Quantum Electronics
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Module No. # 05
Lecture No. # 38
Beam Splitter (Contd.)

Recall that we started looking at a beam splitter and then we looked at what happens to a photon, when it encounters a Mach-Zehnder interferometer.

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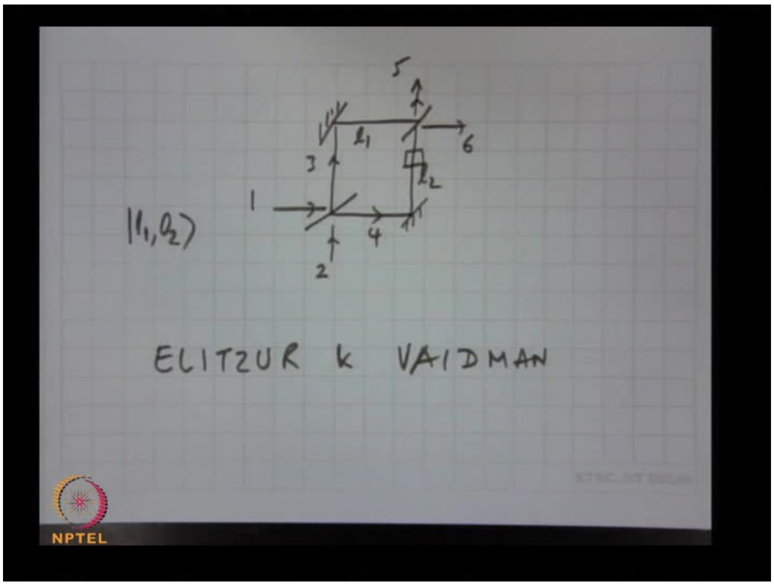

Let me recall, what we did was we had a 50 percent beam splitter generating a superposition of transmitted and reflected states then, they are reflected by 2 mirrors and then recombined another beam splitter.

So there are 2 inputs here 1 and 2, this is 3 and 4, this is 5 and 6 (Refer Slide Time: 00:55). We calculated and showed that depending on the path length, difference l 1 minus l 2, there can be photons can be coming out from port 5 or port 6 or both ports. The result is exactly the same as in classical interference because in classical waves, if I had a single wave coming in from here, it splits into two parts and then I have to traverse it through two different lengths; comes back and interferes after transmitting and reflecting at the beam splitter.

The phase difference between these two paths, depending on the phase difference, I can have constructive or destructive interference in both the arms. Quantum mechanical it is happened even for a single photon. A single photon which is incident on this beam splitter actually splits into a superposition path, superposition of these two paths and then there is interference.

As I mentioned to you, this interference takes place irrespective of the size of this interferometer; also note that if I had a detector here, then this detector will either detect the photon or it will not detect the photon (Refer Slide Time: 02:16), because then you are looking at the particle aspect of light, either it detects a full photon or detects none. If you did not have a detector, you are looking at the wave aspect of light because it goes through both arms and comes out from either 5 or 6.

Now this interferometer is very interesting because tell you that I can use this interferometer to do what is called as interaction free measurement. The question is, is it possible to get any knowledge of an object without knowing the presence of an object? Can I get knowledge of the presence of an object, without knowing anything about the object before and without interacting with it?

Now there were 2 scientists, Elitzur who proposed a very interesting idea way back in 1993 which led to what is what are called as interaction free measurements. The problem is the following; I mean they just amplified the problem to make it more interesting and exciting. I have a larger number of bombs, some of them are duds because there is one sensor which is missing and some of them are true bombs the sensor is present.

The sensor is so sensitive that even if 1 photon hits that sensor if it is a good bomb it will explode, if it is a bad bomb nothing happens. The sensor is such that if there is no sensor I can send a photon through the bomb, across the bomb. These are all closed systems all identical, can I find out even 1 bomb which I know for sure is not a dud. Classically the problem is the moment I have to observe, the moment I try to observe it will explode if it is a good bomb.

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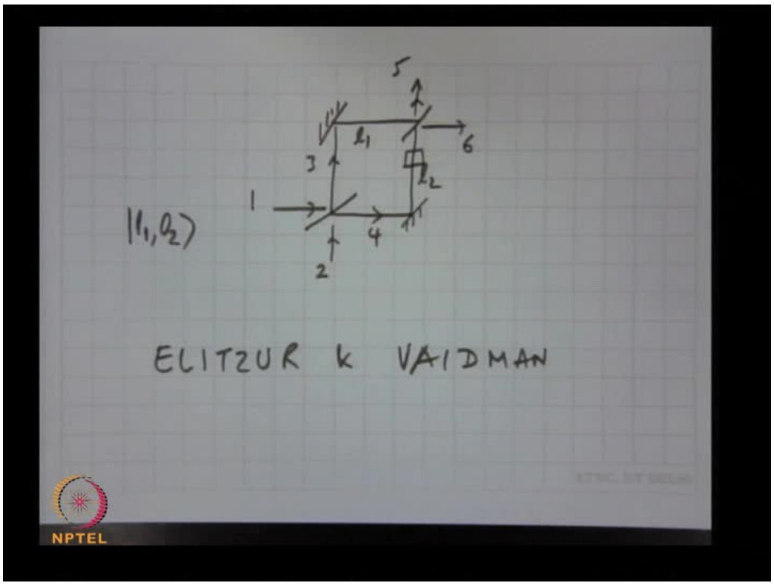

I can use this interferometer to do an experiment and give you a bomb which is 100 percent correct, good and without exploding, the idea is the following. Imagine I set up this interferometer such that as we had done if l 2 minus l 1 is equal to 0, if I send a single photon from here, where does it appear? Only in 6, if l 1 is equal to l 2 because this path and this path you can see are in phase and this path and this path will be out of phase by 5 (Refer Slide Time: 05:15).

In this interferometer, if I set up this interferometer, so that I send a single photon state from here. So I have an input state which is 1 1 0 2 then, the probability of getting a photon in phi is 0 because output state is 0 5 1 6. Now, suppose I place this bomb here there is an opening on both sides and I do not know whether inside there is a good bomb with a sensor, which is active or there is a dud which has in which the photon get through (Refer Slide Time: 05:55).

Now if the bomb is a good bomb is not a dud then, now what happens? The photon comes in, it goes into superposition state this bomb with a sensor is a detector. So in a detector, if you put a detector here, the probability of detecting is half and the probability that the photon is not detected here that means it is in the other arm is also half.

So when the photon is in the superposition path, if this bomb is a dud, the photon can get through and this photon will arrive in 6 it is as if I have not put the bomb. If the bomb is not a dud then, this photon has a half chance of being detected by this bomb in which case everything explodes and you will lose the interferometer or the photon is in this arm and comes to the second beam splitter. Now the second beam splitter, what can happen? There is no other path for interference.

So this has a half path probability of appearing in 5 and a half probability of appearing in 6. If you go back in the analysis and remove the second state, the ket 4 from that analysis you will find that there is a half probability of the photon being detected in arm 5.

When I do the experiment, I send the first photon it may be a dud bomb so the photon arrives here. If the photon arrives here, I know the bomb is a dud or it is a good bomb but, the photon has taken this path; I have no idea, I have no way of distinguishing between these two.

If the bomb explodes everything is gone but, if the bomb does not explode and the photon arrives in 5 it means a bomb is a good bomb because you have destroyed interference because it is like a detector here. Detector has not detected it but, because the photon has arrived in 5, it means there is an object in path 4 and because the photon has been detected in 5 it has not interacted with the object, because if it had followed this path, it would have been absorbed by the object.

A bomb is a magnification I can have a simpler object here; if I have an object here, if I set up the interferometer, so that all photons incident in 1 arrive in 6 and if I put an object here or in the experiment suddenly I find photons arriving here, it only means that this arm or this arm one of these arms is blocked by an object (Refer Slide Time: 09:00). That photon that you have sent has not interacted with the object because if it had interacted, that bomb would have exploded.

Sir, can we destroy the interference effect to lower what the path of the photon might be?

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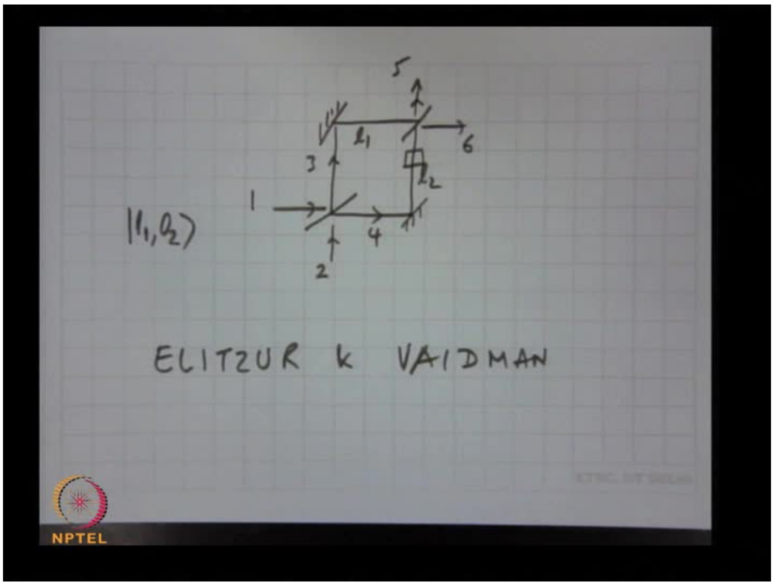

Yes but, please remember here the photon has gone into a superposition path but, it has not been detected by the object. The fact that the photon arrives here means, the object has not detected the photon; the fact that the photon arrives here also implies there is an object in another paths.

So the photon has not interacted with the object but, I know there is an object there. This is a kind of a result of an experiment, which says that if the photon at all along that path it would have exploded the bomb. I get a result which is of a possibility, so this is called interaction free measurements.

Sir (( ))

I have a photon incident from here, let me take two cases. Suppose I send a photon from here and without this object, I will never find a photon arriving in 5. Now for these please remember, it is not 1 photon I should send, I should keep on sending a number of photons every time I find all of them arriving here.

Now somebody does something with the experiment and I see a photon arriving in 5, what could it imply? I have destroyed interference that means in one of these arms there is some object, which has blocked this light.

For example, here (Refer Slide Time: 11:11). Now what I am saying is the arrival of a photon in 5 implies the existence of an object in the arm and the fact that the photon has arrived in 5 implies that the photon has not interacted with the object because if it had come along this path, the object would have observed it. The photon cannot arrive here, if it had followed this path. So it has followed this path to arrive here, which means it actually knows the existence of an object in the other paths.

Sir, this is with probability just 0.25.

Yes but, there are more advanced versions of this where the property can go to very high values. I can modify this, I do not have to just 50 percent reflecting mirrors beam splitter, I can choose different and I can increase the probability significant. So these are called interaction free measurements and may be it can lead to applications. For example, I may have an object in which I do not want to illuminate by even very little; I mean that is for X rays.

So I can actually detect presence of objects without having the x ray at all interacting with the objects. Things like this, there are very interesting consequences of this kind of an experiment and so these are called interaction free measurements and this is a very interesting direction of research.

You see, quantum mechanics within itself with a lot of interesting possibilities and one of them is say I thought would be interesting to tell you is this particular one. So what is actually happening is, what happens in this areas is, if you have information about the path the photon takes in the interferometer, no I mean modify. If there is information in the experiment, which can lead me to the knowledge of the path of the photon in interferometer, you will not observe interference effects.

Please note I may not access that information. If there is information in the experiment which can help me identify the path taken by the photon, then there is no interference. If there is partial information available to me, there is partial interference. If there is complete information available, there is complete destruction of interference. If there is no information available, there is perfect interference.

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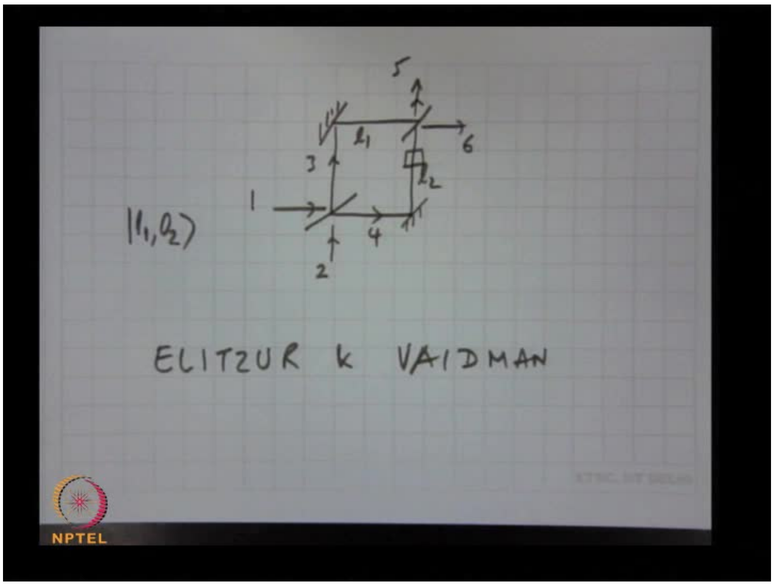

So, it is more to do with information, rather than my act of observation disturbing the system. I do not have to even observe meaning; the existence of information in the system can lead me to destruction of interference in the experiment.

Sir,

Yes

If I observe, photons coming out of port 5,

Yes

I have no way of knowing in which path 3 or 4 as an object in it.

Yes

So I am sort of saying that I put an object in 4, in the area of 4. In the bomb problem for example, I put my bombs in 4.

True

Otherwise, I have no way of identifying whether it is 3 or 4 but, there is an object in this region.

But then, I do not know what path the photon's travelled.

I do not know - I do not need to know.

No, so if I will say that in this region of volume, there is an object; I mean I do not say it is in this region only I can specify, now this entire region there is an object (Refer Slide Time: 14:40). This could be a black box, right? There could be an entry here, entry here and exit here and exit here. All I will say is that initially all the photons are coming here they were all coming out from here, suddenly I see a photon here that means in this volume there is an object.

No, I am asking because you said that the reason, why we are destroying interference because, we have some information about the path that the photon travelled.

Yes

But, I am seeing that we do not know what path the photon took, we just observe it coming out of port 6 port 5.

No for example, suppose let me assume that this mirror, one of the mirrors was wobbling. Then suppose, the presence of wobbling mirror immediately tells me information about the path, because if the mirror wobbles when I send the single photon, I know it has taken this path. If the mirror does not wobble, it has gone through the other path; it is like a detector, it is what is called as a non-demolition experiment. I am not killing the photon but, I am still having the photon but, I observed the path of the photon. The moment I have vibrating mirror in one of the arms, one can show that interference is gone.

So it could be in the situation what I am trying to say. For example, let me assume this is the polarizing beam splitter, if it reflects horizontal polarization and reflect transmits the vertical polarization. When they go through different paths you will not observe interference because two orthogonally polarizing do not interfere. Now, why they do not interfere? Because if I measure the polarization state of the photon and if I find horizontal I know it has taken this path, even in vertical, I know it has taken the other paths.

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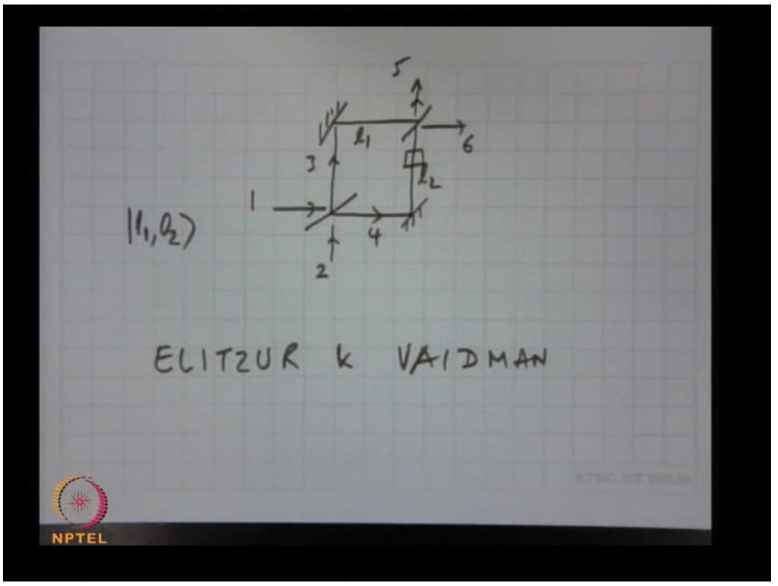

So that information is contained in the experiment. The existence of information in the experiment destroys interference immediately, if I have partial information, partial destruction interference, if I have full information, full destruction interference.

So I do not have to disturb the system, you see many times we say in the electron microscopic explanation for example, you say that if you try to observe which hole the electron passes through you scattered a photon from there and find out etcetera. Those are all right but, there are also experiments in which I do not disturb the system at all and the existence of information on the path of the photon will now destroy my interference because you are not able to observe both the particle and wave aspects simultaneously at the same time.

There are more advanced versions of this experiment using a multiple reflections and so on. There are people who are very interested in these kinds of an analysis and which could be possibly now its envelope.

Now what I want to do is I want to discuss something which I want to use a little later also for discussing. How do I detect light? Normally I have a photon detector. So what does the detector do? When lights falls on it, the light get absorbed and it creates electron hole pairs in a semiconductor or it excites or it ionizes an atom, electron comes out and that will electron is accelerated and I detect this current in the external circuit.

The basic process that is happening is ionization of an atom by the photon. So, a photon detection process is something like a photo ionization of an atom and detecting the electrons that are emitted by the ionized atom.

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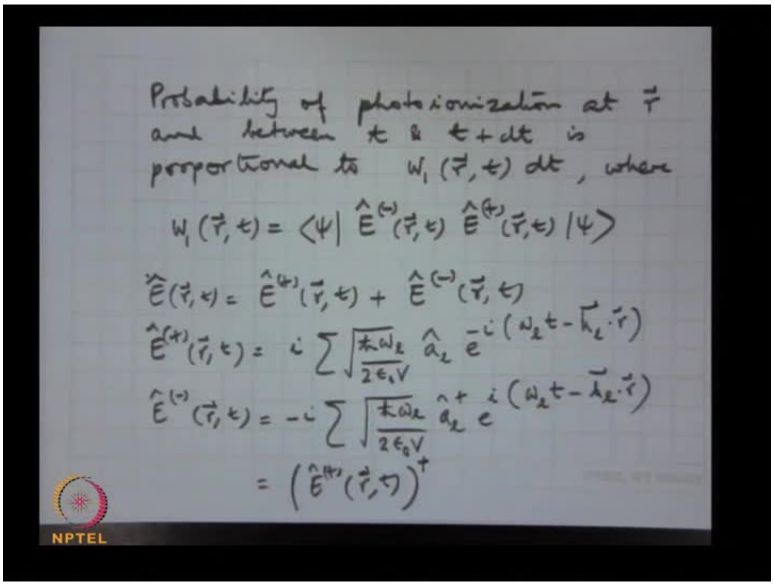

Now there is an analysis which one can do that means this is an interaction of light with atoms, calculating what is the probability of absorption etcetera which I will not go through. But, what happens is the probability of photo ionization at position r and between time t and t plus dt is proportional to what is called as w 1 r t dt, where w 1 r t is expectation value of I will define this E minus r t E plus r t, where E r t is equal to E plus r t plus E minus r t with E plus r t is i sigma square root of h cross omega l by 2 epsilon 0 V a l exponential minus i omega l t minus k l dot r and E minus r t is equal to minus i sigma square root of h cross omega l by 2 epsilon 0 v a l dagger exponential i omega l t minus.

The total electric field operator is split into two parts; one having the destruction operator a l and the other having the creation operator the a l dagger. The a ls are associated with the exponential minus i omega t minus k dot r and this plus implies essentially, this is called the positive frequency part of the field. Although it is expression minus i omega t it is called the positive frequency and E minus is the negative frequency component. As you can see here (Refer Slide Time: 21:16), this is the hermitian conjugate of this E plus dagger is E minus. This is the probability of photo ionization at a position r vector between times t and plus dt that is proportional to this w 1 over r t. Please note that it is not expectation value of E square, intensity is proportional to square of electric field and intensity gives me the probability of detecting light at that point; the higher intensity, the greater the probability of detection or photo ionization.

But this is an expectation value of product of E minus and E plus. Note that E plus contains destruction operators, E minus contains creation operators. So in this product here, all destruction operators are on the right side and all the creation operators are on the left side. All a l daggers will appear here and all a ls will appear here (Refer Slide Time: 22:27), this particular way of ordering these operators is called normal ordering. The number operator a l dagger a l is a normally ordered operator, number operator square is not normally ordered because it is a dagger a a dagger a so, normal order would have been a dagger a dagger a a.

All destruction operators on the right and all creation operators on the left is normal ordering and this is actually, normally ordered product of the two electric fields. This is the total electric field of the electromagnetic field operator, this is the positive frequency part, negative frequency part and the probability of photo ionization at any point r for a given state psi is given by this (Refer Slide Time: 23:26), now this will become clear as we look at some examples.

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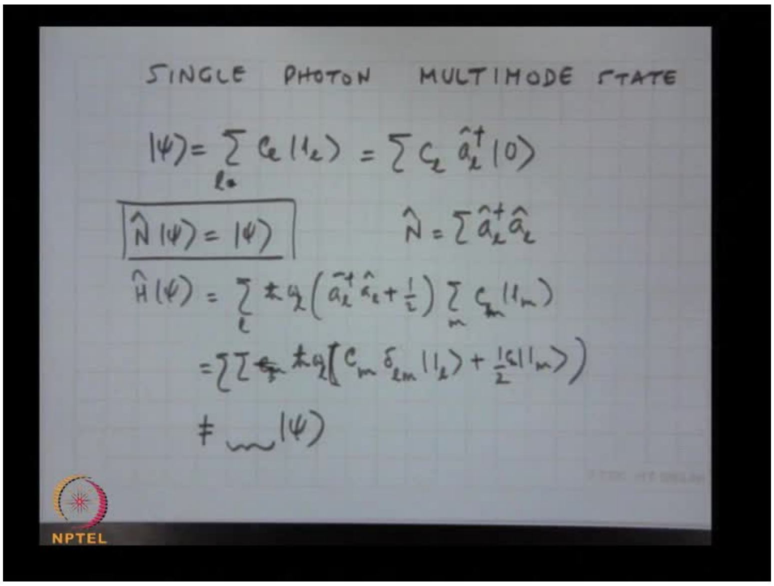

So first, let me start with the simplest example of a single photon multimode state. Remember, we had written this as this summation over l; this is actually some expansion coefficients a l dagger 1 0.

We had already discussed this earlier you can show that, where N is the total number operator which is sigma a l dagger a l. This is an Eigen state of the total number operator which means the total number of photons in this state is exactly 1 (Refer Slide Time: 24:34). This is not an Eigen state of a l dagger a l operator, which means it has not well defined photon in the l th mode because that single photon is in a superposition of various modes. These modes could correspond to different frequencies, different propagation directions, different polarization states whatever it is; this is also not an Eigen state of the Hamiltonian operator.

If you calculate h psi, this will not be proportional to psi because this will be essentially sigma h cross omega a l dagger a l plus half, so this if I call this l, this is m p l 1 m. This will be equal to sigma c m, this is h cross omega, a l dagger a l operating on this will give me a l dagger a l operating on 1 m; if l is equal to m, it gives me 1, if l is not equal to m, it gives 0, so this will be delta l m 1 l plus c m will come there also.

So you see this is not proportional to psi, this is not an Eigen state of the Hamiltonian operator. This is not a well-defined energy state because this photon exists in multiple modes of propagation. This is not an Eigen state of the h operator but, this is single photon state, there is only one photon in the state because of this condition.

Omega l

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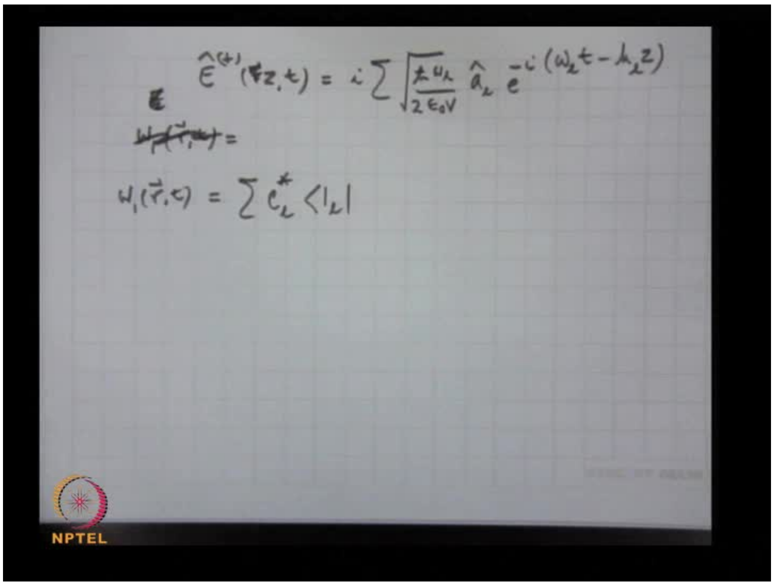

Now, let us look at what is this probability of photo ionization for this state. So this electric field, I must calculate w 1. Let me take these states to be waves propagating in one direction - z direction and same polarization state.

The electric field operator so instead of r t will be z z t because all of them are assumed to be propagating in the same direction, we will have square root of h cross omega l by 2 epsilon 0 V a l exponential minus i omega l t minus k l z. So k dot r I have replaced by k l time z and I am assuming scalar fields which means all fields, all the modes are in the same polarization state otherwise, electric field operators is a vector operator. So, I am assuming that all polarization states are parallel all are polarized in one direction.

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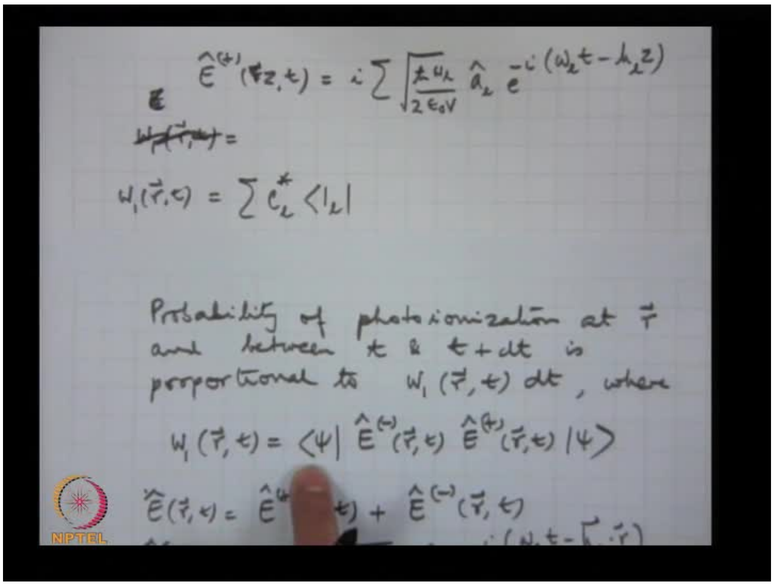

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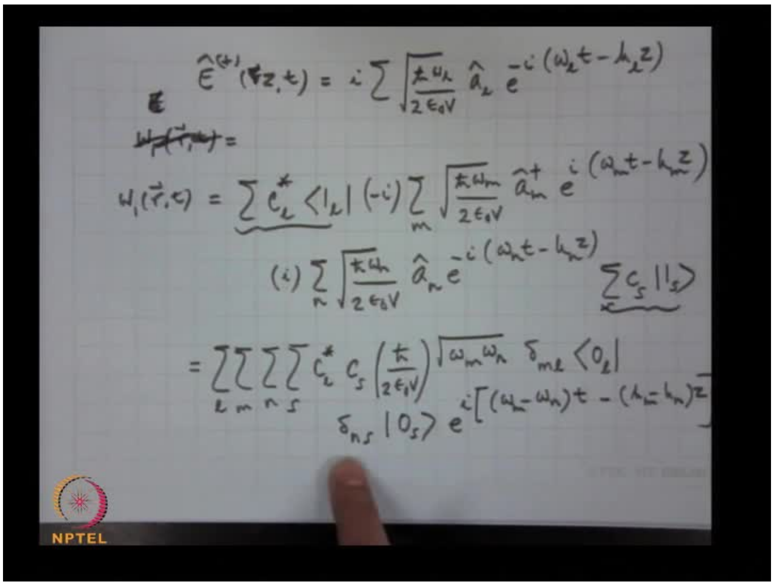

So, the total electric field is simply given by a scalar sum of all the components. To calculate this, let me write here w 1 of r t sigma c l star 1 l; this is this part bra psi is sigma c l star 1 l then, I must multiply by E l minus E l plus which is minus i m. Another index h cross omega m by 2 epsilon 0 V a m dagger exponential i omega m t minus k m z into E plus which is i sigma n square root of h cross omega n by 2 epsilon 0 V a n exponential minus i omega n t minus a n z multiplied by sigma c s 1 s.

So this is bra psi, this is ket psi, this is E minus and this is E plus operator. E minus operator, E plus operator, ket psi, bra psi, now minus i into i is plus 1. Now, let me operate this on the right and this on the left, so can you tell me, sigma l, sigma m, in fact let me write all the sums here sigma n and sigma s; there are 4 sigma's sigma l, sigma m, sigma n, sigma s. So c l star c s I will have h cross by 2 epsilon 0 V square root of omega m omega n.

Now what happens when a m dagger operators on 1 l? If m is not equal to l, it gives me 0; if m is equal to l, a m operating on 1 l.

Ket l delta m l into

L plus 2 0

A dagger operating on ket increases it is a creation operator but, a dagger operating on the bra will decrease, so it will be 0 l. Then this one so I have the 2 exponential, a n operating on this, a n operating on 1 s gives me delta n s 0 s into exponential i omega m minus omega n t minus a m minus k n z it looks big but, it will get simplified very quickly just keep watching.

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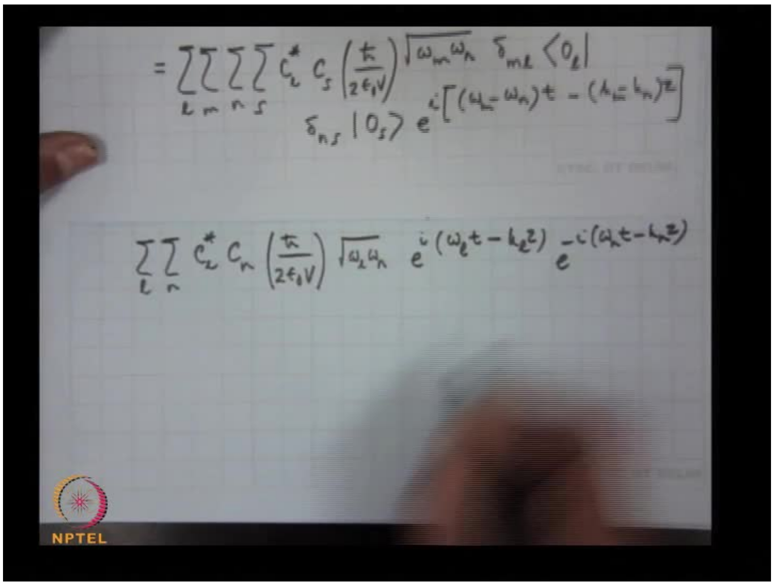

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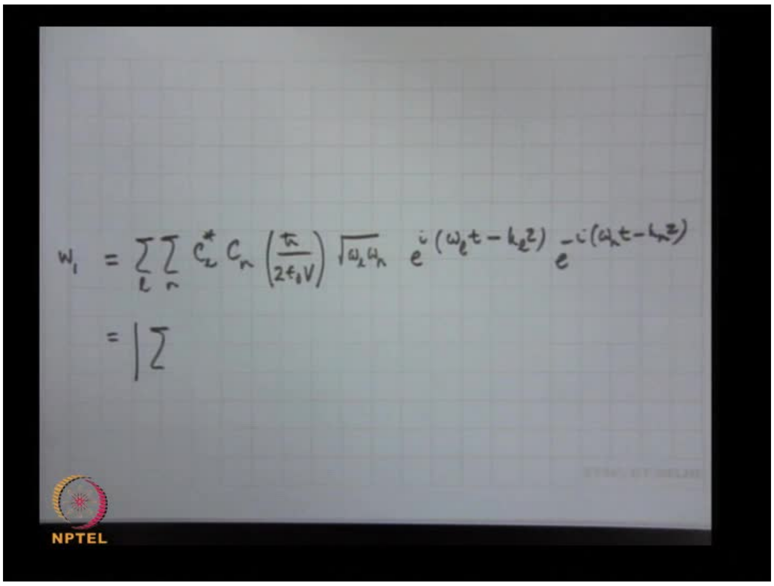

Now I have delta m l, so this m l sum becomes a single sum and delta n s so this also becomes a single sum, so what do I get? I will have sigma l sigma n so, c l star c s becomes c n n is equal to s h cross by 2 epsilon 0 V root of omega l omega n. Now 0 l 0 s 0 l means what? All are in vacuum state because when I write just 1 l it means that the lth mode is in single photon state and all other modes are in vacuum state; so this means it is a complete vacuum state, so this bracket is 1. I will have exponential i let me write omega n becomes l omega l t minus k l z into exponential of minus i omega n t minus k n z.

Sir delta l (( ))

Sorry

Sir there is a delta l s

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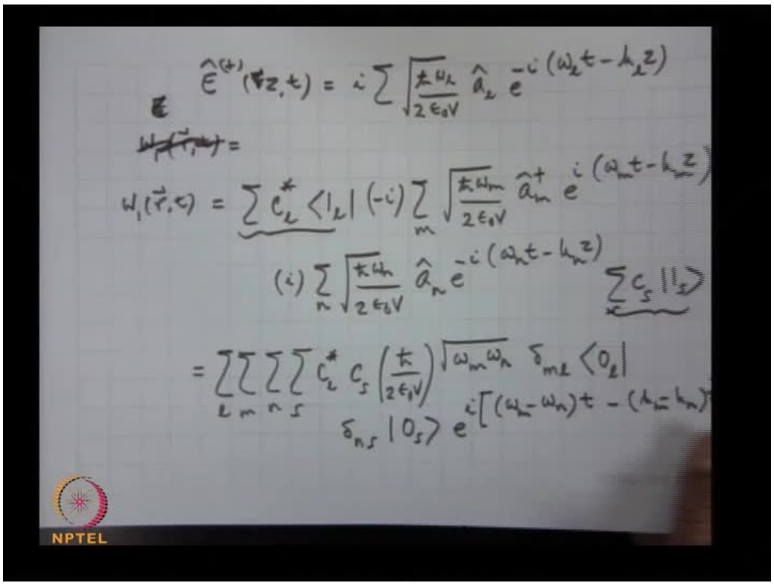

No, this is 0 l is a complete vacuum state all and this is 1, there is no s l because this is 0 s; I am just writing this subscript for the state in which has got destroyed.

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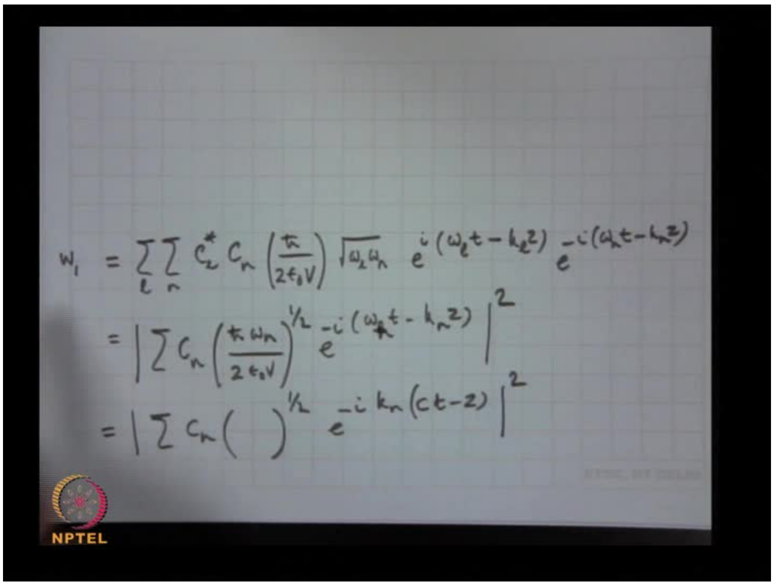

So this is sigma actually, this you see this is nothing but, c n h cross omega n by 2 epsilon 0 V raised to power half exponential i omega l t minus omega n t minus k n z, c n c l star square of this and then you have 2 exponential which are complex conjugate. This is actually a mod square of this, which I can write as raised to power half exponential minus i k n now, what is omega n by k n?

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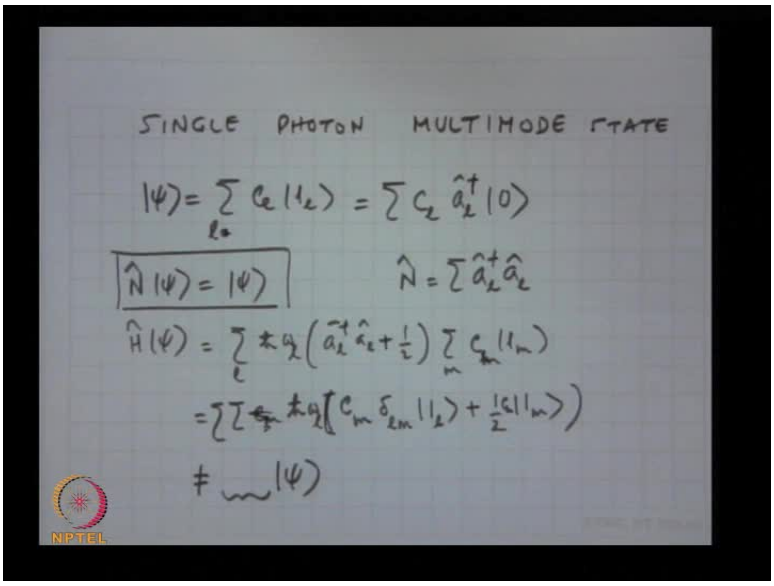

Yes c the velocity of light in free space, (( )) of m all modes are in free space so omega n by k n is a velocity of light in free space. This tells me that the probability of detecting please remember that when I discuss the single photon states, I said that I can have a multimode single photon state. It is like a wave packet depending on the c ls which I choose here in writing this expression here depending on the c ls, which I choose here. I can have a state in which its laterally confined because depending on the frequencies which I add here, I can have a certain spectrum of frequencies which will lead to a wave packet of a finite duration.

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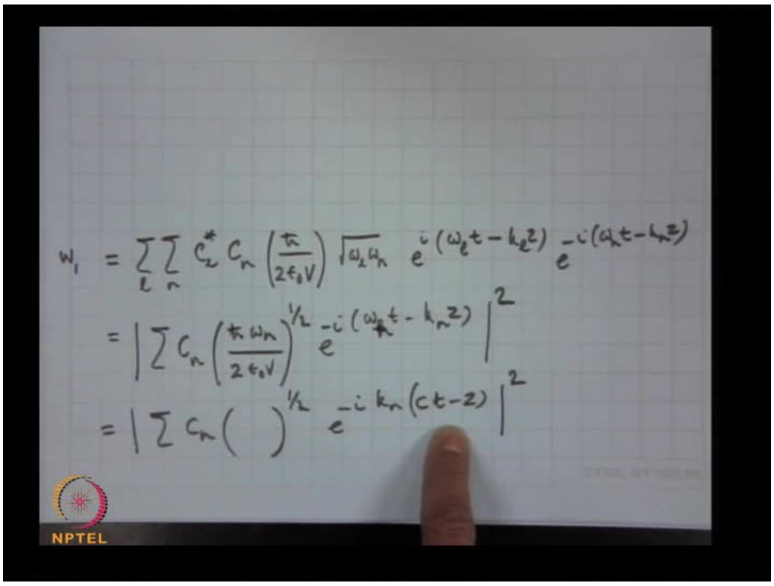

So this implies that this wave packet is moving at the velocity of c because if the probability of finding the photo ionization was maximum at this point at t is equal to 0, at some other time the photo ionization property will become maximum at a different point. It is as if this wave packet is moving in free space at a velocity c, I am only calculating the probability of photon detection.

If I create a multimode state like this and if the probability of photoionization is maximum at a certain time and will drop of on either side depending on the expansion coefficients are chosen. It is exactly like a wave packet but, only consisting of a single photon and this probability wave is actually moving like this in free space at a velocity of light c and if it was here at this time t is equal to 0, it will be here at t is equal to. This is a probability wave, that means the maximum probability of finding the photon or photo ionization was here at t is equal to 0, at a later time it is here, at a later time it is here, later time it is here.

So when we say, when a single photon is incident on a Mach-Zehnder interferometer what we mean is actually it is just kind of a wave packet, it is not a single frequency state; please remember this is not a single frequency state. This contains for confining the probability in the finite region of space, I need to add multiple frequencies exactly like in Fourier transform. It is a Fourier transform, if you take sinusoidal waves they have infinite duration, to make a finite duration wave train I must add many frequencies I am doing exactly the same thing here.

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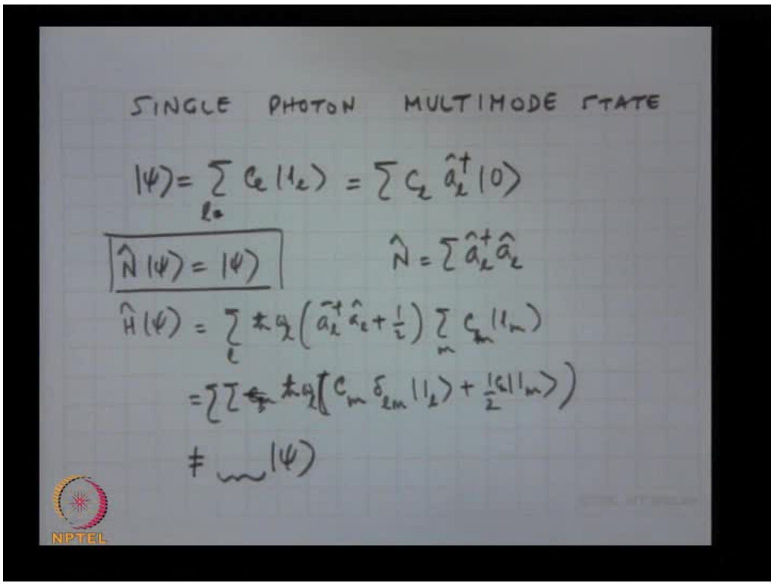

If I put that single photon in a multimode superposition state of different frequencies, I will get a more localized wave packet which will define the region of space in which the photon may be found. Highest probability of finding this photon it is like spontaneous emission, when a spontaneous emission comes out it, comes out in a single photon multimode wave packet.

So when we say a single photon is incident on the Mach-Zehnder interferometer, it is usually this kind of which is coming out from a spontaneous emission or parametric down conversion or whatever it is. A single photon wave packet comes and hits the wave, interferometer gets split into two paths and goes continues.

Yes

Sir, we are trying to interpret the probability of photoionization in terms of the presence of photon at that point.

I am not saying the photon is there, I am just saying the probability of photo ionization. That is all I can say, yes.

That is fine.

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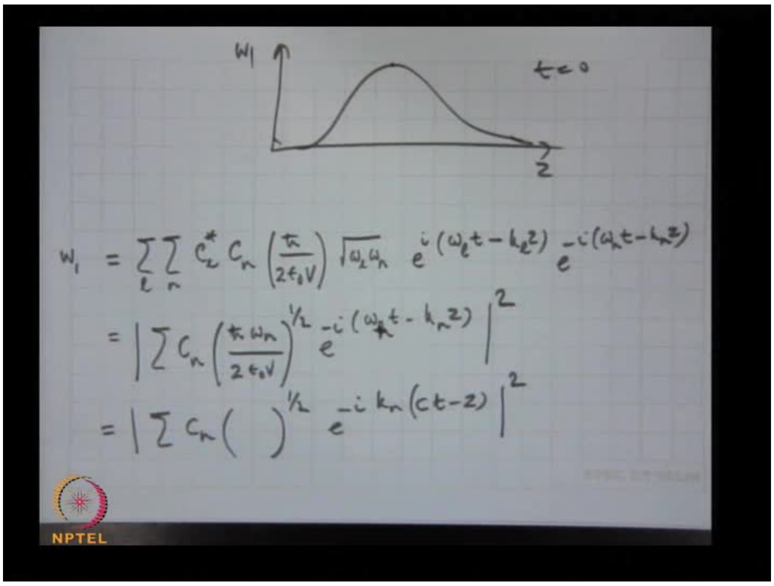

So if the probability of photoionization goes like this, this is w 1 for example, as a function of space for a given time, then I will say maximum probability is here, I am not saying the photon was here.

No

Yes the single photon wave packet defines me a longitudinal distance within which I will most probably find a photon.

So this gives me the probability of photo ionization in (( )).

Yes

If this kind of multimode single photon state is incident on a beam split or a beam splitter Mach-Zehnder interferometer, so how can we say that it will have this kind of probability distribution.

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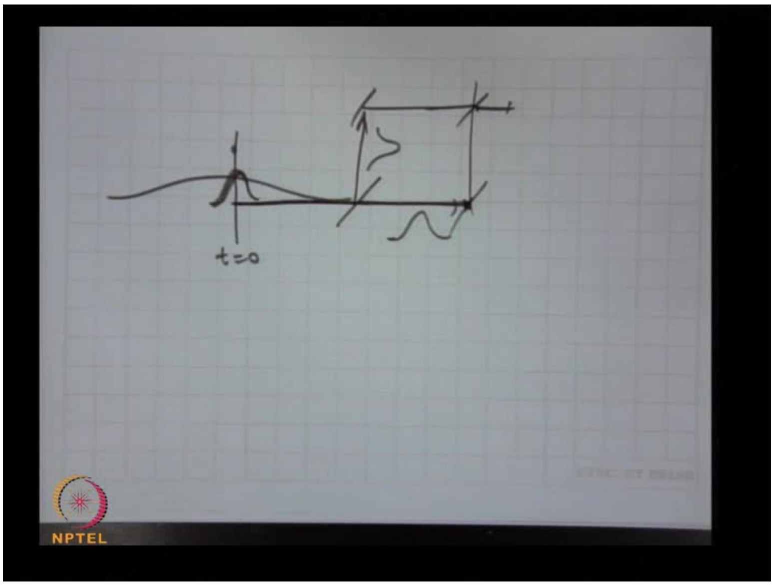

No, if I take this and if I incident from here, this split into 2 parts. If I put a detector here if I launch at t is equal to 0, I will find this suppose I launch a single photon at t is equal to 0 from here, that means I have an atom here spontaneously it emits at t is equal to 0.

This photon then travels I can tell you so, I have a detector here which is trying to detect the photon, I will detect it after a certain interval of time with a probability half and over duration, please remember. For example, if it takes a picosecond to arrive here, there is no point in waiting for a nanosecond because the photon might have gone to the other side anyway.

So if I know this distance and if I know the duration of the photon wave packet, I know when I am expecting to detect the photon here.

Sir, over duration means the variance in the time of duration.

Yes, depends on the wave packet duration.

If the wave packet in much longer then

Yes, now please note that this is related to coherence time it is all inter related.

Yes

I can actually, when we do an experiment with Mach-Zehnder and I say when a single photon is incident, it is essentially stating that the single photon wave pocket comes and split into 2 halves and then continuous and interferes. Please note that the path difference should not become much longer than the spatial extend of wave packet. Then there will no interference because they will arrive at different times of the output beam splitter. That is exactly your condition for interference when you do a classical interferometer and make sure that the path difference is less than the coherence length, it is the same thing.

In a quantum factor, sorry

If the path of the arms it does not the coherence (( ))

Then there will be interference because then they will arrive simultaneously almost

Yes

If the path length difference is much larger than coherence length,

(( )) they will not be in I mean the phase cannot be maintained.

In classical

But, what it means that the probability these two will not arrive at the same time at the beam splitter and they will not interfere. Then you will have finite possibility of detecting the photon here as well as here. So, we will stop here.