Quantum Electronics
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Module No.# 05
Lecture No.# 41
Quantum Picture of Parametric Down Conversion

Today, what I want to do is to look at the quantum picture of parameter down conversion process. Remember, when we were doing parametric down conversion, either degenerate or non-degenerate that means, whether both the output for frequencies are the same which is degenerate parametric down conversion or they are different which is non-degenerate parametric down conversion.

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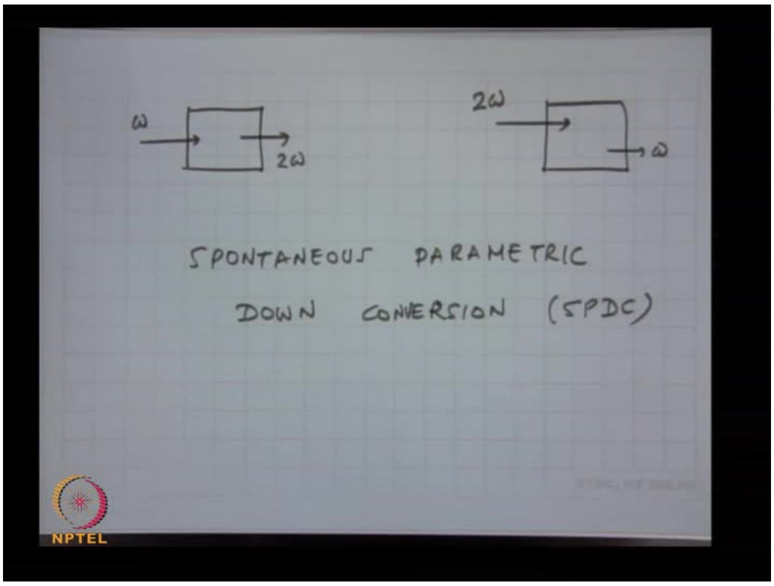

We found that if you had a system in which only wave at the higher frequency was incident, say I call this of 2 omega - degenerate parametric down conversion. If I have only 2 omega incident in a crystal which is capable of second harmonic generation which means I take this crystal, I launch omega and I find 2 omega that means, I satisfy the phase matching condition or the quasi phase matching condition, so that omega generates 2 omega.

If I look at those classical equations and assume that there is a 2 mega input then, I find that no omega get generated in this process, but as I mentioned to you at that time when you conducted experiment you do find omega coming out and this is called parametric fluorescence or spontaneous parametric down-conversion, it is like spontaneous emission spontaneous parametric, short form SPDC. This is a very interesting technique which people use today to generate light having non-classical properties, like squeeze light, squeeze states or entangle states etcetera.

The explanation for this process of parametric spontaneous down conversion is purely quantum mechanical. So, what we will try to do today is, to look at a simplified picture of how the quantum mechanical analysis that we have done can predict this process of spontaneous parametric down-conversion and tell me what are the types of photon that are coming out from this process.

Now, a four quantum mechanical analysis would have to involve writing down the Hamiltonian of the total system in the presence of nonlinearity and using what is called as an interaction picture to find out the properties of the output states of light. So, we will not carry out this process, but we will use a procedure that we had employed for the beam splitter to write the output states from the classical equations, what did we do with beam splitter? We wrote the classical equations connecting the output electric fields to the input electric fields on the beam splitter then, we replace the classical electric fields by operators.

We will do the same thing and so, we arrived at these quantum mechanical equations which we will use to study what the properties are. The derivation of those equations requires solving this problem of writing the Hamiltonian of the system in the presence of nonlinearity and finding out the dynamics of down conversion process and finding out the output states which we will not discuss in detail in the class here.

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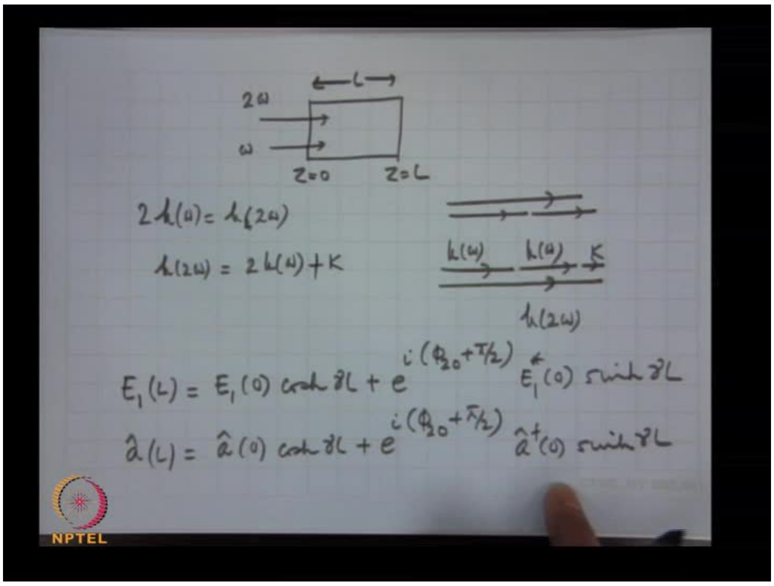

To do this problem, we go back to the classical analysis that we had done. Remember, we had looked to the solving problem, I have 2 omega coming here and omega input. I am satisfying the condition for this process to take place that is, either phase matching that is 2 k omega is equal to k of 2 omega - 2 times k of omega is equal to k of 2 omega - or I satisfy the quasi phase matching condition is equal to 2 k omega minus of K.

This is essentially k omega plus k omega is equal to 2 k omega and this is k omega plus k omega plus k is equal to k 2 omega. So, this is k and this is k of omega and this is k of omega. We had obtained equations for the transformation of this field at omega as you propagate through crystal of length L. Assuming no pump definition approximation that means, we assume that omega 2 omega is very strong and we neglect the loss of power into omega, solve those equations. Let me write down the equation that we had obtained at that time.

The E1 of L is equal to E 10 cos hyperbolic gamma L plus exponential i by 20 plus pi by 2 E1 star of 0 sin hyperbolic gamma L. E 10 is the electric field of the wave at frequency omega at the input, E 1 of L is the electric field at the frequency omega at the output, so this is z is equal to 0 this is z is equal to L, phi 20 is the phase of the pump. Remember, we had written E2 is equal to E2 at z is equal to 0 into exponential i phi 20.

This is the phase of the pump which is the phase of 2 omega frequency and this exponential i pi by 2 is actually i which was stating here, I have taken as the exponential. This is the degenerate parametric down conversion process, otherwise you would have two equations, one for E s and one for E i. You already seen here that it is of the same form as we had written for the squeeze states, so this is the classical expression.

Now, I go into a quantum mechanical picture by replacing these electric fields by the annihilation operators of the corresponding electric fields. So, E 1 I will replace by a of 0 and E 1 star will replace by a dagger of 0 and E 1 of L is a at the output. The quantum mechanical form of this equation, I will write this like as a of L is equal to a of 0 cos hyperbolic gamma L plus e to the power i phi 20 plus pi by 2 a dagger of 0 sin hyperbolic gamma L, a 0 and a dagger 0 correspond to the annihilation creation operators of the field entering the crystal, a of L is the annihilation operator of the field exit in the crystal.

In general, we have studied that in Heisenberg picture the operated evolved with time, so can we evolve with this space quadrants also. This actually time because it takes at certain time for the field to go from z is equal to 0 to z is equal to L, so I can actually replace L by the corresponding time that i equal to take to go from the input to the output plane. It is essentially a picture in terms of evolution of the operators where the time has been replaced by distance in this equation.

Just like I did for the beam splitter case, what I have done is I have replaced the classical expression for the electric field by a corresponding quantum mechanical expression. As I said that this equation can be derived more rigorously by solving the problem of the down conversion process in a complete quantum mechanical picture.

Now, what we will do is you will use this equation to study the properties of the light that is coming out from this picture. So, remember in the beam splitter case - in the later part of the course - what we did was, the expectation values of the output state is given by taking the expectation values of these operators of the output operator expressed in terms of the input operators with the state remaining unchanged.

So, if I input vacuum state here I will have to take the expectation values. If I want to expectation value of the number of photons coming here, I will take the expectation value of a dagger L a L with respect to the input state, where a dagger L and a L are expressed in terms of the input operators a of 0 and a dagger of 0, just like in the beam splitter.

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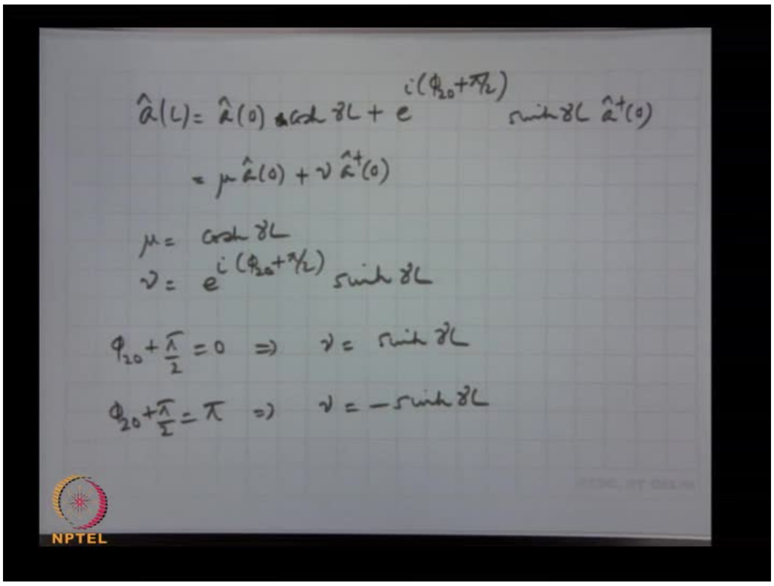

It is a similar equation for beam splitter except that, now it has both a and a dagger here in the expression. Let me write this equation again and you will try to solve this problem, so, a of L is equal to a of 0 cos hyperbolic gamma L plus exponential i by 20 plus pi by 2 sin hyperbolic gamma L into a dagger of 0.

Look at this, I can write this as where mu is equal to cos hyperbolic gamma L and nu is equal to exponential i phi 20 plus pi by 2, what the parametric down convertor is doing is to create an output state with an annihilation operator which is a linear combination of the annihilation creation operator of the input state.

Remember, for squeezing we had written a state with b is equal to mu a plus nu a dagger, it is a same expression, nu is complex in general as we have set earlier also. But, if you take for example phi 20 plus pi by 2 is equal to 0 then, nu is equal to sin hyperbolic gamma L. If you take phi 20 plus pi by 2 is equal to pi then, nu is equal to minus sin hyperbolic gamma L, otherwise nu is complex.

Because we are restricting ourselves to nu and nu real, let us look at these two situations - One can surely do a more general analysis but, to keep the mathematic simple we are assuming - where either nu is plus sin hyperbolic gamma L or nu is minus sin hyperbolic gamma L.

As I think he had asked, you see depending on the value of gamma L the values of mu and nu will change. Depending on the phi 20 the complex amplitude - the complex phase of this - nu will change. So, if I want a larger value of mu and nu I have to make it interact over longer length of the crystal with a larger gamma L. Remember, we had written cos hyperbolic sigma and sin hyperbolic sigma, so sigma is nothing but gamma L here.

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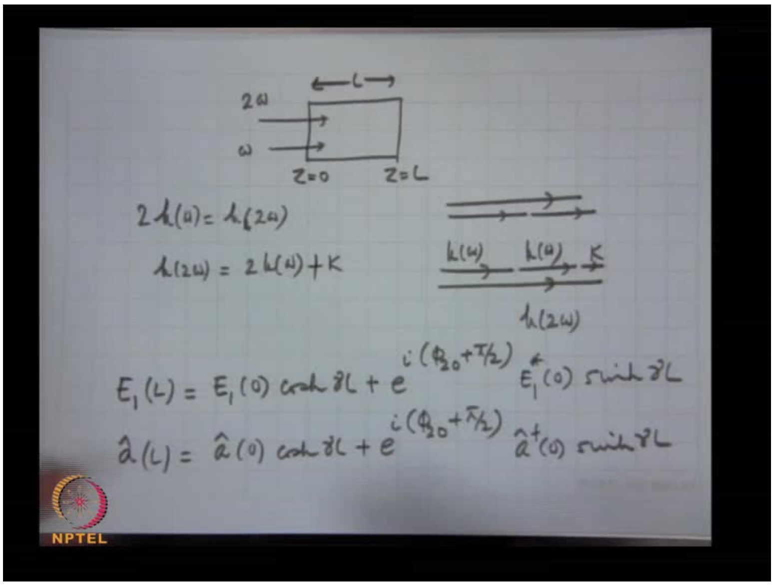

Gamma depends on the nonlinearity of a crystal and the pump power - power at the frequency 2 omega. If you go back and look at the expressions, the gamma which is the gain coefficient depends on the power that is entering at the frequency 2 omega, the nonlinearity of the crystal, the d coefficient that is been used, the polarization states of the light etcetera.

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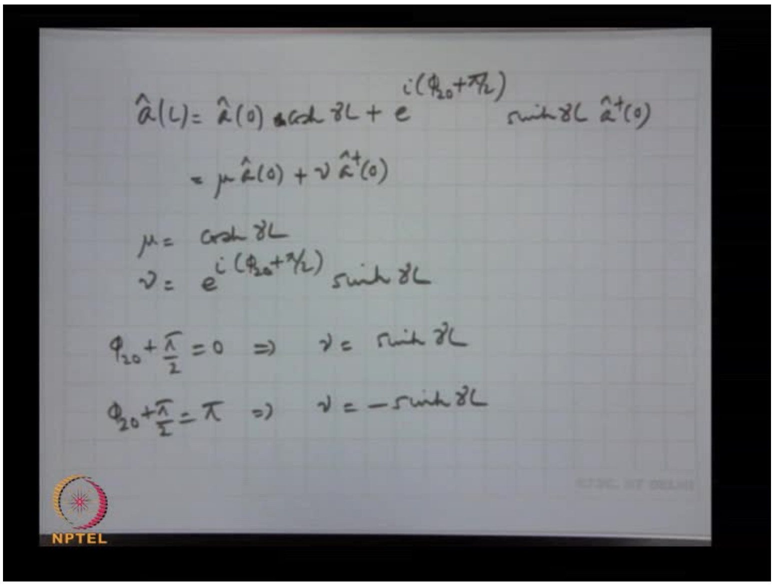

Now, let us look at the following, this is the output annihilation operator related to the input annihilation creation operators. To calculate the expectation of any observable at the output, I calculate the expectation value of that observable expressed in terms of a 0 and a dagger 0 with the states represented by the input states.

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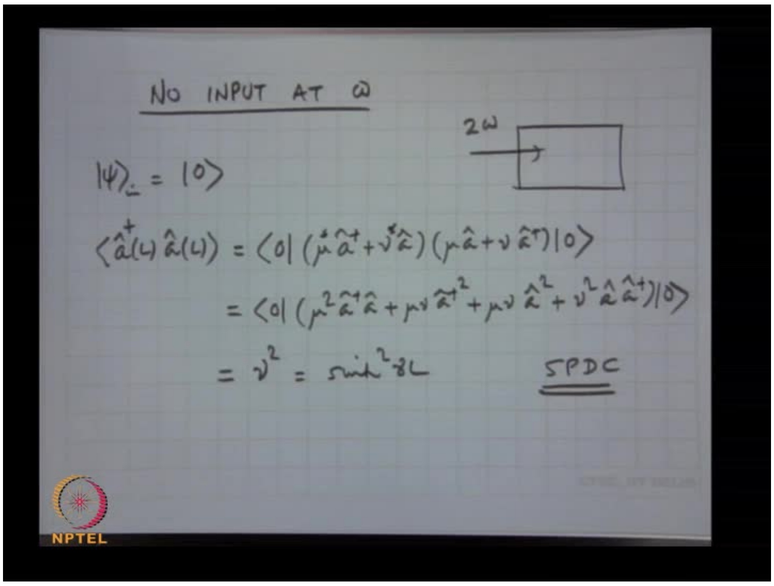

So, let us take the first example that I have nothing incident at omega. No input at omega, what is the input states vacuum. Now, I am writing the only the state for omega because 2 omega, I am not looking at 2 omega at all, I am assuming 2 omega is a classical way undepleted, so I don't care about 2 omega here, I am just writing the states for omega. So, what is the expectation value of the number of photons exiting the crystal? Expectation value a dagger L, a L for a dragger L, a dagger a of the output electric field state is the number operator of the output and the expectation value as we taken with as per the input states.

So, I have a L and a dagger L, so I have mu a dagger plus nu a - now I just do not write 0 a dagger at a means, it is at 0 - mu a plus nu a dagger. I can actually have nu as complex, I just have to write a complex expression here mu star and nu star but otherwise, if I restrict myself to these two values of phi 20 the nu is real. So, this gives me 0 mu square a dagger a plus mu nu a dagger square plus mu nu a square plus nu square a a dagger.

What is the expectation value of a dagger a, 0 a dagger a 0, 0, a dagger square 0, a square a, a dagger 1, because a a dagger is a dagger a plus 1, a dagger is this 0 and that is the spontaneous parametric down-conversion. You have a finite number of photons existing in the crystal at omega frequency even if you do not put any light at omega frequency. That gives for the expectational the number of photons coming out at the exit is of course depends on the gamma and the length of a crystal.

If you know nonlinearity gamma is 0 and there is no output at omega, so that is the first explanation for the fact that there is a finite output even if there is no input at omega frequency, so what is actually happening is the 2 omega photons are spontaneously splitting to generate omega photons at the output through this nonlinearity interaction process.

Sir, but in this case here little know a square so suppose they combine phi 20 plus pi by 2 is not 0 or pi.

It will be mod nu square,

But we are building no mod nu square,

No because I am assuming see otherwise, I have to write star star here that is all is the only difference, so this will be mod nu square it will become. Now, I want to calculate what kind of a state is coming out? Let me calculate the expectation values of the 2 quadrature operators at the exit of the crystal.

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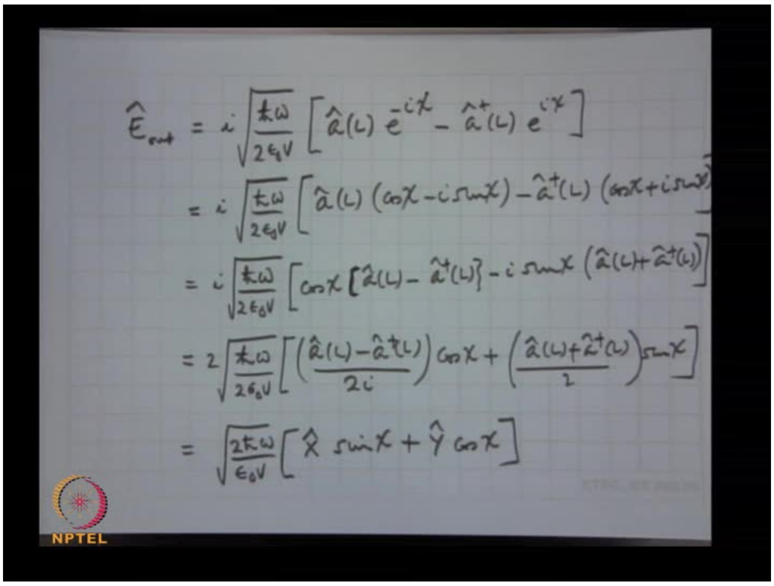

Actually the electric field of the output E out is given by i times square root of h cross omega by 2 epsilon 0 V a of L exponential minus i chi - which is omega t minus k z - minus a dagger of L exponential i chi.

Remember, we had simplified this in terms of the quadrature operator, this is the annihilation operator of the field that is exist in the crystal, crystal is the creation operator and I can write this as a of L cos chi minus i sin chi minus a dagger L of cos chi plus i sin chi which is equal to i times square root of h cross omega by 2 epsilon 0 V, so cos chi into a of L minus a dagger of L minus i sin chi into a of L plus a dagger of L.

If I take the i inside multiplied and divide by 2, I will get a of L minus a dagger of L by 2 i cos i plus a of L plus a dagger of L by 2 sin chi. So, this is square root of 2 h cross omega by epsilon 0 V x a f L plus a dagger of L by 2 is x, a minus a dagger by 2 i is y. Same expression that we had written earlier, expressing E either in terms of a and a dagger operators or in terms of the two quadrature operator.

All I have done is again rederived an expression for the output of the electric field operator in terms of the quadrature operators x and y. So, the x quadrature operator the output is given by a plus a dagger by 2 and similarly, y is a minus a dagger by 2 y.

So, let me calculate what are the expectation values and the variances in the two quadrature operators at the output. I will show this as squeeze state and it is a squeezed vacuum state because you input vacuum and actually the output is.

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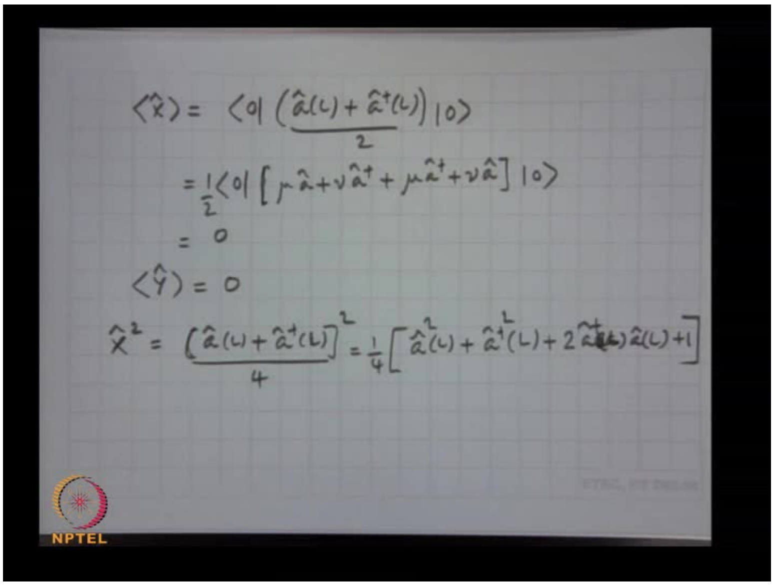

Remember, we derived that in a squeeze vacuum state the number of photons is not 0, it is sin hyperbolic square sigma is nu square is the output of the number of the photons. So, let us calculate what are the expectation values and the uncertainties in the two quadratures. So, x expectation value is 0 a of L plus a dagger of L by 20 which is equal to a of L is mu a plus nu a dagger plus mu a dagger plus nu a.

Again, I am assuming that nu is real, it say the positive or negative, how much is this? 0 because expectation value a and a dagger as 0, so this is 0, similarly expectation value of y is also 0. Now, x square operator is actually a of L plus a dagger of L whole square by 4 which is actually 1 by 4 a square of L plus 2 a dagger a plus 1, x square operator is 1 by 4, a square L a dagger square L plus 2 a dagger a plus 1.

I need to calculate expectation value of a square of L, a dagger square of L and a dagger a of L, where a of L is given by the expression mu a plus nu a dagger and remember, we have done all this when we have discussing squeeze states.

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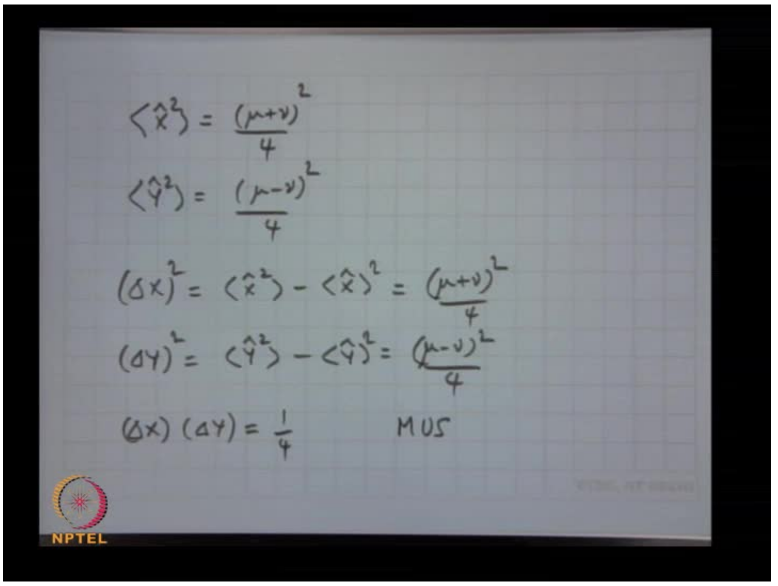

For example, if you go back and look at the expression that we are done earlier, what you will find is that expectation value of x square will be equal to this and expectation value of y square is this. You have to replace a of L and a dagger of L by the corresponding expressions in terms of a of 0 and a dagger of 0 then, find the exception values of d square - it is bit of a algebra - and finally for the expectation values of the input vacuum state.

So, delta x square is equal to x square minus x average square which is equal to mu plus nu whole square by 4 and delta y square is equal to y square minus y average square is equal to this and what is delta x into delta y? nu square minus mu square by 4 and mu square is cos hyperbolic square gamma L and nu square is sin hyperbolic square gamma L. So, this is a minimum and uncertainty state because the product of the uncertainties in the two quadratures is 1 by 4 it is a minimum uncertainty state. Depending on the value phi to 0, if I had chosen phi 2 0 is equal to such the phi 2 0 plus pi by 2 was 0, the nu is plus sin hyperbolic gamma L, mu is sin hyperbolic cos hyperbolic gamma L.

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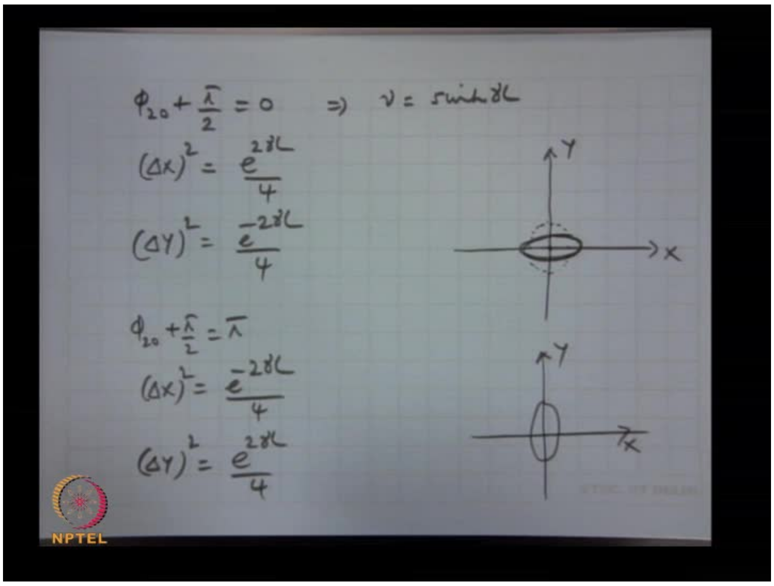

So, if I had chosen phi 2 0 plus pi by 2 is equal to 0 then, delta x square will become equal to - so this implies nu is equal to sin hyperbolic gamma L, so delta x square is - cos hyperbolic gamma L plus sin hyperbolic gamma L whole square by 4 which is this and delta y square, in the phasor diagram this is the state.

Expectation value of x 0, expectation value of y is 0, uncertainty in x is bigger than 1 by 2, the uncertainty in y is less than 1 by 2 and it is a squeezed state, the normal coherent vacuum state would have look like this a circle with certain radius of half here it is an ellipse.

If chose phi 2 0 plus pi by 2 is equal to pi then, delta x square becomes exponential minus 2 gamma L 4 and delta y square becomes exponential 2 gamma L by 4. The corresponding phasor diagram will look like this, it is squeezed in the other direction x is squeezed the y quadrature is not squeezed. Remember, I can show this squeezing by using balanced homodyne detection.

So, if the output coming out of the parametric down convertor is fed into a balanced homodyne detector, I will measure reduced noise in one of the quadratures depending on the pump phase with I have chosen. So, what is coming out from the parametric down convertor with nothing input is actually a squeeze vacuum state.

So, this is one very interesting method of generating squeezed light a parametric down convertor automatically produceswith no input squeezed vacuum.

Sir, in this analysis we have assumed that both the operators, you know when we are calculating the expectation value we assume that we are only operating on the omega wave that the weak single lattice, so a strong signal would not play on its own how can we justify that.

No, two things expectation values of the omega field at the out depend only on the omega field at the output, inside there is mixing taking place but at the output the expectation values of the omega field does not depend on the field of the 2 omega. If I want to measure how many photons are coming in that omega frequency I have to calculate expectation value of a dagger a of a omega field.

The question is whether this equation which I have written are valid and these equations are valid as long as the input omega 2 omega wave a strong enough to be treated in a classical fashion that means, if my 2 omega wave is intense enough -intense a few milliwatts of power is intense enough to be treated completely classical. So, I am assuming no pump depletion which means, 2 omega remains 2 omega continuously but of course some of those 2 omega photons are generating with omega photons.

So, I have lost some photons at 2 omega frequency, if my 2 omega also extremely weak I would have to treat more complex problem of treating both 2 omega and omega in a quantum mechanical passion that is a complete treatment, where both 2 omega and omega are treated as a quantum mechanical operators and output. I have an input state corresponding to say n photon state of 2 omega with vacuum stated omega. Suppose, I put a input state at 2 omega corresponds to a state which is 10 photons coming at omega frequency.

In one fock state corresponding to single mode at 2 omega right, so I would have 10 2 omega 0 omega input. I have to do a quantum mechanical analysis of treating both omega and 2 omega fields quantum mechanically look at what is the output state and then, I will sure see the evolution of 2 omega and omega simultaneously taken.

But, what we are doing is we are treating in the approximation that 2 omega is a classical wave and I am assuming no pump depletion, so 2 omega I am not considering at all. So, if you calculate this classical equation that we had use to operate this to obtain the operator equation in the starting that was obtained using the no depletion condition.

Now, in some other case if we have an analytical expression without using the no depletion approximation then, it would be correct through calculate the expectation values in the same way that we have done - I mean - that would be absolutely correct.

No, what will happen is the expression for E omega at the output will be different from what we are used. It will depend on some fields of the 2 omega frequency also.

No, but in general it is always true that obtaining the classical equation and placing them for the.

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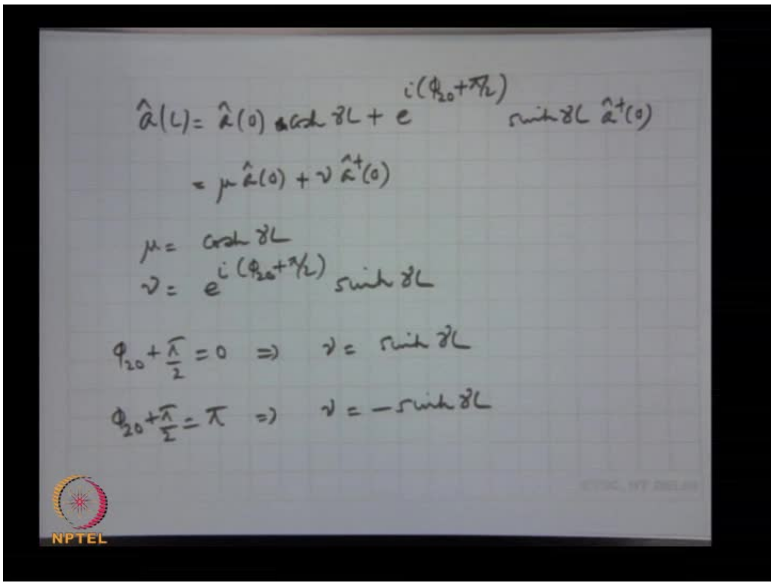

Usually, I will not say always, it is usually true - I am not sure. I think, I am sure there may be cases where I cannot just do it classical to quantum just like we replacing, because I should keep ensuring that for example, I should make sure that how do I make sure this equation is quantum mechanically correct? I need to check whether this operator satisfies the computational relation. So, let me check for example what a computational relationship satisfied by a.

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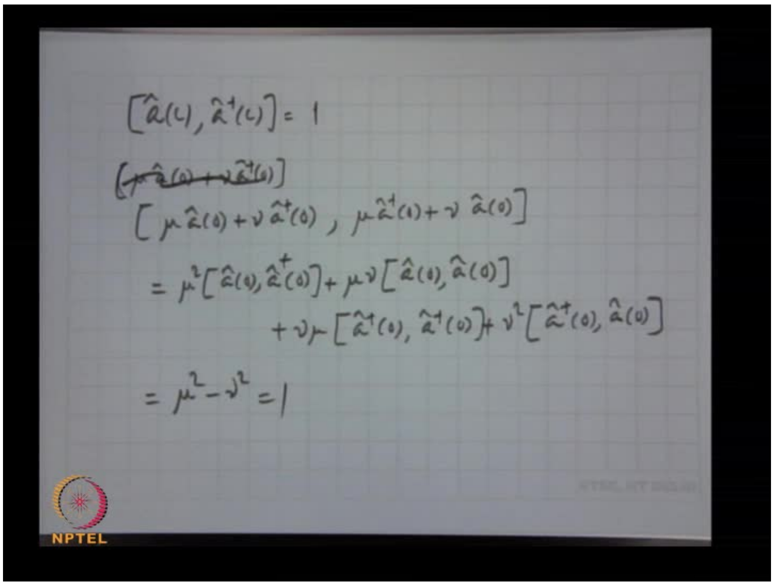

a a dagger must be equal to 1, so nu a of 0 plus nu a dagger of 0; let me write like this, mu a 0 plus nu a dagger 0 and mu a dagger 0 plus nu a 0, so this is equal to mu square a 0 a 0 plus mu nu - sorry a dagger plus mu nu - a of 0 a of 0 plus nu mu a dagger of 0 a dagger of 0 plus nu square a dagger of 0 a 0 what is the value of this, a a dagger 1 second term 0, this is also 0 and this is minus 1.

So, this is right this at least it is not incorrect.

So, an operator corresponding to any observable, if we write it then, like the electric field operator that we wrote it consist of this a and a dagger, so it means that every time we write an operator corresponding to an observable in terms of other operators that they should commute that is the principle that we have.

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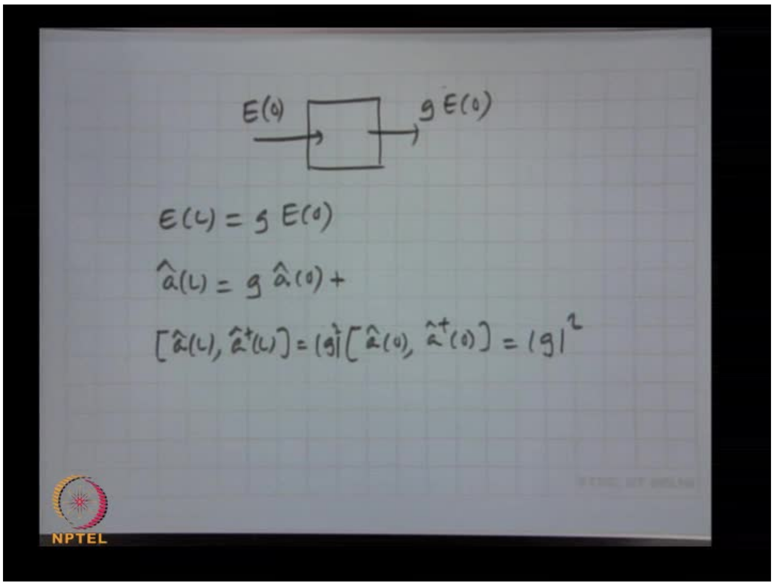

No, for example in the beam splitter case we ensure that a 3 and a 4 satisfy the commutation relations. For example, I will take a situation and tell where the problem is, I could analyze the following problem, I have an amplifier and I input an electric field E of 0, it gets amplified to some gain times E of 0.

So, I will write a corresponding equation, let me try to see I replace this by a quantum mechanical equation, so a of L I will try to write as g of a of 0. Now the problem with this is a of L a dagger of L is equal to g square times a of 0 mod u square actually a dagger of 0 which is equal to mod g square, it is not correct. So, this is not a correct transformation what is missing is, to satisfy this commutation relation I have to add another operator here and that is actually in this amplifier that gives me the noise.

So, it seems that I can just replace by operators and get, but I have to keep ensuring that I am satisfying all the required computational relations and so on. So, I have to add an operator here which will automatically make it sure that commutation relation satisfied. I can actually analyze the amplifier etcetera and show you calculate, what is the noise figure that amplifier etcetera from this just from the simple analysis.

But, in any unknown situation we are trying to use such method classical analysis to quantum analysis we are going. Then what are the operators which should commute I mean the once which commutated earlier.

Now, we are looking at a quantum optical situation where electromagnetic fields are involved, so the commutation relation are a and a dagger must be equal to 1. What are the fields you are taking, wherever you are taking fields those that the annihilation operators integration operators are those fields must satisfy this commutation relation.

Now, I would imagine that given a more complex situation I would have to do to a complete quantum mechanical analysis to find out what the corresponding equations are. As I mentioned in the beginning itself these equations we are writing as just quantum mechanical extension of the classical equations but there is much more deeper basis for getting those equations which can be obtain by complete quantum mechanical analysis.

Sir, suppose here we are getting this a a dagger commutation relation while, so can they exist a situation in a system, then there are two sets of commutation relation both are satisfying the same value 1. You mean 2 transformation equations I do not know, but 1 may be nonphysical.

You see the way I am going is so I am writing the classical equations which are correct and then sort of replacing those fields by the operators, it is a very possible method but I do not know I cannot, I do not say I can guarantee that this is all is got.

Sir, how do we know that whatever transformation we are going to because d only correct

No, because in the classical limit it should reduces the classical equations. In the limit of classical approximation these quantum mechanical equation should reduce the classical equations. So, because I am using the classical equations to obtain the quantum equations and deriving some relationships I except the observables should also say finally satisfying the corresponding classical equations.

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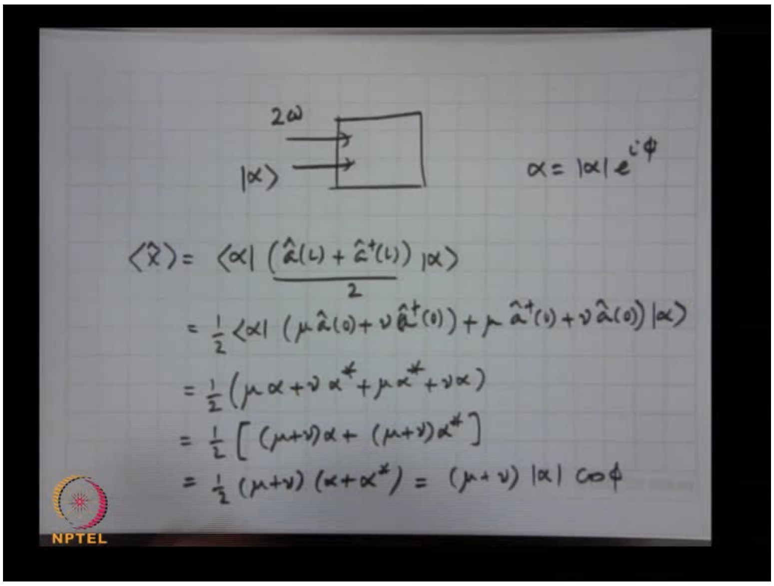

In the limit for example, if I put on a strong field at omega I can also calculate what the output is. In fact instead of putting vacuum, I can do the following; I say that I have a system in which I have 2 omega and I put a coherent state at omega.

For example, what will be the expectation value of x? This will be alpha a of L plus a dagger of L by 2 alpha which is equal to 1 by 2 alpha mu a of 0 plus nu a dagger of 0 plus mu a dagger of 0 plus nu a of 0 alpha. So, this is mod half of mu times alpha plus nu times alpha star plus nu times alpha star, alpha could be complex, so plus u times alpha this is equal to half of mu plus nu into alpha plus mu plus nu into alpha star. So, this is equal to half of mu plus nu into alpha plus alpha star which is equal to mu plus nu mod alpha cos phi.

Assuming alpha is alpha mod exponential i phi input state coherent state is given, then the expectation value of the quadrature operator is now finite, because this equation representing a l and a 0 is a squeezing equation - it is like an squeezing equation - the output will be as squeezed coherent state it will be a squeezed state and depending on phi 2 0 which I choose, I will find squeezing in the x quadrature or in the y quadrature with the input as a vacuum state i found output squeezed vacuum with input coherent state if you continue and calculate x square expectation value using the same expressions except that the expectation values will change because it is not a vacuum state it is a coherent state.

You will find that this squeezed output state this squeezed state. In fact, this an amplifier as you have discussed this is an amplifier and what is interesting is, I do not think we will have time to discuss that you can calculate, you see in amplifier with this a very important parameter of an amplifier. What is it? One is gain and second is, must be bandwidth. Bandwidth is one and then, it is noise signal generated by noise.

The noise generator by the amplifier, so you calculate what is called as signal to noise ratio. The input signal comes with some noise, it has a certain input signal to noise ratio; the output the amplifier amplifies the signal amplifies the noise and adds its own noise. So, usually what happens is the output signal to noise ratio is always worst then the input signal to noise ratio, because you are amplified both the signal and noise and if you did not add any noise then the signal to ratio would have remain the same.

Because you have amplified noise and also added your own noise, the output signal to noise ratio is work done the input signal to noise ratio. If I calculate for this amplifier I will find that output signal to noise ratio and input signal to noise ratio are equal. This phase sensitive parametric amplifier does not add any noise to the signal and in that sense it is a very interesting amplifier because if you have an amplifier with no added noise by the amplifier that is very interesting for any application, if you have detect very weak signals you have to amplify it.

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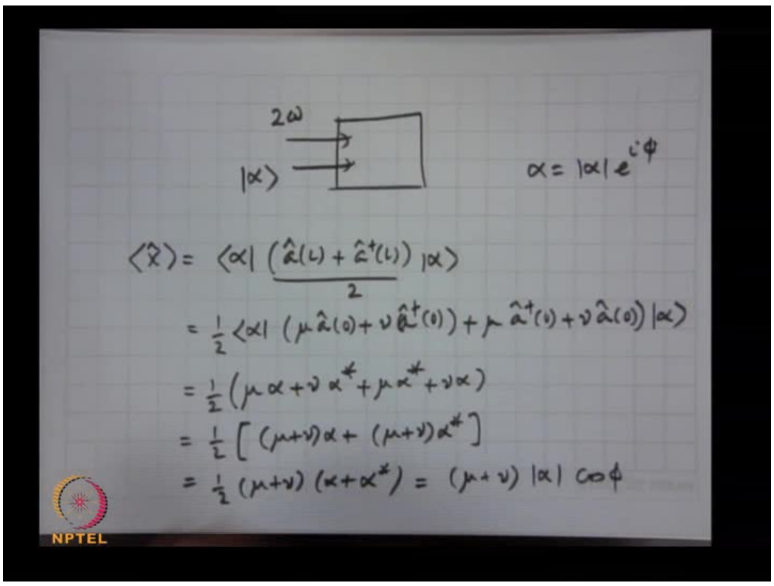

When you amplify you add your own noise, this amplify will not add any noise in principle. So, people have demonstrated the noise reamplification of this amp not completely noise free but much lower than what you except as a typical amplifier.

So, for example you can amplify using population inversion like a laser, you can show that amplifier versions the noise figure at least by a factor of 2, minimum this is quantum limit. If the input signal to noise ratio was 10, the output signal to noise ratio will be half of 10 pi, you go through another amplifier it will be in another half factor of half it goes like this.

This 1 has 10 and 10 at the output of course, the practical amplifiers do not have exactly the same noise at the input and output, but people have shown that the output signal to noise ratio is much better than 3dB which is factor of 2 means, in large scale 3 decibels - 3 decibel is a factor of 2, so this is called the quantum limit.

Quantum limit of noise figure of an amplifier is 3 dB - of a phase insensitive amplifier is 3 dB. People have shown that you can build amplifiers, we using parametric down conversion process where the noise reduction from 3 dB is much very good, I mean you can decrease the noise significantly compare to a phase insensitive amplifier, I think we will end the course here.

What we actually try to do in a course is we extent an initial portion of the course looking at non-linear interactions. We did sub background for that for anisotropic crystals and so on. The non-linear interactions including second harmonic generation, parametric down conversion, parametric oscillation as a chi 2 process and there we found some interesting features of phase incentive amplifier, phase sensitive amplifier we could not discuss at that time the quantum aspects like spontaneous down conversion.

The later part of the course in which we quantize the electromagnetic field and got to understand the bit of the quantum properties of light helped us to appreciate that there can be states of light which are quite different from what we can think of in a classical passion. You can have states of lights in which noise is below what you can except in a standard very well operating laser and you can discuss squeeze states we have not been able to look at entangle states of light which also comes out from this kind of interaction process a parametric down conversion process.

The 2 photons coming out here at the output can have very interesting properties and what are called an entanglement and again the analysis is based on what we have essentially done in the course. So, I hope will give you a sort of introductory concept, how to treat some of this problems and what one can except of course, please note the most general electromagnetic state which we had written c n 1, n 2, n 3, n 4 etcetera n 1, n 2, n 3 acts a huge state space and I think much of it is still unexplored.

Some of those states are coherent states, some of those states are squeezed states, some of those states are entangle states and there may be more hidden states which I may be discovered. What is interesting today I would say is, we are at a time when many of these non-classical features are can be experimentally checked the properties of entanglement for example.

This is all because of the development of technology and understanding of some of these concepts, so people are trying to build sources in which I can get single photons coming out to make a single photon state is not that easy. So, people are trying to make single photon states, 2 photon states and n number of photon states and so on.

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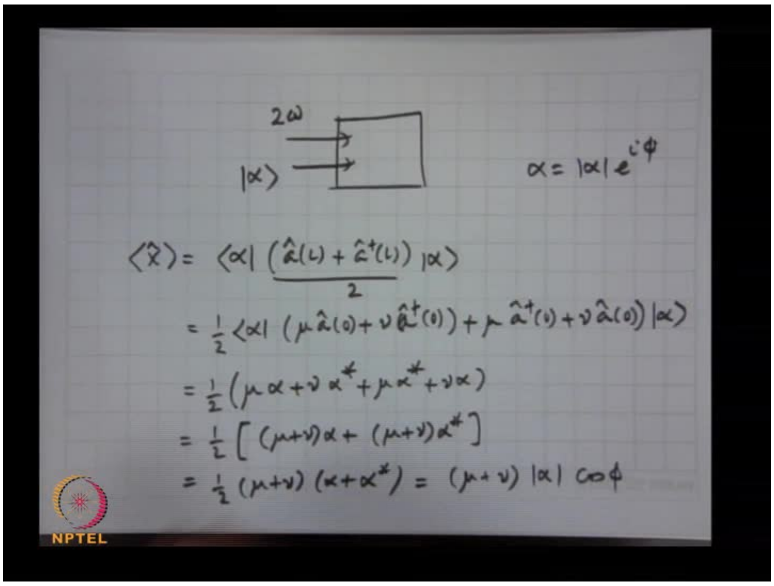

So, there is a lot of interesting experimental and theoretical work in this and we have just had very small glens of the entire field of the quantum optics here, in this latter part of the course. So, I hope you have enjoyed the course and I have enjoyed in this process.

Tomorrow, we will have a class in which we have any question that you have in the course. We can tattle if you do not have many questions then maybe I will briefly tell you about some interesting experiments which can be explained quantum mechanically but which are a bit contra-intuitive, thank you.