Quantum Electronics

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Module No. # 02

Nonlinear Optical Effects Nonlinear Polarization

Lecture No. # 05

Non - Linear Optics

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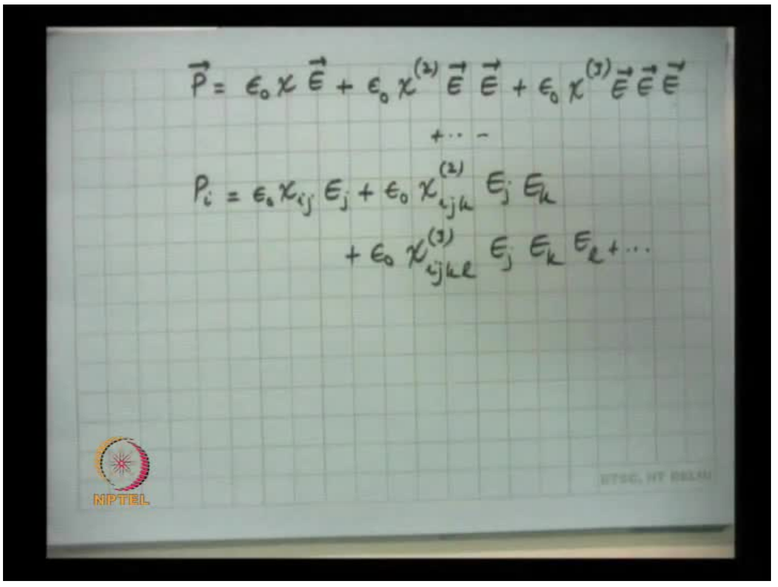

So, we continue with our discussion on non-linear optics. Let us recall that when the electric field of the light wave becomes strong, then the polarization is no more proportional to the electric field - it has also higher order terms - so you have something like epsilon 0 chi 2 E E plus epsilon 0 chi 3 E E E and so on.

Now, this equation is essentially in component form, an equation of the type epsilon 0 chi i j e j plus epsilon 0 chi 2 of i j k e j e k plus epsilon 0 chi 3 of i j k l e j e k e l and so on.

So, please note that, this E E is not a dot product or a cross product, it is just E times E. And this is the short form of representing the summation over chi i j k e j e; k ith component of polarization is proportional to the jth component of electric field through a tensor chi i j k - the linear susceptibility tensor. And these are the non-linear susceptibility tensors; the second 2 here means its E square term, 3 here is E cube term. So, this is responsible for second order nonlinearities, this is responsible for third order nonlinearities (Refer Slide Time: 02.15).

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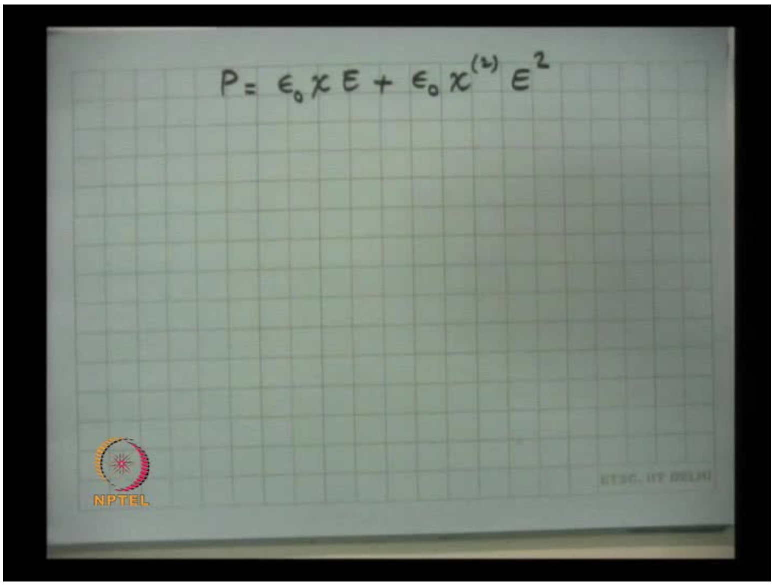

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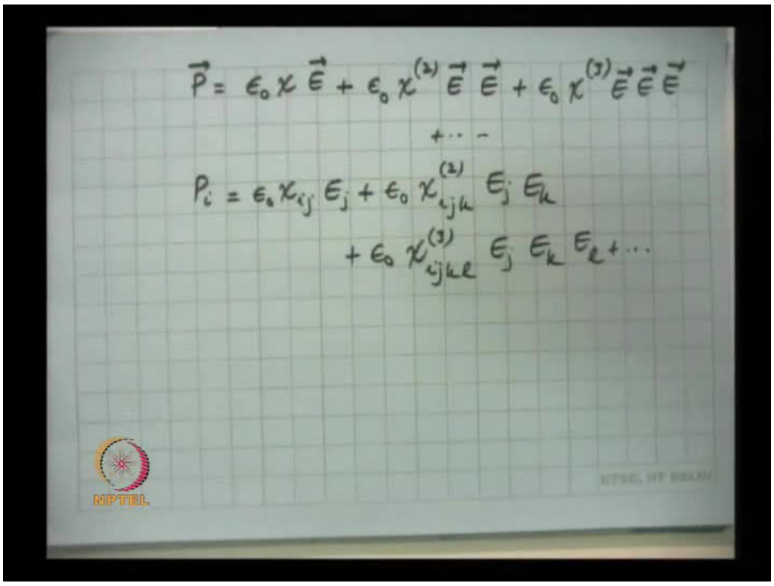

Now, before we discuss the components of these tensors, we started looking at a simplified form of the polarization, to understand how new frequency can get generated.

So, recall that what we had done is, if I write in scalar form, if I write the second order term, I can write this as plus epsilon 0 chi 2 E square; I just write a scalar equation representing the vector equation. We will come back and discuss in more detail, how to write this equation in terms of an effective chi 2 susceptibility and so on.

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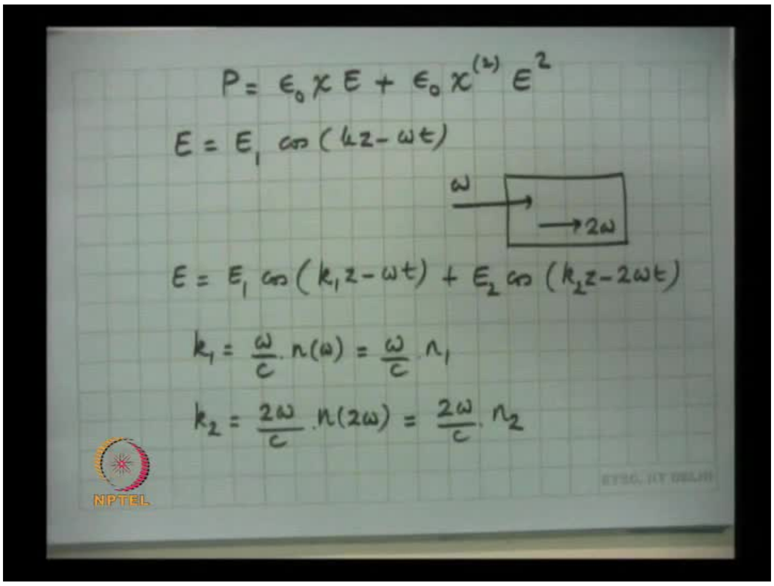

But right now, we want to understand the fundamentals of the new frequency generation and so, we will start with some scalar equation here. And remember, as we discussed last time, if I assume an electric field of the form E 1 cos k z minus omega t, you found that if you substitute this into this equation, there is a linear component of polarization. And E square gives me a component of polarization at 2 omega frequency; and polarization is nothing but dipole moment per unit volume. So, it implies that the dipoles are also oscillating at frequency 2 omega.

So, when a dipole oscillates at a frequency 2 omega, it generates electromagnetic waves at 2 omega frequency; so, that is the origin of 2 omega. But once remember, so if I take a medium a non-linear medium, I launch omega frequency into the medium; so, inside the medium, I will generate 2 omega also.

Now, this equation actually, E is the total electric field. So, once 2 omega starts to get generated inside the medium, now I have electric fields at omega and 2 omega. So, if I for example write the total electric field as E 1 cos, now let me differentiate the 2 frequencies like this; so, I have k 1 is the propagation constant of the medium at frequency omega plus, I have an electric field now corresponding to frequency 2 omega, and k 2 is the propagation constant of the medium at frequency 2 omega.

So, k 1 will be omega by c into the refractive index at frequency omega, which I call as n 1; so, omega by c into n 1. k 2 is the propagation constant of the medium at frequency 2 omega, so 2 omega by c into refractive index at 2 omega, which I call as n 2; so 2 omega by c into n 2.

Please note, that the refractive index of the medium depends on frequency. So, electromagnetic wave at frequency omega and the electromagnetic wave at frequency 2 omega will in general, not see the same refractive index. n 1 is the refractive index of the medium at frequency omega; n 2 is the refractive index of the medium at frequency 2 omega.

So, when I launch inside the medium - a frequency omega, through this E square term I find that electric fields at 2 omega also get generated. So, the total electric field now becomes the sum of these two. So, what will happen if I substitute this into the total electric field here? So, let me look at for example the second term only.

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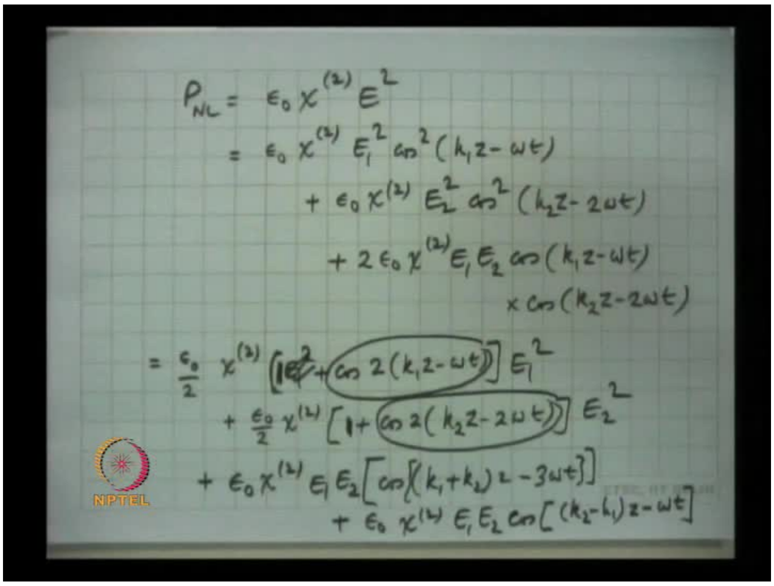

So, let me look at the non-linear polarization term; so, this is epsilon 0 chi 2 into E square; so, I substitute for E from this equation into this E square term here. So, what will I get? I will have epsilon 0 chi 2 E 1 square cos square k 1 z minus omega t plus epsilon 0 chi 2 E 2 square cos square k 2 z minus 2 omega t plus 2 epsilon 0 chi 2 E 1 E 2 cos k 1 z minus omega t into cos k 2 z minus 2 omega t; a plus b whole square.

So, now, I write all the terms in terms of cosines. So, cos square will be in terms of cos 2 theta; and then, this is cos a into cos b in terms of cos a plus b and cos a minus b. so, what will I get? I will get epsilon 0 by 2 chi 2 into E 1 square plus cos times 2 k 1 z minus omega t for the first term; then, I will get epsilon 0 by 2 chi 2 - there is an E 1, yes, E 1 square is outside; this is 1 plus E 1 square - 1 plus cos twice k 2 z minus 2 omega t into E 2 square plus, so this is 2 cos a cos b; so, I can write this as epsilon 0 chi 2 E 1 E 2 into cos k 1 plus k 2 z minus 3 omega t and then I will have one more term, which is plus epsilon 0 chi 2 E 1 E 2 cos k 2 minus k 1 z minus omega t; cos a cos b is cos a plus b plus cos a minus b divided by 2.

So, if you look at this terms now, this 1 here and this 1 here give you optical rectification; that means, some DC polarization term. This is that frequency 2 omega, this term gives you frequency 2 omega, this term gives you frequency 4 omega, because the 2 omega frequency once it is launched inside the medium, generated inside the medium, the nonlinearity at 2 omega frequency will now generate 4 omega frequency - the second harmonic of 2 omega; it also generates a 3 omega term and n omega term.

So, the total electric field now will consist of omega, 2 omega, 3 omega, 4 omega. So, you have to substitute back into the equation and recalculate everything. Now, it so happens as we will discuss, that to generate 2 omega from omega, I must satisfy some conditions called phase matching conditions. If I do not satisfy those conditions, the efficiency of the generation of new frequency is very little. I will explain this phase matching condition mathematically and physically, what is the meaning of phase matching condition, but as we will see to generate 2 omega from omega, I need to satisfy certain phase matching conditions.

Similarly, if you generate 2 omega from 2 omega a frequency 4 omega, I need to satisfy another set of phase matching conditions. Now, usually what happens? Even to satisfy one phase matching condition is not easy. I have to design my polarization states, the propagation direction of the crystal, etcetera, to satisfy even one of the phase matching conditions.

So, normally what happens is, we will be able to satisfy only one of the phase matching conditions; for example, for omega to 2 omega conversion. So, when I satisfy the condition for omega to 2 omega, I may not be simultaneously satisfying the condition for 2 omega to 4 omega term. So, this 4 omega electric field will hardly be generated. This term will not be able to generate electromagnetic wave at frequency 4 omega; this one 3 omega term also requires another phase matching condition; so, usually that is also not satisfied.

But look at this term, this is a term which is generating omega. How is it generating omega? It is mixing the 2 omega and omega terms; it is the difference frequency. Omega frequency and omega frequency add to give you 2 omega frequency; 2 omega frequency and omega frequency mix to give you omega frequency, because omega plus omega is 2 omega and 2 omega minus omega is omega. So, this leads to, as you can see here, a non-linear polarization term at frequency omega. So, the same nonlinearity actually will generate from omega, it will generate 2 omega. And once 2 omega gets generated inside the medium, this 2 omega mixes with omega to generate back omega.

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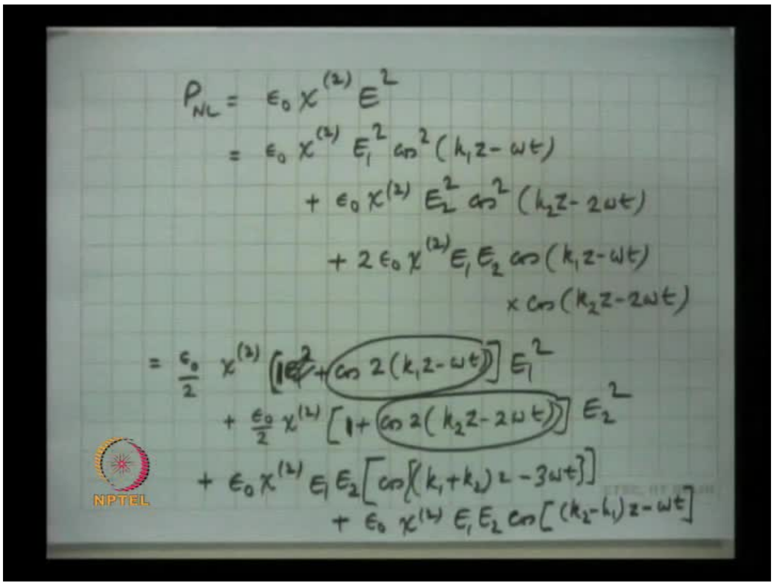

And as I will show you the phase matching condition, for omega to 2 omega conversion is the same as the phase matching condition for 2 omega to omega conversion back. That is, omega plus omega giving you 2 omega and 2 omega minus omega giving you omega, are the same phase matching conditions.

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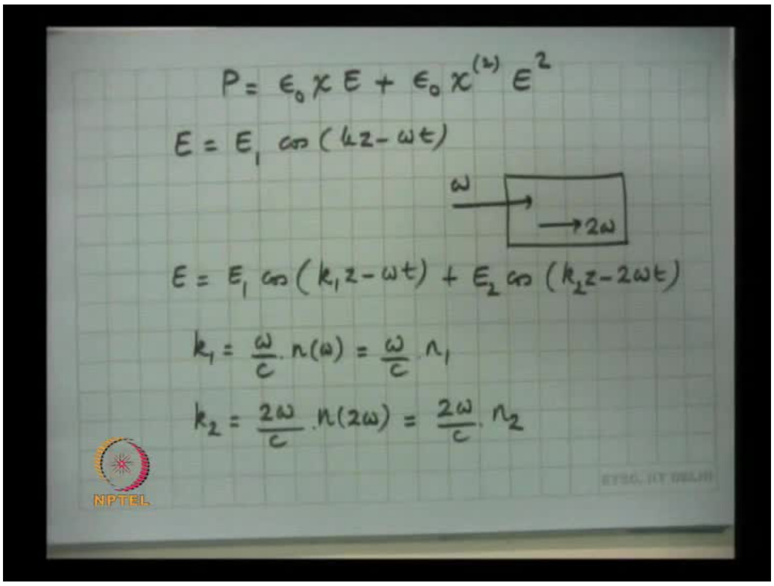

So, what will happen is, if I take this medium and launch 2 omega frequency with omega frequency into the medium, this omega will generate 2 omega; 2 omega and omega mix to generate omega. So, that is a coupling, that is taking place between omega and 2 omega frequency.

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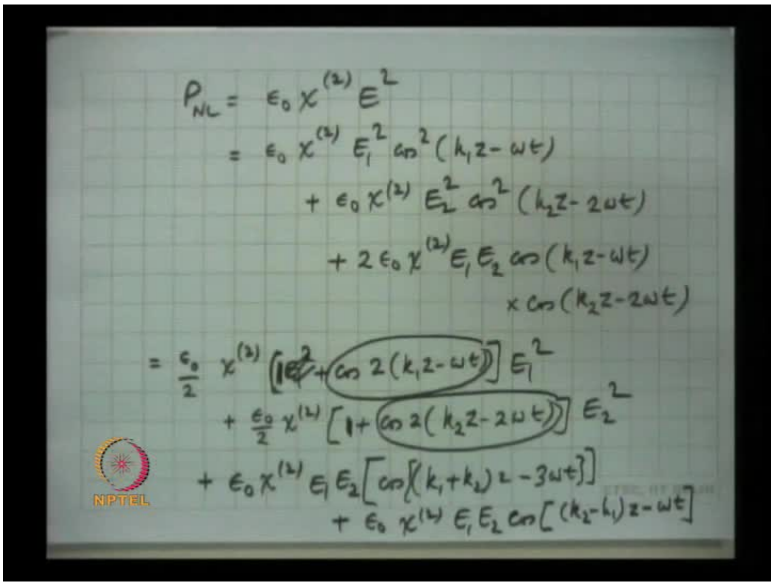

And this process I will show you is efficient only under certain conditions called phase matching conditions. So, although this term is generating new frequencies, it will so happen that the electric field generated at these new frequencies will be negligible.

So, usually, we will neglect all the other kinds of frequencies that get generated in the process and focus on the set of frequencies for which I am satisfying the phase matching condition; and hence, I will only worry about those electric fields.

So, please note that, when I launch an omega into a medium, I will generate non-linear polarization at 2 omega; that non-linear polarization at 2 omega will generate electromagnetic wave at 2 omega. The electromagnetic wave at 2 omega will mix with the electromagnetic wave at omega, to generates waves at omega also.

So, there is a non-linear polarization term at 2 omega, there is a non-linear polarization term also at omega. You cannot have coupling in only one direction; you have omega to 2 omega transfer, omega 2 to omega transfer is also taking place simultaneously.

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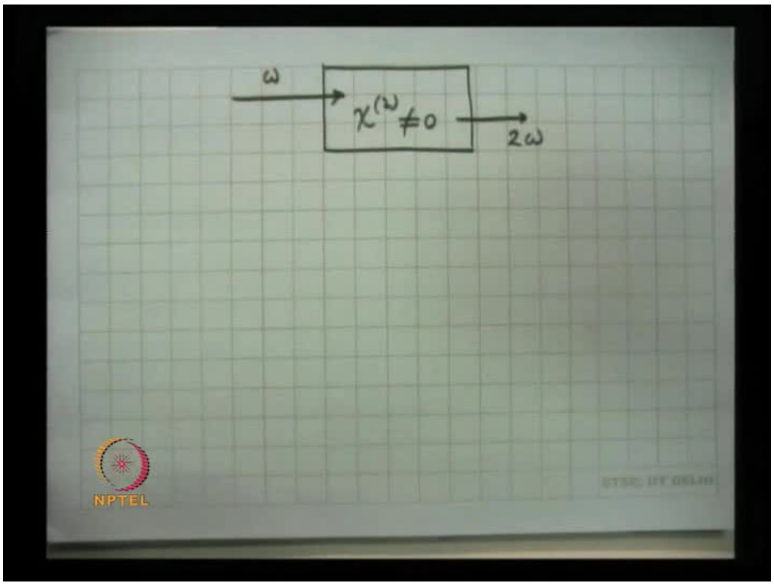

Now, what is the objective now? What is objective? So, problem is, I am given a medium in which this non-linear susceptibility is not 0. As I mentioned to you last time, if the medium has a center of inversion symmetry, the chi 2 tensor is 0; all elements of chi i j k 2 are 0. You cannot have this process taking place in a medium, which has center of inversion symmetry.

So this, let me assume this is a medium which has no center of inversion symmetry and hence can generate second order processes. So, what is my problem? My objective is, I launch a frequency omega into the medium, I want to calculate how much of power am I generating at 2 omega. I launch electric field at omega inside the medium with a certain amplitude, the medium converts omega to 2 omega, so what is the generated power at 2 omega at the output? What does it depend on? How do I maximize this efficiency of conversion? And what are the conditions under which this efficiency will be maximum and how do I achieve those conditions in practice?

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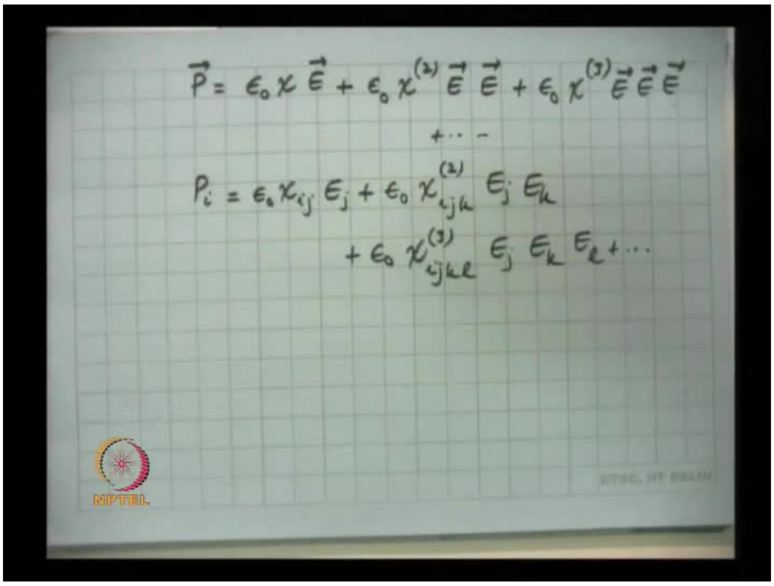

So, the problem is essentially boils down solving Maxwell's equations, with the condition that the polarization now is not just the first term here; it also contains higher order terms. And right now, we are looking at only this term; we will forget about all higher order terms; we will look at the contribution due to the second order nonlinearity in the Maxwell's equations.

Now, to keep the mathematics simple and to understand the physics, what we will do is, we will assume a scalar equation - scalar analysis - in the beginning, discuss the generation efficiencies and so on. And then, I will come back and look at this equation little more carefully, because finally if I give you a medium, I will tell this medium has this tensor - chi i j k elements - I will give you and I will ask you, what polarization of the input should I choose, what is the polarization that comes out of the medium at 2 omega frequency, what condition should I satisfy to maximize this efficiency.

So, this involves little more analysis using these tensor properties of this, but right now, to understand even the basic generation of second harmonic, we will assume a scalar equation and analyze the problem.

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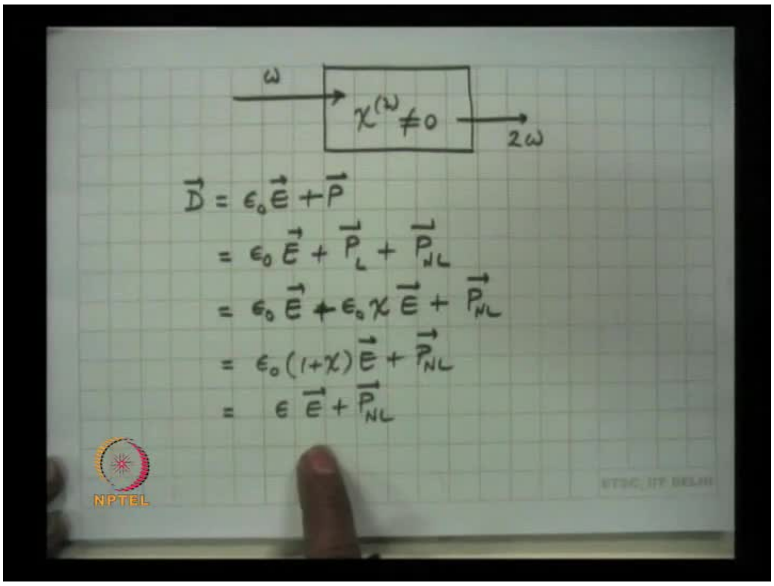

Now, before we do that, I must derive a wave equation in the presence of nonlinearity. So, let me go back and let me write this equation; so, D is equal to remember, epsilon 0 E plus P; this is the defining equation for the displacement vector. Now, I can write this as epsilon 0 E plus P linear plus P non-linear, P linear is epsilon 0 chi E plus P non-linear, this is equal to epsilon 0 into 1 plus chi E plus P non-linear and this is nothing but epsilon into E plus P non-linear; epsilon is the dielectric permittivity of the medium.

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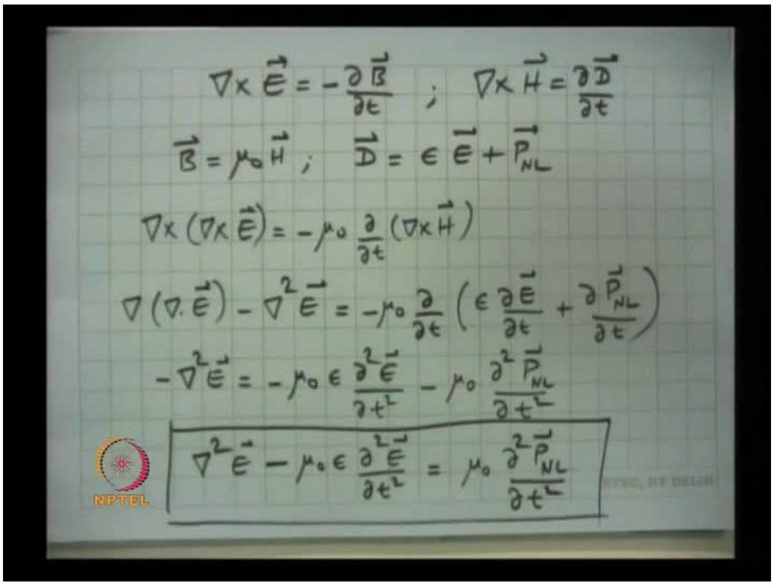

So, the linear part of the polarization has been taken into account in epsilon and this second term is the non-linear term. So, normally, D would have been equal to epsilon E; so, that is the expression for D. So, let us go back and write this equation, so we have del cross E is equal to minus del B by del t and del cross H is equal to del D by del t from the no free currents, no free charges.

Now, we are going to assume, as I mentioned before, B is equal to mu naught H, and D is equal to epsilon E plus P non-linear. So, let me take a curl of the first equation, so del cross del cross E is equal to minus mu 0, now I interchange the time and space derivatives, so I get del by del t of del cross H.

The first the left hand side is gradient of divergence E minus del square E is equal to minus mu naught del by del t of del cross H, which is epsilon del E by del t plus del P non-linear by del t. Now, how about divergence E? Divergence E is 0 in isotropic media.

In anisotropic media, remember, D is perpendicular to k vector, E is not in general perpendicular to k vector divergence; E is equal to 0 implies transfer condition. That means, in anisotropic media divergence E is usually not 0, but again to simplify the analysis here, we are going to forget about divergence E term here, but in principle actually divergence E is not 0 in anisotropic media. But as I left a problem to you, you can check the angle between E and D vector is not very large; usually is very small - it is a couple of degrees.

So, I am going to assume, as a simplification divergence E is equal to 0 and so this is minus del square E is equal to minus mu 0 epsilon del square E by del t square minus mu 0 del square P non-linear by del t square. So, this is my wave equation, del square E minus mu 0 epsilon del square E by del t square is equal to mu 0 del square P non-linear by del t square.

Normally, when you derive the wave equation, the right hand side is 0, because normally you do not take into account any nonlinearity in the medium. So, the wave equation normally you would have obtained is, del square E minus mu 0 epsilon del square E by del t square is equal to 0. The right hand side actually acts as a source for this electromagnetic field; it is a inhomogeneous equation. This P non-linear is the source of the electromagnetic fields; and please note that this equation has to be satisfied for each and every frequencies.

So, the frequency omega will satisfy this equation - for omega frequency, where epsilon will be at omega frequency, E will be at omega frequency, P non-linear at omega frequency. For the 2 omega frequency, the electric field at 2 omega frequency, the epsilon at 2 omega frequency, here P non-linear at 2 omega frequency.

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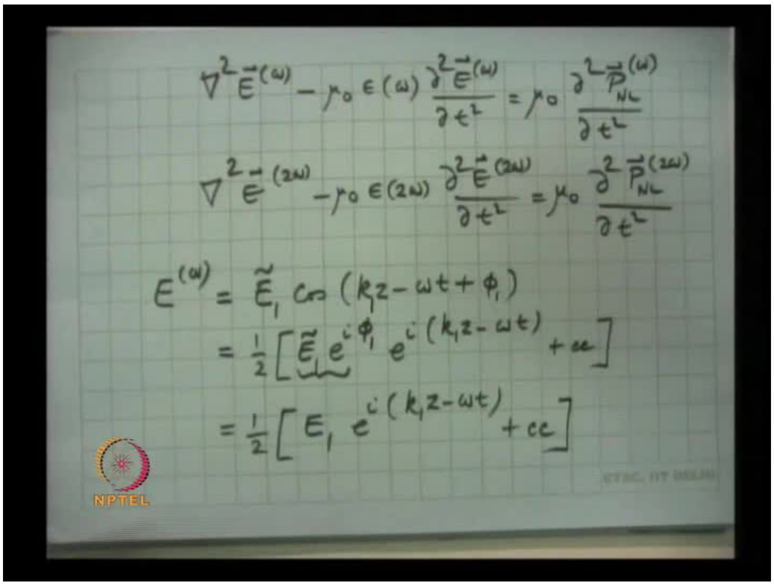

So, for example, I would have to satisfies these equations, del square E at omega minus mu 0 epsilon of omega del square E of omega by del t square is equal to mu naught del square P non-linear at omega by del t square; and del square E of 2 omega minus mu 0 epsilon of 2 omega del square E of 2 omega by del t square is equal to mu naught del square P non-linear at 2 omega by del t square.

I showed you that there is also a non-linear term at frequency omega, because of mixing between omega and 2 omega frequencies. There is a non-linear polarization at 2 omega; there is a non-linear polarization at omega. And these two equations describe the propagation of frequency omega and 2 omega through the medium and the interaction between these two frequencies is brought about by the nonlinearity in the medium.

Normally, if you have a linear medium, if you launch a frequency omega and 2 omega simultaneously, they do not interact; omega frequency propagates independently, 2 omega frequency propagates independently. But when there is nonlinearity, the presence of nonlinearity couples these two frequencies through the non-linear polarization term.

So, in principle, if you launch only one of the waves, it can lead to the generation of the new frequency. So, the condition I have is, I launch electromagnetic field at frequency omega, this electromagnetic field will generate P non-linear at 2 omega, this P non-linear at 2 omega will generate E at 2 omega, E at 2 omega E at omega mix to generate P non-linear at omega, which will then influence the omega frequency.

So, these two equations will give me a pair of coupled equations which I need to solve. I launch omega frequency, this will generate - P non-linear at 2 omega, this P non-linear at 2 omega generates E at 2 omega, E at 2 omega and E at omega mix to generate P non-linear at omega, which will then modify this equation. So, these two equations are to be solved simultaneously.

Now, I want to do a simplified analysis, where I will instead of writing this vector equation, we will try to solve a scalar equation, which means we will assume that, these are waves propagating along z direction, plane wave propagating along z direction and these are linearly polarized electromagnetic fields. So, E is along x direction; for example, it is propagating along some direction z and I will try to solve these equations in a scalar approximation, and later on I will come back and tell you, this approximation where from am I getting this approximation. If I am actually given a vector field which is different, how do I reduce the problem to this problem which I am solving now?

So, remember I also mentioned that, in the presence of nonlinearity, I have to use real fields always; that is why I wrote cosine there. I can actually overcome this, I can go back again to complex notation provided, the total is still real. So, the way I solve it is the following; so, for example, if I write E at omega is equal to E 1 tilde cos k z minus omega t plus phi 1, suppose this is my electromagnetic field at frequency omega.

So, I will write this as half of E 1 tilde exponential i phi 1 e to the power I, let me write k 1 here, k 1 z minus omega t plus complex conjugate c; c stands for complex conjugate. So, I am writing this cosine as this exponential plus a complex conjugate divided by 2. And this one I write as, E 1 e to the power I, where E 1 is the complex electric field at frequency omega; it has also a phase term phi 1. Because it is of the form this plus complex conjugate divided by 2, this is always real. I am still using real notation except that, instead of writing cosine, I am writing exponential plus its complex conjugate.

Yes.

Sir, E 1 tilde is also complex amplitude.

No, in the real equation, E 1 tilde is real. When I write E 1 tilde cos k 1 z minus omega t plus phi 1 E 1 tilde - everything is real, E 1 tilde is real, phi is real, everything is real. But what I am doing is, I am absorbing the phase phi into the E 1 tilde term and defining an E 1 - electric field, which is now in general complex. So, if the phase changes, E 1 will change in complex phase. And when you add the complex conjugate, you will get that phase inside the cosine term again. So, change of phase inside the cosine term will imply that E 1 a changing with z, because phase change is with z.

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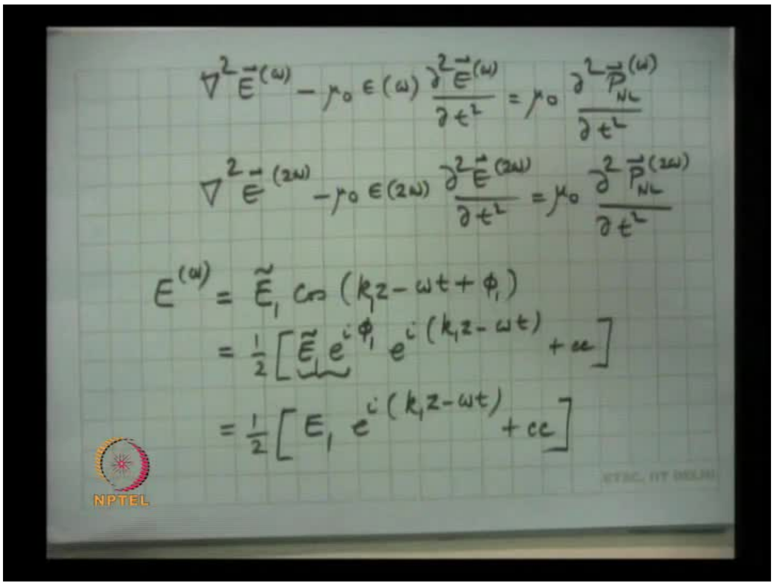

So, whether phase change is with z or amplitudes change is with z, E 1 will change with z. So, E 1 is a complex electric field; and the electric field at omega is written as half of E 1 plus E 1 exponential i k 1 z minus omega t plus its complex conjugate.

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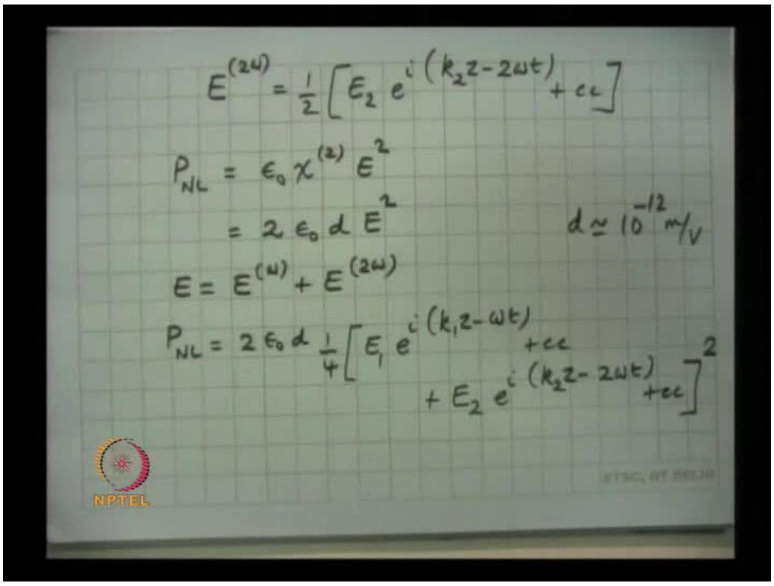

Similarly, I will write for E2, E 2 omega half of E 2 exponential i k 2 z minus 2 omega t plus complex conjugate, where E 2 is the complex electric field at frequency 2 omega; k 2 is the propagation constant of the medium at frequency 2 omega; k 1 is the propagation constant of the medium at frequency omega.

So, if it is an ordinary wave or extraordinary wave, k 1 will have to be taken the correct value. In general, the medium is anisotropic; so, k 1 and k 2 are the two propagation constants of these plane waves propagating along z direction.

Now, let me tell you that this z is the z direction - arbitrary some z direction; this is not the z direction of the crystal. This is an x y z coordinate system in which I am solving the problem and this x y z need not be coincident with x y z of the principle axis system of the crystal; this is some arbitrary x y z system.

So, let me write the P non-linear term, P non-linear is now in non-linear optics notation. There are different kinds of notations used by different people, but what I want to do is, use the following notation in which P non-linear, remember I had written as, epsilon 0 chi 2 E square; I want to write this as 2 epsilon 0 d E square, where chi 2 is represented by 2 d. what will be the units of d in SI units? Look at the dimensions of P non-linear electric field, sorry, 1 by electric field, because P has a same dimension epsilon 0 times E.

So, 1 by electric field means, d will be in units of meters per volt, because E has volts per meter. So, let me tell you the numbers, d is typically of the order of 10 to the minus 12 meters per volt. For most media, the coefficient d has this magnitude about 10 to the minus 12, 10 times 10 to the minus 12, 3 times 10 to minus 12, something like this, these are some numbers of media that are used in non-linear optics.

So, let me substitute now this, what is E? E is the total electric field which consists of E omega plus E 2 omega. So, P non-linear will be 2 epsilon 0 d, so when I add E omega given by this equation and E 2 omega given by this equation and take square, so this factor half will give me 1 by 4 outside; so, I will have E 1 exponential i k 1 z minus omega t plus complex conjugate plus E 2 e to the power i k 2 z minus 2 omega t plus complex conjugate whole square; so, this is E square.

So, from here, let me pick up the non-linear polarization terms at frequency omega and 2 omega. So, first, let me polar the term with non-linear polarization term at frequency 2 omega; P non-linear at 2 omega, so can you tell me what will I get from here, which terms when I take the square, I will have square of this, square of this, square of this, square of this, 2 times this into this, 2 times this into this, you have a plus b plus c plus d whole square (Refer Slide Time: 35.02).

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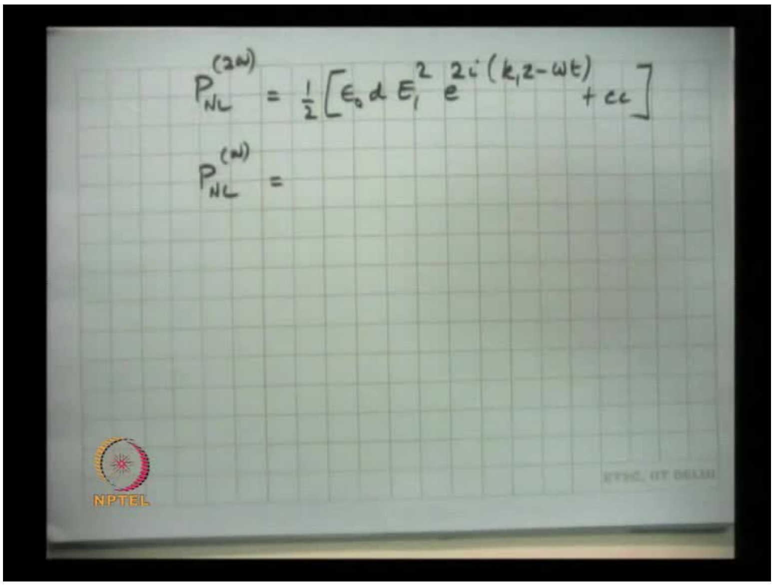

So, can you tell me, which are the terms which will have a frequency 2 omega; this square and this square. Please remember, because the total function is real, for every complex number I get, I will have a complex conjugate. So, I will have, this will be simply, so this factor of 2 and 1 by 4 gives me half, so half; let me write inside as epsilon 0 d E 1 square e to the power 2 i k 1 z minus omega t plus complex conjugate. And there will be no other term ; none of the mixing terms in this square will give me a frequency 2 omega.

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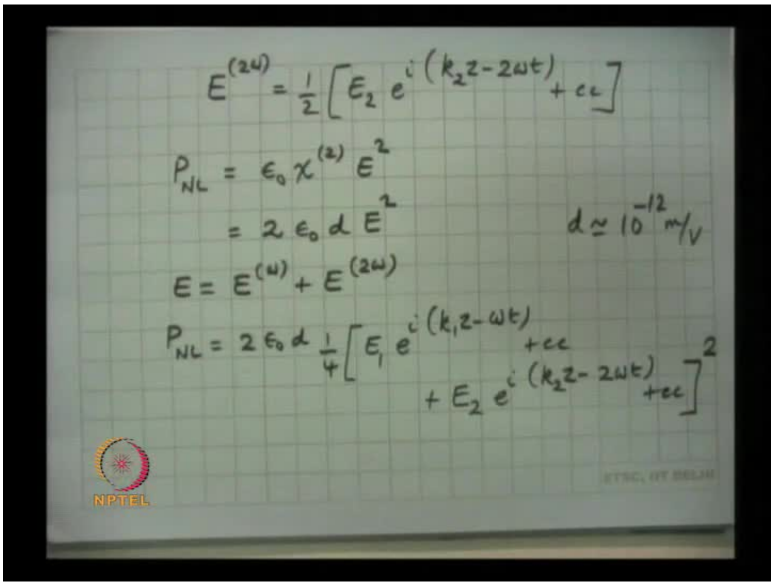

What about P non-linear at omega? So, let me look at this equation again. So, which product will give me omega frequency?

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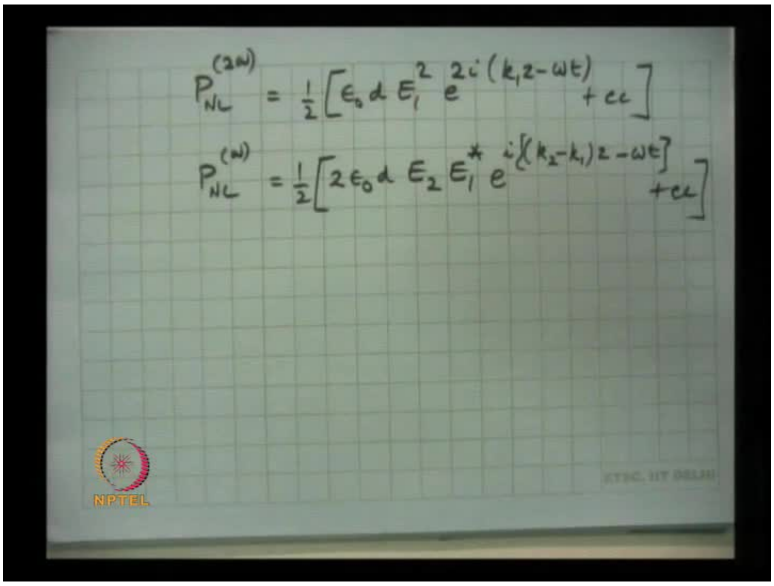

This into the complex conjugate of the first term, because this complex conjugate has exponential plus i omega t multiplied by this, will give me exponential minus i omega i. Similarly, this into its complex conjugate (Refer Slide Time: 36.26), so I will have essentially half of 2 epsilon 0 d E 2 E 1 star; star represents complex conjugate; and what will be exponential term? I will have exponential i k 2 minus k 1 z minus omega t plus complex conjugate.

See, E 1 is the coefficient which multiplies E into the minus i omega t; so, E 1 square will be the coefficient multiplying exponential minus 2 i omega t. So, E 1 square is sitting here for the second harmonic polarization; E 2 is the coefficient of e to the power s minus 2 i omega t; E 1 star is the coefficient of exponential plus i omega t.

So, when I multiply these two terms, I will get exponential minus i omega t, which is what gives me the polarization at frequency omega. Because later on, by looking at this product, you can sort of guess the product term that will appear here and to which frequency that will correspond. For example, E 2 square would correspond to non-linear polarization at 4 omega; E 1, E 2 will give me 3 omega.

So, if you expand this, but you need to worry about these factors which are coming out in the squaring operation here. And so, this will be the non-linear polarization 2 omega frequency; this is a non-linear polarization at omega frequency.

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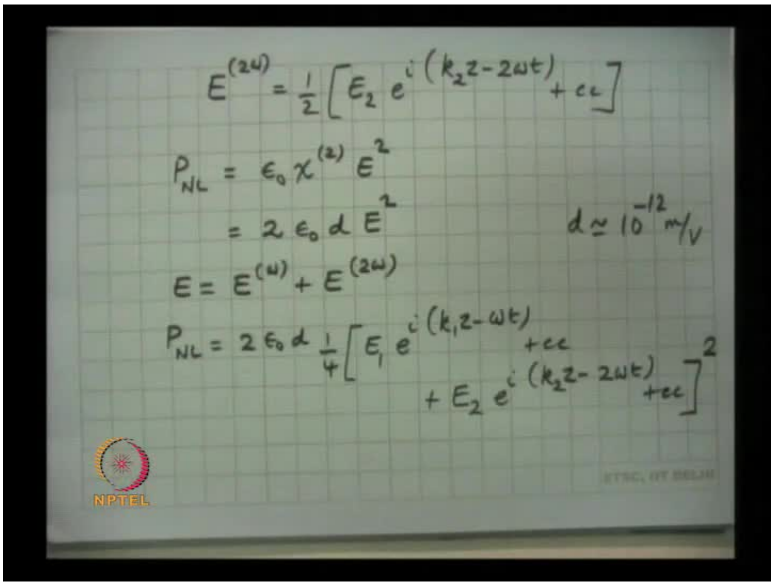

So, first, let me derive the equation corresponding to the frequency 2 omega; so, this is my equation of the electric field at 2 omega. Now, please note now, that because of this generation, the amplitude - complex amplitude - of the second harmonic will now be a function of z; z is the propagation direction, because at the input to the crystal, there was only frequency omega present, but as the wave propagates, the 2 omega frequency gets generated. So, E 2 must have been 0 at the entrance phase of the crystal; and as the wave propagates, E 2 will start to get generated, which means E 2 must be a function of z; z is the propagation direction. And because of this nonlinearity, E 2 will become a function of z.

And if E 2 becomes a function of z, E 1 also has to become a function of z, because you cannot generate second harmonic, unless you lose energy from the fundamental. You are only converting electromagnetic energy at frequency omega, to energy at 2 omega frequency. So, if E 2 is a function of z, E 1 is also a function of z; so, in these equations, I must remember E 1 and E 2 are now functions of z, because of this interaction. because of this nonlinearity.

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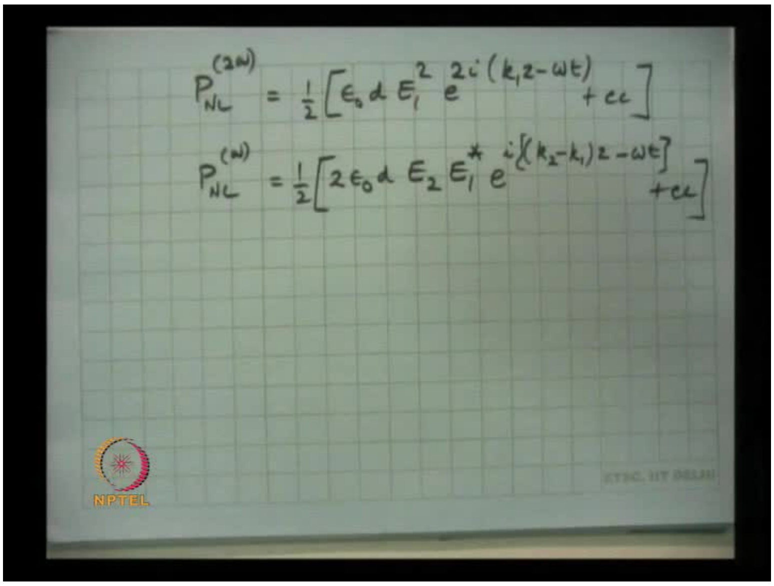

So, the problem is very simple now. This is the expression for E 2 omega; E 2 is a function of z. This is the expression for P non-linear at 2 omega and this is the equation at 2 omega (Refer Slide Time: 40.00).

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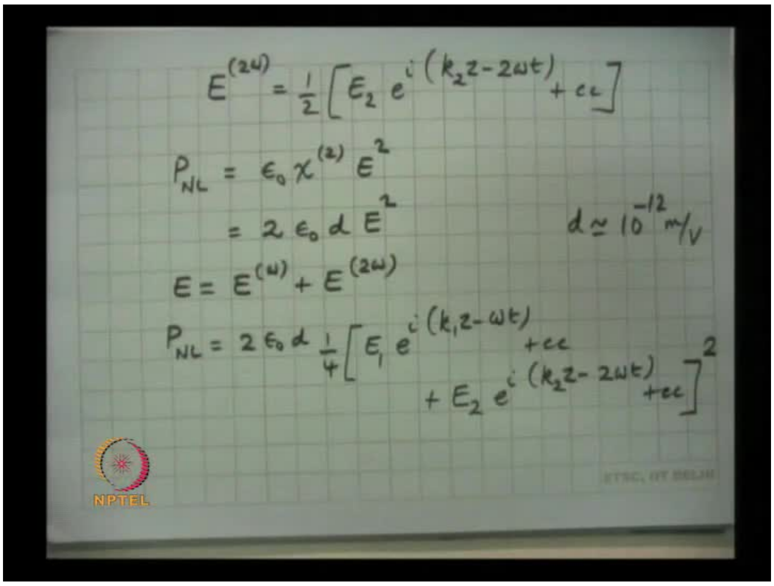

So, if I forget about the vectors, so I have expression for electric field, I have an expression for P non-linear at 2 omega, I must substitute into this equation and simplify. To get an equation for the development of E 2 as the function of z, I will get a differential equation for E 2 of z.

So, let me do this, so remember in this equation - wave equation, there is del square; so, this is differential, del square by del x square plus del square del y square plus del square del z square. So, there is no x dependence, there is no y dependence, so I will be lonely left with the z dependence. So, there will be a del square del z square here, there is a time derivative here, time derivative here.

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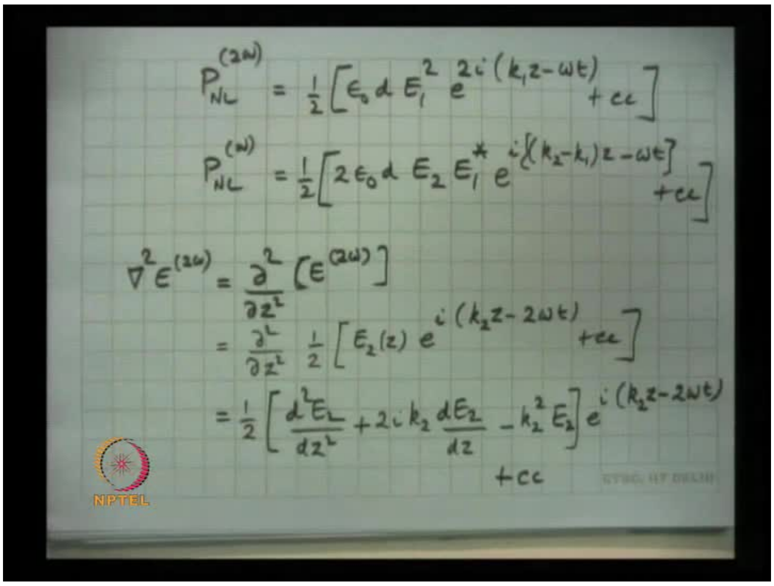

So, let me now calculate, what is the z differential. So, del square E 2 omega will be actually del square by del z square of E 2 omega, which is equal to del square by del z square of half, now let me write E 2 of z e to the power i k 2 z minus 2 omega t plus complex conjugate.

Now, there is a z dependence in E 2, there is a z dependence in exponential i k 2 z; so, this is simple calculation, so let me write down the final expression, I will have d square E 2 by d z square plus 2 i k 2 d E 2 by d z minus k 2 square E 2 into e to the power i k 2 z minus 2 omega t plus complex conjugate; it is a second differential of a product of two functions, you can just differentiate that and you get this these three terms here.

Now, usually, the rate of growth of the second harmonic with z is slow, wavelengths out of the order of micron to 1 micron, but this E 2 will grow in maybe hundreds of micron slowly - it grows slowly as a function of z.

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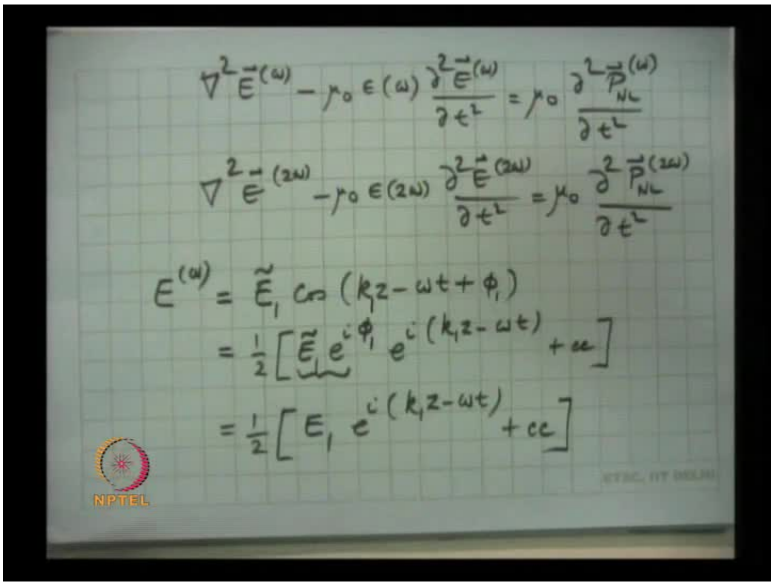

So, I will neglect the second derivative of E 2 with respect to z. Assuming that the growth of the second harmonic slow and so, this is neglected; that means, in one wavelength or a few wavelengths, E 2 does not change significantly, and that is simply neglecting the second derivative. So, I substitute this del square E omega, E 2 omega into this equation, I also calculate this second derivative with respect to time and on the right hand side here.

So, what I need to do is the following, I substitute for this expression into this first term here, then the time derivative is very simple, because it is exponential minus 2 i omega t. So, I will get minus 4 omega square, when I differentiate twice; and P non-linear also will give me minus 4 omega square, with the amplitude term sitting here.

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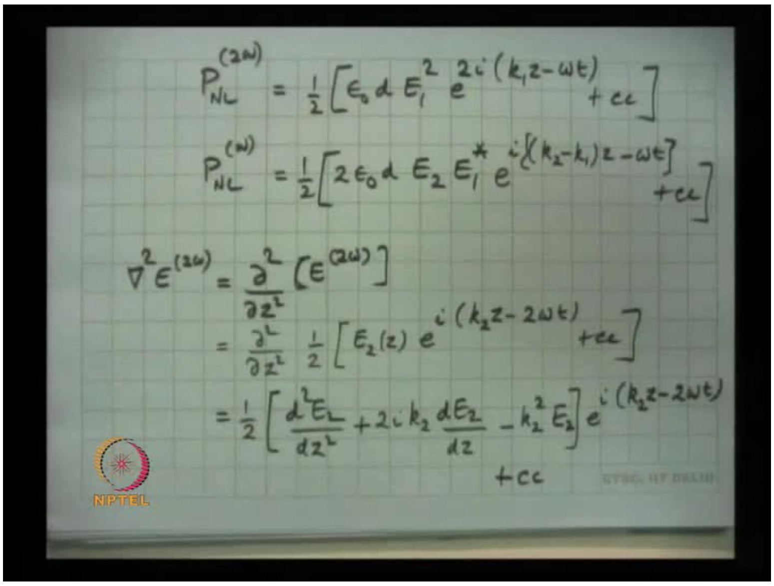

So, now, let me write this equation neglecting this second derivative; so, what you will have? Also another thing, you have every term has exponential minus 2 i omega t and also an exponential plus 2 i omega t. If I need to satisfy this equation for all times, you can show that the coefficient of exponential minus 2 i omega t must be the same everywhere. And similarly, the coefficient of exponential plus 2 i omega t will also automatically be the same on both sides. So, I just do not have to write the complex conjugate term; I only pick up the coefficient of exponential minus 2 i omega t term from each of these terms and write down the equation.

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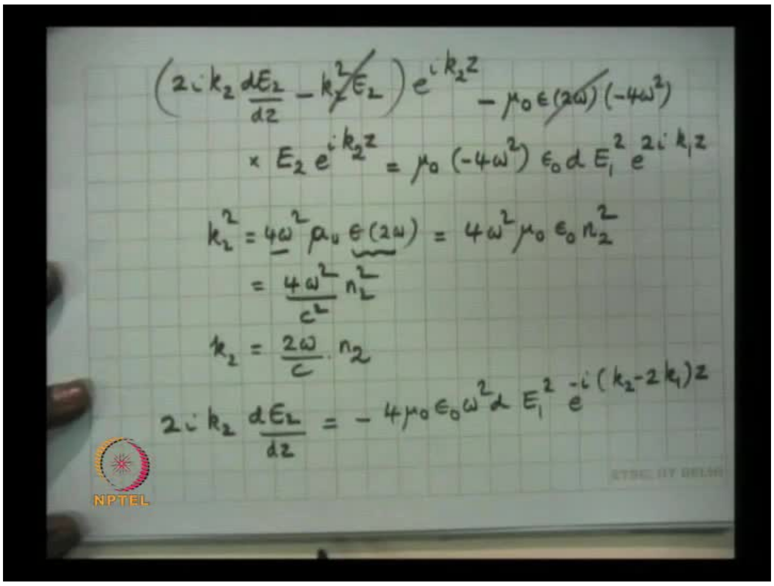

So, what will I get? And remember, there is the factor of half in every term, half also I forget. So, I will get 2 i k 2 d E 2 by d z minus k 2 square E 2 into exponential i k 2 z, that is the first… From the del square term, I am not writing the exponential minus 2 i omega t, because that is I am picking the coefficients of exponential minus 2 i omega t from every term and substituting.

So, this minus mu naught epsilon of 2 omega into, second differential with respect to time, I will get minus 4 omega square into E 2 e to the power i k 2 z, del square E 2 omega by del t square gives me minus 4 omega square into this, is equal to mu naught into, del square del t square of this term factor of half is cancelled off, so I will get, what will I get? Minus 4 omega square epsilon 0 d E 1 square e to the power 2 i k 1 z.

Let me recall, I am substituting the expressions for E 2 omega and P NL 2 omega in this wave equation; in this wave equation, I am substituting E 2 omega and P NL 2 omega. And I am equating terms, which are coefficients of exponential minus 2 i omega t in each of those expressions and I land up with this equation.

Now, can you tell me, what is the relationship between k 2 and epsilon of 2 omega; k 2 is the propagation constant at frequency 2 omega. So, k 2 square is equal to actually omega square 4 omega square mu 0 into epsilon at 2 omega; k is the frequency times mu naught into epsilon of the medium. This is actually, if you substitute this, you will get 4 omega square mu 0 epsilon 0 into n 2 square. Remember, the epsilon 2 omega is epsilon 0 into square of refractive index; and what is mu 0 epsilon 0? 1 by c square. So, 4 omega square by c square and 2 square; so, this implies k 2 is equal to 2 omega by c into n 2, which is what we have assumed.

So, k 2 is related to epsilon at 2 omega by frequency square 2 omega whole square, because the frequency at which the propagation constant is k 2 is 2 omega; so, 2 omega square mu naught epsilon at 2 omega; so this term and this term cancel off. E 2 e to the power i 2 k - that is common, k 2 square; so, you will have, there is a minus into minus is plus; so, mu naught 4 omega square mu naught epsilon into 2 omega; so, these two terms cancel off and this equation will get further simplified.

Now, also let me take this e to the power i k 2 z on the right hand side and simplify this equation; so, what I will get is the followings. Let me write it ,write here itself. So, I get 2 i k 2 d E 2 by d z is equal to minus 4 mu naught epsilon naught omega square d E 1 square exponential i minus i k 2 minus 2 k 1 z; minus 4, epsilon 0, mu 0, omega square d, E 1 square, e to the power, ok

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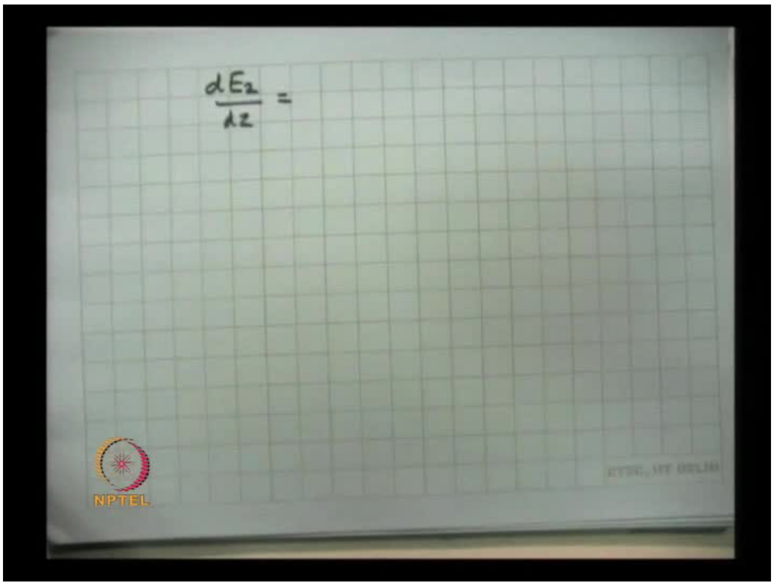

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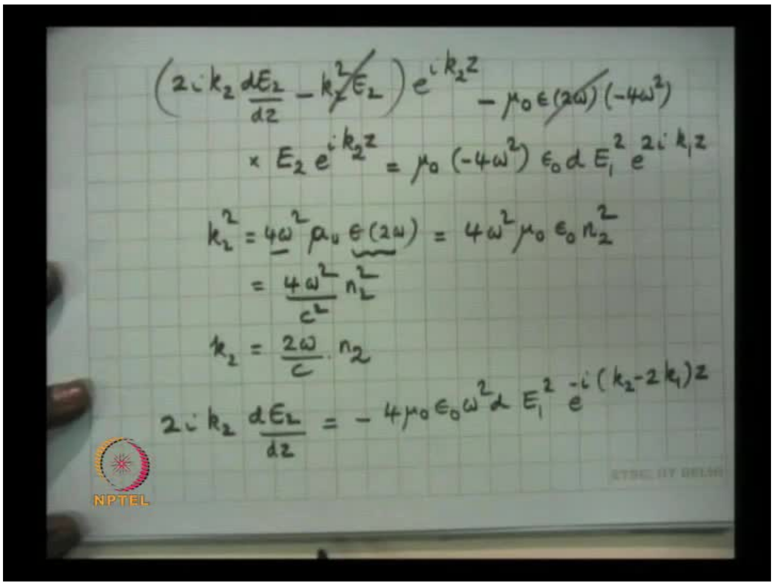

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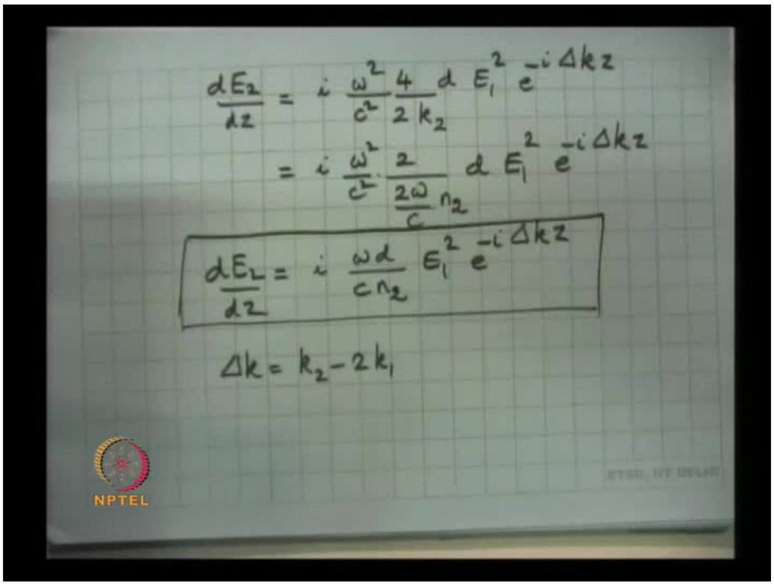

So, let me take this 2 i k 2 on the other side also; so, what I will get is, d E 2 by d z. So, when the i goes to the right hand side, it goes to denominator and because of the minus sign, I take the i in the numerator with the plus sign here. So, I get I, so now omega square epsilon 0 mu 0 is 1 by c square, so I get 4 by 2 k 2 d E 1 square minus I, let me call this delta k z.

Now, I substitute for k 2, so i, so this factor of 2 goes off, so i omega by c into 2 by, now k 2 is 2 omega by c into n 2 d E 1 square exponential minus i delta k z. So, this is i times omega d - omega square by c square – so, omega by omega d by c into E 1 square exponential minus i del k z - a much simplified equation starting from the equation for the second harmonic field.

This tells me, how the electric field of the second harmonic varies with z direction. Please notice that, E 1 is also a function of z in general. So, I cannot solve this equation without writing the equation for E 1.

And delta k is defined as k 2 minus 2 k 1. Now, E 1 is the electric field of the incident wave at frequency omega; E 2 is the electric field of this generated second harmonic field. Now, usually what happens is, in this process of second harmonic generation, the efficiencies are usually very weak; few percent, less than a percent, that is the kind of efficiency, we will calculate some numbers.

So, in most situations, the efficiency for second harmonic generation is not very high. What is approximation I can make? That means, E 1 is almost a constant. If there was a lot of conversion from omega to 2 omega, the energy at omega would have dropped down significantly. If that is the case, E 1 would have varied significantly, but usually the efficiencies are very low. And so, as a first approximation, I can assume E 1 as a constant; and then, I can immediately solve this equation.

So, for example, it may be possible that I take a medium, I launch 1 Watt of frequency omega, and I generate 10 milli Watts of frequency 2 omega. So, the fundamental power will go down from 1 Watt to 990 milli Watts; 10 milli Watts I have generated at second harmonic, this power must have come from the omega frequency. So, the electric field of 1 Watt and 990 milli Watt, which is 0.99 Watts is almost equal; so, that is approximation which I am making here. If the efficiency becomes large, then I cannot make this approximation and I need to solve this equation along with the coupled equation for E 1.

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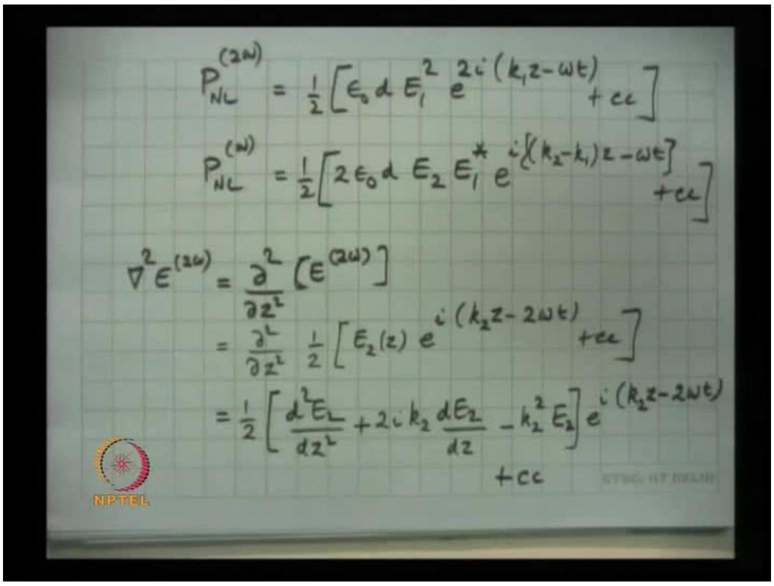

So, before we solve this equation, let me look at this term here and tell you what is the meaning of this. Where is the non-linear polarization term? Look at this equation, this equation tells me the non-linear polarization is also a travelling wave; this is also travelling wave, k z minus omega t. Now, what is the velocity of this wave?

Omega by k.

Omega by k 1, it is 2 omega by 2 k 1, which is omega by k 1 and that is the velocity of the wave at frequency omega. So, omega frequency is propagating, it is generating non-linear polarization, which is also travelling along with it. So, omega by k 1 is the velocity of the wave at frequency of the non-linear polarization term.

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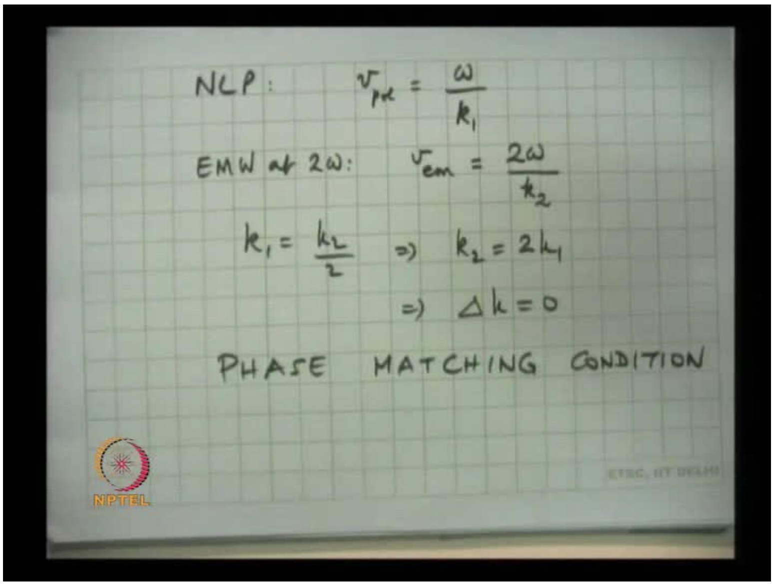

So, non-linear polarization term is propagating a velocity, so polarization is equal to omega by k 1. What is the velocity of the electromagnetic wave at 2 omega? Frequency divided by k 2. The velocity of the electromagnetic wave is the frequency, which is 2 omega divided by the propagation constant at second harmonic. Now, you see this non-linear polarization is the source of this electromagnetic wave, and if the source should keep on continuously feeding energy into the electromagnetic wave, these two velocities must be equal.

If the source weak velocity and the velocity at which it is generating the wave are not at the same speed, they are not travelling together; what will happen is, it will not feed continuously, the energy that is being fed into the omega frequency or 2 omega frequency will not keep on adding constructively.

What is the condition implies? 2 k 1 must be equal to k 2, because if these two velocities must be equal, this implies k 1 is equal to k 2 by 2, which implies k 2 is equal 2 k ,1 which implies delta k is equal to 0; this is very important condition called the phase matching condition.

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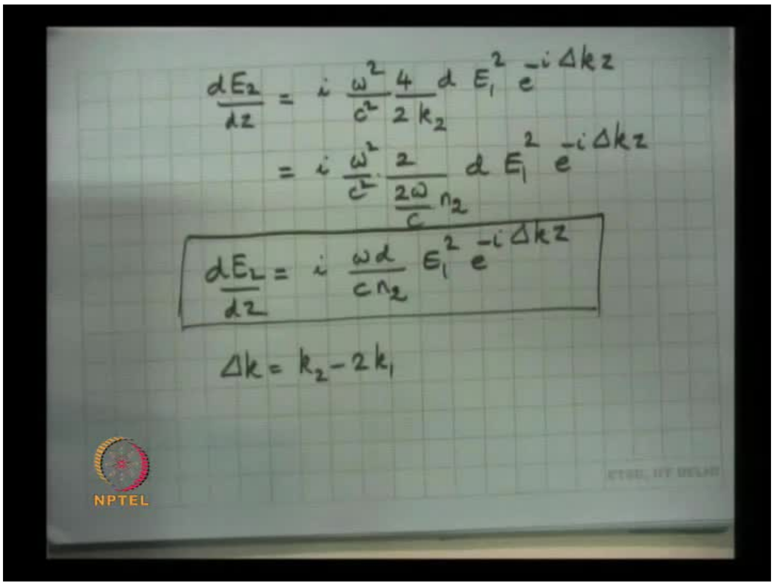

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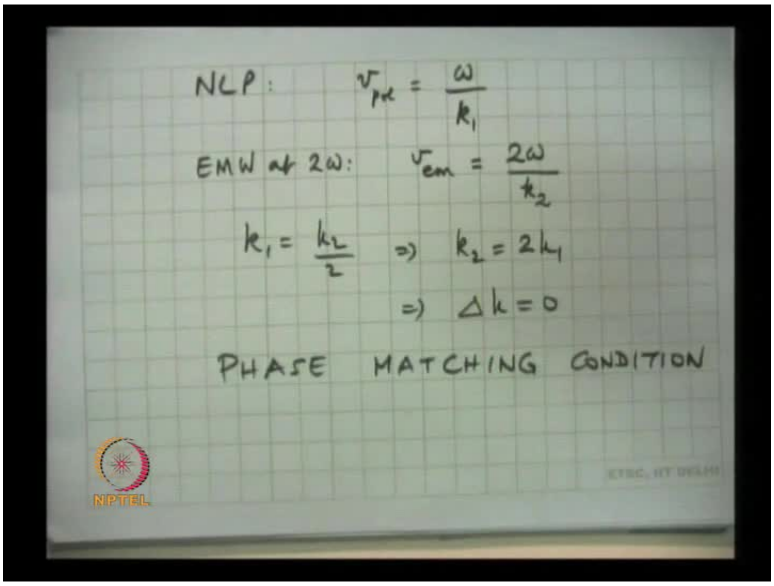

So, what I will do is, we will discuss this phase matching condition little more detail. In the next class, we will solve this equation, when delta k is not equal to 0. And I will show you that this equation tells me, that for maximum conversion, for maximum efficiency, delta k must be 0. And the origin is one of the origins, physically what is happening is the source of electromagnetic field, which is the non-linear polarization. And the electromagnetic field that it is generating must be travelling at the same speed, for the energy to constructively keep on being fed from the polarization to the electromagnetic field.

So, do you have any questions, yes.

Sir, possible to use the conservation, when I assume these equations right (( )) because you ignore 1 term of E omega

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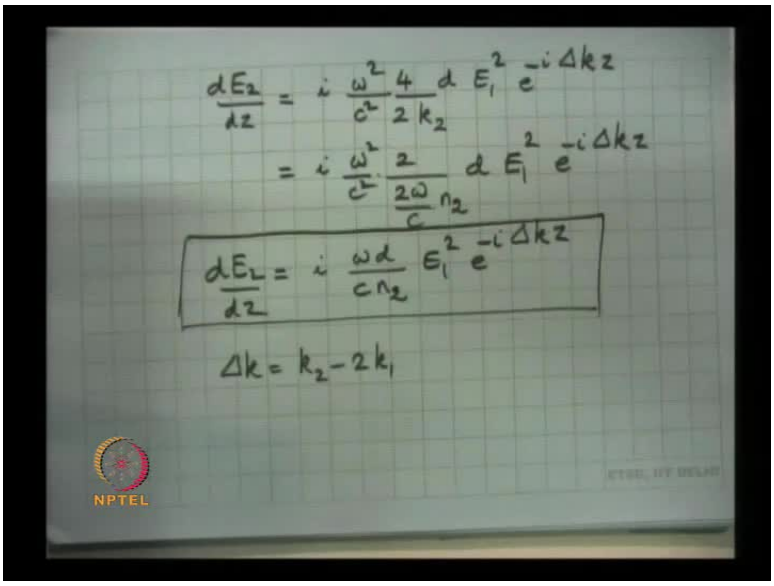

So, the question is, is the conversion energy can be used. Yes, I can use the equations provided, I write this equation and the corresponding E 2 d E 1 by d z equation. Because actually right from the beginning, I am neglecting the generation of all other frequencies; so, I am consistent, I am assuming that is only frequency omega and 2 omega present and those two equations will satisfy the conversion of energy equation.

So, whatever energy I have generated at 2 omega, must have come from omega frequency. So, the sum of the energies of omega and 2 omega must remain constant, as they propagates; that means, I am assuming no other absorption or reflection or scattering; so, the energy conservation is still valid.