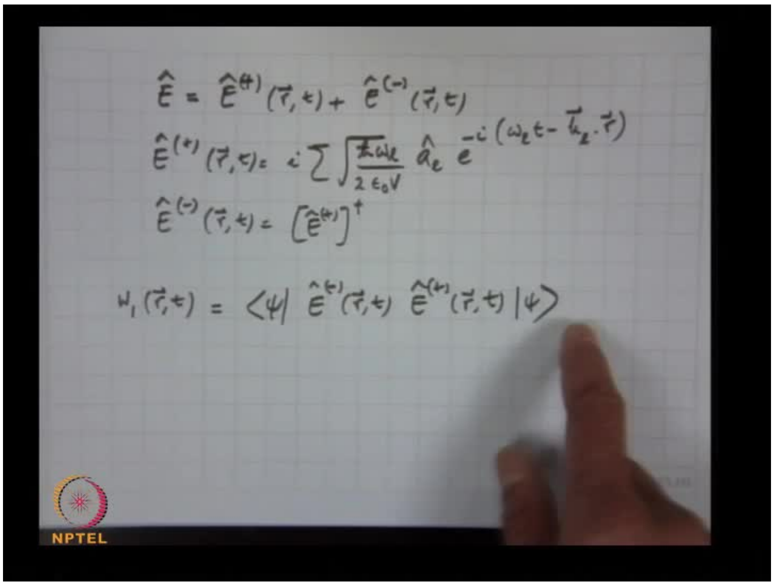
Quantum Electronics
Prof. K. Thyagarajan
Department of Physics
Indian Institute of Technology, Delhi

Module No. # 05
Lecture No. # 39
Beam Splitter and Balanced Homodyning

We will continue with our discussion on the photo ionization expression that we had got last time. We had used for analyzing single photon multi-mode state, I will give you another example of showing single photon interference and then we will move on to this topic of balanced Homodyning.

(Refer Slide Time: 01:03)



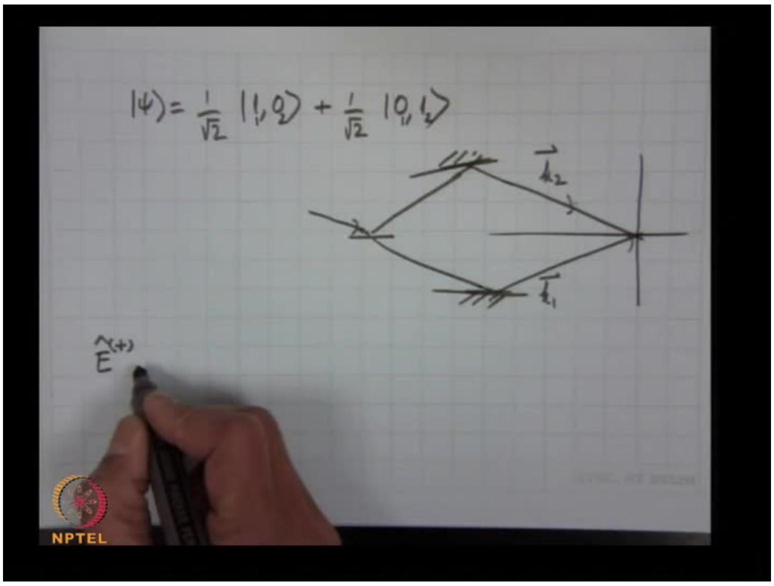
So, do you have any questions on our earlier discussion? Let me recall, we had defined total electric field is actually defined in terms of positive frequency part and a negative frequency part. The positive frequency part is i sigma under root h cross omega l by 2 epsilon 0 V l exponential minus i omega l t minus k l dot r and the negative frequency part is simply the hermitian conjugate of this. We define the photo ionization probability at a point r between times t and t plus d t is proportional to this quantity, where this is defined as expectation value of E minus r t E plus of r t psi.

So, for a given state of the field represented by ket psi, the probability that you will observe photo ionization is given by the expectation value of this quantity. It is a product of E minus and E plus and note that E plus contains a operators and E minus contains the a dagger operators in this expression. Remember this is the total electric field, it consists of the total electric field of all the modes of the system and all the creation operators on the left and all the annihilation operators are on the right, this is called normal ordering. If you would have given a state psi and this is the total electric field to calculate the probability of photo ionization, you need to use this expression and get the probability of photo ionization, which means what is the probability of photo detection.

Now, last time we saw one example where I took a multi-mode single photon state. We look at the probability of photo ionization and found that this probability is actually moving at a velocity given by c. So all it means if you form a wave packet, where the probability of observing photo ionization was maximum at this point, at t is equal to 0; then at a later time, the probability of maximum photo ionization occurs at a different point given by a distance covered at the velocity of light which is exactly the velocity at which this photon wave pocket is moving in free space.

So all it means is that, if I had a wave packet - multi-mode wave packet, single photon wave packet and if I put a photo detector here. That means I have an atom which is spontaneously emitting a single photon, I have n number of such identical experiments each one of the experiments measures the time, this photo ionization takes place here. I will find a distribution because not all of them will detect at the same time because there is a wave packet, single photon wave packet coming; then the maximum probability at certain time and that probability will keep on decreasing as a function of time. This is in a spontaneous emission, it is an exponential decay it is a wave packet which looks like this, and it starts when the spontaneous emission arrives and then decays exponentially.

(Refer Slide Time: 05:03)



Now I want to use this expression to show interference of a single photon and for this I want to consider the following state, a single photon state. 1 by root 2 1 0 plus 1 by root 2 0 1 let me call this, 1 1 0 2 0 1 1 2 so modes 1 and 2. Now remember, these modes are defined by propagation direction, polarization state and frequency. In the earlier examples we looked at, we assume the same polarization state, same propagation direction and different frequencies form a wave packet. Now I want to look at 2 modes which have the same frequency, same polarization but, propagating in different directions.

For example, I could have a situation where this is the screen where I am having a photo detector to measure the photo ionization and I could have one corresponding to this mode and one corresponding to this mode. So this could be the mode 1 with propagation vector k 1, this is mode 2 with propagation vector k 2 and please note that there is only one photon occupying both the modes is something like this. If I had for example, if I could set up a following experiment, I have a mirror here, beam splitter (Refer Slide Time: 06:10).

So I have a beam splitter in which a single photon is incident, this vacuum state from here, this photon produces a superposition state. Then there are two mirrors which direct the beams into two different directions to interfere on a screen. In a Mach-Zehnder, I would have actually interfered them back on a beam splitter but, here I am assuming that these are big mirrors and I can assume these to be plane waves. So, I am assuming that this is a plane wave going in one direction, that is a plane wave going in another direction and I want to find out what happens here.

Classically, what would I see, on the screen? An interference fringe pattern which will be depending on the phase difference, it will be maxima's and minima's on the screen. So what I will expect is quantum, mechanically I will show you that if you have a single photon incident from here and if you could measure the photo ionization probability as a function of position, you will see a fringe pattern. That means, the probability of photo ionization here may be maximum, if you move slightly away the probability becomes 0 depends on amplitudes of these two, again becomes maximum, 0, maximum, 0 (Refer Slide Time: 07:40).

So it is a single photon which is actually coming and creating a state of two modes, each one propagating in a different direction. I will use this expression to calculate and show you that the probability of photo ionization shows me the interference pattern.

Now there are two modes only, so I need to look at the total electric field to consist of only two components. Yes, Mohit

Sir, the probability of photo ionization that we had calculated,

Yes

What does it exactly mean when the photon is incident on an atom, so its probability of photo ionization.

Yes, probability that the ion gets ionized; the atom gets ionized because that will lead to a current in my external circuit.

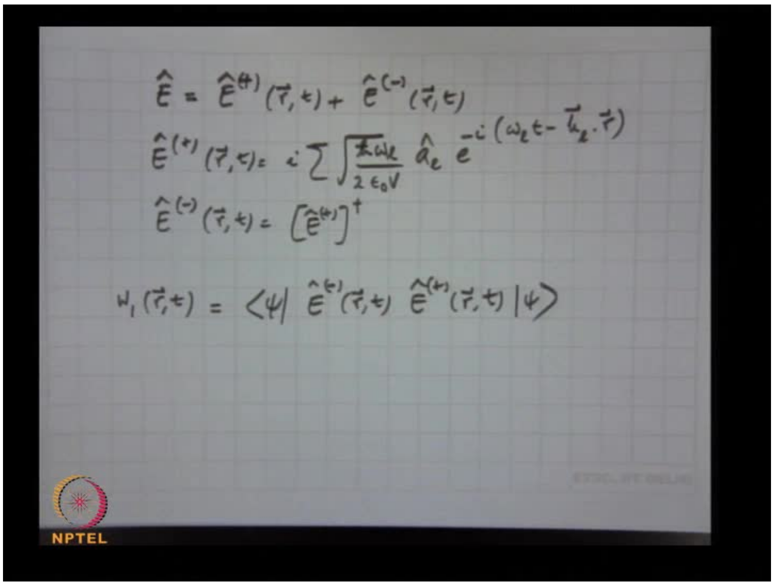
When wave packet consisting of a single photon is incident,

Whatever single photon, any complex this thing, multi photon whatever it is

In our case, what we are analyzing?

Yes

(Refer Slide Time: 08:51)



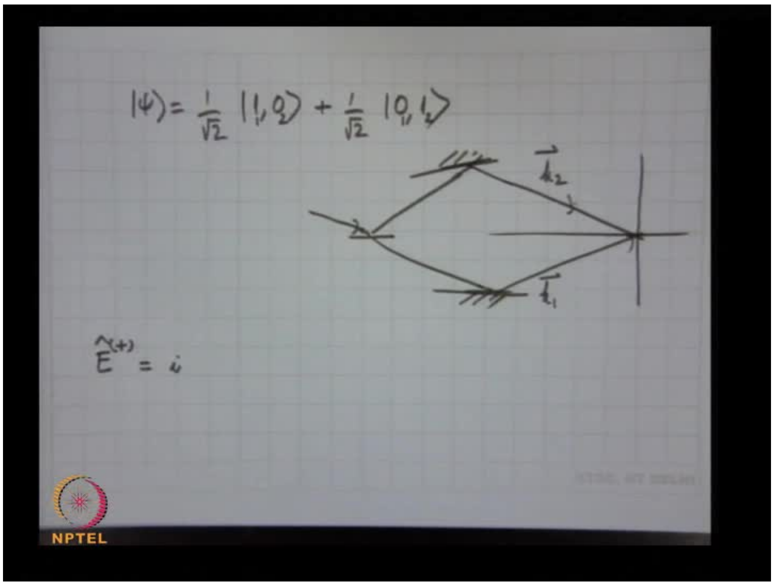
So this is the general expression, where psi could be any general arbitrary electromagnetic state. It could consist of multi-mode states but, different polarization level. Now if it has different polarization actually these are vectors, this has to be dot product here but, we are not going into vector form here, I am assuming that the electric field has one polarization.

Polarization same

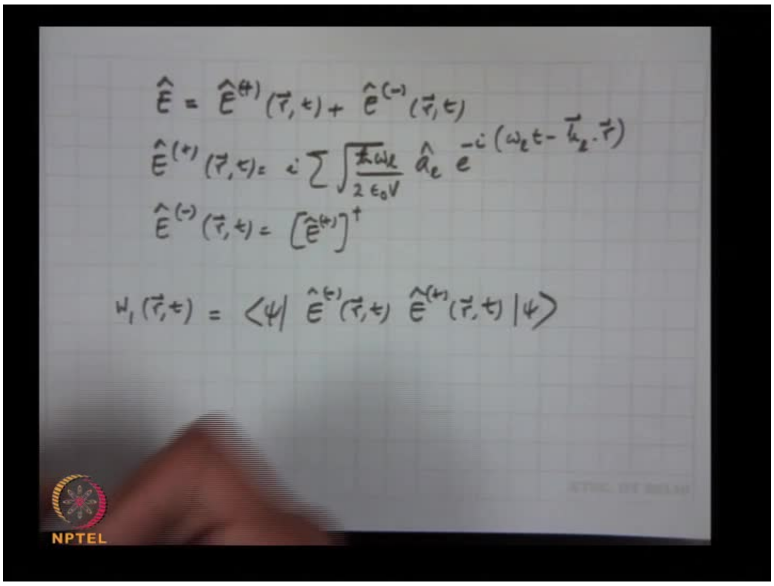
Same polarization, now I had earlier looked at a single photon state occupying superposition of different frequencies.

That is why different modes.

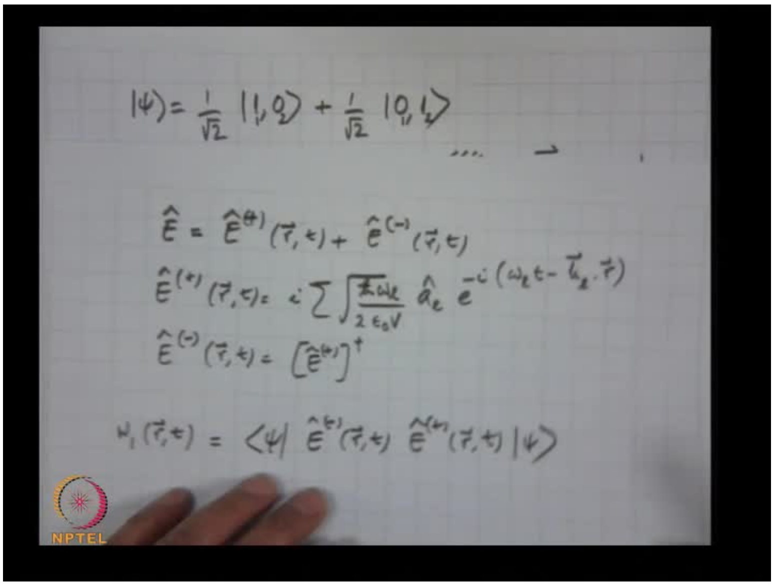
(Refer Slide Time: 09:40)



(Refer Slide Time: 09:49)



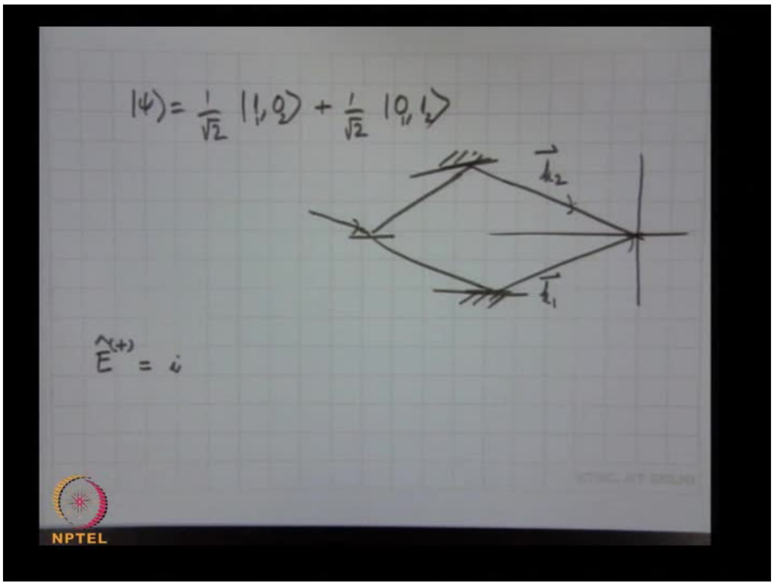
(Refer Slide Time: 10:09)



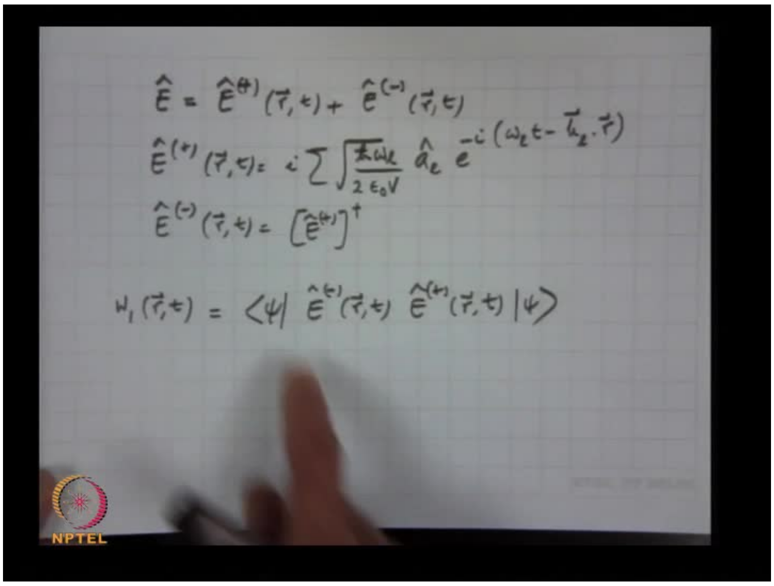
They are all different mode exactly. Now I am looking at a single photon occupying 2 modes at the same frequency but, propagating in different directions, the modes are now different because of propagation direction and not because of frequency. So what will I write for E plus? It will be i so it will consist of 2 terms. Please note that this is the total electric field, I must use the total electric field because the state I am considering is like this. All terms in the sum other than those of the 2 modes will give me 0 when I operate on psi because psi contains 0s for all values of the subscript other than 1 and 2.

So, if in the sum I took a third term omega 3 a l 3, a 3 etcetera when that operates on psi any of these because this is 0 3, this also 0 3, that will be 0. So I do not have to worry about all other terms in this sum, I need to worry only about modes 1 and 2.

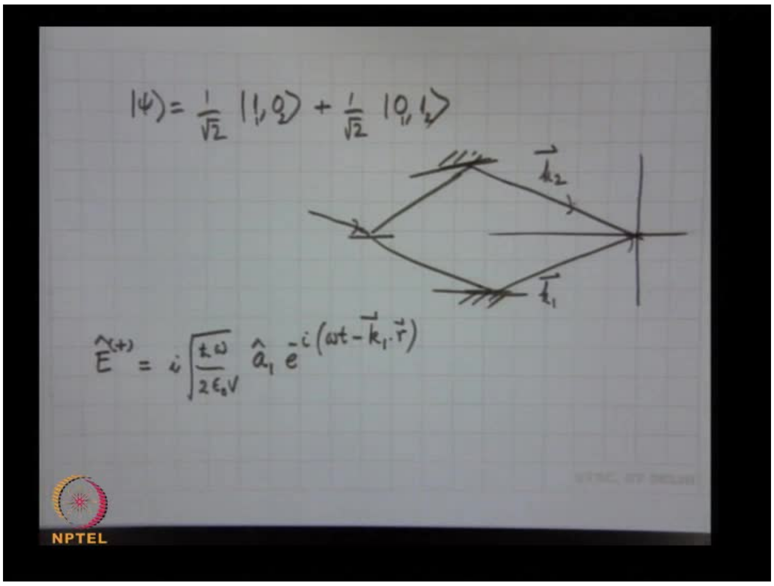
(Refer Slide Time: 10:34)



(Refer Slide Time: 10:38)

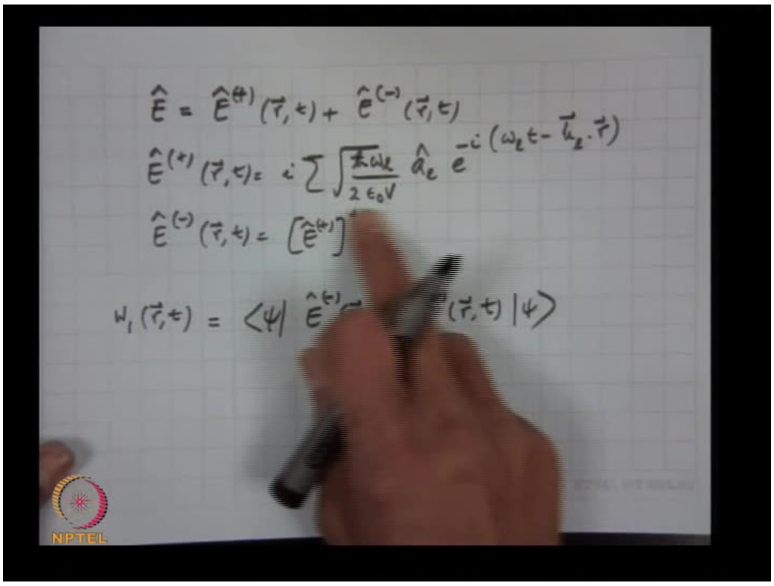


(Refer Slide Time: 10:42)

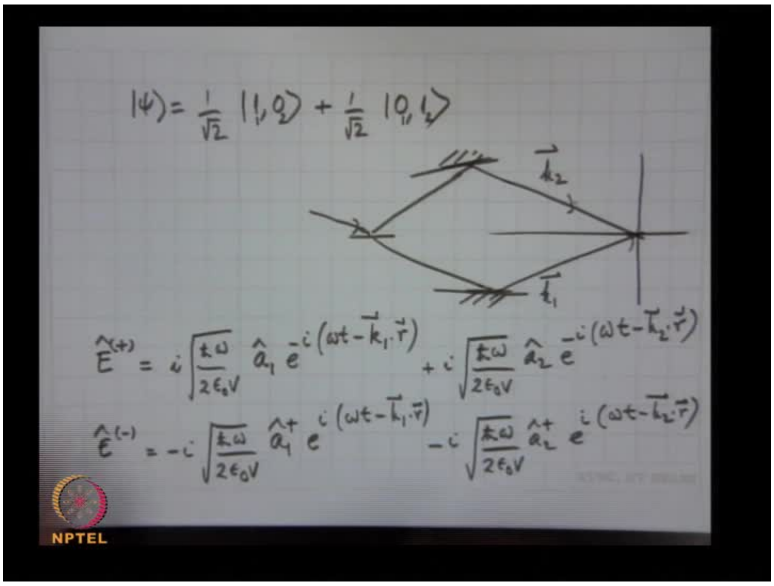


So the total electric field that will be responsible for this, the total electric field that will take part in this giving me finite terms here will consist of two terms. The first one will be i h cross omega by 2 epsilon 0 V, I am assuming 1 frequency a 1 exponential minus i omega t what do I write now, minus k 1 dot r.

(Refer Slide Time: 11:08)



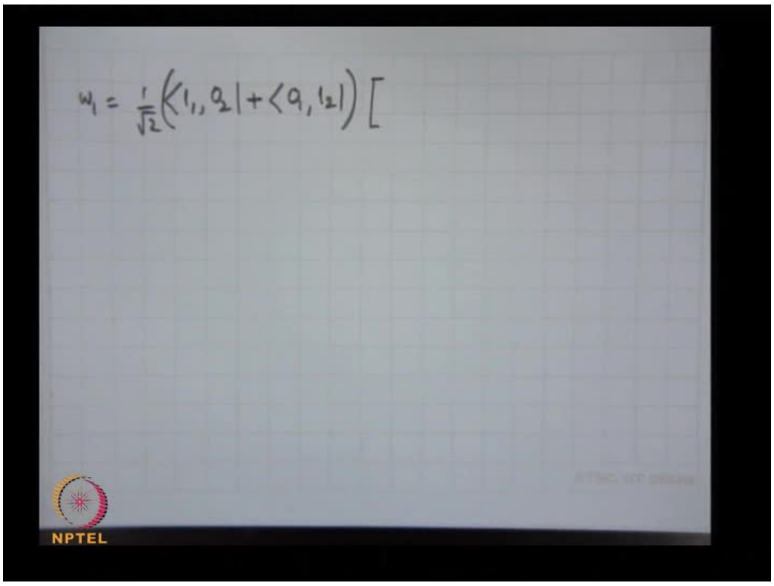
(Refer Slide Time: 11:22)



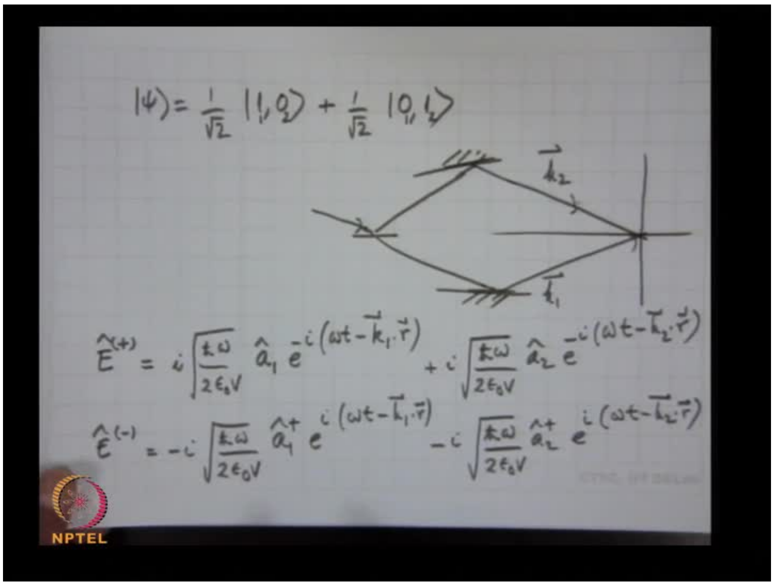
Please see here, this is the electric field, this is one term i square root of h cross omega by 2 epsilon 0 V a 1 exponential minus i omega is the same omega, I just call it and the first mode has k 1. Then, I have i square root of h cross omega by 2 epsilon 0 V a 2 exponential minus i omega t minus k 2 dot r (Refer Slide Time: 11:08).

In the problem sheet, I have given a problem in which the 2 modes correspond to waves propagating in the same direction with the same polarization but, at 2 different frequencies but, you have to follow the same procedure, except that I will have omega 1 and omega 2 case will be the same. E minus will be minus i square root of h cross omega by 2 epsilon 0 V a 1 dagger exponential i omega t minus k 1 dot r minus i square root of h cross omega by 2 epsilon 0 V a 2 dagger exponential i omega t minus k 2 dot r.

(Refer Slide Time: 13:14)

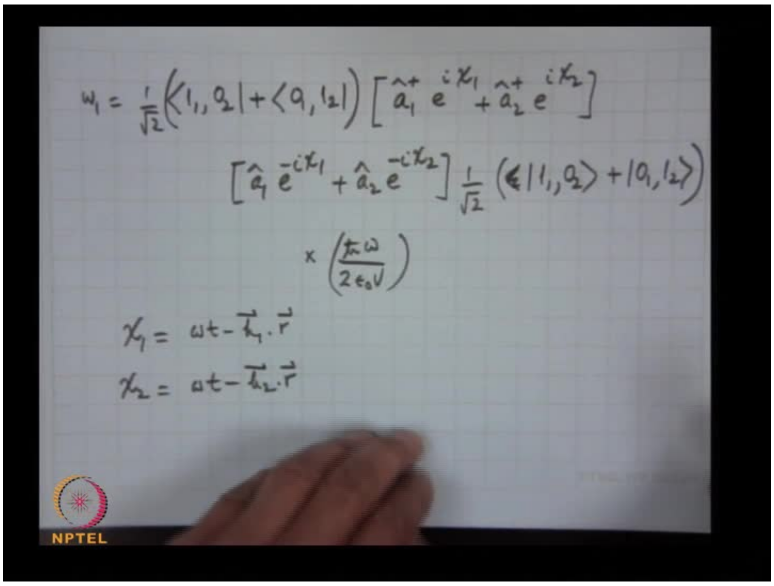


(Refer Slide Time: 13:33)



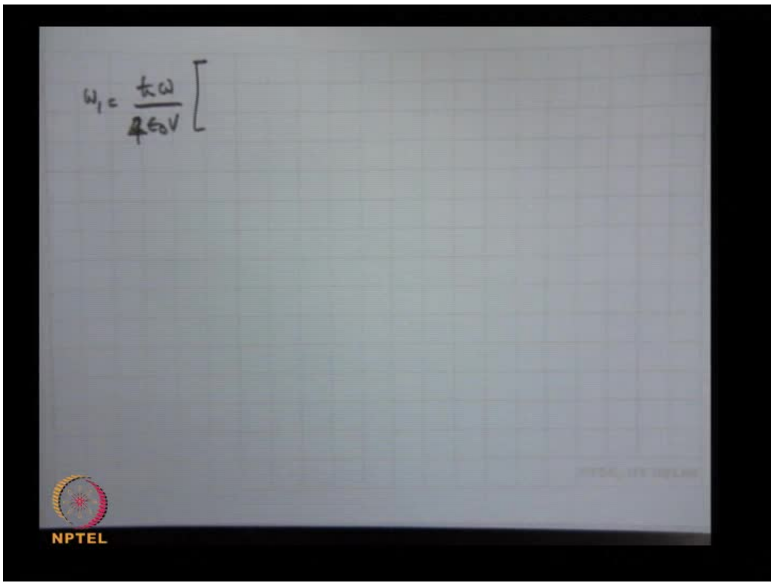
This is the psi, this is E plus, this is E minus; to calculate the probability of photo ionization, I need to calculate the expectation value of E minus E plus in with respect to this state psi (Refer Slide Time: 12:40). Now let me call this chi 1, chi 2 just to make it compact and these expressions outside multiplication factors are the same. So what I will get is w 1 will be 1 by square root of 2 1 1 0 2 plus 0 1 1 2. Now I have this factor, i under root h cross omega by 2 epsilon 0 V so, let me call this as a common factor and this same in E plus and E minus; so I will get an outside factor, which I will write later.

(Refer Slide Time: 13:47)



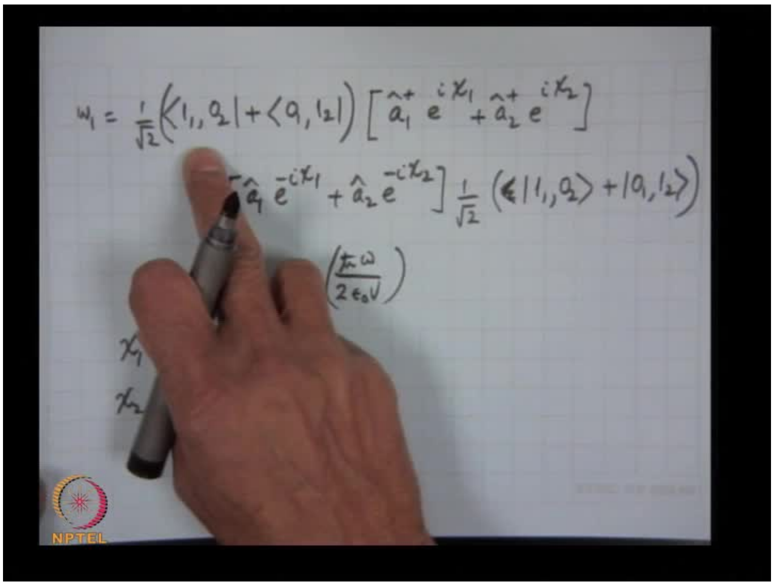
So I have a 1 dagger exponential i chi 1 plus a 2 dagger exponential i chi 2 into a 1 exponential minus i chi 1 plus a 2 exponential minus i chi 2 into 1 by square root of 2 1 1 0 2 plus 0 1 1 2. This is bra psi, this is to be multiplied by so I have i times h cross omega by this thing coming here the minus sign, so I will get h cross omega by 2 epsilon 0 V. chi 1 is omega t minus k 1 dot r and chi 2 is omega t minus k 2 dot r, bra psi E plus E minus E plus into ket psi.

(Refer Slide Time: 15:26)



Now I can operate this on the right side, I can operate this on the left side and ket with expressions, so what happens to this when this operates on this side? Let me operate these two so, I will get now so w 1 will be h cross omega by 2 epsilon 0 V; so there is a 1 by root 2 and 1 by root 2 so that becomes another factor of 2 that becomes 4.

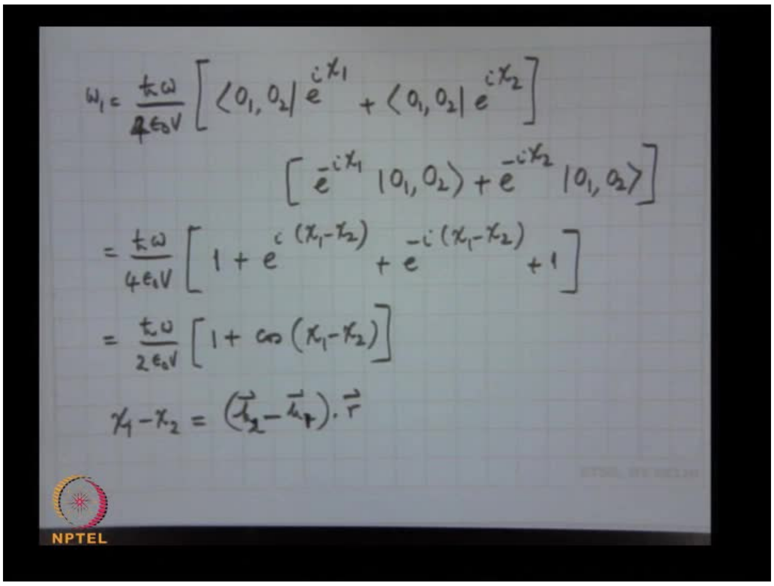
(Refer Slide Time: 15:44)



Now, what is a 1 dagger operating on this state?

0 1 0 2

(Refer Slide Time: 15:52)



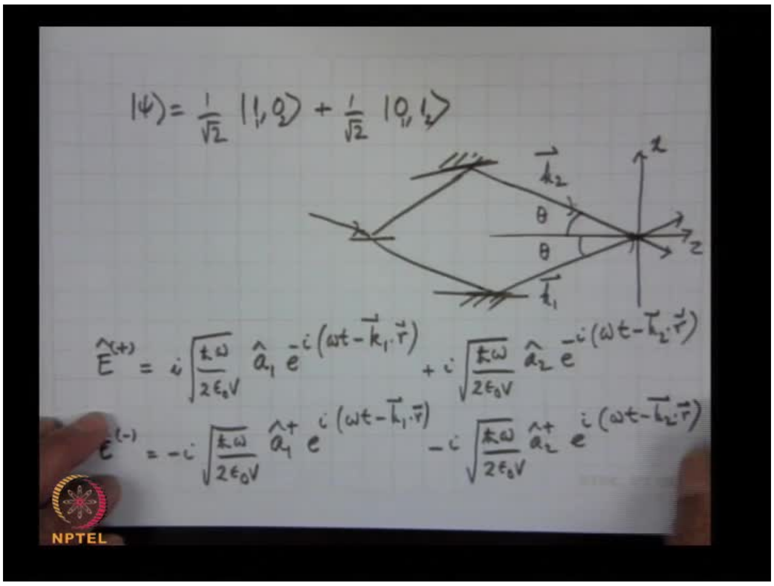
So I will get 0 1 0 2 into exponential i chi 1, a 1 dagger operating on this gives me 0, a 2 dagger operating on this gives me again 0, a 2 dagger operating on this gives me a 1 operating on 0 is 0, a 1 dagger operating on bra 0 is 0. The second one a 1 operating on this gives me exponential minus i chi 1 into 0 1 0 2 a 1 operating on this, second term gives me 0 because this is 0 here, a 2 operating on this gives me 0 because the second mode is unoccupied a 2 operating this gives me 0 minus i chi 2 0 1 0 2. This is actual state, actually all others are anyway 0s; I am not writing them here but, 3 4 5 everything is 0, so this is 0, this is 0; these are all the same states.

So let me open this bracket h cross omega by 4 epsilon 0 V so this into this is 1, this into this is plus exponential i chi 1 minus chi 2, this into this is exponential minus i chi 1 minus chi 2, this into this is 1.

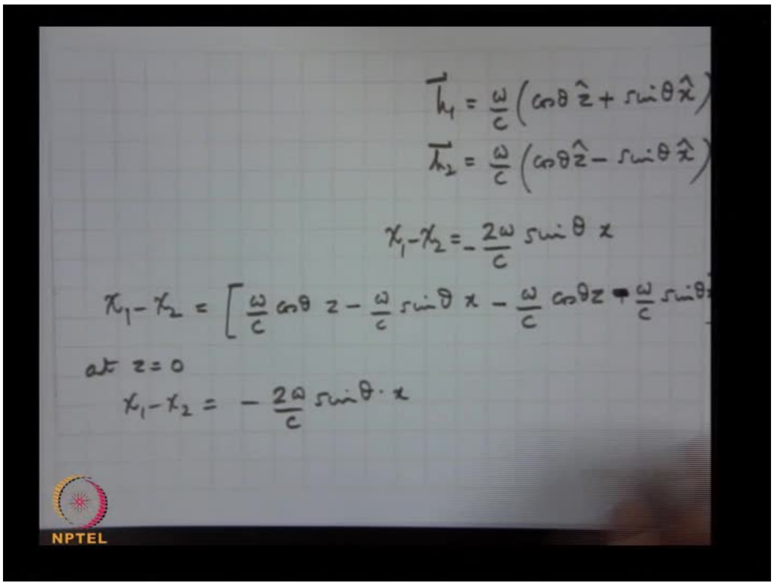
This is equal to h cross omega by 2 epsilon 0 V, I have taken out 2 so, 1 plus cos this is 2 cos chi 1 minus chi 2 and chi 1 minus chi 2 is this is k 1 minus k, chi 1 is minus omega t minus k 1 dot r, chi 2 is omega t minus k 2 dot r sorry, this is k 2 minus k 1, chi 1 minus chi 2 is k 2 minus k 1 dot r.

You see here interference at those values where this is 0, 2 pi, 4 pi etcetera this is plus 1, I get a maximum probability of photo ionization and whenever the cosine term is negative minus 1 the probability becomes 0.

(Refer Slide Time: 19:05)



(Refer Slide Time: 19:46)



For example, in this figure to be more specific let me call this z, let me call this x, let me call this angle as theta and this angle also as theta just to be little more specific (Refer Slide Time: 19:07). So, what is k 1 vector? This is a plane wave going at an angle theta with the z axis like this and this is another plane wave going at minus theta with z axis, so what is k 1 vector?

We have a figure here, so k 1 vector is equal to Omega by c which is the propagation constant into Cos theta z cap plus sin theta x cap and k 2 vector is omega by c into cos theta z cap minus sin theta x cap. If I went measuring the photo ionization probability on this plane, which I call as z is equal to 0, then chi 1 minus what is that chi 1 minus chi 2 becomes equal to which is k 2 minus k 1 dot r, so you get this is at z is equal to 0.

So what you get? k 2 minus k 1 dot r at z is equal to 0, you will simply get 2 omega by c write down so, chi 1 minus chi 2 is equal to Omega by c cos theta z minus omega by c sin theta x this is k 2 dot r minus k 1 dot r which is omega by c cos theta z plus omega by c.

Sorry,

In the last Minus

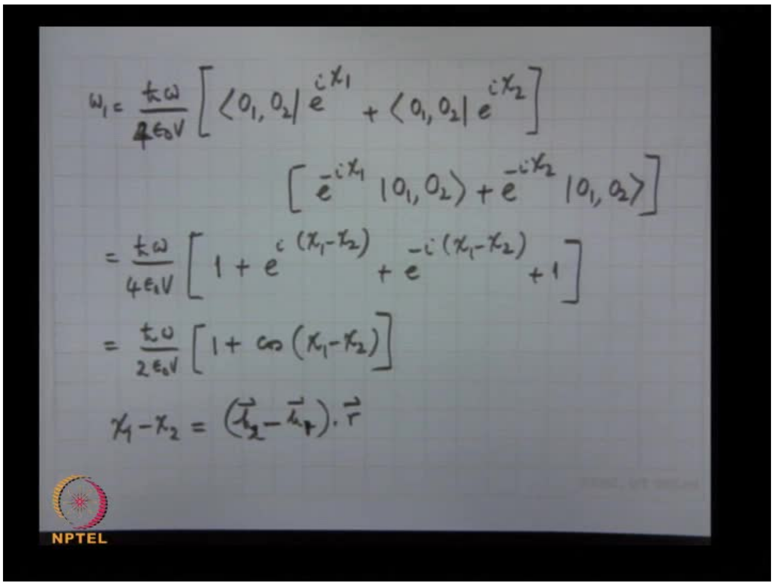
Yes minus. So at z is equal to 0, chi 1 minus chi 2 becomes minus 2 omega by c sin theta into x, so I get for the photo ionization probability.

Sir, why should (( )) equal to at z equal to 0?

I am just on the plane.

On this (( ))

(Refer Slide Time: 22:41)

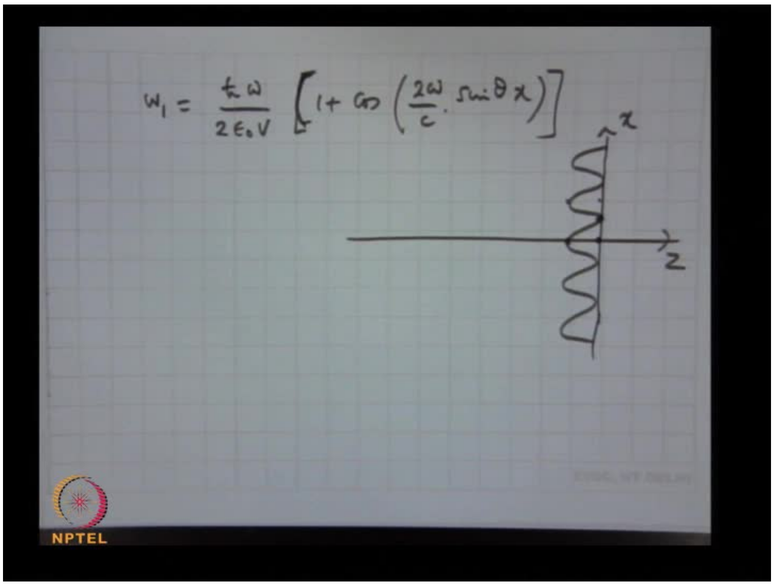


I am just looking at the typical interference pattern that I will observe on a plane. So I am just trying to find out otherwise, this is general expression at any value of x z if you place an ion and look at the photo ionization probability, it is given by this expression.

No, what I am trying to say is that chi 1 minus chi 2 has no electron.

Yes, anyway that is fine because of the type of direction which I have chosen, which means at any plane, I will get the same expression.

(Refer Slide Time: 23:01)



So what I get is w 1 is equal to h cross omega by 2 epsilon 0 V into 1 plus cosine of 2 omega by c on this screen. For example, at x is equal to 0 you have maximum probability if you move away from here, so this argument becomes pi by 2 you get 0 and then when the argument becomes pi it becomes 0, when the argument becomes 2 pi it become maximum again.

So exactly what you will observe in a classical interference pattern between two plane waves, one propagating at plus theta with the z axis, the other propagating minus theta with the z axis, the difference is I have only one photon here. The state psi corresponds to a single photon that is put into a superposition state between 2 modes and because it in a superposition state between 2 modes, when you recombine them and you find that the probability of photo ionization shows interference.

So if you perform your experiment, Young's double-hole experiment with single photons coming in you used to see the interference pattern. Of course, you would not see an interference pattern what you will see essentially is you send one photon; you have a sort of large number of photo detector sitting here. An array of photo detectors, so one of them will click, next you send another photon another one clicks, third time you send another photon another one clicks but, the photo detector sitting here will never click (Refer Slide Time: 25:07).

The photo detectors clicking here will have maximum number of clicks; so it is like plotting a histogram of the number of clicks.

(( ))

Exactly, so it is like a photographic plate which is sitting there and I am sending 1 photon after another photon, the interference pattern gets built up by detecting detector points.

Sir this is I assuming that we have beam splitter and we have actually created the scope which is state but, will we see this (( )) you know, will there be diffraction if I just send a single photon?

Surely, what that would mean is that if you have a slit and if you send a single photon it will arise somewhere on the screen but, if you had n number of photo detectors, large number of photo detectors and you perform this experiment a large number of times, most of the photons will arrive here, at some angle there will be never a photon and that probability of distribution will exactly look like sin square beta by beta square pattern.

My question is that the phase that have you assumed?

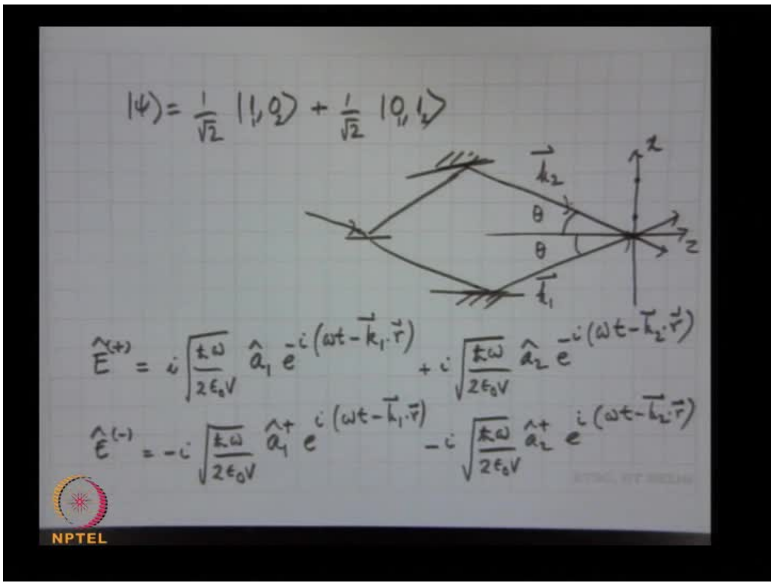
Yes

Will that state result from diffraction also from us if I pass single photon (( ))

Yes it is actually is more than this, from diffraction it is more than this, from diffraction it is a continuum of directions. It is like when I look at the multi-mode single photon state, I looked at different frequencies present simultaneously, a single photon being present simultaneously in many frequencies. What you are talking is single photons present multiply in different directions of propagation simultaneously but, at single frequency.

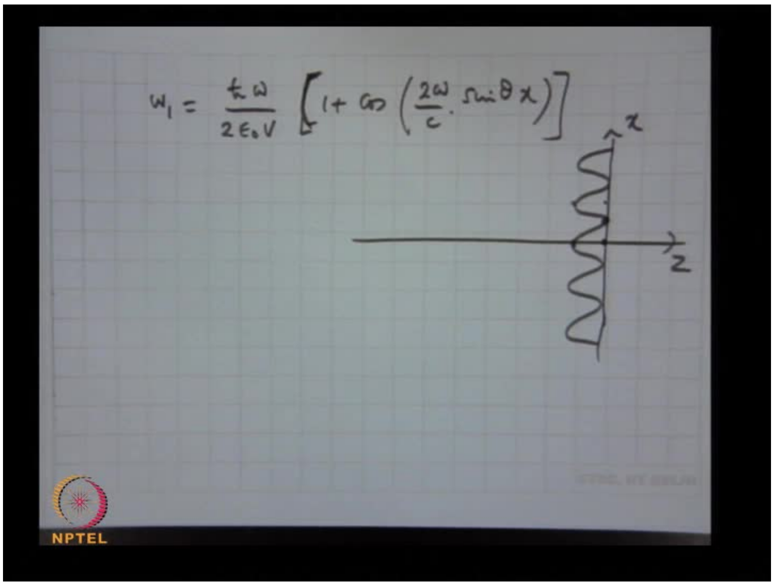
So you get a wave packet in space, a wave packet in time is one which you generate by adding components at different frequencies. You get a wave packet in space by adding waves in different directions and adding them up properly to get a finite beam of size.

(Refer Slide Time: 27:26)

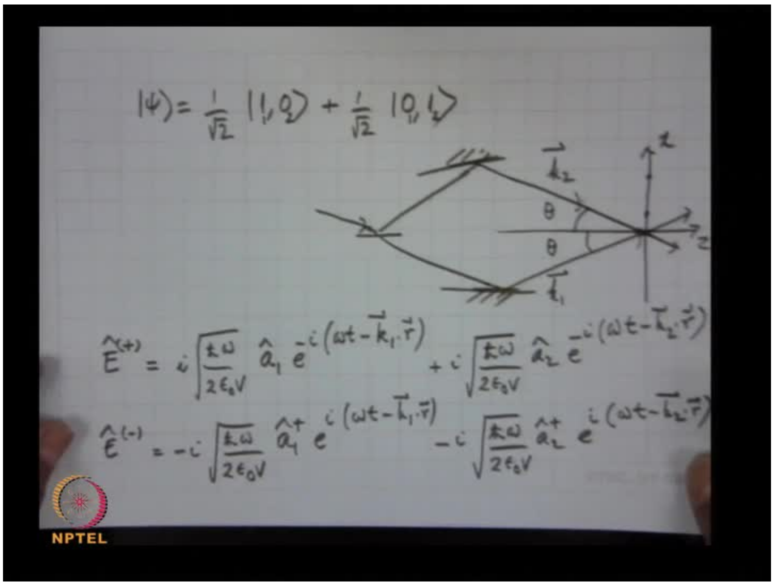


So what you will generate there is much bigger than this because this is just a 2 mode state. Actually, one can do beyond this point and because of lack of time in the course I do not think I will discuss that but, for example, I can calculate things like I put 1 photo detector here, another photo detector here. What is the probability that both of them detect simultaneously? With the single photon there is never possible because there is only one photon but, I can have states in which there are more than one photon there are two more states for example.

(Refer Slide Time: 28:16)



(Refer Slide Time: 28:26)



I can find out probabilities of simultaneous detection, these are called coincidence counting etcetera and the difference between classical and quantum becomes much bigger there because here I do not see any difference from the classical interference pattern, it is the same as classical interference pattern. Although I still have to worry about how a single photon is interfering with itself, single photon which is supposed to be in a superposition state because superposition is a very tricky concept because the photon is in simultaneously in both propagation directions.

Those probability of detecting 2 photon simultaneously at 2 different points or 1 photon here at 1 time, another photon here at another time etcetera is much more interesting but, that will not happen with single photon. The single photon the probability of detecting 2 points is 0 because once you detect at 1 point, it is not there in the second point. So things like Hanbury Brown Twiss experiment etcetera they are all based on those kinds of interference effects which we will not have time to discuss here.

So as I mentioned in the problem sheet, I have given another problem in which you need to take 2 modes with 2 different frequencies but, propagation direction is the same what do you expect?

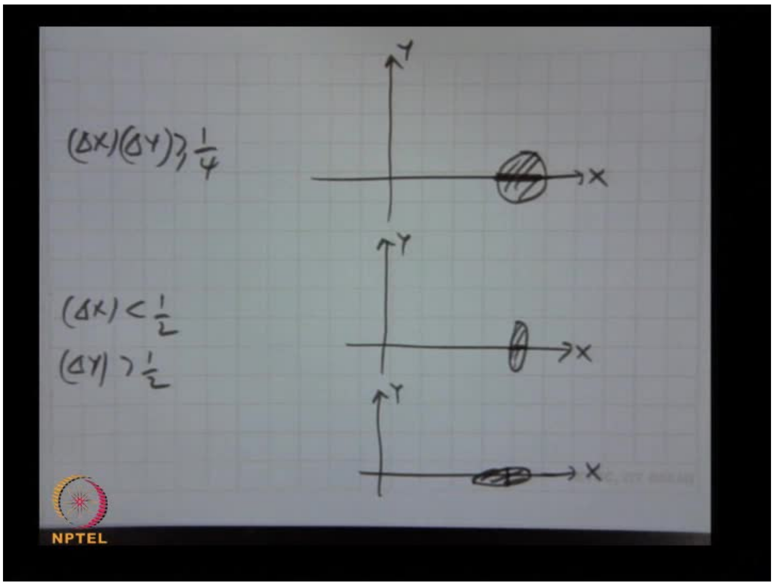
(( ))

Sir omega 1 minus (( ))

Beats, so you will have beats; if you have a detector here, you will see beats. See beats means what? Again it is an addition of a number of measurements because one measurement can only detect 1 photon at 1 time that is all. So that is the beats meaning that I repeat the experiment a larger number of times and I build up a histogram and I find that I will see certain instants of time when there is no intensity at all.

Now what I want to discuss is the beam splitter. Why I did this beam splitter in a little more detail is because first of all, it is very interesting in the sense that I showed you some interaction free measurements possibilities with this and the more interesting thing is that it can be used to detect squeezing.

(Refer Slide Time: 30:43)



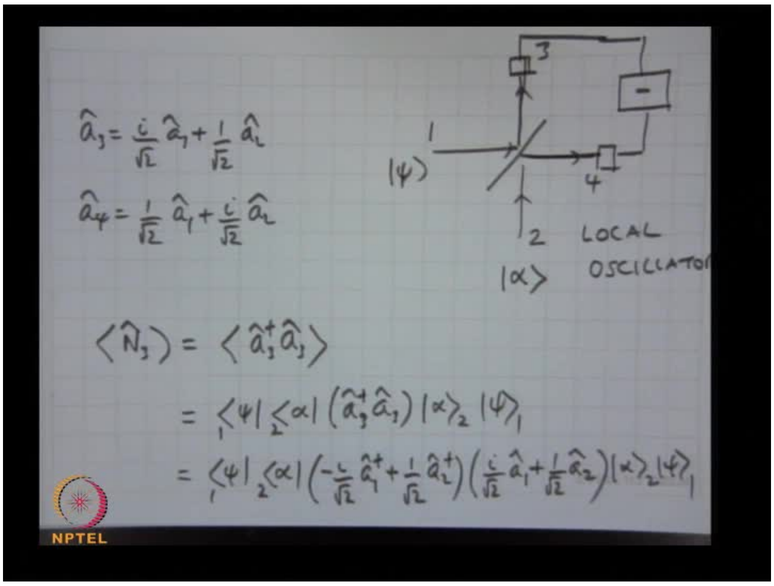
Now please note when we discussed squeeze states, we showed that we wrote the electric field as the sum of 2 quadrature X and Y. In a coherent state, the noise in X and Y are equal, the variances are equal, delta X delta Y is 1 by 4, delta X is 1 by 2, delta Y is 1 by 2. We drew a coherent state like this, I could have a coherent source like this (Refer Slide Time: 30:43). There is an uncertainty principle which tells me that there is a product of delta X and delta Y must be greater than 1 by 4 greater than or equal to 1 by 4 and when it becomes equal to 1 by 4, I get minimum uncertainty states. So X is one quadrature, Y is the other quadrature.

Now in squeeze states which we discussed, we show that it is possible to have a state like this, where delta X is less than half and delta Y is more than half. This product is still equal to half that is 1 by 4, minimum uncertainty squeeze state. I could have a product which is always more than 1 by 4 there is no problem but, minimum is 1 by 4. I can have state like this in which X quadrature is squeezed or I can have a state in which the Y quadrature is squeezed that means the uncertainty in X is less than in a coherent state, the uncertainty in Y is less than in a coherent state in the squeeze state.

Now, when I normally use detectors, I do not measure 1 quadrature; I am measuring the photon is getting detected. I am actually measuring the total electric field and I cannot differentiate between here for example, I cannot differentiate between different phases of electric field.

In order to do this, to be able to measure either the X quadrature or the Y quadrature to show that there is less noise than in the coherent state, I use interference effects. I need a phase reference, the phase has to have a reference and this reference comes from what is called as the local oscillator.

(Refer Slide Time: 32:48)



So what is done is the following: let me look at this set up, so I have a beam splitter. This is input 1, input 2, input 3, input 4, I have 1 detector here, I have another detector here and I subtract the detections from this detector (Refer Slide Time: 32:53). This detector so these are actually giving me expectation values of the photon number, light is coming here continuously and this is detecting.

Now what happens is here what I send is this is called the local oscillator. This sends here a coherent state which means a laser output is a coherent state. Now let me tell you those of you have done electrical engineering would have come across homodyning and heterodyning.

This is a very standard procedure which is used in a single processing where you mix your input signal with another local signal and it can act as an amplifier. It should be amplified the signal and it picks up a particular phase of the input signal, you can pick up a particular phase of the input signal by mixing in electronics for example, a given signal and a signal that you are generating at the local oscillator.

If you have the same frequency between the signal and the local oscillator it is called Homodyning; if the 2 frequencies are different it is called heterodyning. So this is the input state, which I am trying to find out which I am trying to measure. This could be the squeeze state, this could be another number state whatever it is and I am trying to measure, do measurements on this state, and do measurements dependent on phase.

So what I do is I send this state from here on beam splitter, I have a local oscillator which sends in a coherent state. This beam splitter mixes these two beams in this arm and in this arm (Refer Slide Time: 35:00). These two beams which are then falling on 2 detectors, which detect the photons as they arrive here and then I do a processing of subtracting this 4 from 3 or 3 from 4; I will show you that this difference contains information about the phase about the different quadrature's contained in this state psi. I can choose the different quadrature which I am measuring by modifying the phase of this local oscillator.

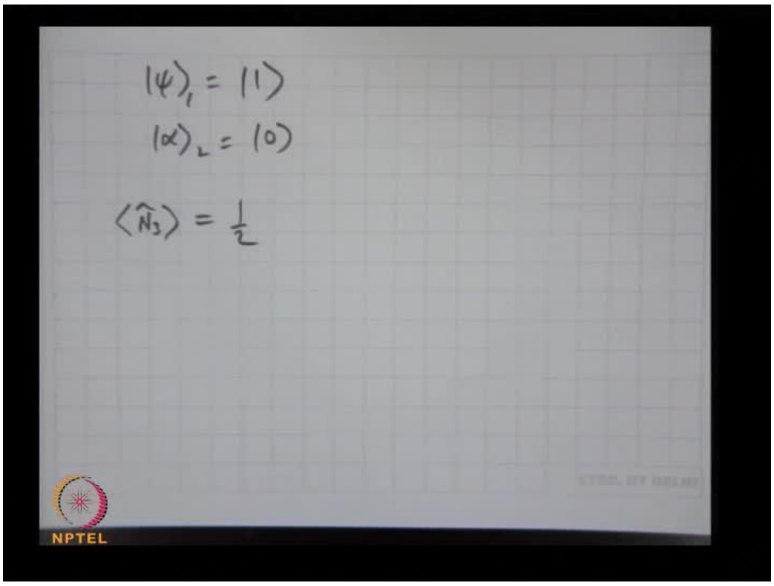
Now, we have analyzed the beam splitter in a particular fashion. Remember, we wrote the expressions between a 3 and a 4 and a 1 and a 2. We wrote for example, a 3 is equal to i by root 2 a 1 plus 1 by root 2 a 2 and a 4 is equal to 1 by root 2 a 1 plus i by root 2 a 2 and then what we did was we assume that the input 0 0 0 1 0 2 goes to 0 3 0 4 and found out the state coming out of the beam splitter. Here we have followed a slightly different procedure and the procedure is as follows. It is a Heisenberg picture, so what I will say is the beam splitter modifies a 1 and a 2 at the input get modified to a 3 and a 4.

So I will represent the state of these systems by the input state and all observables at the output will be calculated by expectation values of these operators with respect to this. For example, suppose I want to calculate, what is the expectation value of the photon number in this arm? So, expectation value of what should I find out? N 3 which is expectation value of a 3 dagger, a 3 I need to know. So what I do is the following: I represent this is psi in the first state and alpha in input 2 a 3 dagger a 3 alpha in state 2 and psi in state 1.

Now I represent the state as at the input and I will write a 3 and a 3 dagger in terms of a 1 and a 2. So what I will have to calculate is psi 1 alpha 2 minus i by root 2 a 1 dagger plus 1 by root 2 a 2 dagger into i by root 2 a 1 plus 1 by root 2 a 2 on alpha 2 psi 1.

Please note the procedure, so what I am writing is the output operators, the operators corresponding to the output state are represented by in terms of the operators corresponding to the input states. a 3 is written in terms of a 1, a 2; a 4 is written in terms of a 1, a 2 and for calculating the expectation values I assume the state that is input into the system.

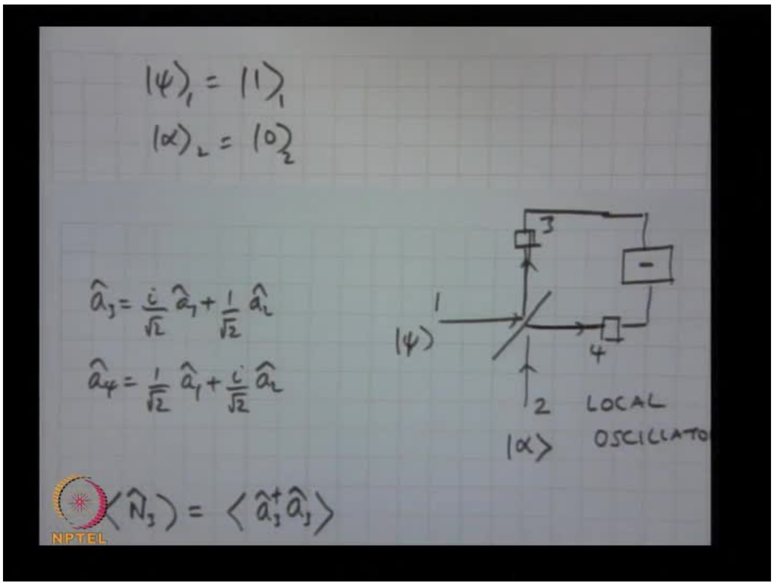
(Refer Slide Time: 39:09)



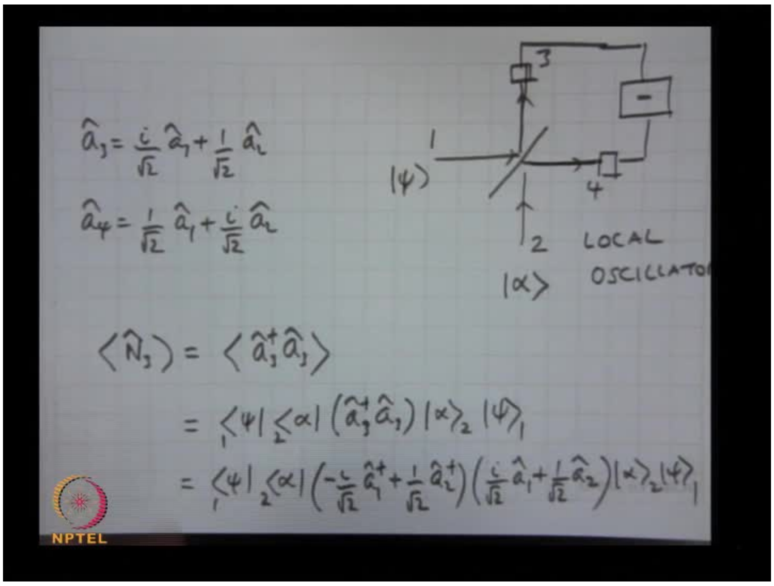
So in this picture, the states do not change but, the operators they first change and I can find all expectation values. Now what do you expect this number to be? Suppose it was 1 photon here and no photon here; I took this example and I assume that psi is 1 equal to 1 and alpha 2 is equal to 0 vacuum state what would I expect this to be?

1 by 2

(Refer Slide Time: 39:24)



(Refer Slide Time: 39:39)



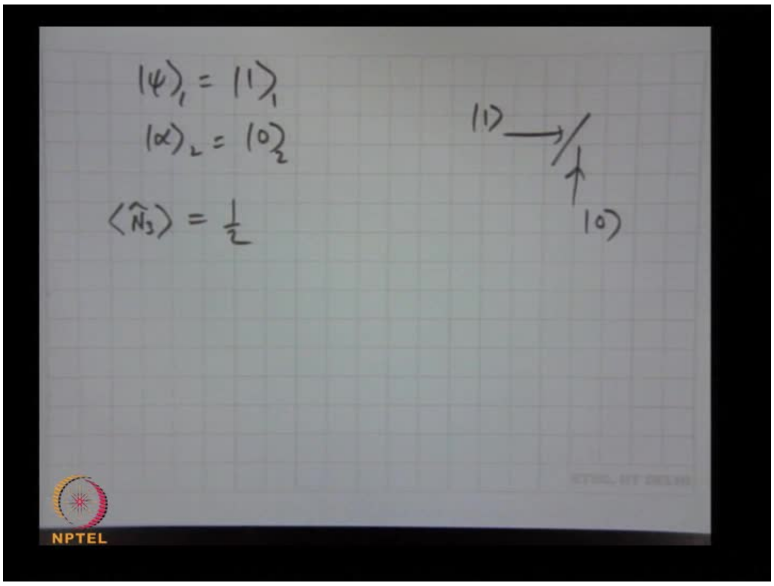
Yes half, so I leave it to you to show. Please substitute psi 1 as 1 1 and 0 the input corresponds to 1 photon in the input arm 1 and vacuum input in the input arm 2 and if you calculate this quantity, you will find half that is the probability of detecting that the average number of photons which will be arriving here. Similarly, N 4 will be half, so this is another picture which is similar to Heisenberg picture where operators have got transformed. I express the transformed operator in terms of the input operators, I keep the state vector the same as that input and then I calculate all expectation values.

If I want to calculate the variance in this photon number measurement here (Refer Slide Time: 40:14), I need to calculate delta N 3, which is N 3 square average minus N 3 average square, so all the procedures exactly the same.

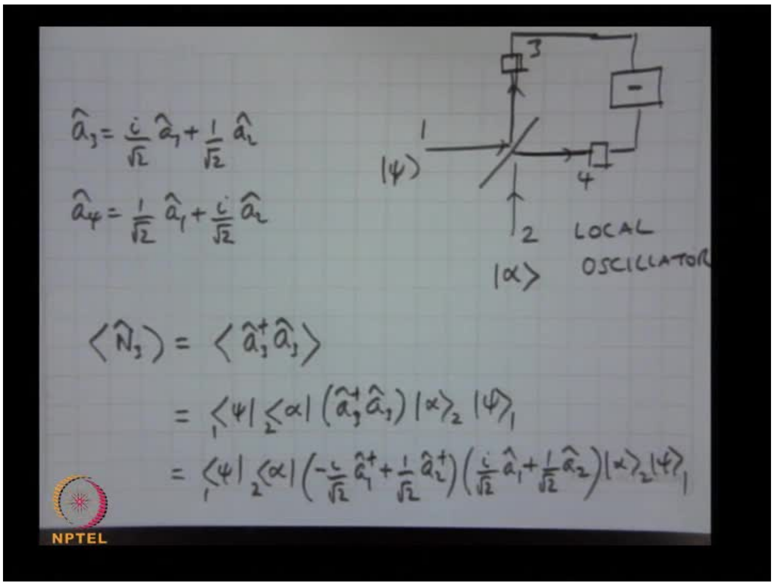
Yes Rajiv

Sir (( )) vacuum state (( )) coherent (( ))

(Refer Slide Time: 40:37)



(Refer Slide Time: 40:52)

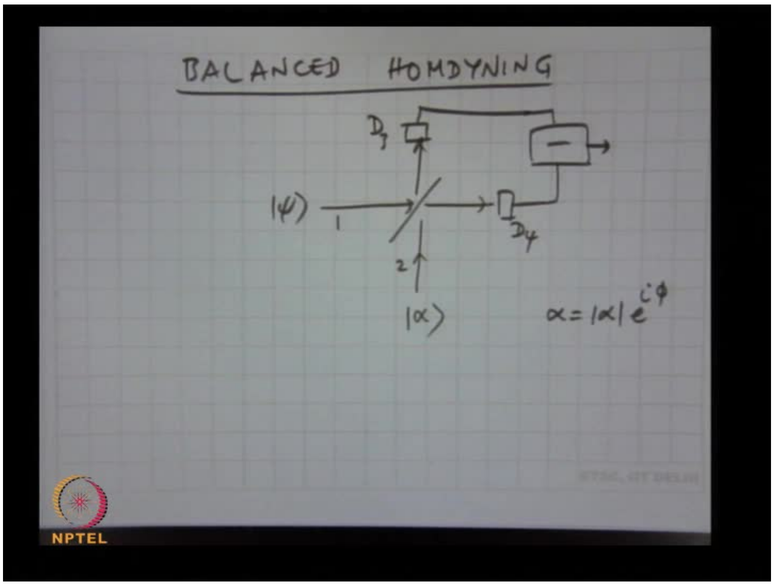


Yes this is homodyning but, I am considering another. Before I consider this, I am considering an example where the vacuum state has a coherent state with the alpha is equal to 0 also right. So I am trying to this is an example I want to use, I want to use this procedure to tell you that this procedure gives me the result that I want in the expectation values.

This is different from what we did earlier because earlier, I use these expressions to calculate what the output state is emerging from the beam splitter. Here I am not calculating the output state, I am assuming the states as if they were at the input states; the operators have got changed, the changed operators I write in terms of the input operators and calculate the expectation values. So if I input 1 photon and vacuum here, I know the probability of detecting photon is half here and half here so the expected number of photon is half here and half there which is expectation value.

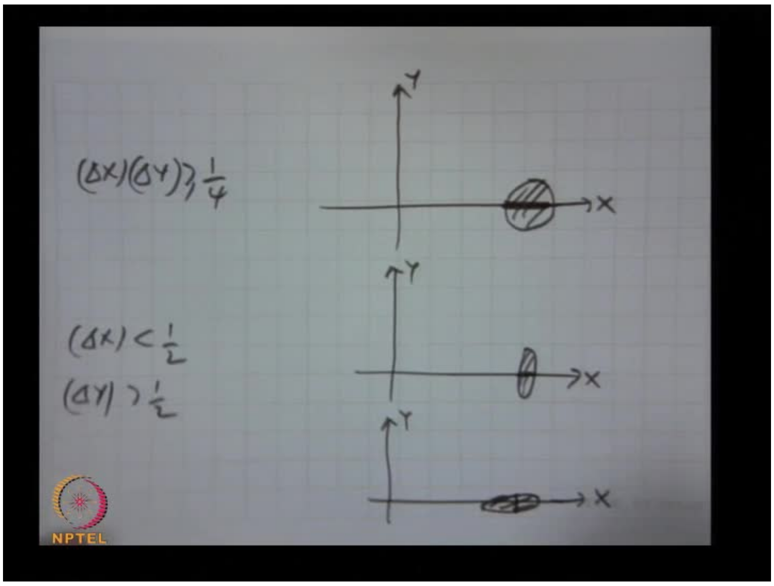
So this is the prelude to what I am going to discuss because I am going to use the same procedure in calculating the expectation value of the photon numbers. Here expectation values of photon numbers here, which I will subtract when I go through the electronic processing here.

(Refer Slide Time: 42:10)

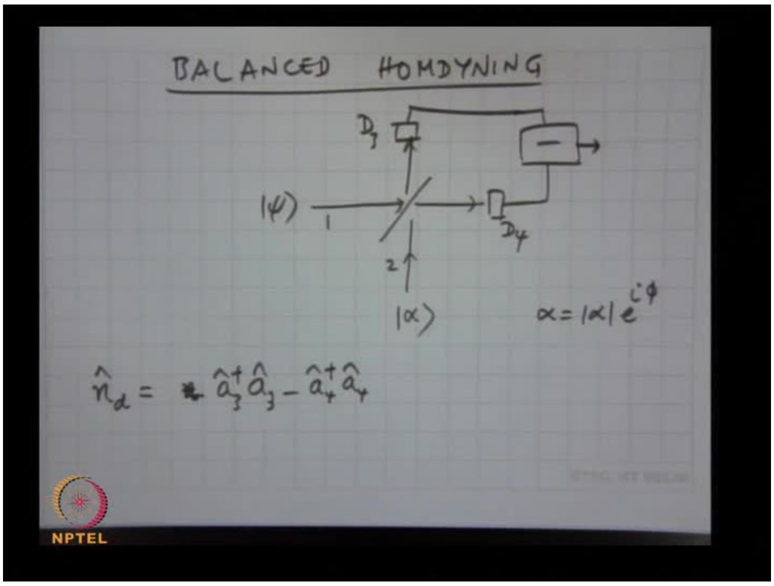


In the Homodyning, so this is what is called as balanced Homodyning, in which we have the unknown state coming from here, we have a coherent state coming in from here. These two outputs are detected and then processed; this alpha has alpha is equal to mod alpha exponential i phi, there is a phase of this coherent state. By varying the value of phi, I will show you that I can pick up X quadrature or the Y quadrature of this. That means the signal what I am getting out from here has the difference in the currents between this detector D 3 and D 4 is proportional to the expectation value of either the X quadrature or the Y quadrature or any combination of them depending on the value phi I choose.

(Refer Slide Time: 43:28)



(Refer Slide Time: 43:47)



So if I wanted for example, if at this input I had launched this kind of a squeezed light I would like to detect X quadrature and I will see that depending on the value of phi I choose, I will either measure the X quadrature or the Y quadrature or actually any other quadrature which I want any other orientation.

Sir what would be an unbalanced version of this?

This is only one that is the standard homodyning in which, they detect one of them and then process that signal.

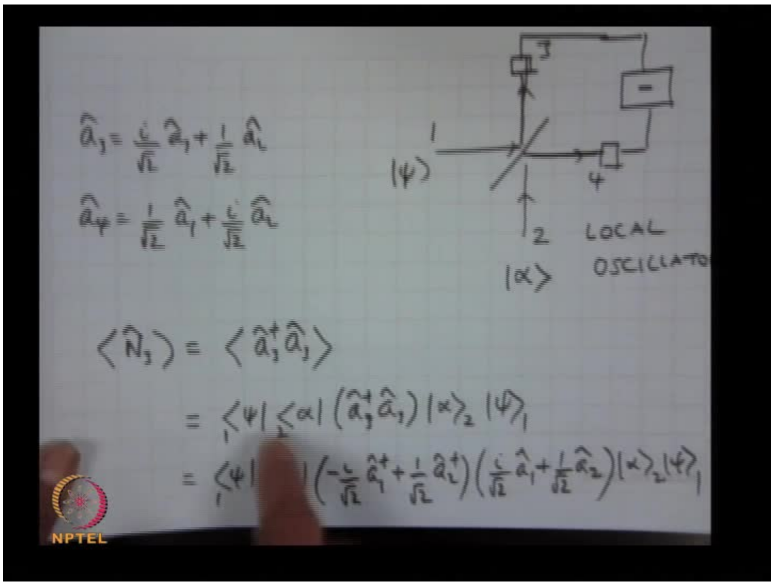
Ok

This is balancing by the other detector, what happens is in this case all noise containing here just cancels off and usually it is assumed, we assume that this is a much stronger field than this one. It is a weak field which is coming for example, squeezed vacuum. I can do measurements on squeezed vacuum coming from here and I can show squeezing in vacuum. Remember, I had shown in between some sets of sets of plots they are all by balanced homodyning techniques (Refer Slide Time: 44:10).

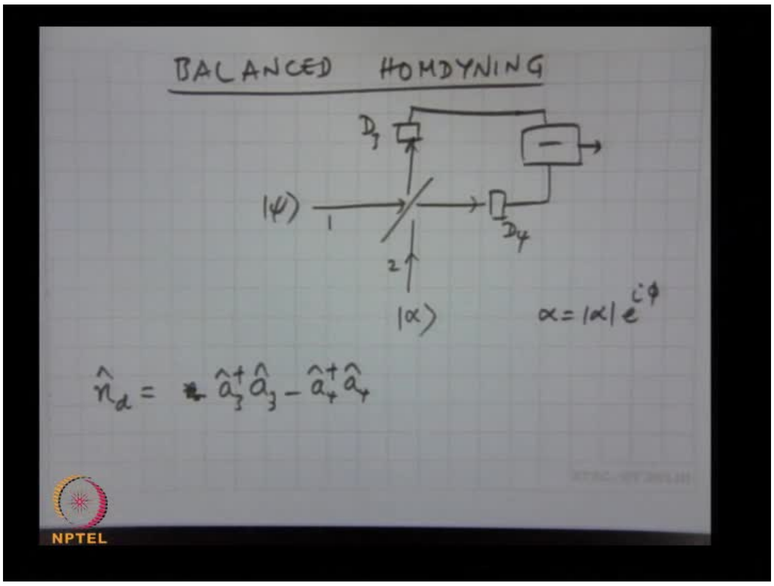
So you are actually measuring currents here but, depending on the phase that you choose here, you are actually measuring an appropriate quadrature of this input field and if it has less noise, you will see less noise here; if it has more noise, you will see more noise here in the difference. I will just give you a few steps and then we will continue in the next class. So what I want to calculate is so - what is this doing? This is measuring the - expected number of photons arriving here there is an expectation expected number of photons here and what I am doing is subtracting the 2.

What I am actually measuring is this quantity, a 3 dagger a 3 minus a 4 dagger a 4. a 3 dagger a 3 expectation value will give me expected value of photon arrival here and similarly, a 4 dagger a 4 here. When I subtract I am actually subtracting and I am actually calculating the expectation value of this quantity because a 3 dagger a 3 expectation value is here, a 4 dagger a 4 expectation value is here and when I subtract I am actually getting this difference.

(Refer Slide Time: 46:05)

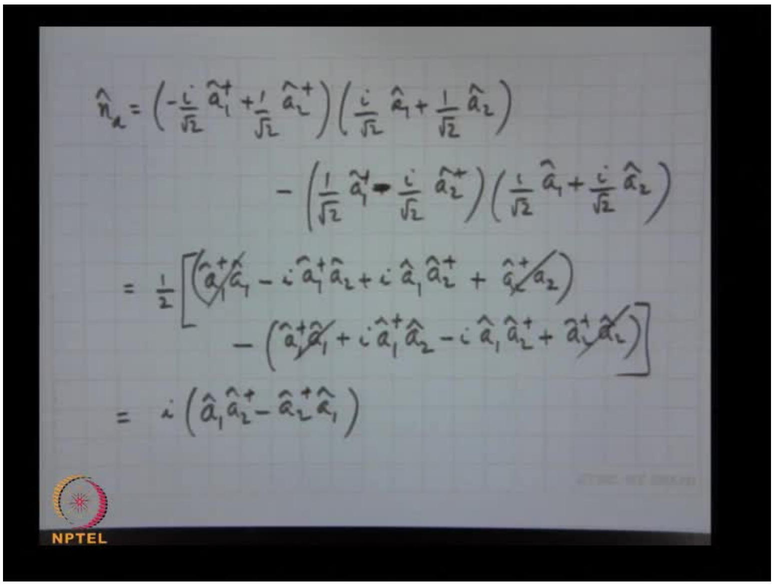


(Refer Slide Time: 46:09)



Let me just write down the expression or just derive an expression for this and then we will stop. What I do is because in the procedure we are going to adopt, we will assume the field at the input to be the representation of the state vector and we will replace these operators by a 1, a 2 operators.

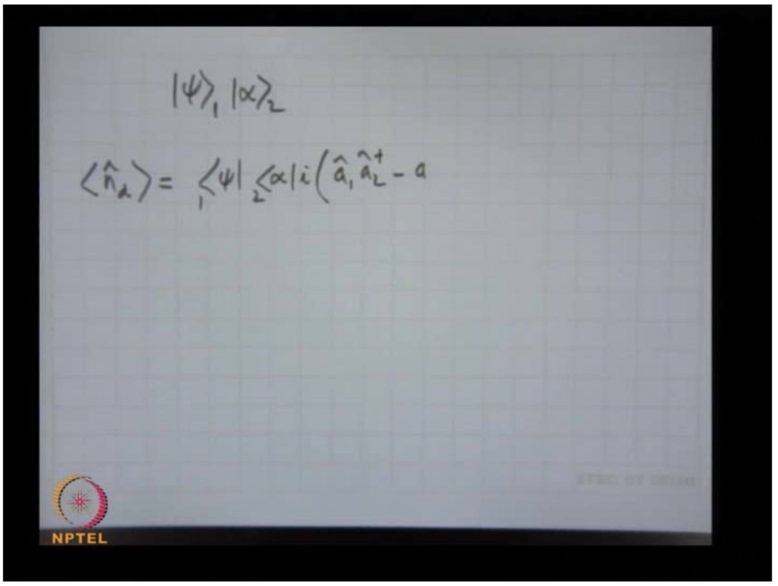
(Refer Slide Time: 46:24)



Let me calculate, what the expression for n d operator is. For this I need to substitute a 3, a 3 dagger and a 4, a 4 dagger. So n d, a 3 dagger a 3 so that is minus i by root 2 a 1 dagger plus 1 by root 2 a 2 dagger into i by root 2 a 1 plus 1 by root 2 a 2, this is a 3 dagger a 3 minus 1 by root 2 a 1 dagger minus i by root 2 a 2 dagger into 1 by root 2 a 1 plus i by root 2 into a 2, a 3 dagger a 3 minus a 4 dagger a 4.

Let me expand this, actually 1 by 2 comes out from everywhere. So, I will have a 1 dagger a 1 minus i a 1 dagger a 2 plus i a 1 a 2 dagger a 1 and a 2 commute they are 2 different modes, so plus a 2 dagger a 2 minus a 1 dagger a 1 plus i a 1 dagger a 2 minus i a 1 a 2 dagger plus a 2 dagger a 2. a 3 dagger a 3 minus a 4 dagger a 4 this (( )) cancels off, this cancels off minus i a 1 dagger a 2 minus i a 1 dagger a 2 plus i a 1 a 2 dagger plus i a 1 a 2 dagger, so this becomes equal to i times a 1 a 2 dagger minus so this operator corresponding to this difference is a 1 a 2 dagger minus a 2 dagger a 1.

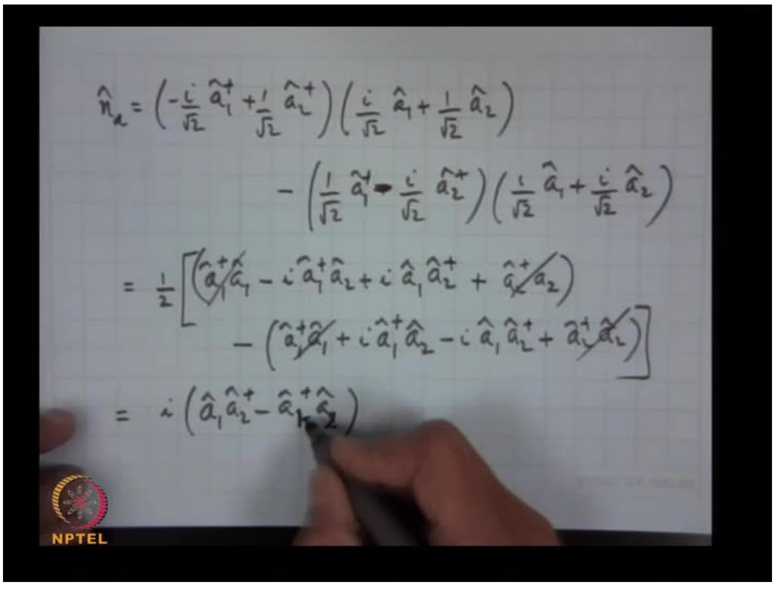
(Refer Slide Time: 49:18)



Now because the input state one of them I have chosen as the coherent state, so what is the state representing input? That state is psi in 1 and alpha in 2, so expectation value of n d is actually psi 1 alpha 2 into i times a 1 a 2 dagger.

It would be a 1 dagger a 2.

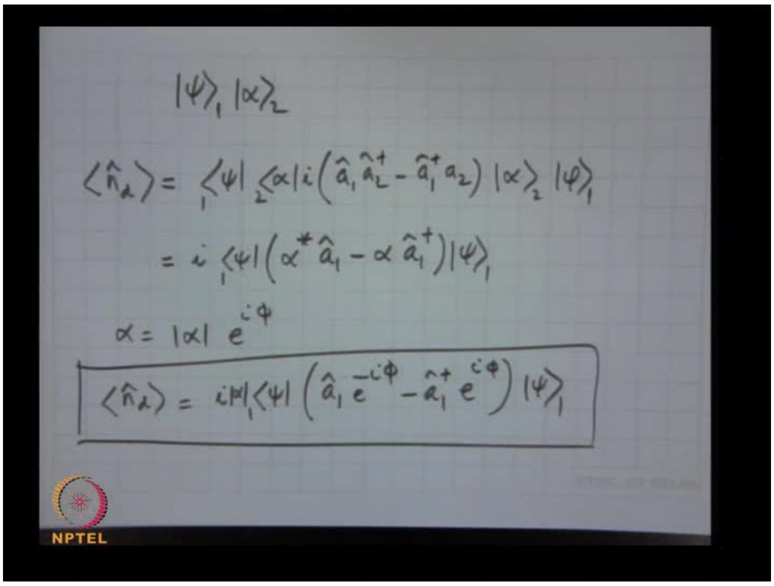
(Refer Slide Time: 49:48)



Yes a 1 dagger a 2 otherwise, they are the same terms yes.

Then are a 2 dagger a 1 and a 1 dagger a 2 dagger a 1 (( )) it should be a 2 dagger a 1. Sir, the first term yes sir a 2 dagger, first term should be a 2 dagger a 1.

(Refer Slide Time: 50:27)



Plus i yes, a 1 a 2 dagger that is right that is what I am writing a 2 dagger and a 1 and a 1 a 2 dagger are the same because a 1 and a 2 commute, because they are corresponding to 2 different modes. So a 1 a 2 dagger and a 2 dagger a 1 are the same, a 1 a 2 dagger minus a 1 dagger a 2.

So a 1 a 2 dagger minus on again alpha 2 I have not specified what psi is. The psi could be an arbitrary state, it could be vacuum state, and it could be squeeze state whatever. Now the a 2's operates on alpha and a 1's operates on psi; psi is input in the arm 1 and alpha is input on arm 2 so i times psi 1. Now what is a 2 dagger alpha bra alpha star into a 1 minus a 2 on alpha, alpha a 1 dagger.

When I open the bracket a 2 dagger is between alpha a 2 dagger alpha, which is alpha star because a 2 dagger alpha bra is alpha star alpha bra. Alpha bra alpha ket is 1 similarly, alpha a 2 alpha is simply alpha and if I write alpha is equal to mod alpha exponential i phi, this is expectation value of n d is actually i times the mod alpha. Let me take out a 1 exponential minus i phi minus a 1 dagger exponential i phi psi. I assume that alpha is a complex number with phase phi and modulus mod alpha, the alpha star is mod alpha exponential minus i phi alpha is mod alpha exponential i phi and so the expectation value of the difference in the signal detected by d 3 and d 4 is given by this.

Now you see if you choose phi is equal to pi by 2 this becomes expectation value of a 1 plus a 1 dagger and that is proportional to x operator. If I choose phi is equal to 0, this will become a 1 minus a 1 dagger and this will be proportional to expectation value of y operator. So by choosing an appropriate value of phi, I can have the signal the difference current which I am measuring either proportional to the expectation value of x operator or the Y quadrature.

So as I tune my phi I will measure the corresponding noise in those quadrature’s. If I choose phi is equal to pi by 2, I will measure the signal corresponding to the X quadrature and the corresponding noise which I will calculate in next class. This is very interesting because I am directly measuring the signal corresponding to 1 quadrature of the sync of this field that I am putting in arm 1.

We will stop here. What I will do next class is to assume that the input state is a squeeze state. We will calculate and I will show you that essentially what we are measuring is the squeeze quadrature side or the corresponding quadrature which has greater noise than the inputs than the squeezed.