builds up and its oscillates like this periodically.

Now, let us look at an example and calculate, suppose I didn’t bother about phase matching, so that will be my efficiency. So, let me calculate this number, let me assume, let we take some typical numbers and calculate what will be my typical efficiency.

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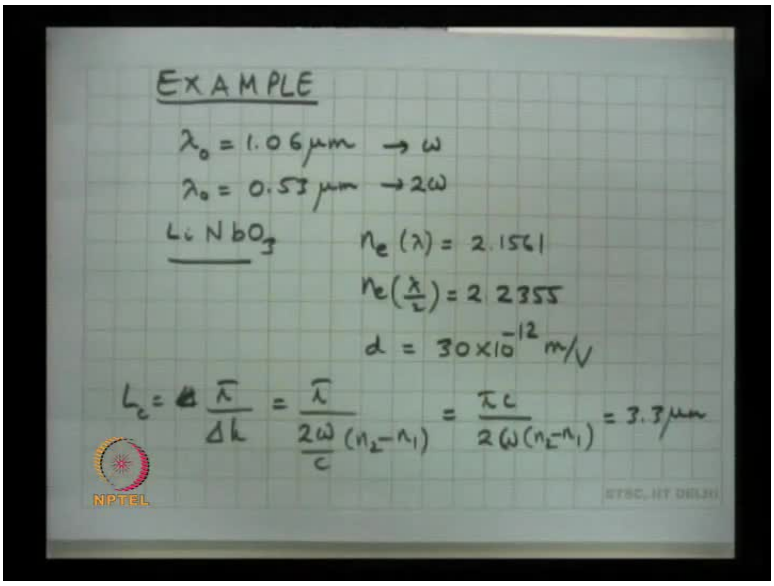

So, here is an example; so, let me take a wavelength of 1.06 microns, what laser is this? Neodymium yttrium laser or Nd glass laser. Neodymium is used in lasers and that is creating a wavelength of 1.06 micron. So, this corresponds to omega. So, what is the wavelength corresponds to 2 omega? 0.53, that is the twice second harmonic.

Now, let me take lithium niobate - L i N bO 3; this is a uniaxial medium. So, let me give you the refractive indices, n e at lambda is equal to 2.1561, n e lambda by 2 is equal to 2.2355, and d is equal to 30 10 to the minus 12 meter per volt.

So, I take lithium niobate and this is the extraordinary refractive index. I also have the ordinary refractive indices, but I will come back to this later. This nonlinear coefficient is the effective coefficient, if you use the extraordinary polarization. So, this I will make it clearer later, but let we assume the refractive indices are given by this, this is nonlinear coefficient, and these are the two wavelengths.

So, let me first calculate what is the Coherence Length. Coherence Length is L c is equal to phi by delta k; so, this is phi by 2 omega by c into n 2 minus n 1, which is equal to phi c by 2 times omega times n 2 minus n 1. So, what is n 1? n eth lambda; n 2? n eth lambda by 2.

Please remember n 1 is the refractive index seen by the wave at the fundamental frequency, which is the wavelength lambda, n 2 is the refractive index seen by the wave at the second harmonic frequency, which is n e of lambda by 2. I know the wavelengths, I can substitute and I get this about 3.3 micrometers - very small Coherence Length. So 3.3 micrometer, maximum; at 6.6 micrometer, it will become 0 again.

If you take a crystal of length 6.6 micrometers of this crystal and launch light, you will get no second harmonic. Although it has been generated inside, it has actually gone back to the fundamental; so, that is the Coherence Length. And now, I need to substitute into this equation.

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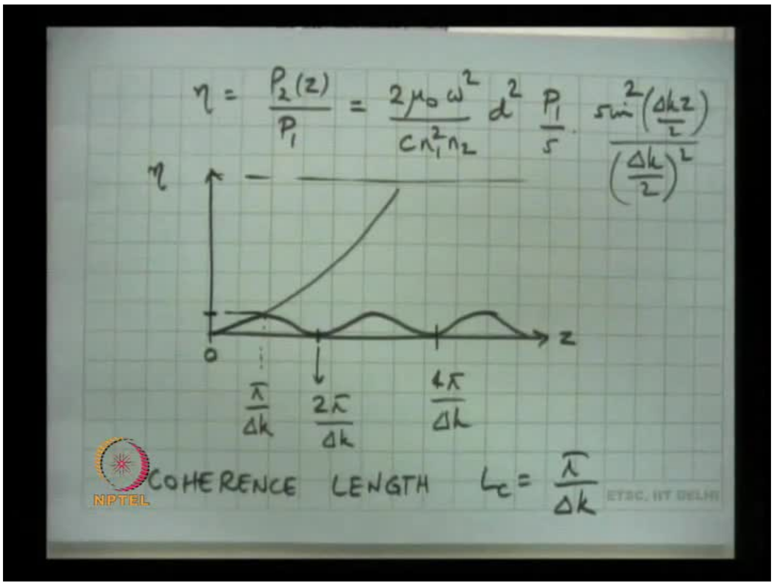

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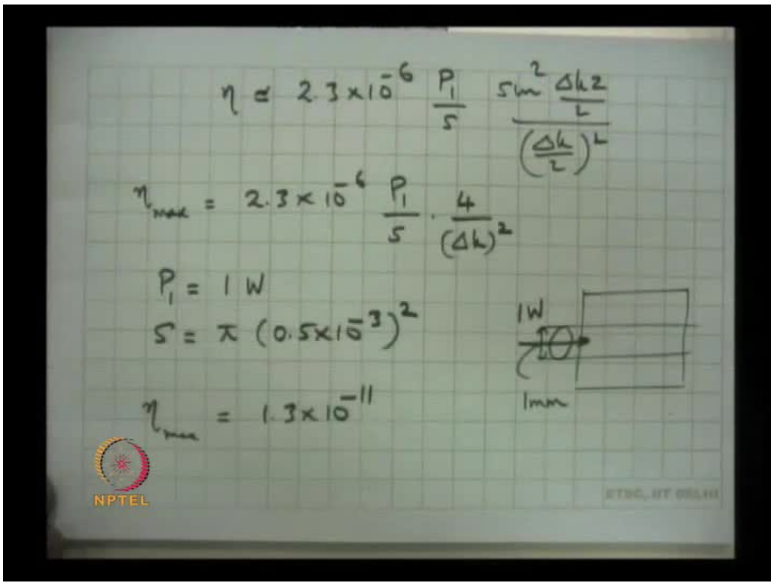

So, I have all parameters, I have mu 0, I have omega, I know c n 1 n 2 d, so let me leave this terms separate, so I can substitute all this. And this is some simple numerical substitution, what you can show is, eta is equal to 2.3 approximately equal to into 10 to the minus 6 P 1 by S into sin square delta k z by 2 by delta k by 2 whole square; you can substitute the values of the constants and that is what you get.

So, first let me calculate what is the maximum efficiency; I will get, so eta max, I will get 2.3 10 to the minus 6 P 1 by S into 4 by delta k square, when the sign function becomes 1. So, let me assume, I have input power of 1 watt and an area of cross section corresponding to phi into 0.5 millimeter square. It is a crystal and the beam has a diameter of 1 millimeter and the power incident is 1 watt; remember, 1 watt is the lot of power at in the laser. So, you have an expression for delta k, you know delta k, P 1, S, everything you know; so, you can calculate eta max comes out to be 1.3 10 to the minus 11.

So, you have launched 1 watt of power at the fundamental frequency and you are generating 13 Pico watts of power, 10 to the minus 12 watts is Pico watts. So, this means essentially that you have a very very low efficiency and that is coming because of this term here. Suppose, I were to achieve phase matching, we will discuss how to achieve phase matching. Let me suppose I were to, I am able to make show delta k is equal to 0, then this term will give me z square.

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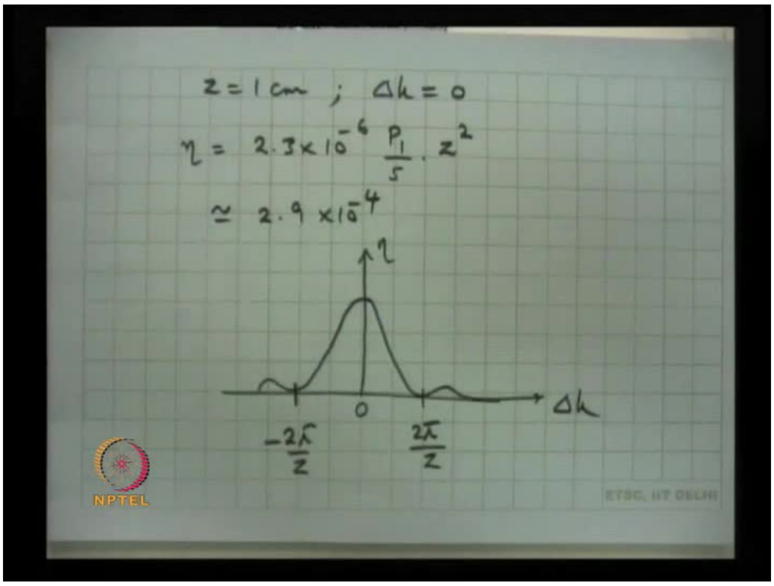

And let me take a crystal of length 1 centimeter. So, suppose I could take a crystal of 1 centimeter and I could achieve delta k is equal to 0, then the efficiency becomes eta is equal to 2.3 10 to the minus 6 P 1 by S into z square. And you can again calculate this number and you will get a value, which is 2.9 10 to the minus 4 - 1 centimeter, an increase in efficiency of 10 million times.

So, this means, essentially with 1 watt of power, you will generate 0.29 milliwatts of power, so still the efficiency is very low. At these efficiencies, the formulation is very accurate; even if you were to solve both equations simultaneously, your numbers will not be different from what you have estimated from here. So, this shows the importance of phase matching.

If you will not take care of phase matching, the efficiencies can be very low. So, in all nonlinear process, you have to ensure phase matching is achieved. We will discuss some processes in which phase matching is automatic; that means, the nonlinear polarization and the wave are travelling at the same speed by definition.

We will see this in third order processes, but you have to make sure that the polarization which is generating the electromagnetic wave and the electromagnetic wave are not travelling at different velocities, to maximize efficiency of conversion. Also note if I were to plot eta verses delta k for a given length, eta verses delta k will be what kind of function? It is like a sync function. So, eta verses delta k if I were to plot, so this is 0; what are these two 0s? This is delta k is equal to when will this be 0 2 phi by z

When the sin function becomes 0, for a given z when the sin function becomes 0, other than delta k is equal to 0. So, you will get, so what is this imply? If you are phase matched, delta k is equal to 0, you have maximum conversion; if you are not phase matched, the efficiency drops down.

By how much can I be of phase matching and still have reasonable efficiencies that depend on the width of this curve? The narrower the curve is, the more carefully I must achieve phase matching; if the curve is very broad, even if I have not phase matched perfectly, it does not matter to me. The longer the crystal length, the more critical the phase matching will become, because this curve will become narrower and narrower, as you increase the length of interaction.

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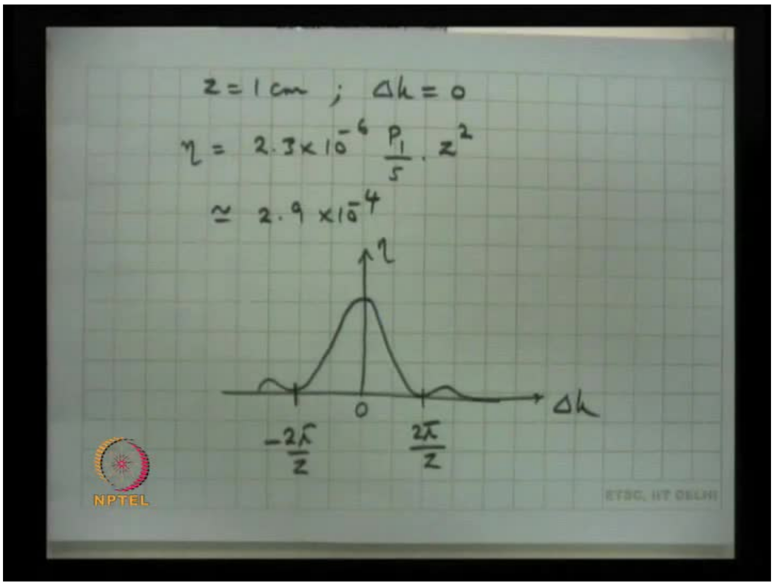

For short crystals, shorter the crystal, the less critical is phase matching, but phase matching is always critical. Please remember the Coherence Length we got for 3 microns. So, normally, this clevia that you are given is not very large; you can be slightly… even if you are not perfectly phase matched, its still alright, you get some efficiency. But lower than when you are phase matched, by how much lower it depends on this width?

So, what we will now do is have a quiz. So, what I will do in the next class is discuss beyond this point and I will let me… Before I finish, let me just give you an interpretation of this process itself; this second harmonic generation process is a process in which one photon at frequency omega and another photon at frequency omega merge to form a photon at frequency 2 omega.

In a quantum mechanical picture, the second harmonic generation is a process in which a photon at frequency omega and a photon at frequency omega merge to form one photon at frequency 2 omega.

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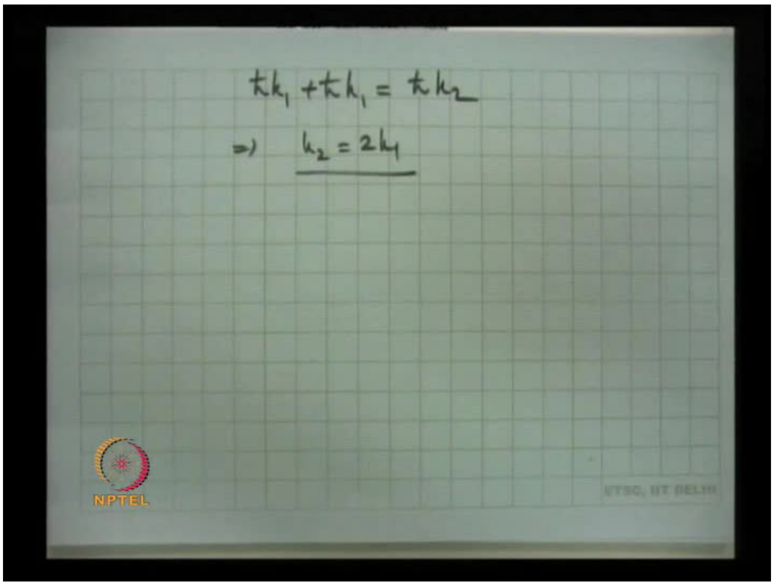

When this process takes place, you need to satisfy two conditions: the energy conservation condition, which is h cross omega plus h cross omega is equal to h cross 2 omega; and momentum conservation, what is the momentum conservation? It is matching condition, because h cross k 1 plus h cross k 1 is equal to h cross k 2; the momentum of 1 photon, so you have 1 photon, another photon, the sum of the momentum must be equal to momentum over the output photon and this is nothing but phase matching condition.

So, phase matching condition is nothing but the velocities of the polarization and waves should be equal, is nothing but mathematically I get maximum conversion efficiencies, is nothing but saying that I am conserving momentum in this process.

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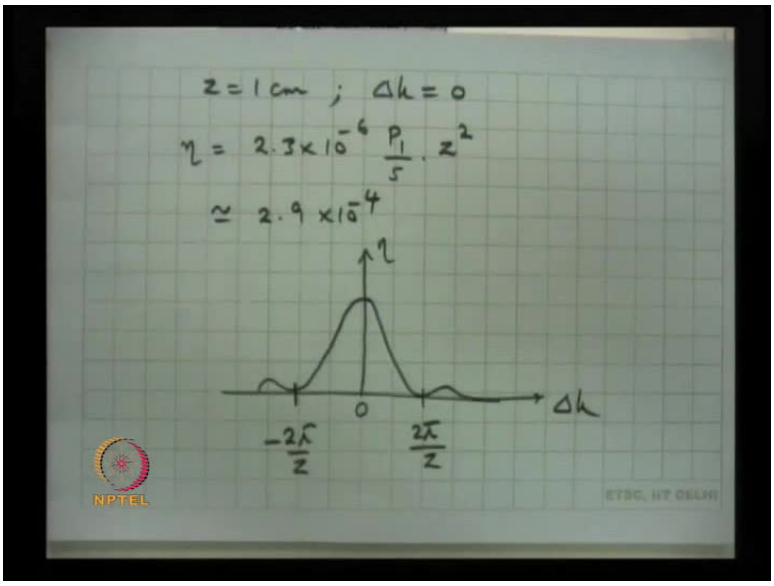

Sir, if they are not phase matched, we saw that we had some efficiencies, it should not be possible.

Question is even if I am not perfectly phase matched, there is still some conversion, how am I not satisfying the momentum conservation? Remember, quantum mechanically momentum has an uncertainty; when there is an uncertainty in momentum, you can still satisfy the conservation within that uncertainty.

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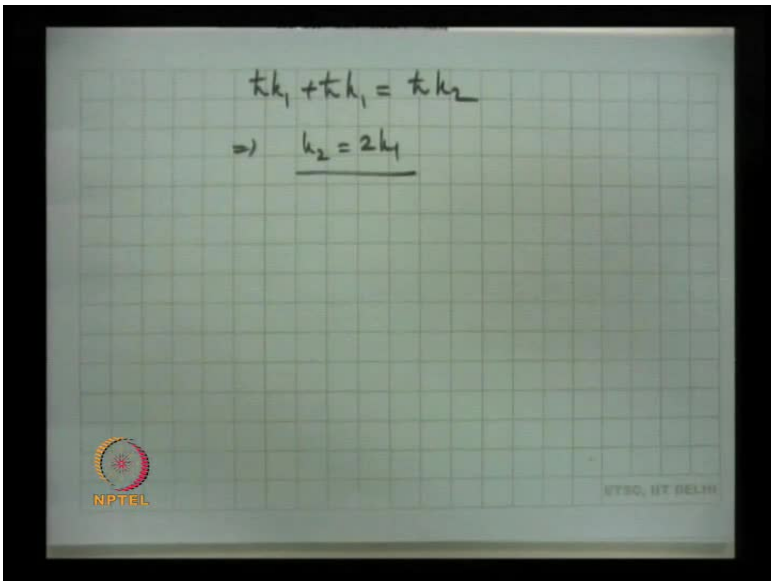

So, the longer the length of the crystal, the smaller is the uncertainty in momentum and the narrower the phase matching curve is; so, they are all consistent. So, this is simply coming because of the fact, that you have uncertainty in momentum. In energy also, there could be uncertainties, but because the time of interaction is so large, the uncertainty in energy is extremely small.

So, effectively it is coming as the energy conservation and this phase matching condition is nothing but momentum conservation. So, this we will, I will try to explain more classically right now through those equations. And later on towards the later part of the course, this will come out from quantum mechanical principles itself.

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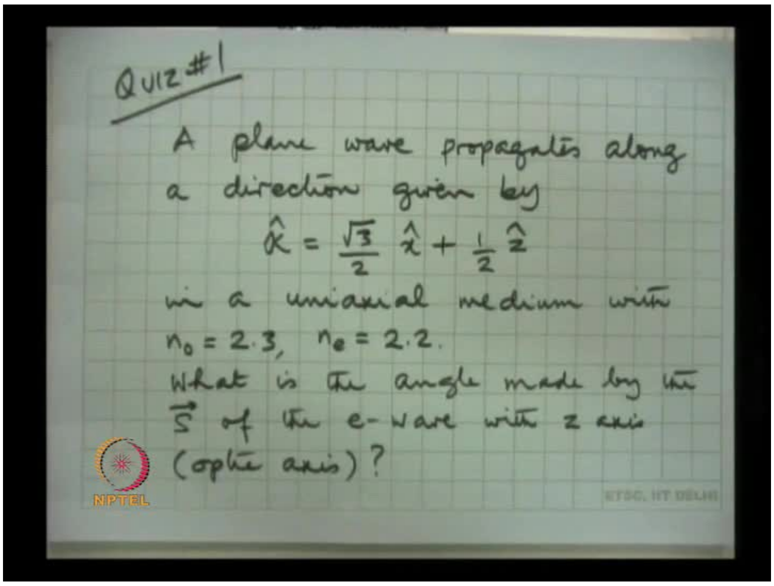

We will now have a quiz; so, what I will do is, I will write down the question here, so please pull out a sheet of paper and write the answer and give it to me.

So, the question is a plane wave is propagating in a direction given by this unit vector, in uniaxial medium with these two refractive indices: ordinary index 2.3, extraordinary index 2.2.

The question is - what is the angle made be the S vector of the e- wave with z axis? Even if you get an expression finally without actual calculations of values is fine, but final result should have just numbers and inverse functions or sin functions, tan functions whatever it is, but you do not need to take the inverse values, but it must have the numbers final.