Quantum Electronics
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Module No. # 05
Lecture No. # 37
Beam Splitter

We started looking at effects of a beam splitter on input states. Now do you have any questions? So, let me recall what we did was, problem we are looking at is you have a symmetric beam splitter.

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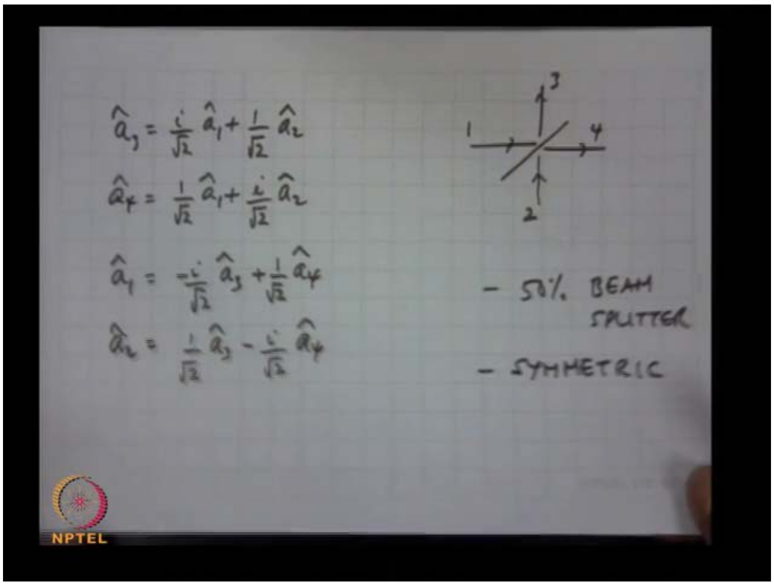

There are 2 inputs ports called 1 and 2, there are 2 output ports called 3 and 4. If it is a symmetric beam splitter, then we used energy conservation between 3 and 4 and 1 and 2 and got some relationships between the aptitude reflection and transmission coefficient. And what we found is a classical relationship between the electric fields of the light emerging in 3 and 4 with respect to light entering at 1 and 2. And then, we replace those classical electric fields by annulation operators and got these equations, a 3 is equal to I by root 2 a 1 plus 1 by root 2 a 2 and a 4 is equal to 1 by root 2 a 1 plus I by root 2 a 2.

These can be inverted and we obtain the following equations of the inverted, I mean expressing a 1 a 2 in terms of a 3 and a 4. So, a 1 is equal to minus I by root 2 a 3 plus 1 by root 2 a 4 and a 2 is equal to 1 by root 2 a 3 minus i by root 2 a 4. This is assuming 50 percent beam splitter and symmetric. So, the reflection coefficients from 1 to 3 is the same as from 2 to 4 and similarly the transmission coefficient from 1 to 4, which is same as 2 to 3. So, these relationship relate the annulation creation operators and 3 and 4 with respect to 1 and 2, and we started looking at using these to understand what happens when certain states of light are incident at the input.

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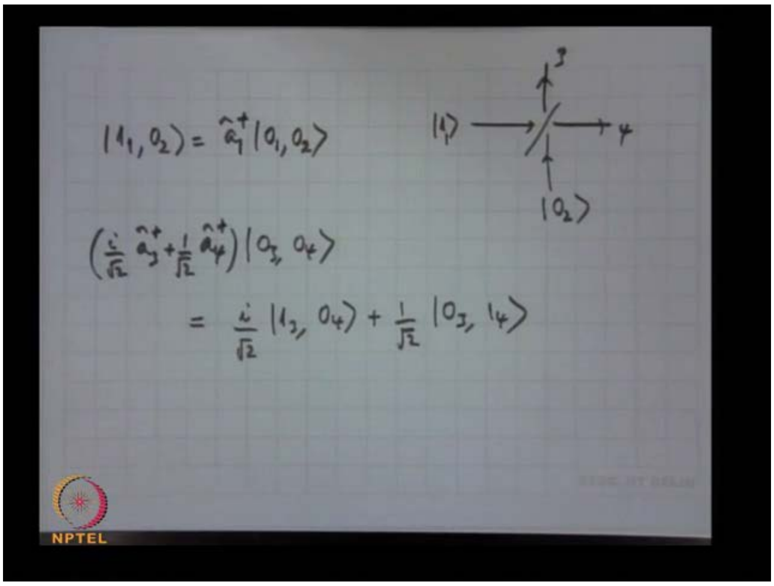

So, what we did was in the last class we looked at the following situation, that I have a single photon state incident on 1 and nothing incident on the port 2. So, as I mentioned last time, we cannot neglect the presence of this second input port in the calculations because if you assumed a 2 was 0. So, even if I have light incident on 1 only, I cannot neglect 2 because if I forget about a 2, then the output relations a 3 and a 4, do not satisfy the commutation relations. a 3 a 3 dagger is not equal to 1 a 4 a 4 dagger is not equal to 1. So, I need to always remember that even though port 2, I may not be actually illuminating by any light I still have the vacuum state entering.

So, the first example we looked at was, a single photon incident on port 1 and nothing incident on port 2. So, we said that input vacuum states in 1 and 2 give me output vacuum states at 3 and 4. So, we wrote the input state which is given by this as a 1 dagger 0 1 0 2. So, the output will be, I will to get the output I express a 1 dagger in terms of a 3 dagger a 4 dagger and use the fact that vacuum incident at 1 and 2 gives out vacuum at the output 3 and 4, so 3 4 and I have a dagger. So, I by root 2 a 3 dagger plus 1 by root 2 a 4 dagger. And this is the output state which gives me I by root 2 1 3 0 4 plus 1 by root 2 0 3 1 4. And this is a state in which it says that this is a superposition of a photon coming out in 3 and no photon in 4 and another state in which there is a photon coming out in 4 and there is no photon in 3. So, this is a superposition state and these 2 paths 3 and 4 are actually entangled by this operation here. This cannot be written as a product of states in 3 and state in 4.

Sir, how it will be go from 0 1 0 2 to 0 3 0 4? You did not attend last class. If I have no light states incident in 1 and 2, because this is a linear operation and vacuum coming out at 3 and 4. So, 0 1 0 2 gives me 0 3 0 4 output and a 1 dagger gets modified to a combination of a 3 dagger and a 4 dagger. Because of this relationship, the a 3 and a 4 are related to a 1 and a 2, if I invert these equations a 1 and a 2 are related to a 3 and a 4 and the way I analyze is I write the operator a 1 in terms of a 3 and a 4 and use the fact that vacuums in 1 and 2 gives me vacuum in 3 and 4.

Sir, but we did not operate anything to get off like when. So, if I have vacuum input in 1 and vacuum input in 2, what will be output 3 and 4? 0 3 0 4. So, this is the output in the absence of any allied input. With input 1 photon the input state I write as a 1 dagger 0 1 0 2, now to get the output state, I represent a 1 dagger in terms of a 3 and a 4 the output states. So, a the 1 dagger is given by I by root 2 a 3 dagger plus 1 by root 2 a 4 dagger, 0 1 0 2 is given by the usual 0 3 0 4 at the output and now if I have operate this I get the output state. What I will do little latter is use another formulation where we will use these operators to represent the operators at the output and we will get the measured measurements, any observables measured at 3 and 4. For example, what is the probability of detecting a photon here or what is the electric field coming out expectation electric field coming out here etcetera by a slightly different procedure, where we will assume, it is something like an eigen, but picture we will use this to obtain the representation, but here this gives me a simplified picture of what kind of output state I expect from this inputting the single photon in state in one of the input ports and vacuum in the other port. Now, let me go to another situation, where I have 1 photon coming in 1 and 1 Photon coming in 2. Assume these have the same frequency and same polarization etcetera. Now, so the input is given by 1 1 1 2, which is equal to a 1 dagger a 2 dagger 0 1 0 2. a 1 dagger operating on 0 1 gives me 1 1 a 2 dagger operating on 0 2 gives me 1 2.

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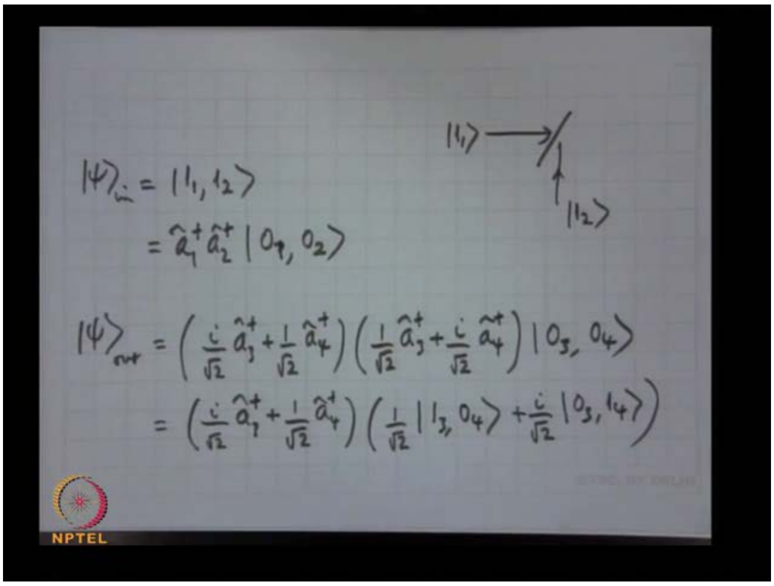

So, the output state, I can represent by writing a 1 dagger and a 2 dagger in terms of a 3 dagger a 4 dagger and using the fact that 0 1 0 2 gets transformed to 0 3 0 4. So, a 1 dagger is I by root 2 a 3 dagger plus 1 by root 2, a 4 dagger 1 by root 2 a 3 dagger plus I by root 2 a 4 dagger. Now, let me operate this on this. So, this is I by root 2 a 3 dagger plus 1 by root 2, what do I get 1 by root 2 a 3 dagger operating on 0 3 0 4. a 4 dagger operating on 0 3 0 4 gives me 0 3 1 4. a 3 dagger operating on 0 3 0 4 gives me 1 3 0 4. a 4 dagger operating on 0 3 0 4 gives me, it is a remember, these are creation operators a 3 dagger operating on 0 gives me 1 3, a 4 dagger operating on 0 4 gives me 1 4.

So, now I have to use the second now bracket. So, let me open up this bracket further. So, what do I get this is equal to, so, I by root 2 a 3 dagger operating on the first state gives me let me write it here a 3 dagger operating on 1 3 0 4 into actually 1 by root 2 plus i by root 2 i by root 2 a 3 dagger operating on 0 3 1 4 plus 1 by root 2 into 1 by root 2, a 4 dagger operating on 1 3 0 4 plus 1 by root 2 into i by root 2, a 4 dagger operating on 0 3 1 4. So, this is i by 2 a 3 dagger operating on 1 3 0 4. Will there be any multiplication factor? Yes. So, square root of 2 2 3 0 4 minus 1 by 2 a 3 dagger on this thing will give me 1 3 1 4, this is plus half a 4 dagger operating on this gives me 1 3 1 4 minus I by 2 into root 2 0 3 2 4. A 3 dagger operating on 1 3 gives me square root of 2 2 3; that means, there are 2 photons in mode 3 the output 3, this is 1 photon in port 3 and 1 photon in port 4, similarly 1 in 3 and 1 in 4 and 0 in 3 and 2 in 4 and these 2 cancel off.

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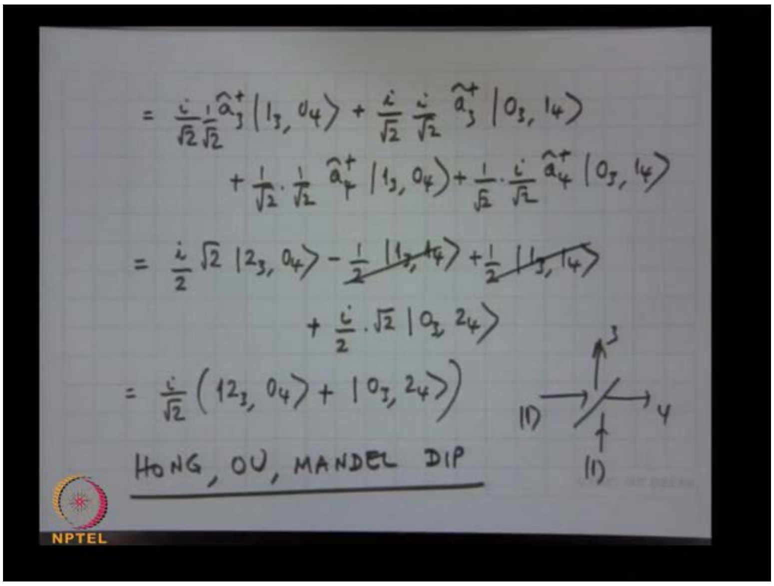

So, what does it imply? So, this state implies that when I have a 1 photon state here and 1 photon state here, either both of them go to the third port or both of them go to the fourth port. The probability of getting 1 photon in port 3 and 1 photon in port 4 is 0. It is something like an interference. What is actually happening is, you see whenever there are multiple ways of reaching from point a to point b and these are all indistinguishable, I must add the probability amplitudes and then square them because interference. The probability of getting 1 photon in 3 and 1 photon in 4 there are 2 ways. Both the photons get transmitted or both photons get reflected, because the photons are in the identical state, there is no they have distinguishing at the output, whether the port 3 is eliminated from 2 or 1 and similarly for port 4. And I will leave it you to calculate to show that these 2 processes are exactly out of phase because of a pi by 2 reflection coefficient here. Remember the reflection coefficient is phi by 2 out of phase with respect to the transmission coefficient.

So, what happens is, the probability of getting 1 here and 1 here through both transmitting or both reflecting the totally probability becomes 0, because there is interference. This is a different kind of interference compared to what is called as single photon interference. So, what will happen is, I will get either both of the photons here or both photons. So, if you have a photon detector here and a photon detector here and look at what are called as coincidence counts, that means, what is the probability of getting a photon, a detection in both the photon detectors simultaneously, you will be 0. Because either this photon detector detects 2 photons and this none or this detects none and this detects 2 photons and each as half a probability, because of a 1 by root 2. Here the probability that 2 photons appear in 3 and none appear in 4 is mod square of I by root 2, which is 1 by 2. Similarly the probability that the photons come in 4 and none of them appear in 3, is also how? The probability of getting 1 in 3 1 in 4 is 0, and let me tell you these experiments are conducted ,so you have detectors at both terminals both outputs and you look at what are called as coincidence counting; that means, you find out you send photons after photons then find out whether you have any detections in both the detectors simultaneously and you find its 0, I mean it is very close to 0.

How did you explain to delta n, so, this is an inert term? There are 2 ways of getting 1 photon ,there is 1 way of getting both photons in 3, and none other than 4, because this 1 gets reflected this 1 get transmitted, there is only 1 way. There is only 1 way of getting both of them here and none of them here, but to get 1 here and 1 here, there are 2 possibilities. One either this 1 transmits and this 1 transmits simultaneously or this 1 reflects and this 1 reflects.

So, the probability of getting at 3 and 4 1 each is the sum of the probability amplitudes of getting both of them transmitted or both of them reflected and that becomes 0. So, second of a 2 photon interference, this is a completely non classical picture. In this you cannot in classical if I illuminate from here and here, there is always a possibility that there are waves in both arms and this experiment which is this. So, when you when people measure for example, you need to measure the detection between these 2 outputs 3 and 4 as a function certain delays because you need to detect at the same time and these experiment is conducted and is called the hong ou mandel dip.

If you do not do them simultaneously, you still can get detected detection at both, because sometime both the photons are arriving here, sometime both the photons are arriving here. So, they do a measurement and this has been experimentally verified and this is one way of detecting or actually doing an experiment to find out whether these 2 photons are identical. Now, this is a picture in which we have not taken wave packets and done any done any analysis with simplified analysis, but I could have taken a wave packet single photon wave packet here a single photon wave packet here and then I would have got essentially the similar kind of a conclusion that either both the photons will arrive in 3 or both photons will arrive in 4. And this is a completely non classical effect.

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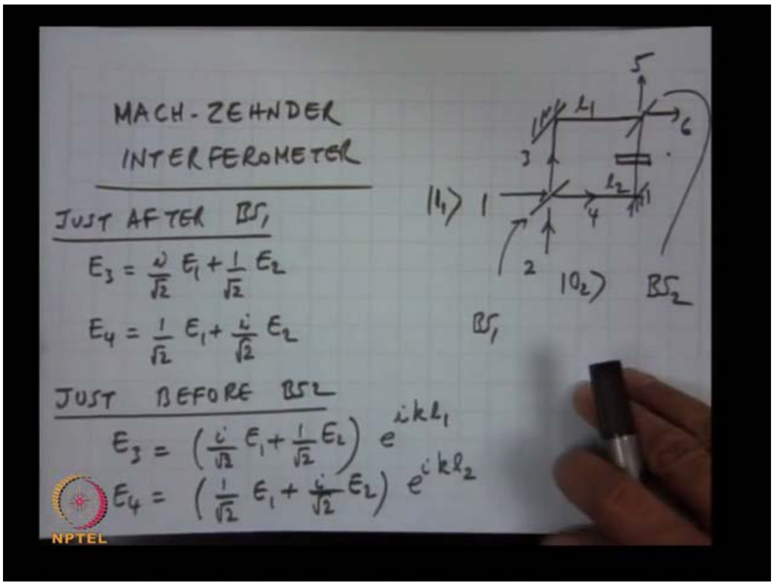

Now, let me go to the another experiment, where I can have I build an interferometer. So, I have. So, this is 1 2 3 4 5. Let me assume that I have a phase shifter here. For example, I could have a different path length on this on compared to this. In a classical interferometer, this is a classical mach zehnder interferometer.

So, light is incident in usually one of the ports, classical in a classical picture, the wave gets transmitted and reflected with equal amounts and then they travel through different paths combined back and a second beam splitter and depending on the phase difference between these 2, they will interfere constructively in 5 or 6, or you will like both in 5 and 6. And as you change the phase difference between these 2 arms, you will see light coming in 5 or 6, because 1 of them is reflected here and reflected this is transmitted and transmitted this is reflected and transmitted transmitted and reflected. So, in 5 for example, you have wave coming from this path reflected from here and reflected from this beam splitter ,there is other 1 on 5 here transmitted here and transmitted here. Both are them arrive in 5 depending on the phase difference you will have constructive or destructive interference here in arm 6 you have reflected here transmitted here transmitted here reflected here and again depending on the phase difference you will have constructive or distractive interference then because of energy conservation if you have complete constructive interference here you must have complete destructive interference here and similarly the other way around and if you have an arbitrary phase difference you'll have might coming in both arm and you can actually calculate the intensity here or 5 or 6 as a function of the phase change will be sinusoidal it is a 2 beam interference very simple 2 beam interference

Now, the question is I send a single photon into the interferometer what am I expected to get at the output 5 and 6, if I had used a classical picture of a photon like a particle and I say that it is either reflected or transmitted, then if it is reflected it comes here, so, it is either reflected or transmitted similarly if it is transmitted here it could be either reflected or transmitted, so, I should have 50 percent chance of getting light here and 50 percent chance of getting the photon here. Normally please remember, it is not either reflected or transmitted it goes into a superposition state of both being reflected and transmitted. And so, these 2 paths of the photons actually will now interfere. So, probability of the photon taking this path and this path these 2 probability amplitudes interfere and will lead to an output either on 5 or in 6.

So, let us I can do the calculation. So, what we will do is, we will relate 1 and 2 to 5 and 6 classically and then just replace the classical electric fields by annulation operators in 5 and 6. So, what is e 3? e 3 is I by root 2 e 1 plus 1 by root 2 e 2 and e 4, this is classical electric field now. So, 1 by root 2 e 1 plus I by root 2 e 2. Now this is just after the beam splitter 1, this is just after b s 1. So, let me call this b s 1 and this is beam splitter 2. So, the field 3, suppose this length is l 1 this path length is l 2, optical path lengths l 1 in arm 3 and l 2 in arm 4. So, just before b 2 b s 2, what will be the fields? e 3 will be I by root 2 e 1 plus 1 by root 2 e 2 into the phase change suffered which is I times k times l 1. The phase change suffered by the wave in going from this beam splitter to this beam splitter is k times l one. So, the phase now is exponential i k l 1. Similarly I can write for e 4, where it arrives on beam splitter 2, so e 4 will be equal to 1 by root 2 e 1 plus I by root 2 e 2 into exponential i k l 2. So, l 2 is the optical path length from b s 1 to b s 2 along the path 4. So, this is the field as it enters these points. So, what will be e 5 and e 6? So, now, just like I had e 3 and e 1 in terms of e 3 e 1 and e 2, e 3 and e 4 in terms of e 1 and e 2, I can write e 5 and e 6 in terms of e 3 and e 4.

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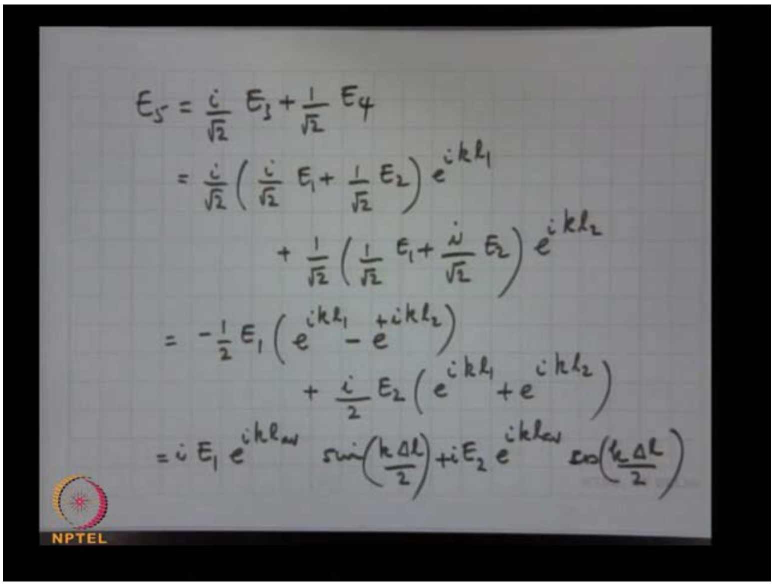

So, e 5 will be, I by root 2 e 3 plus 1 by root 2 e 4. e 3 is incident from here reflected to 5, e 4 is incident from here transmitted to 5. So, just like 1 goes to 3 and 2 goes to 3 here 3 goes to 5 and 2 goes to 4 goes to 5 and so, there is a I by 2 I by root 2 into e 3 plus 1 by root 2 into e 4.

So, let me replace e 3 and e 4 in terms of e 1 and e 2, so I will get this is equal to I by square root of 2 into e 3 is I by square root of 2 e 1 plus 1 by square root of 2 e 2 exponential i k l 1 plus 1 by square root of 2 1 by square root of 2 e 1 plus I by square root of 2 e 2 exponential i k l. So, this is equal to minus 1 by 2 e 1. So, there is minus 1 by 2 e 1 and there is also a 1 by 2 e 1 here. So, exponential I k l 1 minus exponential minus plus I k l 2. So, this is minus 1 by 2 e 1 exponential i k l 1 plus 1 by 2 e 1 exponential i k l 2. So, there is a minus sign here, there is a minus sign here and then I have plus I by 2 e 2 into exponential i k l 1 plus exponential i k l 2.

So, I can take things common out here and write in terms of sin, and this 1 in terms of cosine. So, what do I get? So, I will have now let me write this here straightaway. So, e 1 exponential i k l average sin k delta l by 2, I will defined these quantities plus e 2 exponential i k l average sin cos a delta l by 2. l average is l 1 plus l 2 by 2 and delta l is l

1 minus l 2 l 2 minus l 1 by 2. So, what I have done is you take out exponential i k l 1 plus i k l 2 from here common i k l 1 plus i k l 2 by 2 and then you will have exponential i k l 1 minus l 2 by 2 and l minus of l 1 minus l 2 by 2. So, this becomes a sin function with I, there will be I here, there will be I here . There will be I here from the sin function there will be I here from the cosine function, because cosine does not have I here is a I sitting. So, this I have actually eliminated e 3 and e 4 by writing e 5 and e 6 in terms of e 5 in terms of you ready to similarly you can write e 6 in terms of e 1 and e 2.

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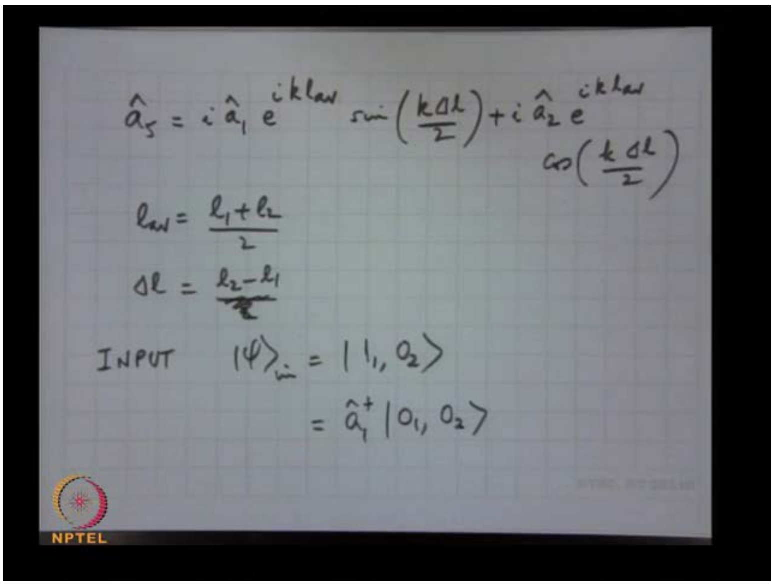

So, now I can replace these electric fields by the corresponding annulation operators. So, for example, I will have a 5 is equal to i a 1 exponential i a l average sin k delta l by 2 plus i a 2 i k l average, let me define these quantities, cosine k delta l by 2. So, l average is l 1 plus l 2 by 2 and delta l is equal to l 2 minus l 1. This surges operators. So, now let me assume an input of , so this is, actually I must get the other relations for a 6 also, but let me do the following.

If this similar to this should be I a 2 and expression plus I a 1.

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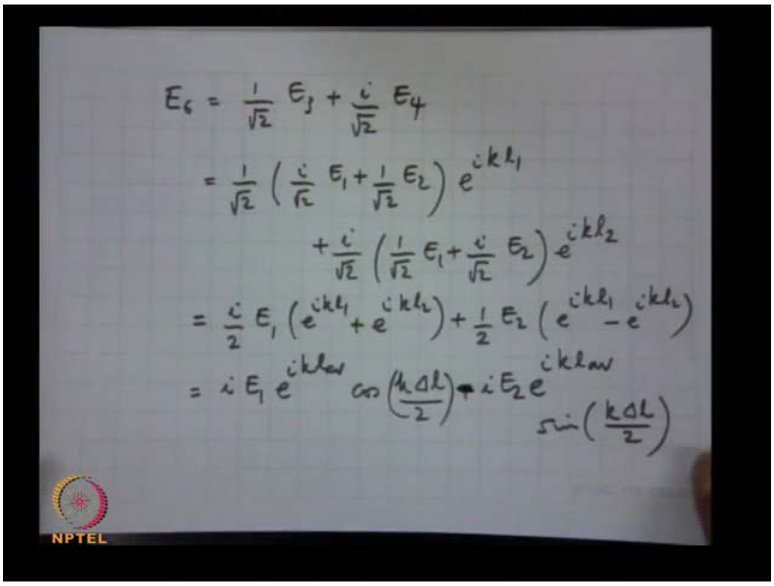

The only difference would be this i by root to becomes 1 by root 2 and this in this expression here. So, e 6 will be 1 by root 2 e 3 plus I by root 2 e 4. So, what will I get? 1 by root 2 plus I by root 2 1 by root 2 e 1 plus i by root 2 e 2 exponential i k l 2.

So, this is i by 2 e 1 exponential i k l 1 plus exponential i k l 2 plus 1 by 2 e 2 exponential i k l 1 minus exponential i k l 2. So, this gives me i e 1 exponential i k l average cosine k delta l by 2 plus i e 2 exponential l i k l average sin a delta l by 2. The cosine and sin have got the interchanged between the 4 5 and 6. In port 5 e 1 was multiplied by sin and e 2 was multiplied by cosine function and in port 6 we have a e 1 getting multiplied by cosine function and e 2 getting multiplied by sin function.

Sir there is a minus sign? There will be minus here l 2 minus l 1 because I have defined a delta l is l 2 minus l one. So, let me write these 2 equations here. e 5 is equal to, so, i e 1 exponential let me take out this common. So, e 5 is equal to i times exponential i k l l average into e 1 time k delta l by 2 plus e 2 cos a delta l by 2 and e 6 will be I exponential i k l average into e 1 cos k delta l by 2 minus e 2 sin. So, let me invert this equation and get e 1 in terms of e 5 and e 6. So, what do I have to do? I multiply this by sin this by cos and add. If I multiply this by sin and this by cos and add this will be cancel of. So, I will get e 5 sin k delta l by 2 plus e 6 cos k delta l by 2 will be equal to i exponential i k l average into e 1. I multiply the first equation by sin, second equation by cosine and I add these 2 equations. Sin square plus cos square is 1 these 2 cancels of and I get this.

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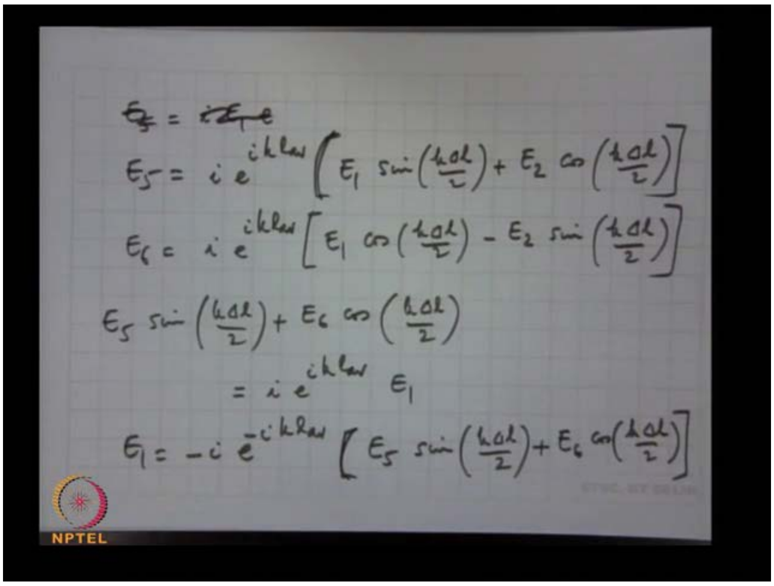

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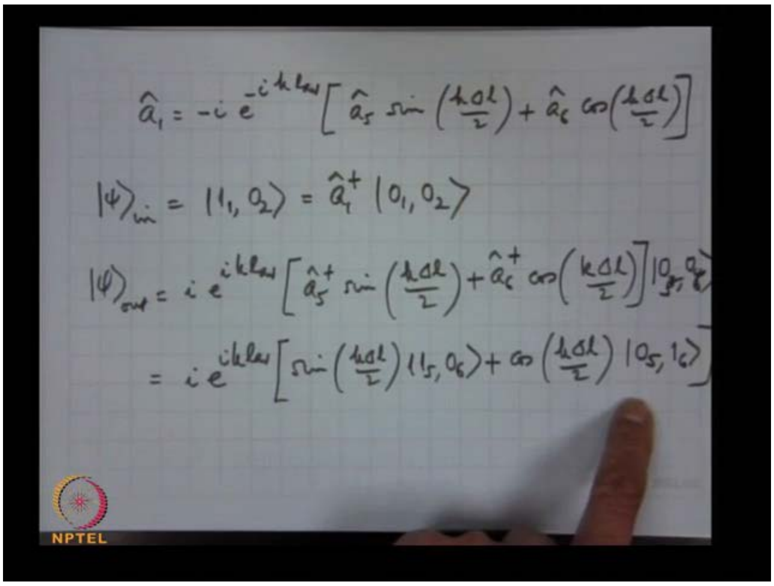

So, actually this gives me e 1 is equal to minus i exponential minus i k l average into e 5 sin k delta l by 2 plus e 6 cos k delta l by 2. Now, I can actually replace these electric fields by the annulation operators and I can write a 1 in terms of a 5 and a 6. So, a 1 becomes minus I exponential minus i k l average into a 5 sin a delta l by 2 plus a 6. So, now I can go to the input states. So, psi in, which is 1 1 0 2 is actually a 1 dagger 0 1 0 2. So, I can write psi out is equal to, so I write the dagger of this, so, I exponential i k l average, a 5 dagger sin a delta l by 2 plus a 6 dagger cos k delta l by 2 operating on 0 5 0 6.

So, this will be I exponential i k l average into sin k delta l by 2 1 5 0 6 plus cos k delta l by 2 0 5 1 6. Output is also a superposition state of a 1 photon appearing in 5 and noting appearing in 6 and 1 in 6 and 1 in nothing an appearing in 5 and it depends of course, on delta l which is the difference in path lengths between in 2 arms. So, if delta l is such that this cosine function is 0, if k delta l by 2 is equal to phi by 2, then this cosine function is 0 the sin function is 1. The probability of finding the photon is only in 5, there is no photon appearing in 6.

If the sin function is zero; that means, if k delta l by 2 is 5 or 0, then the photon appears only in 6. Phase mod, there is only 1 photon entering and it is interfering with itself, in a sense that the photon has 2 ways of reaching 5 or 6, 1 along path 3 or 1 along path 4 and you have your way to differentiate at the output ,whether the photon is coming from path 3 or path 4. And so what I need to do is I need to do add the probability amplitudes of its coming here via 3 or via 4 and then calculate the total probability. So, the probability amplitudes are actually interfering and so, even though there is only 1 photon, you find interference effect between the probability amplitudes of both the paths.

So, there is a situation for example, that you have a photon entering here and if you ensure for example, delta l is equal to 0, photon entering in 1 will always come out of 6. So, in wave analysis, it is fine. We say. it is 1 photon interference in waves also if have amplitude here certain light entering here. I can adjust the interferometer to ensure the all the light is coming out of from 6. But so, this is actually 1 photon enters and in both arms, please now note that this rectangular which I have drawn could be millions of miles of size, millions of miles size, which means I do not have to have these 2 paths close together. I could have to travel may be a million miles before I come back to the main interferometer here in principle. And it seems it is as if the photon is actually sensing both arms on interferometers simultaneously, but if you have to put a detector for example here, you will see it here or you will not see it here. That is like a particle. So, part it is like a particle which either gets reflected or transmitted if I put a detector here, there is a probability half of detecting the photon on this arm. If I put the detector here, there is the probability half of detecting the photon here, but if I build up this interferometer, then I find that there is interference effect between the 2 paths and I can adjust so that the photon always arrives in 6 and never arrives in 5 for example.

Now, this has a very interesting sequence and I will briefly discuss this in the next class because, I can do what are called as non-interaction measurement, interaction free measurements. I can do an experiment ,in which I can tell you there is an object in arm 3, without the photon having gone through the arm 3 at all. Without having interacted with the object I can tell you there is an object in arm 3.

For example if I put something here, if I block this arm, so either the photon then, it is like a detector, you see blocking means what, I put an opaque object here, that is like a detection. I am not detecting it, I am not making any measurement, but I stop the photon from continuing further. I have actually detected it. So, the moment I detect it, there is a finite probability of the photon coming in both arms. I will discuss this in the next class, a very interesting aspect of this experiment of this mach zehnder interferometer, where I can do interaction free measurements and there are some very interesting counter intuitive concepts of this interferometer itself, which I will discuss in the next class, but first we will have a quiz.

Sir, if we look in this part when this delta l is apparent and l is 0, If delta l is 0, Why the photon is always coming out of the mode sections? Because the photon can arrive in 5 or 6 along 2 indistinguishable paths. One is this path and one is this path. If the 2 are indistinguishable, I must calculate the probability amplitudes of these 2 arms of the these 2 paths and add that probability amplitudes before I calculate that total probability which is the mod square. What is actually happening is the probability amplitudes are interfering destructively, because suppose the path lengths were exactly equal, this probability there is a pi by 2 phase shift and there is another pi by 2 phase shift. This one there is no pi by 2 phase shift at all. So, they are out of phase here completely. The probability amplitudes cancel each other. This one they will add, but I can change the path length and make sure that there is constructive in 5 and destructive in 6.

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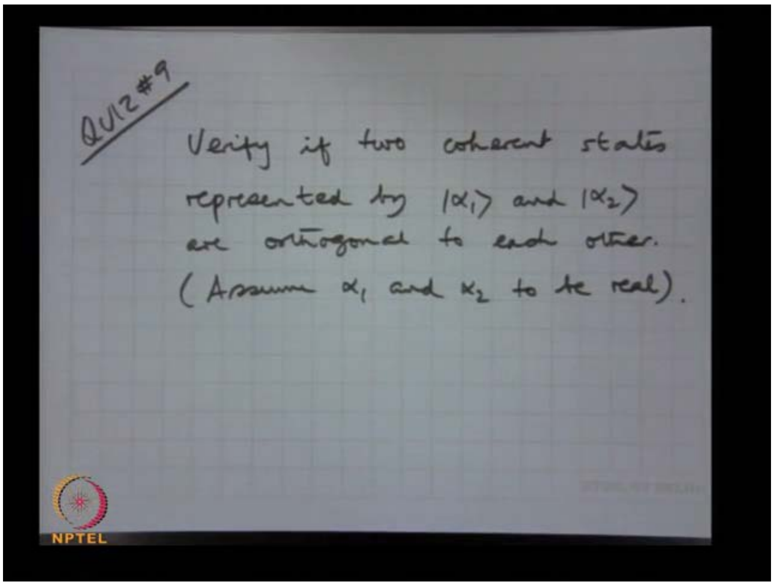

So, the problem is you have to verify whether 2 coherent states represented by alpha 1 and alpha 2 are orthogonal to each other for example, 2 fock states represented by n is equal to 1. So, for a fock state 1 and fock state 2 are orthogonal to each there. So, is alpha 1 and alpha 2 are they orthogonal to each others.