Quantum Electronics

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Module No. # 01
Lecture No. # 04
Brief review of electromagnetic waves;
Light propagation through anisotropic media
Anisotropic Media (Contd.)

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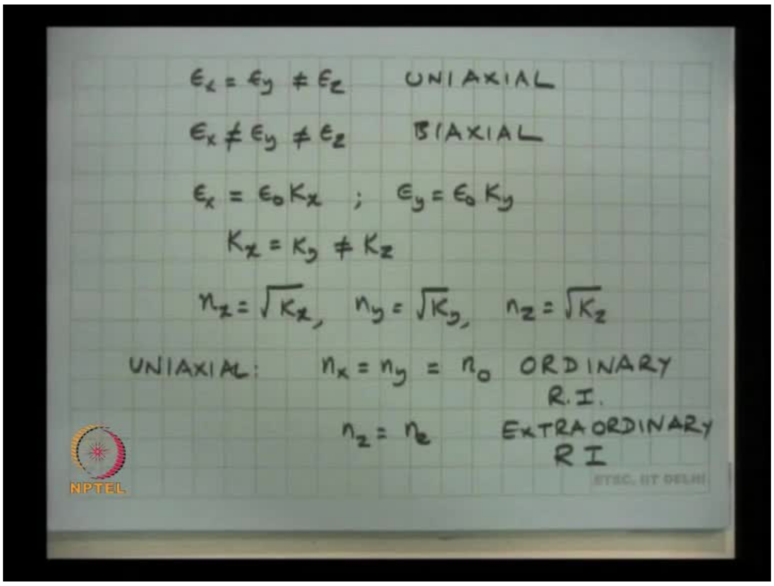

We continue with the discussion on anisotropic media. Let us discuss what we have been looking at; so, we are looking at plane wave solutions in anisotropic media and we found that there are uniaxial media, for which epsilon x is equal to epsilon y is not equal to epsilon z; this is uniaxial; and there are biaxial media, in which epsilon x is not equal to epsilon y is not equal to epsilon z; and if all the three are equal, then it is isotropic.

Now, in uniaxial, epsilon x is equal to epsilon zero K x and because epsilon y is defined as epsilon 0 K y. In uniaxial media, K x is equal to K y is not equal to K z and because the dielectric constant is quite of refractive index. We define the three principle refractive index is as we have defined earlier, n x is equal to square root of K x; n y is equal to square root of K y; and n z is equal to square root of K z, this is general; and so, for uniaxial media, n x is equal to n y, and this is called n o; this is called the ordinary refractive index; and n z is called n e - the extraordinary refractive index.

In general, for biaxial media, n x, n y, n z are the three principle refractive indices. For uniaxial media, n x is equal to n y, it is called n o the ordinary refractive index; and n z which is the third refractive index, is called n e - the extraordinary refractive index.

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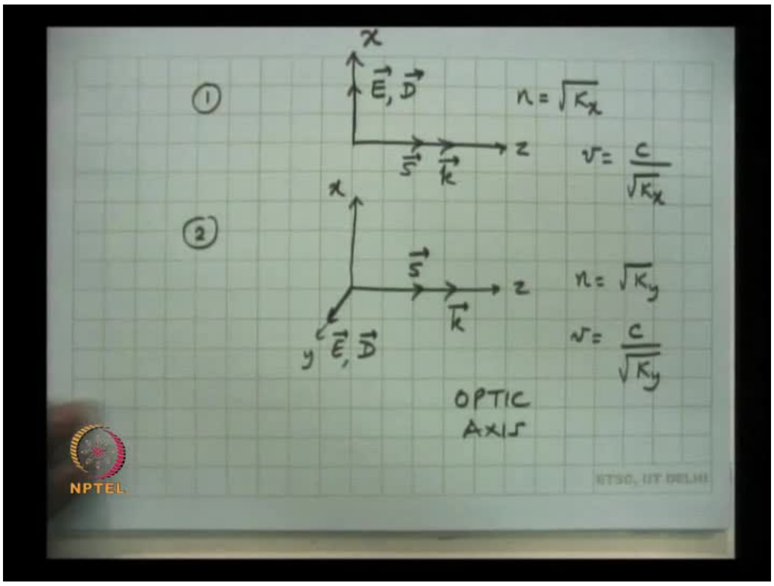

Now, what we did was, we looked at propagation of plane waves and we found an Eigen value equation, which contained e vector and the refractive indices; so, for a given propagation direction, which means kappa cap direction, I have to solve those three equations to get the values of n; I will find two values of n, which has called the two principle refractive indices - two Eigen refractive indices - and correspondent to those refractive indices, I will find two electric vectors, two displacement vectors which will correspond to that waves; so, these are two modes, Eigen modes of propagation.

So, for every direction of propagation, we would have two Eigen modes; the D vectors of those two Eigen modes will be a perpendicular to the k vector of the propagation and the corresponding D vectors will give me the corresponding e vectors and everything all vectors will be defined.

So, for every propagation direction, which means for every kappa cap direction or for every k vector direction, I will find two Eigen solutions corresponding to two refractive indices and two polarization states; and both polarization states are linear.

So, as an example, first when we looked at is, if I propagate along the z axis; now, let us also recall that the x, y, z axis, we are looking at is the principle axis system in which the epsilon tensor is diagonal; half diagonal elements are 0 in this coordinate system.

So, if my propagation direction is k along this z direction; we got two solutions, the first solution with an electric vector pointing along x, the corresponding displacement vector also pointing along x, because if E is along the principle axis, the D vector is also along the principle axis and because E is now perpendicular to k S, is also perpendicular to D and S and k are parallel.

So, the energy propagation direction and the wave propagation directions are superposed; they are the same; E and D are parallel to each other; and the refractive index of this wave was square root of K x; this is one solution. The second solution, we obtained is propagation along z direction again, so if I call this y, E vector is along y, D vector is along y, and D vector is also parallel to k now with the refractive index, which is square root of K y.

This is one Eigen mode; this is another Eigen mode; this Eigen mode propagates with a refractive index n is equal to square root of K x; this Eigen mode propagates with the refractive index n is equal to square root of K y; so, the phase velocity of this wave will be square root of C by square root of K x; and phase velocity of this wave will be C by square root of K y. Now, note that in uniaxial media, K x is equal to K y and so both the velocities are equal.

So, this is the special direction in uniaxial media in which the two Eigen modes have the same velocity of propagation and this is called the optic axis. If you have a biaxial medium, K x is not equal to K y and so the velocities of the two waves are different. Now Mohith, you had a question.

No sir.

So, in these two modes, the S vector and k vector are parallel to each other and E and D are parallel to each other and I have 1x pole rise mode, so the propagation is like this; 1x pole rise mode travels with velocity C by square root of K x; the y pole rise mode travels with the velocity C by square root of K y; and so, these are two Eigen modes.

If you consider any other polarization state propagating along the z direction, I would have to break it up into the x Eigen mode and the y Eigen mode; the x component will propagate with this velocity C by square root of K x; the y component will propagate with a velocity C by square root of K y; and if the velocities are unequal, then they will have an addition of phase difference and so the polarization state will change as it propagates.

But in uniaxial media, because K x is equal to K y, the two velocities are equal and the propagation will not change the polarization state and that is the special direction called the optic axis; and in uniaxial media, there is only one such direction.

If K x is not equal to K y is not equal to K z, that means, in biaxial media there will be two directions where this will happen; the two Eigen modes will have the same speed, they are not coincident with x y or z, it is some other direction; there are two other directions in which the velocities of the two Eigen modes will become equal, but we will not look into more details of biaxial media, but we will concentrate more on uniaxial media.

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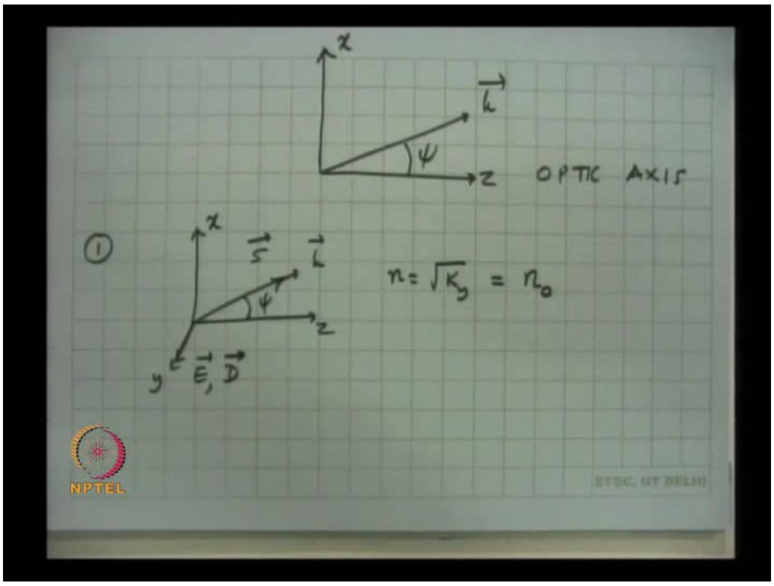

The second case, we looked at is, when you have propagation in the x z plane; so, the k vector makes an angle psi with z axis; so, in uniaxial, it is making an angle psi with the optic axis z axis - is the optic axis; this is also the optic axis.

So, it makes an angle psi with the optic axis. We found two solutions. The first solution was, so let me draw the figure here - again here; so, this is z; this is k vector; psi; this is x; one polarization had a E vector like this along the y direction; D vector also parallel to the y direction because when E is along principle axis; D is parallel to E and then S vector is also like this; and the refractive index seen by this wave is square root of K y; and in uniaxial media, it is n o; and this refractive index is independent of psi.

So, this wave propagates as if it is propagating in an isotropic medium in which the refractive index or the velocity is independent of the direction of propagation. In an isotropic medium, no matter what direction I propagate, the refractive index is the same, the velocity is the same.

So, this particular polarization propagates with the same velocity C by n o irrespective of the angle psi that you make with the optic axis. This is one particular polarization. Note that, this polarization is perpendicular to the k vector and to the optic axis; so if I restrict myself to uniaxial media, this is the optic axis, this is the propagation direction, the propagation direction is in the x z plane; this polarization, which means the D vector of this wave is perpendicular to the k vector and z vector and the z direction which means the optic axis.

So, the ordinary wave or ordinary polarization, this is called the ordinary polarization, is perpendicular to the optic axis and to the propagation direction represented by the k vector; the only angle I need to specify is psi, because epsilon x is equal to epsilon y, there is complete symmetry in x and y directions.

So, no matter what orientation I propagate as long as I make an angle psi with the optic axis. Suppose, my optic axis is like this; if I propagate at 30 degrees to the optic axis here or 30 degrees here or 30 degrees here, the refractive index will be always n o for this particular wave, which is called the ordinary wave.

And so, suppose, my optic axis is here and propagation is here, this polarization has to be perpendicular to the optic axis and k vector; and this will be like this; so, this will propagate like this; this polarization and the k vector and S vector of this wave are parallel to each other.

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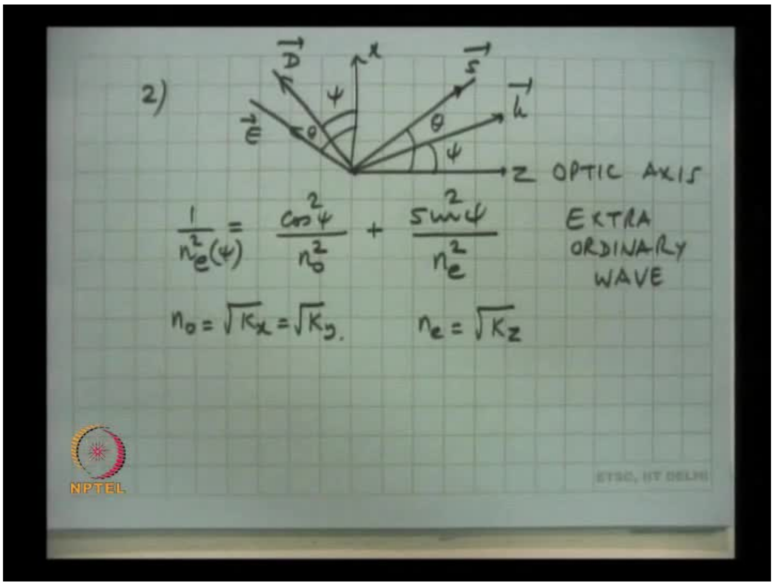

The behavior is very similar to an isotropic medium and hence the name ordinary refractive index. The other solution that we obtained was, this is z axis, this is x axis, this is k vector again; note that no matter what direction I propagate, no matter what medium it is, D is always perpendicular to k vector.

So, we found the second solution to have its D vector in this plane; so, this is psi, we found this to be psi and so this is S vector, this is the E vector; so if this is theta, this is also theta, because the D vector is not along the principle axis; E vector and D vector are not parallel to each other and hence the S the pointing vector and the propagation vector k are not parallel to each other.

The velocity of this wave satisfies this equation. So let me, put a subscript here, just to I will explain this n e square of psi is equal to cos square psi, remember you had K x, which is n o square plus sin square psi by K z, which is n e square.

Please note, n o is by definition square root of K x is equal to square root of K y; and n e by definition is square roots of K z. These are constants of the medium, they do not depend on the propagation direction. If I give you a medium like Lithium Niobate, which is a very important uniaxial crystal, I can give you the value of n o and n e for a particular frequency - for a particular wavelength.

This n e has an n e of psi; this e stands for, subscript e is for extraordinary wave; this solution corresponds to an extraordinary wave and the velocity of this extraordinary wave depends on psi through this equation; for this wave the S vector and k vector are not parallel and hence the name extraordinary it is not like a isotropic medium, it behaves in a different fashion and so it is called an extraordinary wave; and n e of psi - please differentiate n e of psi and n e - n e is a constant, n e of psi when I write it means the refractive index as seen by the extraordinary polarization, when it propagates with its k vector making an angle psi with the optic axis; it is the optic axis k vector makes an angle psi with the optic axis.

The first solution has its polarization along y and has the refractive index n o independent of psi called the ordinary wave; the second solution has a refractive index depending on the angle psi according to this formula and it is called an extraordinary wave and has its S vector and k vector not parallel to each other.

So, this equation gives me the refractive index as seen by the extraordinary wave which is now a function of psi; and as you can see from here, if psi is equal to 0, then the propagation is along the optic axis and n e of psi becomes n o and that is the definition; the optic axis, this direction when the refractive index seen by the ordinary wave and the extraordinary wave become equal if psi is 90 degrees, then I propagate perpendicular to the optic axis, then n e of psi becomes n e.

So, n e is the refractive index of the extraordinary wave if it propagates perpendicular to the optic axis; so, if I propagate perpendicular optic axis along x, one polarization will be y, which propagates with the refractive index n o; the other polarization is z propagating with the refractive index n e.

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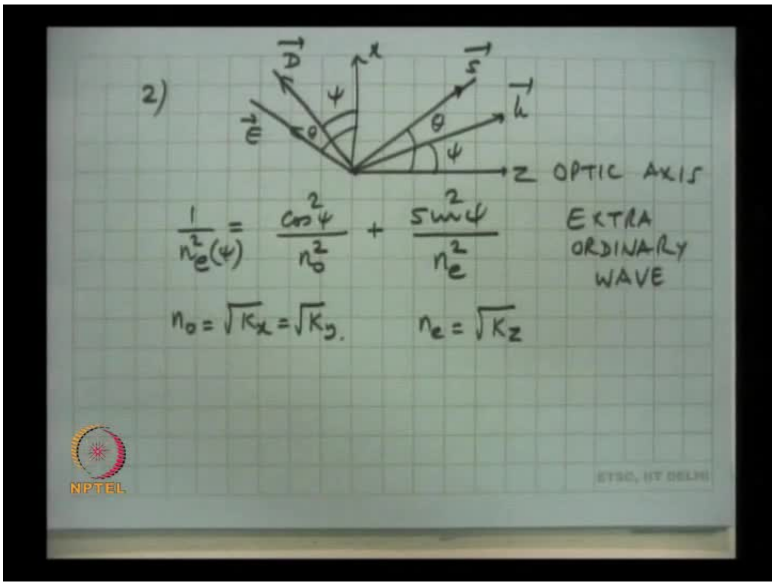

So, note that, the maximum difference of velocities between ordinary and extraordinary waves will appear when you propagate perpendicular to the optic axis; at any other direction the value of this quantity will be between n o and n e; so, maximum difference, if you want a maximum difference between propagation speeds between the ordinary and extraordinary waves, you need to propagate perpendicular to the optic axis for a uniaxial medium.

So, I leave it to you to find out what direction should I propagate. Suppose, I give you n x, n y, and n z values say n x is less than n y is less than n z; so, let me assume then what direction should I propagate to maximize the difference in speeds between the two polarization states.

It is y direction, because n x and n z, as I have defined n x is less than n y less than n z, so when you propagate along y, one will be n x, the other will be n z and that is the maximum difference that you can get in the medium.

Also note that, for the ordinary wave, S and k are parallel; but for the extraordinary wave, in general, if you do not propagate along the optic axis or perpendicular to the optic axis, then S and k are not parallel; so as I showed you on the first day, you may have a wave front looking like this, the wave front is along this k vector direction, but the energy propagation direction is oblique.

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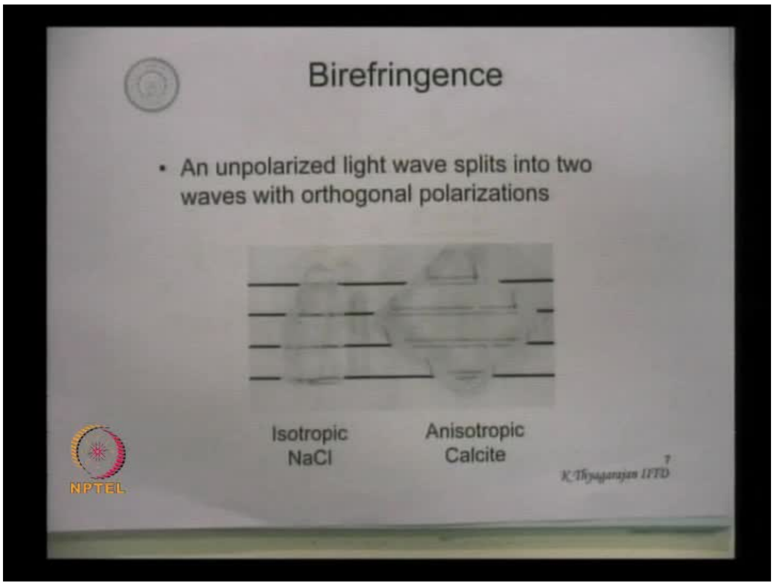

So, the wave front is looking in this direction, but it is propagating like this, energy is propagating obliquely and this is the reason for what is called as birefringence; in certain crystals, if you can use the certain crystals to see for example, double images; so, this is a normal isotropic crystal - sodium chloride - and that is an anisotropic crystal calcite.

So, you can see two lines, two images coming through this, because light is un-polarized, it comes out through the crystal; one of these lines corresponds to the ordinary wave; and the other one corresponds to extraordinary wave; the refractive indices of the crystal are different for the ordinary and extraordinary waves.

So, the refraction inside the crystal and refraction outside the crystal is different giving you two images and that is called birefringence, because there are two refractive indices. And this particular property is used in constructing many polarization devices like half wave plates, quarter wave plates, and so on.

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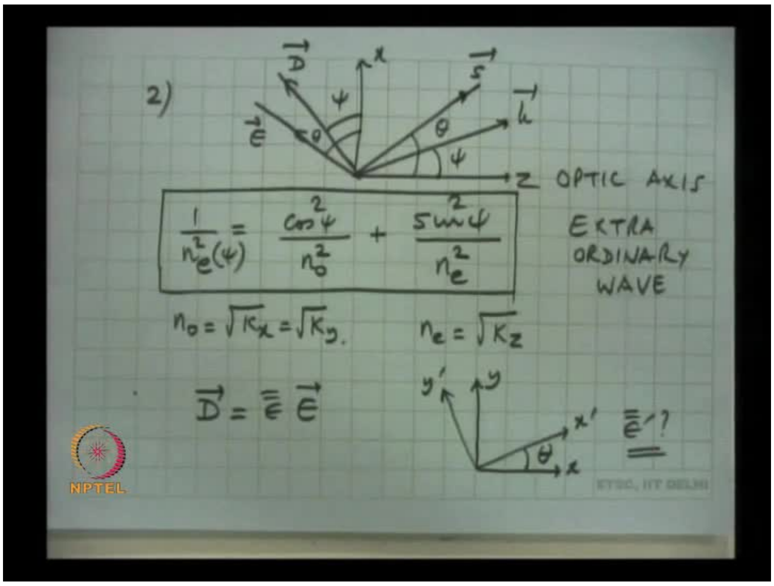

Now, I want to just give you one problem; and that is, suppose, I have this equation, please remember we have written this equation D is equal to epsilon E and epsilon is assumed to be in the principle axis system. Now what will happen if I rotate my coordinate system and go to another system.

So, let me give you a simple problem. So, let me look at x y coordinates; so, this is x and y, these are principle axis system; suppose, I rotate the coordinate system around the z axis and go to another coordinate system x prime y prime, suppose this is theta, what is epsilon in this coordinate system? Let me call it as epsilon prime.

How does the epsilon matrix transform itself when I change my coordinate system and I want you to show that, if the medium is uniaxial, which means if epsilon x is equal to epsilon y, the epsilon matrix will remain the same; in general, if epsilon x is not equal to epsilon y is not equal to epsilon z, then this rotation changes the epsilon matrix, it may no more remain diagonal; but if it is a uniaxial medium when epsilon x is equal to epsilon y, the epsilon in the x y z coordinate system, and the epsilon in the x prime y prime z coordinate system are the same; and that is why I said, this equation is the most general equation for extraordinary waves in a uniaxial medium, because as I told you, if you give me a principle axis system and if you say I propagate like this, suppose this is z axis x y and z; So, if you say I propagate like this, I will rotate my coordinate system, such that, my k vector lies in the x z plane and epsilon in this rotated coordinate system and the original system are the same, this is what I have asked you to show.

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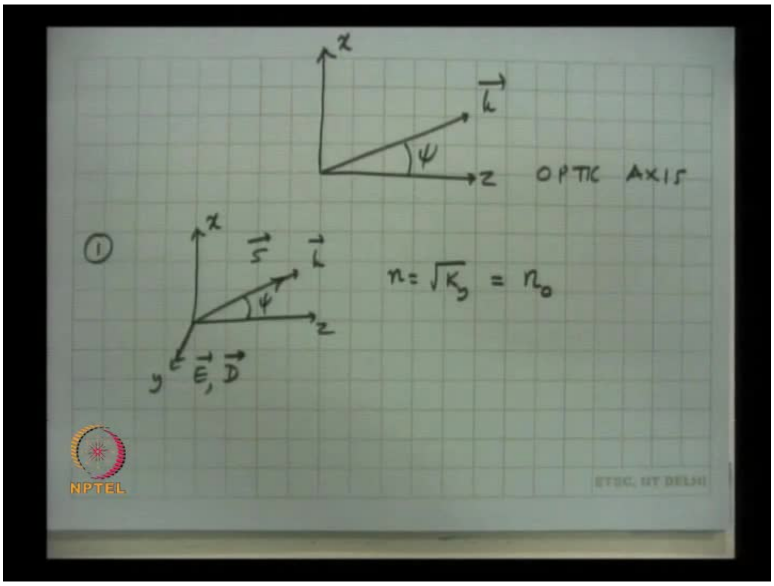

So, all that is important in a uniaxial medium, is what is the angle made by the k vector with the optic axis; so, the conclusion of all this analysis is simply the fact that, if I take a uniaxial medium with optic axis like this and propagation is some arbitrary direction, I have two Eigen modes, the first Eigen mode called the ordinary wave has its D vector perpendicular to the optic axis and k vector; look at this solution, this is the ordinary wave, the E vector and D vector of this wave are perpendicular to the optic axis and to the propagation direction k vector.

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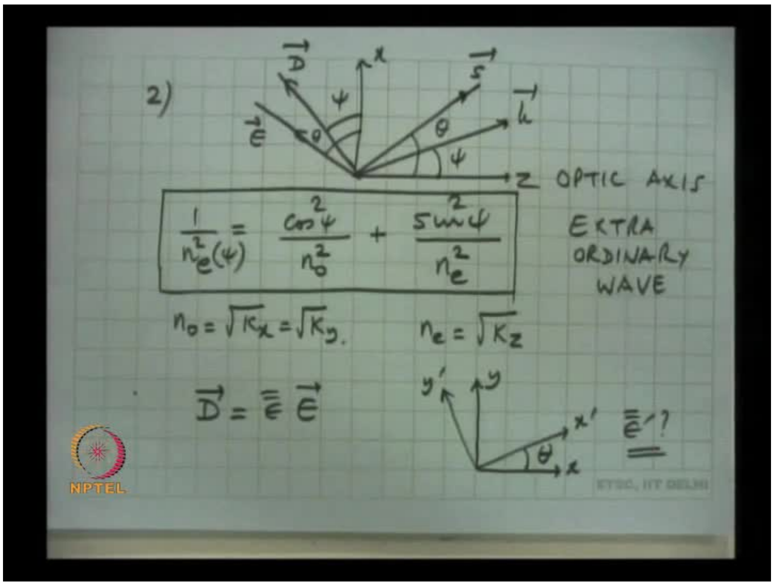

The refractive index as seen by this is always n o, independent of psi. The second solution is an extraordinary wave with its D vector lying in the plane containing the optic axis and the k vector and refractive index as seen by this wave satisfies this equation.

So, if I give you medium and I tell you n o is so much, n e so much and if I say I propagate at 30 degrees to the optic axis, you can find out what is the velocity of this wave; so, please just show this, because these you can derive this from the transformation properties of the vectors - D and E vector; you know how a vector transform on rotation, so using this transformation, you can find out the transformation of epsilon tensile.

Also please note that, these refractive indices are functions of frequency, which is called dispersion; the ordinary index changes with frequency; the extraordinary index itself changes with frequency; so, how does the refractive index vary? If I increase the wavelength, will the refractive index decrease or increase, normally decrease, as you increase the wavelength, the refractive index decreases, which is called normal dispersion.

So, n o and n e, both of them will have dispersion, both of them will have a frequency dependent values; so, this needs to be kept in mind, because when we look at non-linear interactions, we will look at different frequencies and the n o and n e values are themselves functions of frequency.

Here, please note that, there is an angle between k and S vector - let me call this alpha; so, alpha is theta minus psi; now, under what conditions will alpha be 0; for what values of psi is alpha 0 - 0 and pi by 2; if psi is 0, I propagate along the optic axis, k and S are parallel; if psi is 90 degrees, k and S are again parallel; and in between alpha is finite, so there must be a value of psi at which alpha must be maximum; so, what I would like you to do is, please find out the angle alpha; the information you have is, for a given psi, D vector makes an angle psi with the x axis; if k vector makes an angle psi with this z axis, D vector makes the same angle psi with x axis; the D vector and E vector are related through D is equal to epsilon E.

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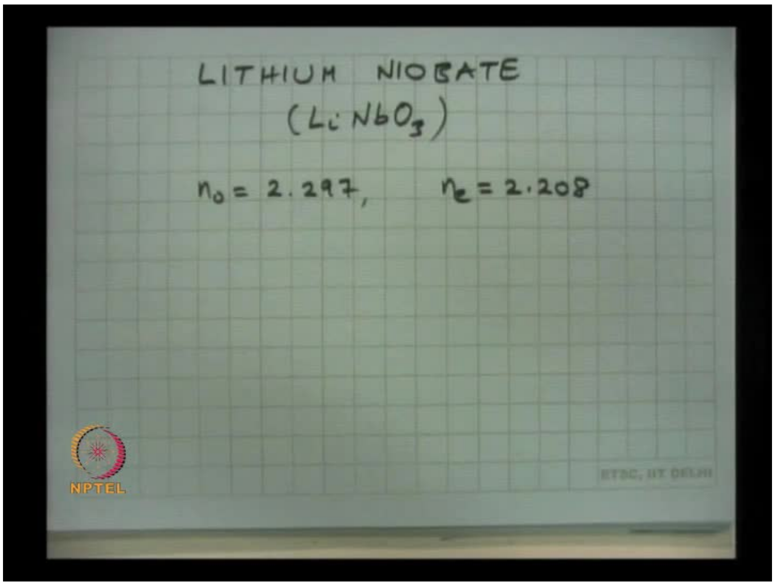

So, you can find the direction E vector. If you know the direction of E vector, you know the direction of S vector, because S vector is perpendicular to E vector; so, you can find out theta, so you know theta and psi, you know alpha, so what I would like to do you - to do - is to find out an estimated value of alpha, so just for us to know what these numbers are. So, let me give you a specific example. Let me take Lithium Niobate as an example. Lithium - this is LiNbO 3; this is a very important medium in optics just like silicon is in electronics; this has a very high non-linear coefficient; it has very high acoustic-optic coefficients; it has a very high piezoelectric coefficient; it is a very interesting crystal very nice crystal and its transparent somewhere around from 350 nanometers to about 4 microns or so - so - huge transparency band; and its used in many applications.

So, let me give you at some specific wavelength. The ordinary index is given by 2.297, and extraordinary index is equal to 2.208. So, in uniaxial crystals, you can have n o more than n e or n o less than n e; if n o is less than n e, the ordinary wave will travel faster than the extraordinary wave always; the extraordinary wave refractive index varies between n o and n e; the ordinary refractive index, ordinary wave always has the refractive index n o.

So, if the crystal has n o less than n e, the ordinary wave is always traveling faster than the extraordinary wave and such a crystal is called a positive uniaxial crystal; just by convention, positive uniaxial means n o less than n e with the ordinary wave travelling faster than the extraordinary wave.

If n o is more than n e, the extraordinary wave travels faster than the ordinary wave and a crystal is called negative uniaxial crystal. So, this is an example of a negative uniaxial crystal. And you can see the difference, the difference is not very large, it is about 0.09; it is a strongly anisotropic crystal and this difference will lead you to a difference between k and S vector.

So, what I would like you to do is, say take psi around 45 degrees and try to find out what kind of values of alpha you will get ,is it 10’s of degrees, or is it a fraction of degree, or what numbers are these; and why do not you try to find out what is the value of psi at which alpha will be maximum; there must be some value of psi, some propagation direction k vector where I will get maximum difference between the S vector and k vector, that means, maximum difference between the propagation direction of the wave front and the energy.

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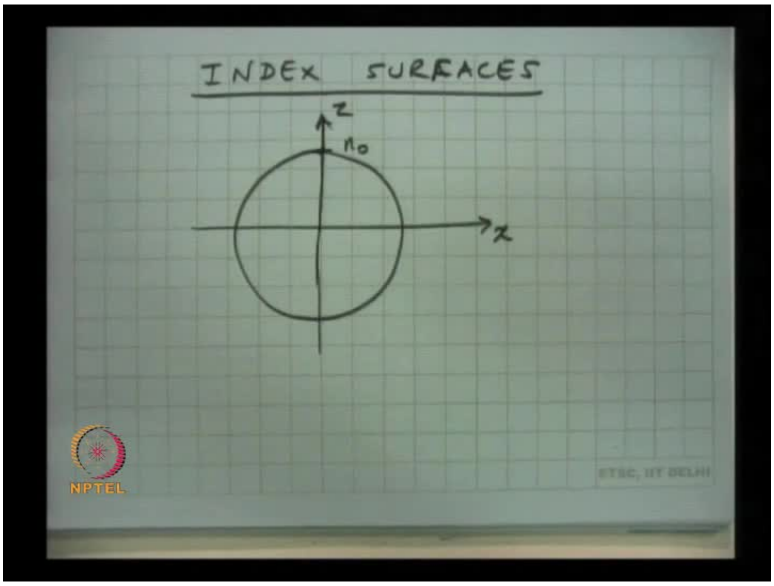

Now, what I want to introduce is a another very important concept before we go to the non-linear effects and this is called index surfaces. In anisotropic media, actually, if you go through some books, you will find a lot of surfaces, there are ray surfaces, there are index surfaces, there is optical index ellipsoid, etcetera, a lot of surfaces; so, when you look at a surface in a book, you need to be careful to find out what is it representing. Now, this is called an index surface; it is a plot of refractive index variation of the ordinary wave and the extraordinary wave as a fraction of angle.

So, if I plot a figure, so for example, this is the figure which I want to plot, this is z axis, this is x axis, remember psi is the angle made by the k vector with the optic axis; and here, this is the optic axis, I am looking at uniaxial media, so this is the optic axis; so, I am going to plot how the refractive index of the ordinary wave changes as I change psi, so I will take, that means, the distance from here will represent the refractive index of the wave as seen in that direction.

So, suppose, I propagate like this in this direction what will be the refractive index seen by the ordinary wave n o? So, I have to plot a point here n o, what about different other directions? Its always n o, the ordinary wave always in - is the - same refractive index n o, no matter what angle I propagate.

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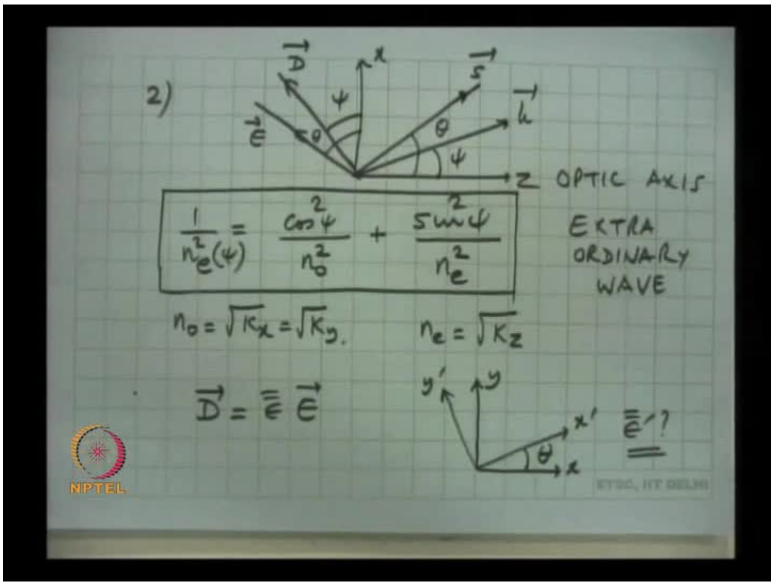

So, what is the surface I will get, what is the curve I will get, I will get a circle; this circle will have a radius of n o; all what it means is, if you propagate in different directions, the distance from the center to this curve is the refractive index seen by that wave; so, this is the ordinary index surface; I must plot another one for extraordinary index surface and remember the extraordinary refractive index has a formula like this as a fraction of angle.

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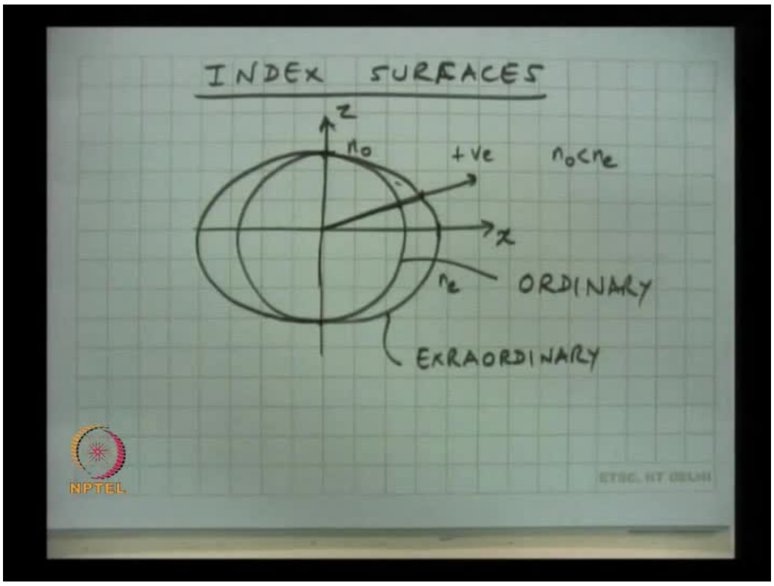

What is this equation? It is equation of an ellipse; n e of psi versus psi is equation of an ellipse with two - major - semi major and semi minor axis n o and n e; I will leave it you to show that, this the equation of an ellipse and so I will get an ellipse. Now, the question is, how will the ellipse be sitting with respect to the circle?

Will the ellipse touch this circle at any point?

Yes sir.

Where?

z axis.

On the z axis. So here and here, the ellipse will touch, because n e of psi will become n o, if psi is equal to 0; will the ellipse lie inside the circle or outside the circle? Depends up on n o and n e. So, suppose, I take a positive uniaxial crystal, so positive means, let me plot for positive, positive means, n o less than n e; so, the ellipse will be lying outside.

So, it will be like this; this distance is n e; so, this is the extraordinary - this corresponds to the extraordinary wave; this is the ordinary wave.

Because of symmetry in the x and y directions, the index surface can be obtained by revolving this curve around the z axis; so, you will get a sphere for the index surface of the ordinary wave; you will get an ellipsoidal revolution for the index surface of the extraordinary wave; these are the intersections of these two surfaces with this x plane and z plane.

In uniaxial media, because epsilon x is equal to epsilon y, you can obtain the index surfaces by revolving this curve around the z axis; so you will get a sphere for the ordinary wave index surface and you will get a an ellipsoidal revolution for the extraordinary index surface.

So, all this means is, if you propagate in this direction, the refractive index as seen by the ordinary wave this distance, the refractive index as seen with extraordinary wave is this distance, that is all; it means, it is just a figurative representation of the two equations for the ordinary surface and the extraordinary indexes. Please note that, these numbers are frequency dependent, n o and n e are frequency dependent, so if I plot the index surfaces at different wave lengths, they will have different sizes the radii of the circle and the ellipse will be different at different frequencies; and we will use this representation to understand certain features in the non-linear interactions.

So, a sort of complete our discussion on anisotropic media because we will primarily be using the fact that, I have two refractive indices that are available to me in an anisotropic medium.

One of the refractive indices is controlled by this angle psi, because for non-linear interactions I will see that, I need to satisfy some conditions for efficient generation of new frequencies and that will need some kind of a dependence of refractive index - some more refractive index values - and I can use this n e depending on psi to achieve this efficient interaction process; and we will use this index surfaces plots to understand some of those characteristics.

Do you have any questions in anisotropic media; otherwise, I will introduce the concept on non-linear polarization and look at some non-linearity. So, let me start looking at how do I introduce this non-linear optics essentially.

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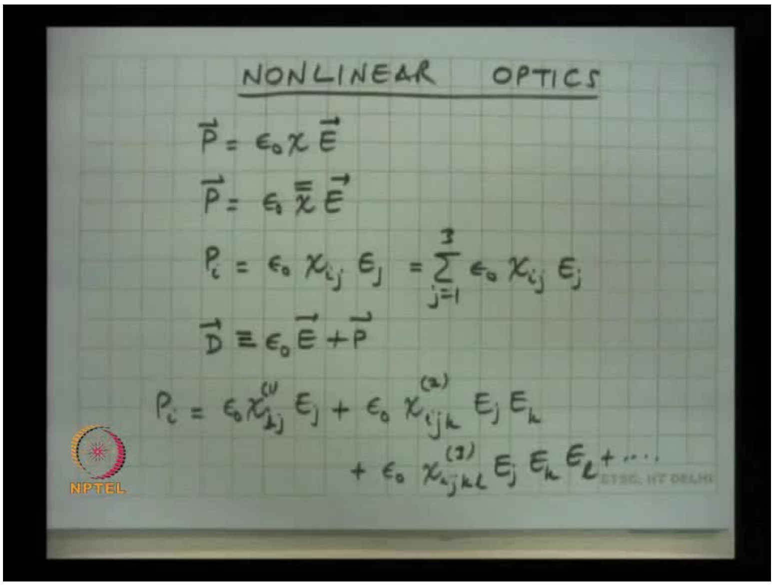

So, remember, we wrote this equation P is equal to epsilon 0 chi E; this is for a isotropic medium, if chi is a scalar; if chi is a tensor in anisotropic medium, the same equation will be P is equal to epsilon 0 chi tensor into E; chi would have…; in the principle axis system, there are three diagonal elements, which may not be equal to each other; for uniaxial, two of the first two elements will be equal; the 1 1 and the 2 2 elements will be equal; in a biaxial all three elements will be unequal.

In component form how do I write this equation, I can write this as P i is equal to epsilon 0 chi i j E j; then I am assuming summation over repeated indices, because j is repeated on this side, this is a short form of this equation this epsilon 0, j is equal to 1 to 3.

Now, this is an equation representing the medium. Please remember, this equation when I write D is equal to epsilon 0 E plus P is the definition the displacement vector; there is no approximation in this; this is an approximation; this is relating how the medium responds to an applied electric field; and this approximation is a linear approximation, I have written terms up to E.

Now, if you go back and look at how do I derive a microscopic theory or expression for chi; you represent the atomic dipoles as thermionic oscillators and obtain this expression under the approximation that the displacement of the dipole is very small and the oscillation is harmonic; this is true as long as the electric fields are not very large; when the electric field becomes large comparable to the interatomic electric fields in the medium, then this approximation fails; the oscillators are no more harmonic; so, the restoring force is no more proportional to displacement; and I need to modify this equation and that is where the concept of nonlinearity comes in.

So, instead of this equation, what I will have to write is, for example, so let me write here P i is equal to epsilon 0 chi i j E j and I will have additional terms like, for example, epsilon 0 chi i j k, so let me write a 2 here to represent it is a second order term, E j E k plus epsilon 0 chi i j k l - this is third order term - E j E k E l plus etcetera.

Suppose, we have 1 here which is not written, if I do not write 1, it is a linear susceptibility; this is the linear susceptibility; this is called the second order susceptibility, because P is proportional to E square; this corresponds to third order susceptibility, P is proportional to E cube and so on.

So, I must replace this equation, which is a linear equation by this once the electric fields inside the medium becomes strong; so, if I take a light wave with a sufficiently high intensity value, the electric fields of that light will be strong and I will start to see effects of these terms in my analysis.

So, this term is proportional to E square, it leads to second order effects; this term is proportional to E cube, it leads to third order effects; second order effects include things like second harmonic generation, which means, I launch a light of frequency omega inside a medium and this term can lead to the generation of light of frequency 2 omega second harmonic.

If I launch 2 waves of frequencies omega 1 and omega 2 inside a medium, because of this term I can generate omega 1 plus omega 2 and omega 1 minus omega 2 frequencies; so, I can generate some frequency, I can generate different frequencies; and this terms so leads to all kind of second order effects in which two frequency can get mixed up.

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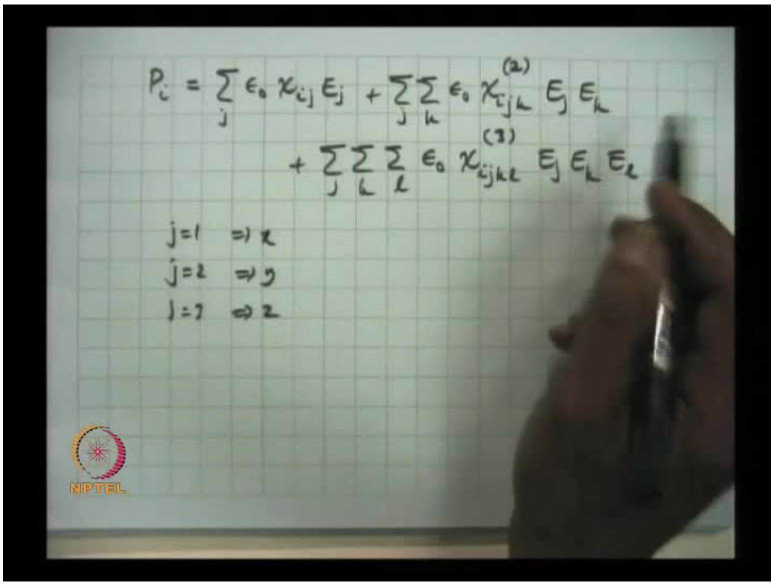

Remember, in linear case, if E has a frequency omega, P will also have a frequency omega only, it cannot have a new frequency; but because of this term, because of this product here, if E has the frequency omega, this E square can have a frequency 2 omega and that leads to the generation of second harmonic some frequency etcetera; so, these are all called the second order effects; this leads to the third order effects.

Yes

What is…

This is the kth component of E vector, where j k l go from 1 2 3; so, this is actually shortened form of this equation; so, P i the ith component of the polarization is, sigma epsilon 0 chi i j E j sum over j plus sum over j sum over k epsilon 0 chi i j k 2 E j E k plus sum over j sum over k sum over l epsilon 0 chi i j k l 3 E j E k E l; so this could be E 1 E 2 E 3, which is E x E y E z; so, j is equal to 1 is x; j is equal to 2 is y; and j is equal to 3 is z; so, this will have terms like E x E y E z, E x E x E z, E x E x E y - all kinds of combinations.

So, the point is, what is happening is, the linear expression told me that P only depends on E linearly, but when they applied electric fields becomes strong then I can have additional terms in this expansion; that means, P i can depend on product of two electric fields E j E k.

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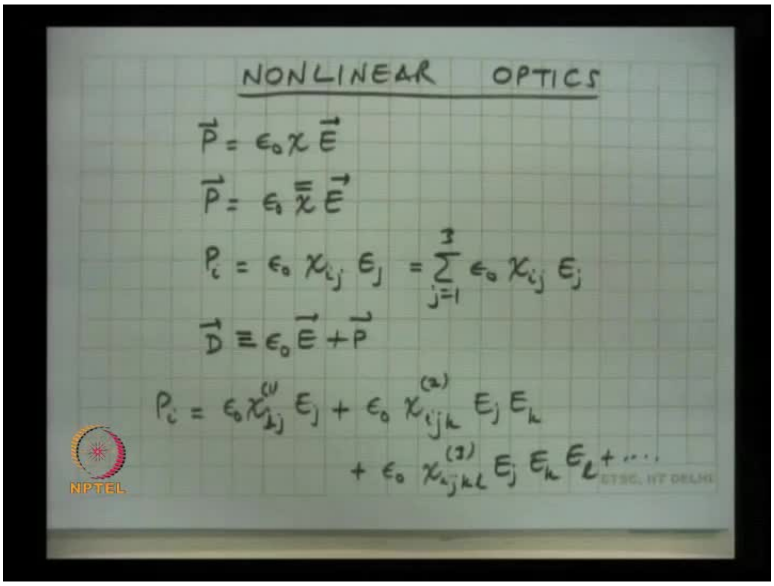

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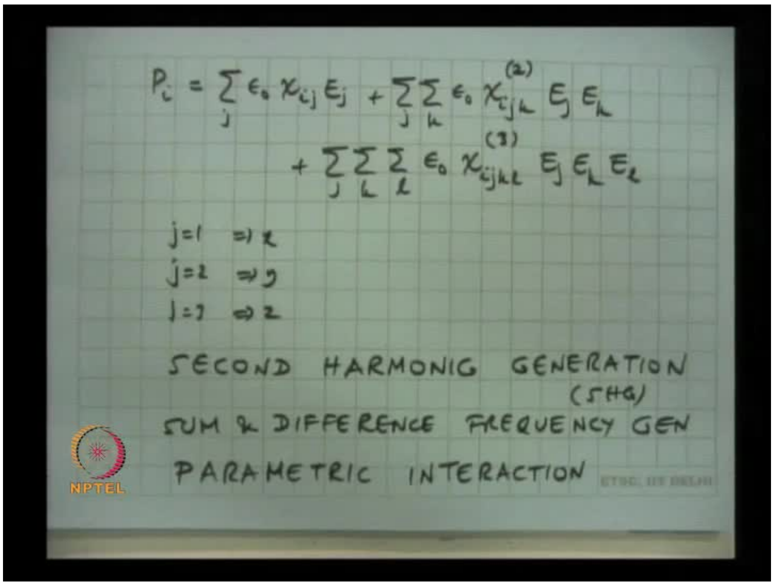

Similarly, with the third order effects, I can have products of three electric fields, this lead to what are called the second order effects; so, this is what we will normally write; we will assume repeated indices are summed over, so j is summed over k is summed over l is summed over from 1 to 3; j is equal to 1 is x; j is equal to 2 is y; j is equal to 3 is z; similarly, k is equal to 1 to 3, l is equal to 1 to 3; so, this is called the second order susceptibility, third order susceptibility. The third order terms…

Can we observe these non-linear effects in isotropic medium? It will depend on this matrix. Now, let me tell you, I will show you later on this particular, how many elements are there in chi i j k?

27

3 into 3 into 3; I can go from 1 to 3; j can go from 1 to 3; k can go from 1 to 3; so 27 elements. If your medium possesses a centre of inversion symmetry, all chi i j k 2 elements are 0. Media possessing centre of inversion symmetry, what is the meaning, I position myself at a point I, replace all atoms with coordinates x y z by minus x minus y minus z, so I take this atom put it here, take this atom put it there, take this atom put it here.

I replace all atoms with coordinates x naught y naught z naught by minus x naught minus y naught minus z naught; I get a new medium now. If this medium is the same as the first medium, this medium possesses a centre of inversion symmetry; if the medium possesses a centre of inversion symmetry, all 27 elements of chi i j k 2 are 0, so this medium will not exhibit second order non-linear effect.

There are cubic crystals which have a non-zero chi i j k; there can be crystals which are isotropic; there are isotropic crystals which means chi i j is a scalar, but chi i j k is non-zero; gallium arsenide is a material in which this is finite; but if you take for example, an amorphous medium like glass, it is completely amorphous, so this is not there in glass; you cannot have a second harmonic generation normally in a glass medium.

There is no such restriction in the third order term; this is present in all media whether it is a gas or a liquid or a solid is present always; so, glass will possess this; water will possess this; there are material which possess this; this all materials will possess this, but materials which have this strongly; actually, this starts to effecting the propagation much before this one, because these are actually is like a perturbation expansion and each the next term is supposed to be smaller than the earlier term.

So, when you take a light propagating through an optical fiber, it is made of glass, so this is not there, this is a first term that appears and this controls how light propagates through an optical fiber over long distances.

Now, this leads to things like what I have defined in the course self phase modulation, cross phase modulation, etcetera, which means, as I will show you this term leads to a change of refractive index depending on the intensity of the light wave.

I am sorry.

Depending on the intensity or electric field of the light wave, the refractive index of the medium will change; this will lead to generation of mixing of three frequencies now; you see there are three electric fields here multiplied, so there are three frequencies getting mixed so etcetera; so, we will come to this, but there is the expansion; so, first what you will do if you will discuss this, the effect of this term and then once we discuss then this, discussion will go on to this term.

So, second order effects which is what first we will discuss; and in second order effects we will look at second harmonic generation called SHG - sum and difference frequency generation and something which is very interesting called parametric interaction. You can actually amplify light using this process; and the optical parametric oscillator, which is the very important coherent source uses these three waves of interaction to generate light at new frequencies.

So, these effects can lead us - can help us - to generate light at new frequencies given one particular frequency; if you give me a red laser beam with 800 nanometer wavelength, the second harmonic will give me what wavelength?

400

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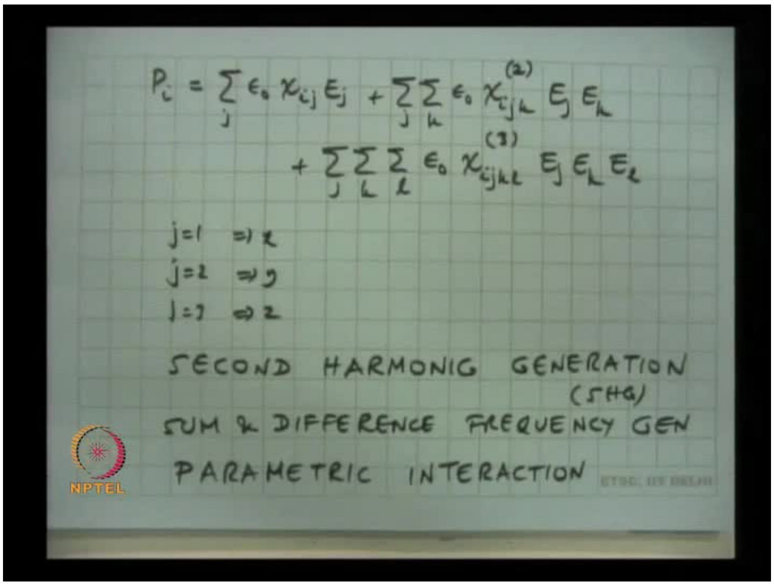

wavelength will get halved as frequency with gets doubled so I can generate a blue laser out of a red laser; you may have seen green laser pointers and some of them have a red laser followed by a non-linear crystal which converts the red into green and that is a very easy way to convert light from between one wavelength to another wavelength, but you need to satisfy some interesting conditions that we will discuss.

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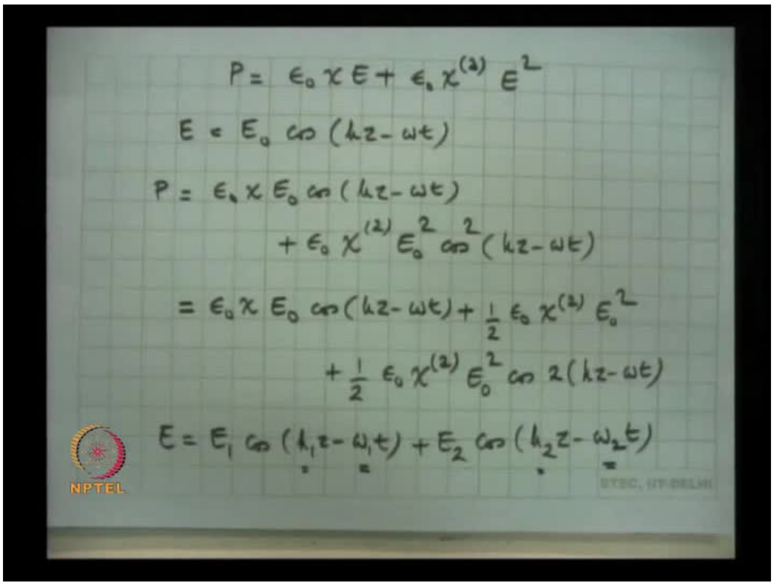

Now, before we go into analyze this, let me look at what is the effect of this second term by taking a scalar equation; what this equation tells me is, P is proportional to E square some product in terms of E x E y etcetera; but for the moment to get a physical understanding, let me look at this following equations normally, I would have written P is equal to epsilon 0 chi E plus let me write this as some epsilon 0 into chi 2 E square.

Let me look at a scalar equation just to understand what is going to happen.

Yaeh.Yaeh.Yes.Yes.

So, the question is, when I convert from red to blue, the frequency has increased, so the energy of the photons has increased; the material does not do anything other than helping in converting from omega to omega; so, if you are able to convert all the input red light to the corresponding second harmonic, the energy that you are inputting is equal to the energy that is exiting; so, what does it mean?

(( ))

Number of photons, if you have 10 to the power 10 photons incident in the crystal corresponding to 800 nano meter, there will be half of 10 to the power 10 photons coming out at the frequency at the wavelength of 400 nanometers, so there is complete energy conservation.

So, number of photons is not being conserved, I am not conserving the number of photons; so, what as.., you look at the picture 2 photons and frequency omega actually merge to form 1 single photon at frequency 2 omega, that comes out, so the medium only helps in this process of conversion, it has not absorbed or give any energy; it is a completely reactive system; it just allows this interaction to take place and this is what I will show you how this happens here.

So, the medium does not give any energy from itself or does not absorb any energy. So, let me look at this equation, before I do this please note now that, because the equation is non-linear; I should not use complex expressions, because as I told you the beginning, the sum of real parts of two complex numbers is equal to the real part of the sum of the two complex numbers; but if I take products, that is not true; the real part of the product of two complex numbers is not equal to the product of the real part of the complex numbers, so all the time I need to use real electric fields, real polarization, real, etcetera.

But we will still use complex by judicious choice of the expressions, we will come to that later. So, let me assume, I have an electric field of the type E is equal to E zero cos kz minus omega t, it is a wave going in the z direction in the medium and it has an expression cos kz minus omega t; so, what is the polarization that I will generate, epsilon 0 chi E 0 cos kz minus omega t plus epsilon 0 chi 2 E 0 square cos square kz minus omega t.

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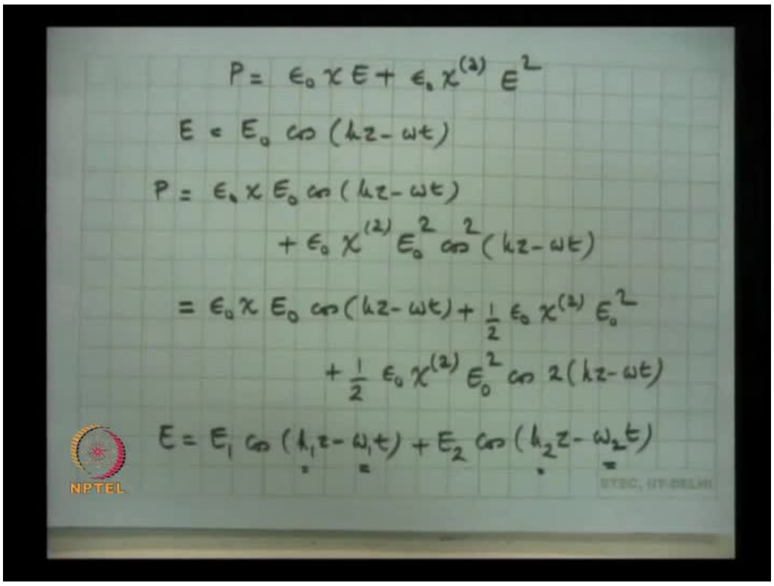

Now, let me express cos square theta in terms of cos 2 theta; so, this would be epsilon 0 chi E 0 cos kz minus omega t plus - what is cos square theta, cos 2 theta is 2 cos square theta minus 1 - so this is half epsilon 0 chi 2 E 0 square plus half epsilon 0 chi 2 E 0 square cosine of 2 times kz minus omega t; I have written cos square kz minus omega t has half of 1 plus cos 2 times kz minus omega t.

Now, recall what is polarization? Polarization is dipole moment per unit volume; so this polarization represents dipoles; this term gives me polarization oscillating at frequency omega; this term represents a polarization independent of time.

So, if a medium has a time independent polarization, what does it mean? If you polarize a dielectric, there is accumulation of charges of the surfaces, so this will lead this term will leads to an accumulation of charges on the surface of the crystal; this term is called optical rectification; it is like a rectification process in electronics, where starting from omega you launched light at frequency omega and you have generated a constant a polarization term which corresponds to a displacement of charges, which does not vary now, it is constant.

The last term has a frequency 2 omega; so, this polarization now has a 0 frequency component, a component frequency omega, and a component at frequency 2 omega; and because polarization are oscillating dipoles; this means, that now the dipoles are oscillating not only at frequency omega, but also at a frequency 2 omega; it is a combination, they are oscillating in a combination of 0 frequency, omega frequency, and 2 omega frequency; and when the dipole oscillates at frequency 2 omega, it will generate radiation at frequency 2 omega, because oscillating dipoles radiate electromagnetic fields.

So, this particular term immediately tells me that because of the chi 2 there is a component in the polarization oscillating at frequency 2 omega and that oscillation at frequency 2 omega will now generate electromagnetic waves at frequency 2 omega; and that is the origin of the second harmonic, which comes out of the crystal, it is this term of the polarization which is responsible for the generation of 2 omega from the medium.

If you take, for example, E is equal to E 1 cos k 1 z minus omega 1 t plus E 2 cos k 2 z minus omega 2t, that means, if your light wave has two frequencies omega 1 and omega 2, propagation constants k 1 and k 2, if I substitute into this equation what will I get, I will get the product of these two cosines in the non-linear term and those product can be written as cos a plus b plus cos a minus b.

So, you will have one term at frequency omega 1 plus omega 2 and one term at frequency omega 1 minus omega 2; so, the component of polarization at omega 1 plus omega 2 generates some frequency; the component of polarization corresponding to omega 1 minus omega 2 generates the different frequency.

So, this particular term is responsible for the second harmonic generation, some frequency generation, difference frequency generation, and this is what we will study in more detail; so, the problem we will have to address is, under what conditions will this process be efficient; suppose, I launch 1 milli Watt of red light, how many nano watts or micro watts of power of the second harmonic will I generate.

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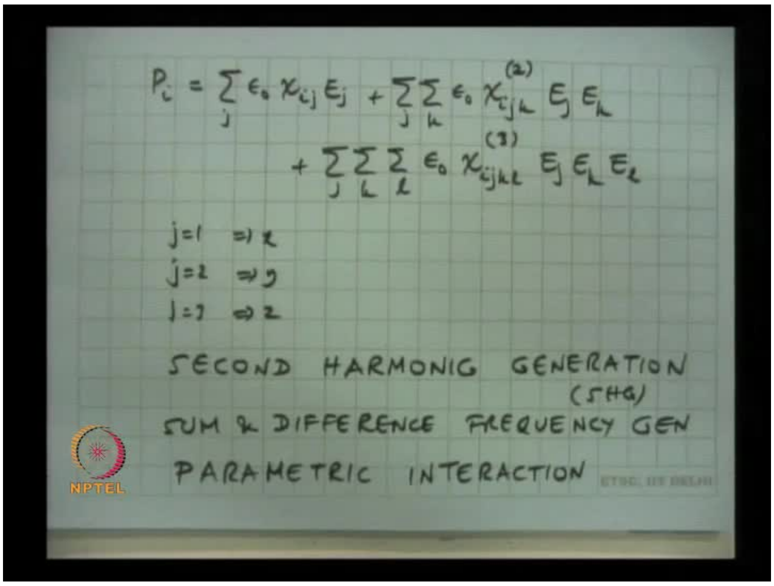

How do I maximize this conversion process? And what is the efficiency of this process? So, this is what we will address in the next class; and what I will do in the beginning is, have a brief discussion of this tensor, which is sitting here, it is a 27 elements tensor here; we will use some interesting concepts or contraction of indices and reduce this matrix of 27 elements to 18 elements; and as I will show you many elements are 0 in materials etcetera and you will starting from this equation, we will go back to Maxwell's equations and analyze the problem of generation of second harmonic.

So, is there any question?

Yes, all elements chi i j k 2 are 0.

I will discuss that in next class. I will tell you that, if the medium possess center of inversion symmetry; if I revert the direction of electric fields, it is the same as before, because there is center of inversion symmetry; so, P will also get reverse our sign; if I change the sign of E j and E k and change sign of P then unless chi i j k is 0, I cannot satisfy this equation; so, it is just the fact that inversion symmetry means immediately that all elements must be 0.