Quantum Electronics
Prof. K. Thyagarajan
Department of Physics
Indian Institute of Technology, Delhi

Module No. # 05
Lecture No. # 40
Balanced Homodyning

We were discussing balanced homodyning in the last class. So, today we will discuss little more details of balanced homodyning and look at some examples of how this process helps me to differentiate between squeezed states and any state which is not squeezed like, the coherent state is not a squeezed state. So, this particular method is a very, very, useful method to demonstrate and to measure certain non-classical features of light; do you have any questions from what we have discussed in the last class?

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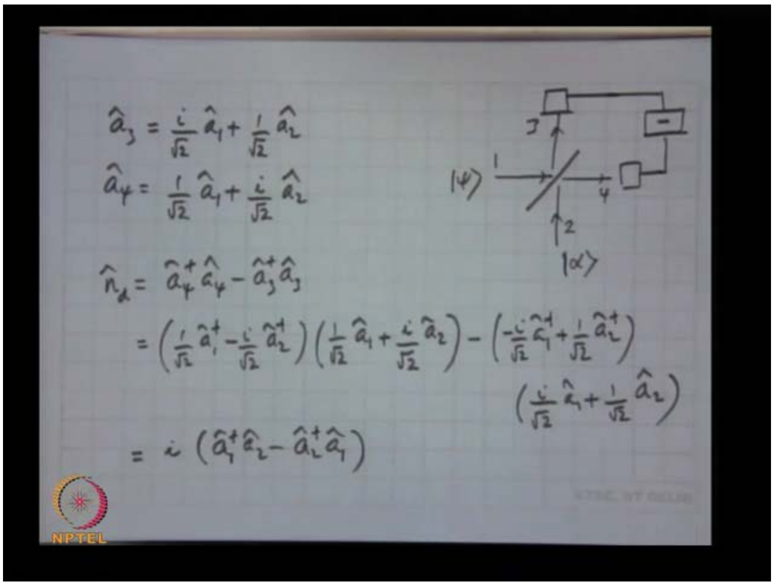

So, we continue, let me recall, in balanced homodyning what we are having is, we have a 50 percent beam splitter, the arm 1 is the input state which is an unknown state; in arm 2 we put in a coherent state which is represented by alpha, which means it is like a wave coming out of a laser, a good stabilized laser will produce a coherent state. Then, in arms 3 and 4 we have light which is reflected from state 1 and transmitted from state 2, and similarly, in arm 4 we have 1 which is transmitted from 1 and reflected from to 4, reflected from 2.

Then, we have 2 detectors here which measure these intensities coming here, and then we do a subtraction and it is this signal that we are looking at, which I will show you contain interesting features of the state which is coming from here. So, let me write down again the annihilation operators in arm 3 and 4 as related to 1 and 2. So, if you recall, this is i by root 2 a 1 plus 1 by root 2 a 2, similarly, a 4 is equal to 1 by root 2 a 1 plus i by root 2 a 2. So, what we are actually detecting is the expectation value, the number operator here and here, and then subtracting them.

So, the expectation value the number operator here is expectation value between a 4 dagger and a 4, here it is a 3 dagger and a 3, and we are subtracting, so what we are trying to do is, a measurement of this operator which is a 4 dagger a 4 minus a 3 dagger a 3. So, in the last class, we substituted these expressions here, so let me just repeat that, so 1 by root 2 a 1 dagger minus i by square root of 2 a 2 dagger into 1 by square root of 2 a 1 plus I by square root of 2 a 2 minus minus i by square root of 2 a 1 dagger plus 1 by root 2 a 2 dagger into i by square root of 2 a 1 plus 1 by square root of 2 a 2.

So, if you open these brackets up what you can show is, this comes out to be i times a 1 dagger a 2 minus a 2 dagger a 1; did I do n 3 minus n 4 last time? Yes sir, yeah, so anyway, so difference between these 2, so what these two detectors are detecting is, expectation values of a 3 dagger a 3 and a 4 dagger a 4 and subtracting the signals coming from detectors 3 and 4. This signal which is being detected also have noise, because in all these I am interest also in how much is the noise that is being detected, because it is in the noise properties of these states which are coming here which is what I have to show.

There is squeezing for example, a certain quadratures of this field there is squeezing, so I also need to calculate what is the variance in n d? The difference in the number of photon being detected in 3 and 4 and for that I need the expectation value of n d square, because the variance in n d which is the difference in photon numbers delta n d square will be expectation value of n d operator square minus expectation value of n d operator square.

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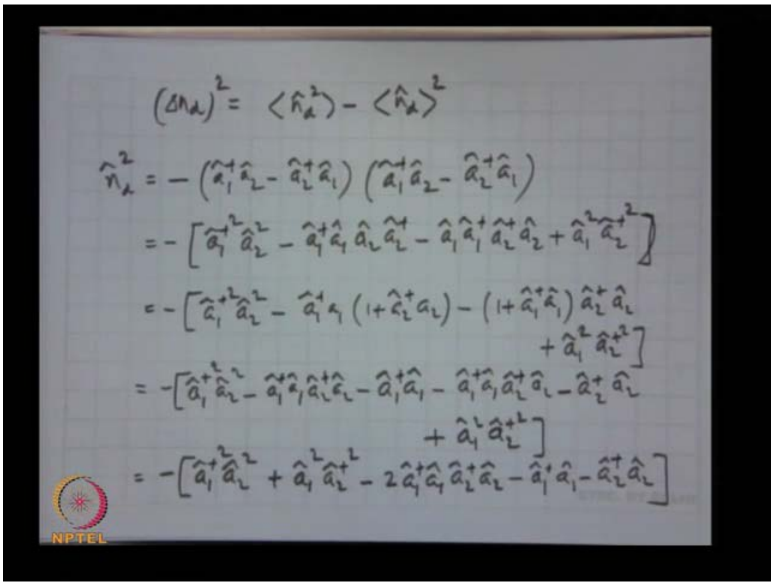

The variance of an operator is defined by expectation value of the square of the operator minus expectation value of the operator square. So, I need to calculate n d square, so n d square is - this is the expression for n d which you have calculated - n d square will be square of this, so which is minus a 1 dagger a 2 minus a 2 dagger a 1 into a 1 dagger a 2 minus a 2 dagger a 1. So, this is minus a 1 dagger square a 2 square minus a 1 dagger a 1 a 2 a 2 dagger minus a 1 a 1 dagger into a 2 dagger a 2 plus a 1 square a 2 dagger square.

We have to keep track of the ordering of the operators, the inputs are 1 and 2 are assumed to be statistically independent, so there operators commute, so I can just interchange or I can just exchange the positions of a 1 and a 2. Now, all ordering you remember is with the dagger operators in the left and the non-dagger operators on the right, so I want to replace this a 2 a 2 dagger by what? 1 plus a 2 dagger a 2, and similarly, this a 1 a 1 dagger I will replace, so this is minus a 1 dagger square a 2 square minus a 1 dagger a 1 into 1 plus a 2 dagger a 2 minus 1 plus a 1 dagger a 1 into a 2 dagger a 2 plus a 1 square a 2 dagger square.

This is equal to minus a 1 dagger square a 2 square minus a 1 dagger a 1 a 2 dagger a 2 minus a 1 dagger a 1 minus a 1 dagger a 1 a 2 dagger a 2 minus a 2 dagger a 2 plus last time which is a 1 square a 2 dagger square. So, let me simplify this, this is minus a 1 dagger square a 1 a 2 square plus from here a 1 square a 2 dagger square from here, then I have minus 2 times a 1 dagger a 1 a 2 dagger a 2, these 2 terms and I have minus a 1 dagger a 1 minus a 2 dagger a 2, a 1 dagger square a 2 square plus a 1 square a 2 dagger square, these two are identical terms, so of minus 2 a 1 dagger a 1 a 2 dagger a 2 minus a 1 dagger a 1 minus a 2 dagger a 2.

So, I have the two operators representing the difference in the photo currents being generated by the 2 detectors, and n d square which will help me to calculate the variance in the signal that is being detected as a difference between the detected 3 and 4. I need to calculate expectation values. As I told you what we use is the fact that, the operators transform as represented by these equations and we assume the states to remain the same.

To calculate the expectation values of n d and n d square I use the input states. So, the input state consist of a product of this state import 1 and alpha import 2, this is given to be as a local, it is called the local oscillator, it is a field which I am providing, and this is unknown state which I am trying to characterize. So, unknown state is input in arm 1 of the of the beam splitter; in arm 2 I input a coherent state and we will assume that this coherent state is a strong state which means alpha square is large compared to the expectation values of a dagger a for example, in this state; that means, the intensity of this is much higher than the intensity of this, in a standard homodyning technique, it is exactly the same, the local oscillator is a much more powerful beam than the signal, and actually the homodyning lets you amplify this signal. So, what this is doing, is amplifying the quantum fluctuations of this state into a classical level which can be measured by photo detectors.

So, I need to calculate the expectation values, so what is the expectation value of n d and expectation value of n d square, so let me calculate the expectation value of n d.

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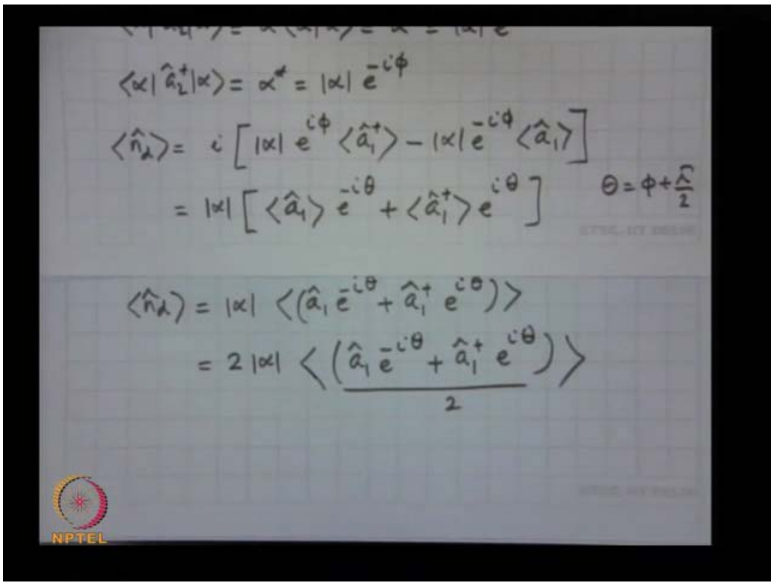

This is psi and alpha in this state 2 of i times a 1 dagger a 2 minus a 2 dagger a 1, i times, now a 1 and a 2 a 1 and a 1 dagger operate on the size, a 2 and a 2 dagger operators on the alpha, because a 2 corresponds to arm 2 where alpha state is incident and a 1 corresponds to arm 1 where psi is incident. So, I will have psi a 1 dagger psi into alpha a 2 alpha, this is 2 minus alpha a 2 dagger alpha into psi a 1, what is the value of this alpha a 2 alpha? This is a coherent state, so operating on the coherent state gives me alpha, so this is alpha, alpha alpha which is alpha.

Coherent state is defined by a times alpha is equal to alpha times alpha, similarly, this 1 alpha a 2 dagger alpha will be equal to alpha star, so I can write this as mod alpha exponential i phi, where phi is a phase this is mod alpha minus i phi. So, expectation value of n d becomes I times, so this is mod alpha exponential i phi, let me just write expectation value of this, this is means that with respect to the input state minus mod alpha e to the power minus i phi a 1.

This quantity, I am just writing as expectation value of a 1 dagger, this quantity I am writing as expectation value of a 1, alpha a 2 alpha is mod alpha exponential i phi and alpha a 2 dagger alpha is mod alpha exponential minus i phi. So, let me take i inside and mod alpha out, so this becomes actually a 1 exponential minus i theta plus, I will define this, a 1 dagger exponential i theta the theta is defined as phi plus pi by 2.

So, i is exponential i pi by 2, so when I take i inside, this is exponential i pi by 2, so exponential i phi plus pi by 2 is i theta, this minus I is exponential minus i pi by 2 which along with this gives me exponential minus i theta So, just take an i inside combined with the exponential i phi term and I have got expectation value like this. So, actually this is nothing but the expectation value of a 1 to power minus i theta plus a 1 dagger exponential i theta.

So, let me write this as mod alpha, this is expectation value of a 1 minus i theta plus a 1 dagger e to the power i theta. So, let me write this, now I guess 2 times mod alpha times by multiply and divide by 2. Now, what is this quantity here when theta is equal to 0? The quadrature x, remember, x was defined as a plus a dagger by 2, if you have the electric field if you go back and look at the electric field operator, it is in terms of a and a dagger.

Then, we wrote the electric field in terms of x and y quadrature operators, x quadrature operators is a 1 plus a 1 dagger by 2, so theta is equal to 0, this operator is x operator and theta is equal to pi by 2, this is minus i, this is plus i, if I take i in the denominator, it becomes a 1 minus a 1 dagger by 2 i, which is the y operator, so this let me define this as this, so x of 0 is x operator, x of pi by 2 is y operator.

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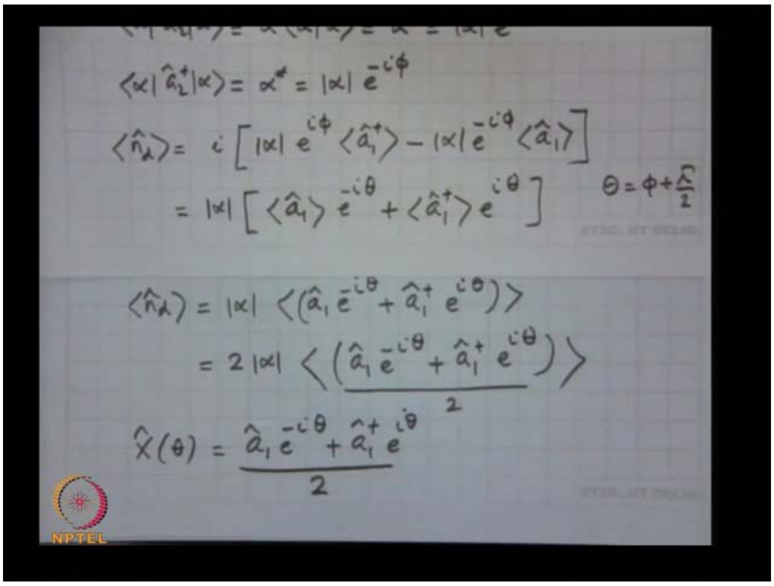

So, already you see in the detector signal which is being detected as the difference between n 3 and n 4, that signal will either give me expectation value of x or expectation value of y depending on the value of theta I choose, and theta is nothing but phi plus pi by 2, and phi is what the phase of the coherent state that is coming in arm 2.

So, if I appropriately choose the phase of the local oscillator which is an arm 2, which means if I choose appropriately phi, then I can vary my theta, and depending on the value of theta I choose, the signal that I am measuring is proportional to the expectation value of x or expectation value of y.

What is happened is, because I have given a reference phase which is the local oscillator, I am able to measure either the x quadrature or the y quadrature. So, the expectation value of the signal that I am measuring is proportional to expectation value of x if I choose theta is equal to 0, and if I choose theta is equal to pi by 2 I measure the expectation value of the y operator of the y quadrature.

And this is interesting, because now I will show you that the noise in n d square, that is, variance in n d is also proportional to the variance in the x quadrature and the y quadrature depending on the value of theta I choose. So, for that I need to calculate expectation value of n d square, so I have already got expression for n d n d square, so I need to calculate expectation value of n d square, here it is n d square - let me write it down here is n d square.

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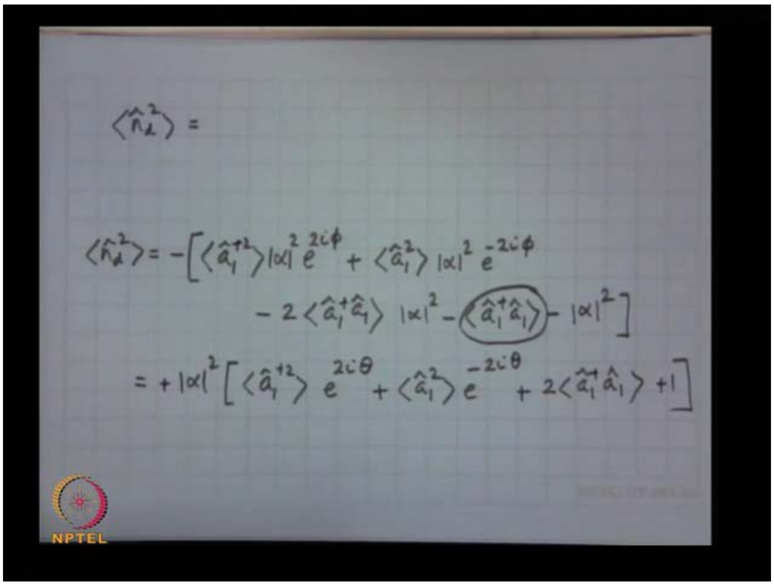

So, expectation value of n d square minus, now this is expectation value of a 1 dagger square, what is expectation value of a 2 square for the coherent alpha square, so mod alpha square exponential 2 i phi, because alpha is alpha mod exponential i phi plus expectation value of a 1 square into mod alpha square exponential minus 2 i phi, minus twice expectation value of a 1 dagger a 1, what is expectation value of a 2 dagger a 2 for coherent state? Mod alpha square.

Mod alpha square minus expectation value of a 1 dagger a 1 minus mod alpha square, these expectation values are being taken with respect to the state incident on arm 1 which I am not defined here, it could be vacuum, it could be squeezed vacuum, it could be coherent state, it could be squeezed state, it could be number state, whatever state, so this expectation values have to be taken with that state.

Now, let me assume that this, what is this? This is the expectation value of the number operator for the input state 1, in arm 1, if my coherent state is very strong, this will be negligible compared to mod alpha square. So, I neglect this term, and I get minus mod alpha square into let me take the minus sign also inside, so this is a 1 dagger square, this is 2 i theta, because please note minus 1 is exponential i pi and that is exponential 2 times i pi by 2, and if I add with 2 i phi I get exponential 2 i theta, plus a 1 square exponential minus 2 i theta minus 2 times a 1 dagger a 1 minus 1.

I have neglected this term, and I have taken out mod alpha square out, so these are plus and plus sorry. I have taken the minus sign inside, this minus with exponential 2 i phi gives me exponential 2 i theta, this minus 1 with the exponential minus 2 i phi gives me minus 2i theta, and this is 2 a 1 dagger a 1 ok. So, I have got expressions for the expectation value of n d and expectation value of n d square, and from here I can calculate the variance, which is expectation value of n d square minus expectation value of n d whole square.

Now, let us look at some examples, so the first example I want to consider is that in arm 1, I do not put anything, I do not have any incident wave in arm 1, so what do I represent psi as, so the first example is where I have, here is alpha coming, so what is input state here? Vacuum state, so psi and this vacuum state satisfy this equation, a operating on 0 is equal to 0 that is a vacuum - coherent vacuum state; standard vacuum state, where none of the modes are occupied, because after this you will consider this squeezed vacuum state, ok.

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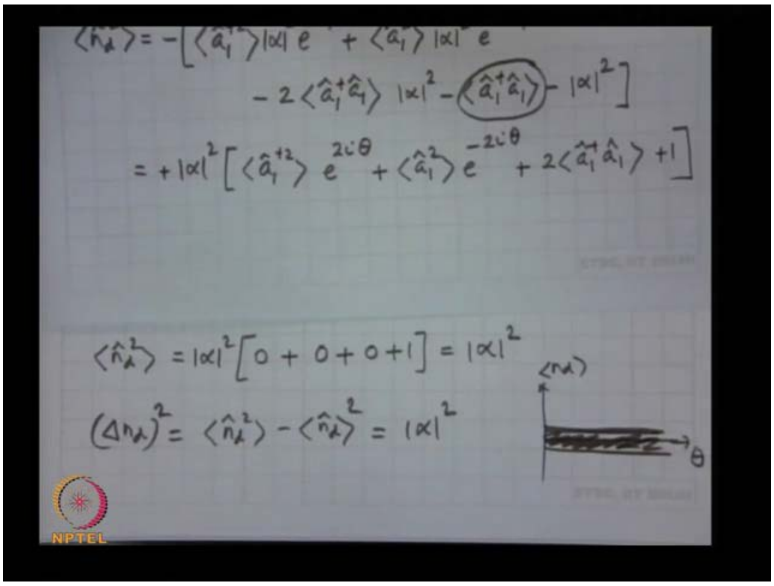

So, now, let us calculate expectation value of n d, and expectation value of n d square and from there the variance. So, what is the expectation value of n d? This is 2 times mod alpha times expectation value of a 1 exponential minus i theta, so this is 0 plus a 1 dagger e to the power i theta by 2 0, how much is this? a 1 operating on 0, a 2 dagger operating on 0 0.

The difference signal is 0, there is no difference signal, there is signal in arm 3 and there is signal in arm 4, but the difference signal average is 0, because this is expectation value, on an average you are getting 0 signal at the difference between this and this. See, classically you will expect this because you have light coming in 1 of the beam splitter, you have identical detectors here, you subtract the signals you will get 0, that is what is happening actually. Expectation value of n d square, now n d square is here, so these mod alphas square, what is the expectation value of a 1 dagger square? 0 0, yeah, tell soon.

Sir, in this statement this expectation value of n d square is equal to 0 can't we get x square? This is related to, this is actually the expectation value of x theta square.

Sir, suppose, if we put theta is equal to…

Yeah.

Can't we get x square

Expectation value of, yeah, I am just keeping theta in general and I will tell you what happens, theta is equal to 0 is x square, theta is equal to pi by 2 is y square automatically. So, I am not I am I could done it here, but I continue with theta and later on, for example, for coherent square state you will see theta does not come into the picture at all whatever theta you choose its independent of theta, but in squeeze states it may depend on theta. So, you can choose different values of theta to calculate. So, what is a 1 dagger square expectation value? This is 0 a 1 dagger square 0, which is 0; second term a 1 square expectation value 0 a 1 dagger a 1 also 0, there is no signal, but there is a variance.

So, delta n square delta n d square is n d square minus n d square which is, and this variance is actually coming from the coherent state actually input in the beam splitter. So, if you were to plot as a function of theta, so what I want to experimentally is, choose 1 theta value measure many times that signal that I am getting, that signal can be positive or negative, with this variance I change my theta, this is independent of theta. So, as I change my theta, if I can scan my theta for example for time, I will measure constant variance; that means, the signal that I am getting as the difference signal is fluctuating and the fluctuation is given by this variance.

What is the variance of a coherent state? Mod alpha square, this is exactly the variance of a coherent state that I am measuring, in this example because I am not putting any signal on the other arm - arm 1.

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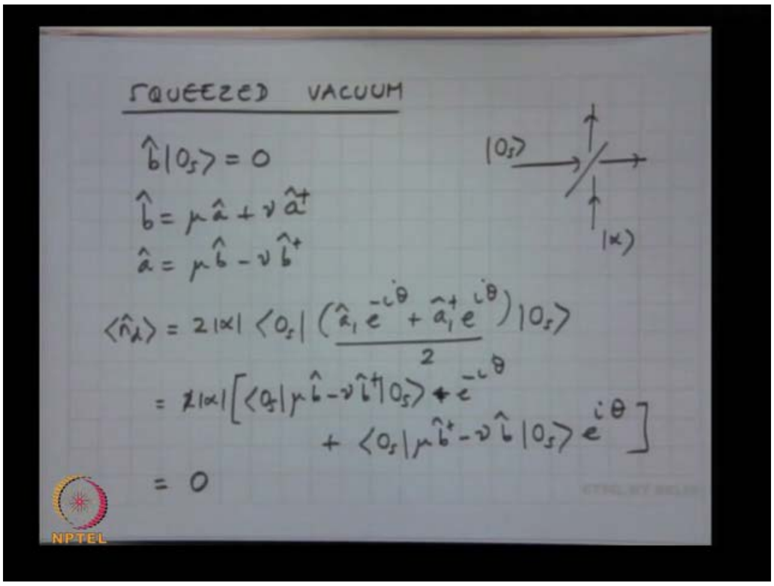

Now, let me go to a squeezed vacuum state.

Sir, coherent vacuum state would have no variation.

No, no variation meaning, only a coherent vacuum state.

Yeah, in the number of photons, but not in the electric field if you calculate the electric field there is variance, yeah. How do I define squeezed vacuum? Where b is defined as, it is the state which satisfy this equation where these on operator given by mu a plus nu a dagger, next class I will show you that the output from a parametric down converter - degenerate parametric down conversion process - with no input at omega generates this kind of a state. Now, from here you can actually calculate which we have done earlier, mu b minus nu b dagger, because we need expectation values of the various quantities.

So, first let us calculate, so what I am getting is, now this is squeezed vacuum state coming from here, and this is alpha state and I am measuring in 3 and 4, expectation value of n d 2 mod alpha 0 s a 1 e to the power minus i theta plus a 1 dagger e to the power i theta by 2. Anyway, let me cut this 2 off, I will have 0 s a 1 is mu b minus nu b dagger plus , sorry, into the e to the power minus i theta plus 0 s mu b dagger minus nu b 0 s e to the power i theta.

I am assuming mu and nu to be real just for simplicity, what is the value of this? b times 0 s is 0, b dagger times 0 s is 0, b dagger times 0 s is 0, b times 0 s is 0, so this is 0 - no signal. The difference signal gives me 0, this is squeezed vacuum please note we have discussed earlier, squeezed vacuum is not empty, there are photons in squeezed vacuum, because if you calculate the expectation value of a dagger a in the state it is finite. Now, I need to calculate expectation value of n d square and for which I would need, so expectation value of n d square for which I will need, expectation value of a 1 square expectation value of a 1 dagger square expectation value of a 1 dagger a 1.

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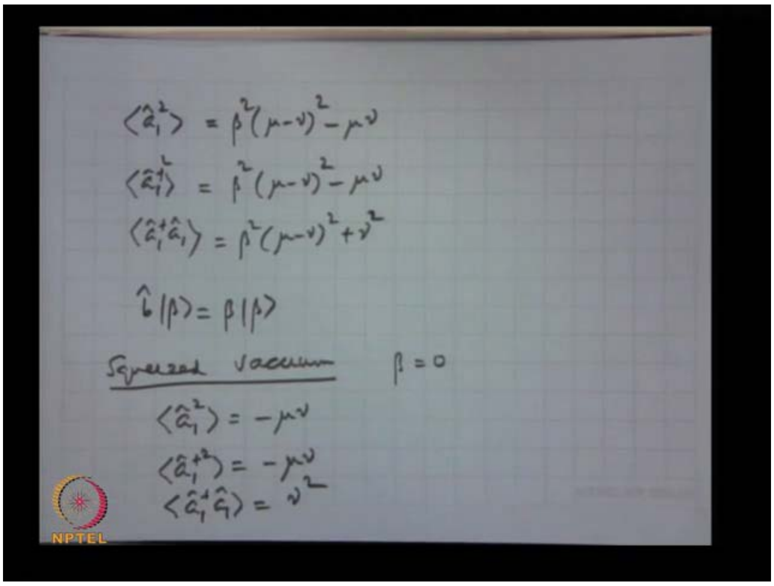

Now, we have done this before, so let me just recall and give you the expressions which we will use here to simplify, if I let me give you for which we have done before, for a general squeezed state, this is equal to beta square mu minus nu whole square minus mu nu this is equal to beta square into mu minus nu whole square minus mu nu, this is equal to beta square into mu minus nu whole square plus nu square, where b beta is the eigen value of this equation and for squeezed vacuum beta is 0.

So, for squeezed vacuum, we have derived these equations in an earlier class, for squeezed vacuum beta is equal to 0, so expectation value of a 1 square is minus mu nu, expectation value of a 1 dagger square is equal to minus mu nu, and expectation value of a 1 dagger a 1 is equal to nu square. These are the three quantities which I would need to calculate the expectation value of n d square. So, let us calculate now the expectation value of n d square, this is the expression.

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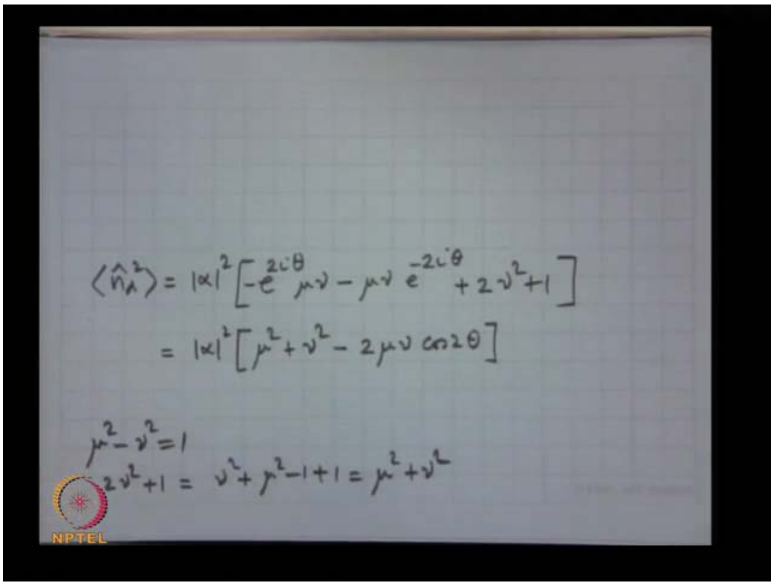

So, expectation value of this is expectation value of n d square, mod alpha square exponential 2 i theta, expectation value a 1 dagger square is minus mu nu we have just now calculated, a 1 square expectation value is also minus mu nu plus 2 nu square plus 1, A 1 dagger a 1 expectation value is nu square, a 1 square expectation value is minus mu nu, a 1 dagger square expectation value is minus mu nu, and this is equation. So, this is mod alpha square, now I can replace 1 of the nu square, what is the relation relationship between mu and nu?

Mu square minus nu square is equal to 1

So, nu square is 1 plus mu square. So, I will get.

Mu square minus nu square is equal to 1

Mu square minus nu square is equal to 1.

Mu square is nu square minus 1

Mu square minus 1, so this becomes mu square plus nu square, 2 nu square minus 1 is mu square plus nu square, mu square minus nu square is equal to 1, so 2 nu square plus 1 is equal to nu square plus mu square nu square plus mu square minus 1 plus 1.

Simple algebra, so this is mu square plus nu square minus 2 mu nu cos 2 theta, if I sum these 2 I get 2 times cos 2 theta. So, that is n d square expectation value of n d is 0, so, what is the variance in the signal that is coming out from the difference I will get delta n d square is equal to n d expectation value square n d square expectation value minus n d expectation value square which is equal to mod alpha square into mu square plus nu square minus.

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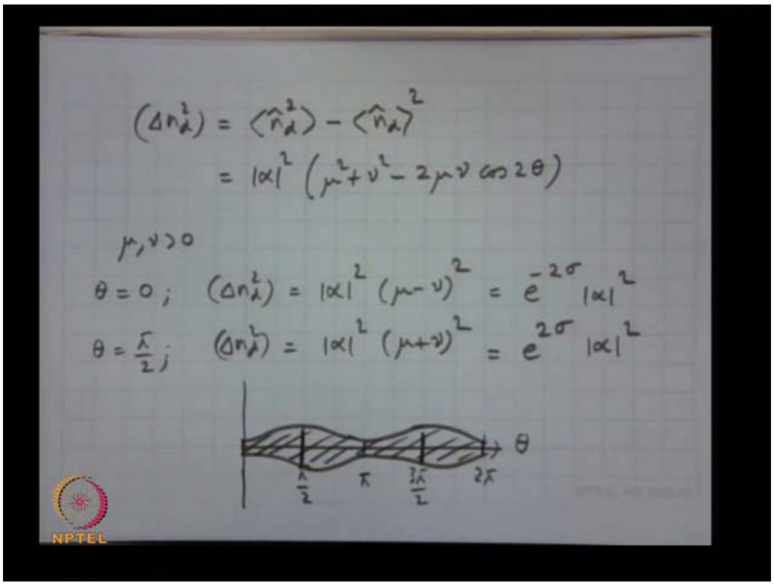

You saw similar a expression in squeezed state, so suppose let me assume that mu and nu are both positive, then if theta is equal to 0 delta n d square becomes mod alpha square into mu minus nu whole square. If theta is equal to pi by 2 delta n d square becomes mod alpha square into mu plus nu whole square and remember because mu square minus nu square is equal to 1 we had written mu as cos hyperbolic sigma, and nu as sin hyperbolic sigma, so this is actually, if I replace mu by cos hyperbolic sigma and nu by sin hyperbolic sigma I can replace, I can take care of an equation mu square minus nu square is equal to 1.

And I get this kind of a relationship exactly what you see here is what it implies is that the variance in the signal that should measuring will now depend on the value of theta that I choose. If I choose my theta to be 0, the variance is less than mod alpha square, if I choose theta is equal to pi by 0 the variance is more than mod alpha square, so if I were to plot the signal that I am getting as a function of phase.

The expectation value is 0, I just now showed in squeezed vacuum state expectation value of n d is 0. So, the average will be here, and then as a function of time, if I vary theta from 0 to 2 pi at 0 theta is equal to 0 the variance is so much, so let me call this as pi by 2, pi, 3 pi by 2, and 2 pi. So, at pi by 2, I have greater variance, at pi again a small variance, 3 pi by 2 again greater variance, 2 pi small variance, so far a similar figure I had shown from a research paper.

So, what it implies is that if you were to lock your local oscillator phase to be 0 theta is equal to 0; that means, phi is equal to minus pi by 2, because phi plus pi by 2 is theta, so by an appropriate choice of the local oscillator phase, I can reduce the noise in my detector signal, much below what I would have got, had I just put nothing in the beam splitter. Please note, if I put nothing here, and if I put only alpha here, this particular one has a variance of mod alpha square, because nothing is vacuum state - the coherent vacuum state. If I put in squeezed vacuum state here, then what I will see is, these 2 depending on the phase which I choose, will have a noise which is much mu mod alpha square depending on the mod of squeezing which depends on the value of sigma. If I have greater squeezing larger value of sigma, there is greater reduction in the noise that I can achieve.

So, let me tell you experimentally, people have achieved more than 10 decibels of noise reduction which is ten times.

What is the difference between a vacuum state and the squeezed vacuum state?

Sorry,

What is the difference between a normal vacuum state and a squeezed vacuum state?

The normal vacuum state satisfies a 0 is equal to 0, it has no photons in any of the modes, I am looking at a single mode, there is no photon occupied in that. This particular squeezed vacuum state satisfies this equation, where b is an operator given by this, this particular state has photons in it, and this state is generated as I will show you in the next class, when I take an optical parametric process and shine in only pump light.

If I take a crystal and I launch only frequency omega and if I satisfy the phase matching condition for omega to omega by 2 conversion classically I do not get any output, but I will show you quantum mechanically that even if you do not put anything, I will get an output which is the spontaneous parametric down conversion. And that light which is coming out actually is squeezed it is called squeezed vacuum, it not vacuum any more, there are photons in it, but it is a state, its special state satisfies satisfying this equation.

So, of course, you have got reduced noise in 1 phase as the cost of increased noise in the correspondingly opposite phase - different phase. So, depending on the value of nu and mu, and you can actually have squeezing at different times and this is squeezed vacuum, 1 can actually go into a squeezed state which is not a squeezed vacuum but a general squeezed state, where the expressions which I have given here for expectation value of a square a square a dagger square and a 1 dagger a 1 is given.

And if I do not put beta is equal to 0, I will still get I will get a figure like this. In fact, what I will be able to show you is, if you take a general squeezed state the expectation value will be not 0, but it will go like this, it will be a larger noise here, smaller noise here, larger noise here.

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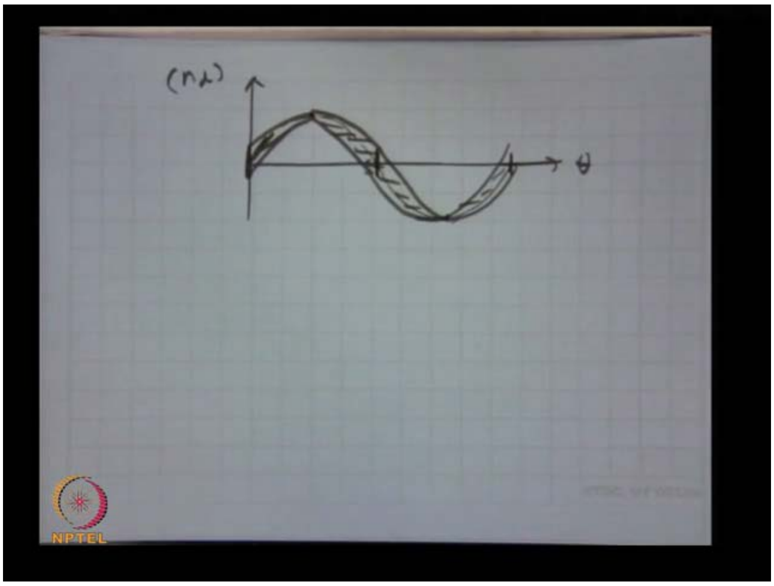

So, there is a finite signal instead of being 0 as in vacuum, it is a sinusoidally oscillating signal, but depending on the phase theta which I choose, I can actually detect much reduced noise. So, squeezed states are very interesting from this point of view and they do find applications in essentially, interferometry or for detecting various small changes of phase and so on and so forth, because there is much reduced noise from a simple laser beam that is coming out of a laser which is a coherent state.

The beam from a laser is a coherent state I can generate squeezed states by a parametric down conversion process, and that squeezed state has reduced noise in 1 quadrature compared to the others and that reduced noise I can use for signal processing application yeah.

Yes.

If a is consequently omega t minus k z.

No, this is only the phase of the alpha, omega t minus k z is a separate term, this is the eigen value of a, so that a which I am using there is the time independent part of the annihilation operator. If you go back and look at the discussion on squeezed state, sorry, in coherent state that we had, we had written a alpha is equal to alpha, alpha is without the time dependent part.

So, in the Heisenberg, where the operators are varying with time, but this particular one, this is the eigen value of a at t is equal to 0 for example, this is not the time dependent part. So, that is the fixed phase of the local oscillator that I can choose by varying my input. Sir, how can this reduced noise we used when discussed is to characterize the input rate from if we have a reduced either let us that is it theta is equal to pi by 2.

See for example, in this mach zehnder interferometer, I input only light in one of the arms, but the other arm there is vacuum - I put squeezed vacuum. Now, what is going to happen is, the interference is going to take place between your input state and the squeezed vacuum, and by using balanced homodyne detection at the output, I can use this reduced noise that is present in 1 quadrature of the squeezed vacuum which is coming at the input of the mach zehnder.

So, I can have much better sensitivity of phase changes detection, suppose, I have in one of the arms of the mach zehnder I have a device which whose phase is changing with with some parameter, I can detect that phase change much more accurately by using squeezed vacuum in the input of the.

At a particular phase of the input coherent state.

Yes, I have to choose parameter, so that I detect light corresponding to the phase with reduced noise.