

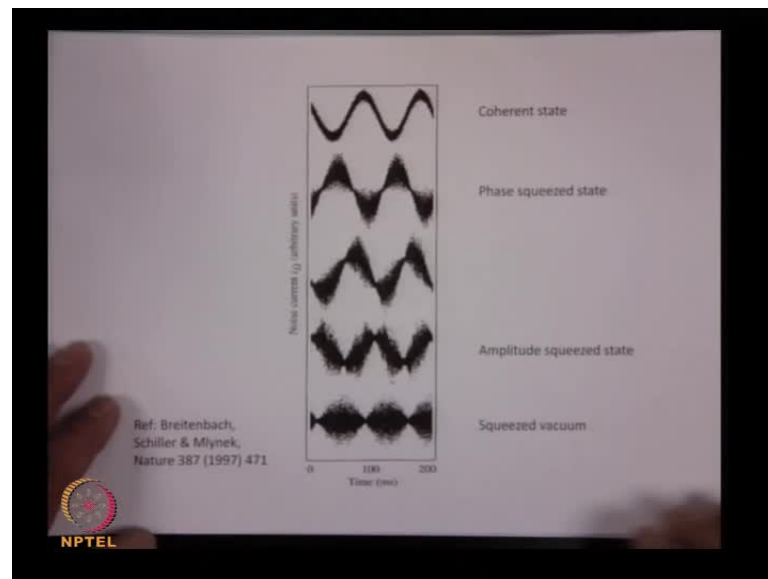
Quantum Electronics
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Module No. # 05
Lecture No. # 35
Quantum States of EM Field (Contd.)

We will continue with our discussion on quantum states of electromagnetic field. Remember, till now we have been discussing single mode states. We have looked at fock states, we have looked at coherent states and we have looked at some squeeze states.

Today we will expand the discussion and go to multimode states. In particular we will look at 2 mode state. But, before we start discussing the 2 mode states I want to present some interesting experimental results, on measurement of the quantum states of electromagnetic fields in these various states.

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These are some results, from this reference which is quoted here Breitenbach Schiller and Mlynek. Which appear in nature in 1997 in which we have been able to measure the electric field with the noise in different states of light. Now the electric field in electromagnetic wave is oscillating very rapidly, usually when we do measurement you are only measuring intensity levels. You are typically measuring the number of photons arriving in a certain interval of time.

We are not able to measure the phases or the quadrature amplitudes of electromagnetic. There is a very interesting technique called balanced homodyning by which you can measure specific quadratures of the electromagnetic field. Later in this course we will discuss balanced homodyning but, essentially it involves mixing. The given electromagnetic field, which you want to measure with another field, is called the local oscillator. In fact homodyning is a very old technique which is used in signal processing in microwaves etcetera.

What is done here is using a beam splitter we mix the given state of the field which you are trying to measure with another field. This is created by a laser which is under coherent state. We mix these 2 fields and measure the signals coming in the reflected arm and the transmitted arm of the beam splitter. After doing some signal processing between these 2 signals coming out, we can actually measure the electric field and the noise coming out in different quadratures.

Now, the signal coming out in these 2 detectors depends on the phase of the local oscillator. By varying the local oscillator phase you actually measure the x quadrature or the y quadrature electric field or any other quadrature you have interested in. These are some results of measurements given in that paper. I would urge you to read that paper; it is a very interesting paper. (Refer Slide Time: 01:13) You remember we had been drawing figures like this when we are discussing various states of light.

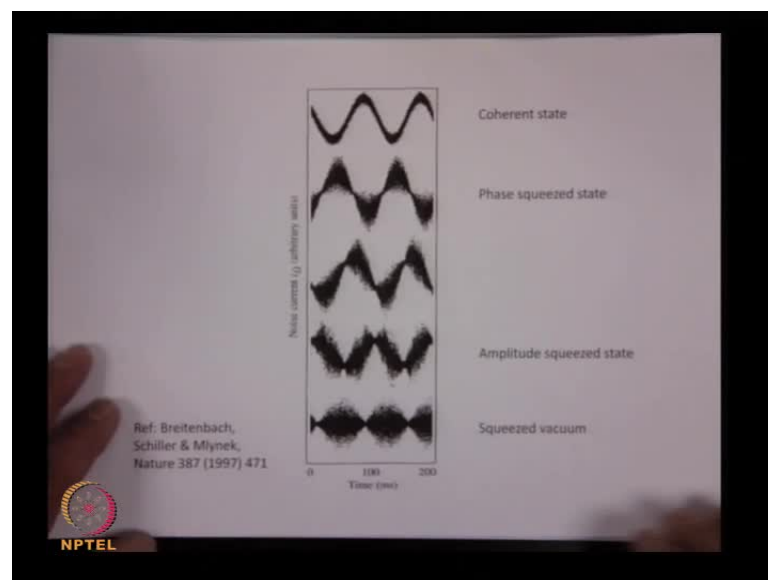
For example, in the coherent state we had drawn this kind of a representation where, the centre of this thick line is actually the expectation value of the electric field. The width of these lines represents the noise. By varying the electric field corresponding to different phases of the local oscillator, you can actually measure the electric field and obtain the noise that is present in the different quadrature.

This is the field variation of coherent states; you can see that the noise is constant with time. The expectation value is following something like classical electromagnetic field. Remember, it was $\sin(\omega t - \theta)$ that we are discussed. The second curve corresponds to a phase squeezed light. We can still see the amplitude is sinusoidally varying but, there is also a noise. You can see here, the noise at these points of intersection with the axis is very much reduced compared to the noise that is on the amplitude points. Here, the noise is much more than in a coherent state. The noise at

these intersection points is much less than in a coherent state. If actually distributed the noise between the 2 quadrature by decreasing the noise in 1 quadrature and increasing the noise in the other quadrature.

That is a phase squeezed state we are discussed earlier. This is an amplitude squeeze state where you see here, the crossings are very broad but, the peak electric field amplitudes are having much lower noise compare to that in vacuums in a coherent state. That squeezing in a 1 quadrature, this is squeezing in another quadrature; this is actually squeezing in some arbitrary quadrature.

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Finally, this one is squeezed vacuum very similar to what we had actually drawn looking at the expectation value of the electric field and the expectation value of the square of the electric field. From which we estimated the noise, the variance in the electric field and we showed that in a squeezed vacuum state the variance is time dependent. There are intervals of time where the noise in the squeezed vacuum state is lower than in normal vacuum coherent state vacuum. If you had a similar measurement of a coherent state coherent vacuum state then you would have got a constant amplitude variation. But, noise which we had estimated earlier, that noise is independent of the phase of the field. And you would have a constant amplitude noise.

These are very similar to our representation. We had drawn looking at the expectation value of the electric field, the variance in electric field in different states of light. The coherent state, this squeezed vacuum state that we had plotted earlier and these are actually showing that it is possible by measurement techniques. Which you called balanced homodyning techniques. It is possible to bringing the quantum fluctuations present in electromagnetic field into a classical level. That you can measure, this is very interesting because this provides us with much more vivid pictures of the actual states of quantum fields.

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TWO MODE STATES

$$|\psi\rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \dots \sum_{n_k=0}^{\infty} C_{n_1, n_2, n_3, \dots, n_k, \dots} |n_1, n_2, n_3, \dots, n_k, \dots\rangle$$

With this we will now moving to discussing multimode states of light. Do you have any questions on the signal mode states? We will like to discuss what are called as 2 mode states. Now remember, we had written earlier, that the most general state of an electromagnetic field is represented like this. Summation over these $C_{n_1, n_2, n_3, \dots, n_k, \dots}$ etcetera, $n_1, n_2, n_3, \dots, n_k, \dots$ what this implies is that, there are n_1 photons in mode 1, n_2 photons in mode 2, n_3 photons in mode 3 etcetera.

The corresponding amplitude function which is multiplying this ket state is $C_{n_1, n_2, n_3, \dots, n_k, \dots}$. Each 1 of these numbers actually can vary from 0 to infinity. The number of photons in mode n_1 can vary from vector 0 to infinity. Similarly, in mode 2, mode 3 so on. This is a very general state; they are both generous multimode state of an electromagnetic field. Instead of discussing this both general multimode state what I

would like to do is, to discuss 2 mode states in which only 2 modes are occupied. Remember all our earlier discussions, where based on single mode states, which means only 1 mode was occupied. Now, 1 mode means a given propagation direction, a given frequency and a given polarization state. You could have a vertically polarized light at frequency ω propagating in this direction. That is 1 mode, so our earlier discussions of fock states, coherent states, squeezed states, these are all single mode states in which only 1 particular mode, 1 polarization state, 1 frequency, 1 k vector direction are populated by photons, all others states been empty.

Now we are generalizing this to a state in which now there are 2 modes which are populated. These 2 modes could correspond to in general 2 propagation directions, 2 different frequencies and 2 polarization states, which are different. So, I could have for example, 2 different modes which correspond to the same frequency, same propagation direction but, 2 orthogonal polarization states. So, one could be vertically polarized light at frequency ω propagating along z. The other could be horizontally polarized light at the frequency ω propagating along z, these are 2 different modes.

I could have 1 vertical polarization, propagating in the z direction but, at frequency ω_1 . The vertical polarization mode propagating along the same direction z but, at frequency ω_2 , these are two different modes. I could have 1 polarization state, 1 frequency but, 2 different propagation direction. These 2 modes are in general specified by different frequency, different polarization states or different propagation direction. Beyond the single mode states you want to look at 2 mode states in which there are 2 modes, which are in general populated. What are the properties of such states? To discuss these 2 modes states let me write this equation. What happens of this equation when I consider only 2 modes?

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$$|\psi\rangle = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} C_{n_1, n_2} |n_1, n_2\rangle$$

$$|\psi\rangle = \alpha |0, 0\rangle + \beta |0, 1\rangle + \gamma |1, 0\rangle + \delta |1, 1\rangle$$

$$|\psi\rangle = \underbrace{\sum_{n_1=0}^{\infty} C_{n_1} |n_1\rangle}_{\text{mode 1}} \underbrace{\sum_{n_2=0}^{\infty} C_{n_2} |n_2\rangle}_{\text{mode 2}}$$

When I have a 2 mode state, I will have essentially sigma n 1 is equal to zero to infinity, sigma n 2 is equal to 0 to infinity, C n 1, n 2, n 1, n 2 when I write this, I am assuming that all other modes are in the vacuum state. This actually contains 0 3, 0 4, 0 5 up to infinity. They are all in the vacuum state; these 2 modes which are called 1 and 2 are occupied. This is superposition of these 2 mode states.

For example, if I assume that there are 2 modes but, I can only have the maximum of 1 photon per mode. Then the state will look like this, I can have alpha 0 photon in mode 1, 0 photon in mode 2 plus, 0 photon in mode 1, 1 photon in mode 2 plus, 1 photon in mode 1, 0 photon in mode 2 plus, 1 photon in mode 1, 1 photon in mode 2 forces possibilities. This is C 0 0, this is C 0 1, this is C 1 0, this is C 1 1.

That is a general state restricted to the fact that there can be only a maximum of 1 photon per mode. Now, please note that this psi in general cannot be written as a product of mode 1 and mode 2. For example, the psi for some situations could be written like this. This is only dependent on mode 1, this is only dependent on mode 2 and this is a product state. This is much more general than this product state. This is only a subset of the states, this is much more general. For example, in general if alpha beta gamma and delta are different cannot be written as a product of these 2 single wave that means product of state 1 into a state 2. That leads to a very interesting consequence because you can have

states quantum states which have very strange properties of correlation between the measurements done on the 2 modes.

This is not always possible and this is a more general 2 mode state where it is a linear combination of states which containing different photon numbers in the mode 1 and mode 2. In general this cannot be factorized into a product of 2 kets. One corresponding to mode 1 and the other corresponding to mode 2.

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The image shows a handwritten derivation on a grid background. The equations are as follows:

$$\begin{aligned}
 |\psi\rangle &= |1, 1, 2\rangle \\
 &= |1, 1, 2, 0_3, 0_4, \dots, 0_L, \dots\rangle \\
 \hat{N} &= \sum_L \hat{a}_L^\dagger \hat{a}_L = \hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2 + \hat{a}_3^\dagger \hat{a}_3 + \dots + \hat{a}_L^\dagger \hat{a}_L + \dots \\
 \hat{N}|\psi\rangle &= (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2 + \hat{a}_3^\dagger \hat{a}_3 + \dots) |1, 1, 2, 0_3, \dots\rangle \\
 &= 1 |1, 1, 2, 0_3, \dots\rangle + 1 |1, 1, 2, 0_3, \dots\rangle + 0 + 0 \dots \\
 &= 2 |1, 1, 2, 0_3, \dots\rangle = 2 |\psi\rangle
 \end{aligned}$$

In the bottom left corner of the grid, there is a circular logo with a sun-like pattern and the text "NPTEL".

Let us start to discuss some specific examples and things to become clear. The first state I want to look at is the following state, $|1, 1, 1, 2\rangle$. This implies all other states are in the vacuum state. Normally I would not write this because, when I write this, I am looking at only 2 mode states. These are the 2 modes which are occupied, all other modes are empty.

Let me try to calculate is this an Eigen state of the total photon number operator? Let me try to calculate if this is an Eigen state of the total photon number operator. Recall that the total photon number operator is $\sum a_l^\dagger a_l$. This is equal to $a_1^\dagger a_1$ plus, $a_2^\dagger a_2$ plus $a_3^\dagger a_3$ plus, $a_l^\dagger a_l$ and so on. It is an infinite series.

Now please note that when I operate with \hat{N} on this state. All $a_l^\dagger a_l$ with l not equal to 1 and 2 will give me 0 because the a_l operating on this with l not equal to 1

or 2 because, these are in vacuum states will give me 0. I need to only operate with the $a_1^\dagger a_1$ and $a_2^\dagger a_2$ on the state. Because, any other operator $a_1^\dagger a_1$ will give me 0, when you operate on the state ψ .

For example, Let me calculate $\hat{N}\psi$, this is equal to $a_1^\dagger a_1 \psi$ plus, $a_2^\dagger a_2 \psi$ plus, $a_3^\dagger a_3 \psi$ plus so on. Operating on $|1, 1, 2, 0, 3\rangle$ etcetera, $a_1^\dagger a_1$ operating on this. This is the number operator for mode 1 that operates on the part corresponding to mode 1 and that gives me 1 into $|1, 1, 2, 0, 3\rangle$ etcetera; plus $a_2^\dagger a_2$ operating on this. This operates on the 1 2 state here and that gives me 1 into $|1, 1, 2, 0, 3\rangle$ etcetera; plus $a_3^\dagger a_3$ operating on this gives me 0; $a_4^\dagger a_4$ operating on this gives me 0 etcetera. This is equal to 2 times $|1, 1, 2, 0, 3\rangle$ etcetera. This is equal to 2 times ψ .

This state ψ which I have written here is an 8 state of the total number operator and the Eigen value is 2. This implies that there are 2 photons occupying this state, 1 photon is in the mode 1 and 1 photon is in mode 2. If you have an ensemble of such states, if you measure the total number of photons in all these states, each 1 of the states will give you a number 2 value, 2 because it is an Eigen state of the total photon number operator.

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The image shows a handwritten derivation on a grid background. At the top, it states:
$$\hat{a}_1^\dagger \hat{a}_1 |1, 1, 2, \dots\rangle = 1 |1, 1, 2, \dots\rangle$$

$$\hat{a}_2^\dagger \hat{a}_2 |1, 1, 2, \dots\rangle = 1 |1, 1, 2, \dots\rangle$$
A horizontal line separates this from the next part, which defines the state:
$$|\psi\rangle = \frac{1}{\sqrt{2}} |1, 0, 2\rangle + \frac{1}{\sqrt{2}} |0, 1, 2\rangle$$
Then, it calculates the action of the total number operator:
$$\hat{N}|\psi\rangle = (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2) \left(\frac{1}{\sqrt{2}} |1, 0, 2\rangle + \frac{1}{\sqrt{2}} |0, 1, 2\rangle \right)$$

$$= \frac{1}{\sqrt{2}} (1 |1, 0, 2\rangle + 0 + 0 + 1 |0, 1, 2\rangle)$$

$$= \frac{1}{\sqrt{2}} (|1, 0, 2\rangle + |0, 1, 2\rangle) = |\psi\rangle$$
In the bottom left corner of the grid, there is a small circular logo with a star and the text 'NPTEL' below it.

Let me check whether this is also an Eigen state of the number operator corresponding to each mode. If I want to calculate this, let me operate this with $a_1^\dagger a_1$ which is the

number operator corresponding to mode 1. This is equal to $1 \times |1\rangle_1 |1\rangle_2$; this is also an Eigen state of the number operator of mode 1 and similarly, for mode two. $a_2^\dagger a_2 |1\rangle_1 |1\rangle_2$ will be equal to one times $|1\rangle_1 |1\rangle_2$ etcetera.

In the given ensemble of states if you measure the number of photons in mode 1, you will always find 1. If you measure a number of photons in mode 2, you will always find 2. These are Eigen states of the individual photon number operators. This is also an Eigen state of the total photon number operator because, as we will see later on. This is not always true; there can be states in which they may not be Eigen states of the number operators.

This state is a state which has actually 2 photons and each photon is occupying different mode please note that this is different from a fock state. In which there are 2 photons occupying 1 mode. There both the photons are in the same mode but, they could have different. Here, the 2 photons are occupying 2 different modes; 1 photon in mode 1, 1 photon in mode 2. This could as I mentioned this could correspond to the 2 modes being 2 different frequencies propagating in the same direction, same polarization state or 2 different propagation directions. Each mode being occupied by 1 photon.

Let me come to another example, let me look at this state ψ is equal to $\frac{1}{\sqrt{2}} |1\rangle_1 |0\rangle_2 + \frac{1}{\sqrt{2}} |0\rangle_1 |1\rangle_2$; This is a superposition state of mode 1 being occupied by 1 photon, mode 2 being empty, mode 1 being empty, mode 2 being occupied by a photon. This is a state which cannot be written as a product of mode 1 and mode 2. Let us find out whether this is an Eigen state of the total number operator. If it is an Eigen state I will come to know how many photons are there in the state. Let me operate with $n \psi$, this is equal to $a_1^\dagger a_1 \psi + a_2^\dagger a_2 \psi$ operating on $\frac{1}{\sqrt{2}} |1\rangle_1 |0\rangle_2 + \frac{1}{\sqrt{2}} |0\rangle_1 |1\rangle_2$; $a_1^\dagger a_1$ operating on $\frac{1}{\sqrt{2}} |1\rangle_1 |0\rangle_2$ gives me $1 \times \frac{1}{\sqrt{2}} |1\rangle_1 |0\rangle_2$ plus, $a_1^\dagger a_1$ operating on $\frac{1}{\sqrt{2}} |0\rangle_1 |1\rangle_2$ gives me 0; because the mode 1 is not occupied I get 0 $a_2^\dagger a_2$ operating on $\frac{1}{\sqrt{2}} |1\rangle_1 |0\rangle_2$ gives me 0; because mode 2 is unoccupied in this part plus $a_2^\dagger a_2$ operating on this gives me one time $\frac{1}{\sqrt{2}} |0\rangle_1 |1\rangle_2$;

This is equal to $\frac{1}{\sqrt{2}} |1\rangle_1 |0\rangle_2 + \frac{1}{\sqrt{2}} |0\rangle_1 |1\rangle_2$, which is equal to ψ $|1\rangle_1 |1\rangle_2 + |1\rangle_1 |0\rangle_2$ square root of 2. This state is an Eigen state of the total number operator

and how many photons are there? There is just 1 photon because n psi is equal to one times psi.

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$$\hat{a}_1^\dagger \hat{a}_1 (\psi) = \hat{a}_1^\dagger \hat{a}_1 \frac{1}{\sqrt{2}} (|1, 0\rangle + |0, 1\rangle)$$

$$= \frac{1}{\sqrt{2}} |1, 0\rangle$$

$$\langle \psi | \hat{a}_1^\dagger \hat{a}_1 (\psi) = \frac{1}{\sqrt{2}} (\langle 1, 0| + \langle 0, 1|) \frac{1}{\sqrt{2}} |1, 0\rangle$$

$$= \frac{1}{2}$$

Now this state is a such a state in which there is only 1 photon but, it is in a superposition state of either being in mode 1 or being in mode 2. Let us check for example, whether this is an Eigen state of the number operator for 1 mode. How do I check again a 1 dagger a 1 operating on psi is equal to a 1 dagger a 1 by root 2, 1 1, 0 2 plus, 0 1, 1 2; which is equal to 1 upon root 2, 1 1, 0 2; because, a 1 dagger a 1 operating on 1 1, 0 2 is 1 1, 0 2; a 1 dagger a 1 operating on 0 1, 1 2 is zero.

This is not an Eigen state because this is different from psi. This is not an Eigen state of the a 1 dagger a 1 operator. What is the expectation value I can calculate? The expectation value of the number of photons that I will measure in mode 1. There I can calculate by calculating the expectation value. What do I get?

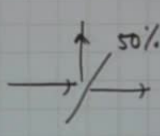
That is psi a 1 dagger a 1 psi, which is equal to 1 by root 2, 1 2, 0 2; plus 0 1, 1 2; 1 by root 2, 1 2, 0 2; which is orthogonal to 0 1 and 1 1 This gives me 1 by 2, 1 1, 0 2, 1 1, 0 2; which is 1, so that is half.

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$$\begin{aligned}\hat{a}_1^\dagger \hat{a}_1 |1, 1, 2 \dots\rangle &= 1 |1, 1, 2 \dots\rangle \\ \hat{a}_2^\dagger \hat{a}_2 |1, 1, 2 \dots\rangle &= 1 |1, 1, 2 \dots\rangle \\ &— \\ |\psi\rangle &= \frac{1}{\sqrt{2}} |1, 0_2\rangle + \frac{1}{\sqrt{2}} |0_1, 1_2\rangle \\ \hat{N} |\psi\rangle &= (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2) \left(\frac{1}{\sqrt{2}} |1, 0_2\rangle + \frac{1}{\sqrt{2}} |0_1, 1_2\rangle \right) \\ &= \frac{1}{\sqrt{2}} (1 |1, 0_2\rangle + 0 + 0 + 1 |0_1, 1_2\rangle) \\ &= \frac{1}{\sqrt{2}} (|1, 0_2\rangle + |0_1, 1_2\rangle) = |\psi\rangle\end{aligned}$$

What is it imply? If you take an ensemble of such states in this superposition state, if you measure the total number of photons in this state. You will always find 1; if you try to measure the number of photons in mode 1 you will get an expectation value of half. What you are finding is essentially, it will be either 1 photon or no photons the probability of finding. For example, the probability of finding 1 photon in the state is I can take a projection of this into this state.

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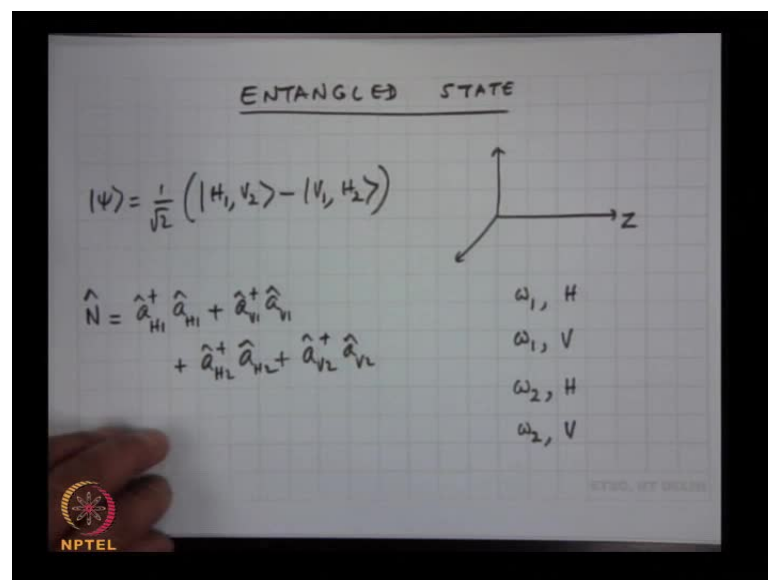
$$|\langle 1, 0_2 | \psi \rangle|^2 = \frac{1}{2}$$


I have $|1, 0\rangle$; which is equal to $|1\rangle$ by 2 because $|1, 0\rangle$ is orthogonal to $|0, 1\rangle$; The probability of finding 1 photon in this state is half similarly, the probability of finding no photons in this state is half. This is not an Eigen state of the individual photon number operator but, it is an Eigen state of the total photon number operation. We will later on find the state in the problem of beam splitter. In fact we will later on look at this problem, if you take a beam splitter.

A beam splitter which is 50 percent reflecting and 50 percent transmitting. If you send 1 photon from here, it goes into a state of superposition state of these 2 modes. 1 mode going like this another mode going like this. This is the kind of state that we will be prepared, when we send a single photon and a beam splitter, we will look at this problem a little later and find out more interesting features of the beam splitter itself.

This is a superposition state unlike the earlier state of 1 photon occupying either mode 1 or mode 2. Because of this $1/\sqrt{2}$ there is equal probability of being occupying mode 1 or mode 2. Please note that I can have different amplitudes here. All I need to have a ψ , ψ is equal to 1. The ψ has to be normalized.

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Now, I want to look at another very interesting state called the entangled state. To discuss the entangled state let me look at the following situation I have. Let me assume that I look at modes, which are propagating along the z direction. Let me assume there

are 2 frequencies ω_1 and ω_2 , 2 polarization states horizontal and vertical. From looking at 4 modes, I can have ω_1 frequency in the horizontal polarization state. I can have ω_1 frequency in the vertical polarization state. I can have ω_2 polarization in the horizontal polarization state. I can have ω_2 in the vertical polarization state. 4 possible modes, I can have these are 4 modes this is ω_1 horizontally polarized, ω_1 vertically polarized, ω_2 horizontally polarized, ω_2 vertically polarized. Now I want to look at one particular state which is like this, ψ is equal to $\frac{1}{\sqrt{2}}(H_1 V_2 - V_1 H_2)$;

Let me explain to you what this implies? The symbol this is a state in which ω_1 horizontally polarized state of ω_1 is occupied by 1 photon. Simultaneously, the vertically polarized mode of ω_2 photon is also occupied. This is a state in which the vertically polarized mode of the ω_1 photon is occupied by 1 photon. The horizontally polarized mode of the ω_2 photon is also occupied by 1 photon.

When I write like this, you know it implies that all other modes are in the vacuum state. This implies that there is no photon which is horizontally polarized at the frequency ω_1 . The mode corresponding to vertical polarization of ω_1 frequency is also unoccupied all other frequencies, all other propagation directions etcetera. Now such a state is called an entangled state. First thing is cannot write this as a product of the different states here. Let me look at, how many photons are there in this state? Again like before we need to look at the total number operator which now because, there are 4 modes I have to have a H_1 dagger, a H_1 plus, a H_2 dagger, a V_1 plus, a H_2 plus, a V_2 dagger, a v_2 .

The number operators corresponding to ω_1 horizontally polarized state. Here, number operator corresponding to vertically polarized state of ω_1 . The number operator corresponding to horizontally polarized ω_1 state. The number operator corresponding to vertically polarized ω_1 state.

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$$\begin{aligned}\hat{N}|\psi\rangle &= (\hat{a}_{H1}^\dagger \hat{a}_{V1} + \hat{a}_{V1}^\dagger \hat{a}_{H1} + \hat{a}_{H2}^\dagger \hat{a}_{V2} + \hat{a}_{V2}^\dagger \hat{a}_{H2}) \\ &\quad \frac{1}{\sqrt{2}} (|H_1, V_2\rangle - |V_1, H_2\rangle) \\ &= \frac{1}{\sqrt{2}} (|H_1, V_2\rangle - 0 + 0 - |V_1, H_2\rangle + 0 - |V_1, H_2\rangle \\ &\quad + |H_1, V_2\rangle - 0) \\ &= \frac{1}{\sqrt{2}} (2|H_1, V_2\rangle - 2|V_1, H_2\rangle) \\ &= 2 \frac{1}{\sqrt{2}} (|H_1, V_2\rangle - |V_1, H_2\rangle) = 2|\psi\rangle\end{aligned}$$

The total number operator actually contains many other infinite number of terms but, because all those states are in the vacuum state. In these given states that are taking those operators will give me zeros. When I operate them on the psi state, I am not even writing that. Let us calculate what the value of n times psi is. n psi is equal to a H 1 dagger, a H 1 plus, a V 1 dagger, a V 1 plus, a H 2 dagger, a H 2 plus, a V 2 dagger, a V 2 operating on 1 by root 2, H 1 V 2 minus v 1 H 2.

I want to check, what I am doing is, I am trying to check whether this is an Eigen state of the total number operator. If so, I will be able to find out how many photons are there in the state. This is 1 by root 2, a H 1 dagger, a H 1; on this gives me 1 because, the H 1 state is occupied a H 1 dagger, a H 1 occupied acting; on this will give me 0 because, H 1 state is not occupied a V 1 dagger, a V 1 acting on this will give me 0 because, V 1 state is not occupied a V 1 dagger, A v 1 operating on this will give me 1; a H 2 dagger H 2 operating on this will give me 0 because, H 2 state is not occupied a H 2 dagger, a H 2 operating on this will give me 1. a V 2 dagger, a V 2 operating on this will give me 1; a V 2 dagger, a V 2 operating on this will give me 0; This is equal to 1 by root 2, H 1 v 2, H 1 V 2; Two times H 1 V 2 minus, V 1 H 2, V 1 H 2; two times V 1 H 2 which is equal to two times 1 by root 2 of H 1 V 2 minus, V 1 H 2; which is equal to two times psi.

Because, this is the state which I am looking at the state is psi, is equal to 1 by root 2, H 1 V 2 minus, V 1 H 2; This is 1 by root 2, H 1 V 2 minus, V 1 H 2; That is 2 psi. How

many photons are there? There are 2 photons in this state. This is an Eigen state of the total photons, number operator. If you take ensembles of such states and measure the total number of photons in this state you will always get a value 2.

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$$\hat{a}_{H1}^+ \hat{a}_{H1} |\psi\rangle = \frac{1}{\sqrt{2}} \hat{a}_{H1}^+ \hat{a}_{H1} (|H_1, V_2\rangle - |V_1, H_2\rangle)$$

$$= \frac{1}{\sqrt{2}} |H_1, V_2\rangle$$

$$\langle \psi | \hat{a}_{H1}^+ \hat{a}_{H1} | \psi \rangle = \frac{1}{\sqrt{2}} (\langle H_1, V_2 | - \langle V_1, H_2 |) \frac{1}{\sqrt{2}} |H_1, V_2\rangle$$

$$= \frac{1}{2}$$

Let me look at whether this is also an Eigen state of an individual photon number operator. For example, is this an Eigen state of a H 1 dagger a H 1. This is 1 by root 2 a H 1 dagger a H 1; H 1 V 2 minus V 1 H 2 which is equal to 1 by root 2 of H 1 V 2 this acting on this gives me H 1 V 2 into 1; this acting on this gives me 0.

This is not an Eigen state of the individual photon number operator because, this is not psi. What is the expectation value of the horizontal polarization state? We have this state that you are looking at actually is the linear combination of two possibilities mode 1. The horizontally polarized omega 1 being occupied. Simultaneously the vertically polarized omega 1 being occupied. This is a vertically polarized omega 1 being occupied. Simultaneously, a horizontal polarized state. Which is at omega 1 being occupied.

This is not an Eigen state of the individual photon number operator a H 1 dagger, a H 1 or a V 1 dagger a V 1. What is the expectation value? I can calculate psi a H 1 dagger a H 1 psi. What will I get if I pre multiply by psi, psi bra? I will have 1 by root 2, H 1 V 2 minus, V 1 H 2; 1 by root 2 H 1 V 2 which is equal to half. The probability of detecting the mode corresponding to omega 1. At horizontally polarized state. Detecting the

photon in that state is half. Similarly, you can calculate what are the probabilities of finding the photon in the vertically polarized omega 1 state or the horizontally polarized omega 2 state or the vertically polarized omega 2 state.

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$$\begin{aligned}
 |H\rangle &= \frac{1}{\sqrt{2}} |+\rangle - \frac{1}{\sqrt{2}} |-\rangle \\
 |V\rangle &= \frac{1}{\sqrt{2}} |+\rangle + \frac{1}{\sqrt{2}} |-\rangle \\
 |H_1, V_2\rangle - |V_1, H_2\rangle &= \frac{1}{\sqrt{2}} (|+\rangle - |-\rangle) \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle) \\
 &\quad - \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle) \frac{1}{\sqrt{2}} (|+\rangle - |-\rangle) \\
 &= \frac{1}{2} (|+\rangle |-\rangle - |-\rangle |+\rangle + |+\rangle |-\rangle - |-\rangle |+\rangle)
 \end{aligned}$$

Now this is an interesting state because actually the state of polarization of neither the photon 1 nor photon 2. At these 2 frequencies omega 1 and omega 2 are defined. For example, I can write the same state in a different basis. Let me try to look at another basis, let me call this horizontal, vertical let me call this plus and this is minus. Let me assume pi by 4; 45 degree, in fact I leave the general problem of theta to you.

This is an expression representation of psi in the horizontal vertical polarization state. Let me try to express psi in the basis state corresponds to plus or minus. From this equation I can see that H is actually 1 by root 2 plus state, minus 1 by root 2 of minus state. The vertically polarized state is 1 by root 2 plus state, plus 1 by root 2 minus state. Let me calculate H 1 V 2 minus V 1 H 2, this is equal to H 1. I have 1 by root 2 plus 1, minus, minus 1; This is H 1 V 2 is 1 by root 2 plus 2 plus minus 2. This is H 1 V 2 minus 1 by root 2 plus, 1 plus minus 1 into 1 by root 2 H 2; so, that is plus 2 minus, minus 2.

Now you see plus 1 plus 2; This plus 1, plus 2 that we cancel off with the minus sign here. minus 1, minus 2 with the minus sign, minus 1, minus 2 with the plus sign cancels off. What I left with, this is half of plus 1 minus, 2 minus, minus 1 plus 2.

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$$|H_1, V_2\rangle - |V_1, H_2\rangle = |+1, -2\rangle - |-1, +2\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|H_1, V_2\rangle - |V_1, H_2\rangle)$$

$$= \frac{1}{\sqrt{2}} (|+1, -2\rangle - |-1, +2\rangle)$$

So, plus 1 plus 2 has canceled off minus 1, minus 2 has canceled off. Then I have minus, there is a minus here, I have plus 1, minus 2, minus into minus is plus; Then you have minus, minus 1, plus 2, the plus 1 minus 2, plus 1, minus 2, minus 1 plus, 2 minus 1, plus 2; Two times and this two characters cancels off, what I have got is essentially $H_1 V_2$ minus $V_1 H_2$ is equal to plus 1, minus 2; minus, minus 1 plus 2; The state ψ which I wrote earlier as square root of $H_1 V_2$ minus $V_1 H_2$ can also be written as, 1 by square root of 2 plus, 1 minus 2 ; minus, minus 1 plus 2 . This is the linear combination of the states represented in a horizontal vertical basis. Here, we written the plus minus basis.

In this state we are writing it as a linear combination of this polarization states. Here, we are writing as the linear combination of these 2 polarization state. This is plus state, this is minus state, this is vertical state, horizontal state, this is vertical state. In fact I believe it you have to calculate if you take a θ here instead of $\pi/4$. Some arbitrary θ you can show that still the state is represented in the same expression. What you are now seeing is the polarization state of the 2 photons are not defined at all but, what will happen if I were to pass some? Because, these 2 photons have 2 different frequencies I can actually split them and make them go in different directions

If ω_1 frequency passes through the horizontally polarized analyzer then at that means, it has jump into the state H_1 and automatically the ω_1 frequency photon jumps into the V_2 state. If ω_1 photon does not pass through this horizontally

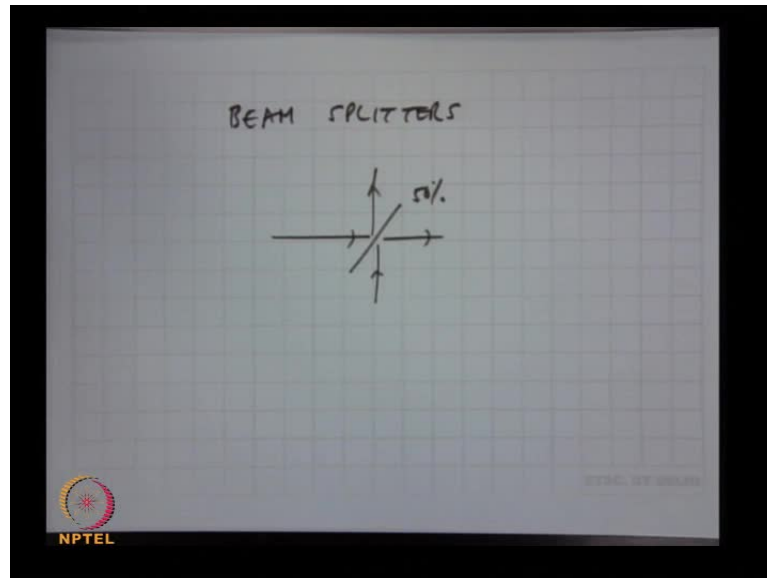
polarized analyzer, it has jumped into the V_1 state. The second photon automatically jumps into the H_2 state not only that, if I were to orient my analyzer at 45 degrees to the horizontal, if the photon passes through, if ω_1 photon passes through this analyzer it jumps into the $+$ state. Automatically the ω_2 photon has jumped into the $-$ state

It is projected into the $-$ state, if photon 1 at ω_1 does not pass through the $+$ state. This oriented analyzer the ω_1 photon has jumped into the $-$ state. The ω_2 photon automatically jumps into the $+$ state. Please note that this happens irrespective of the physical separation between the photons at ω_1 . These two very interesting, very strange quantum correlations between these two photons. Such entangled states very, very interesting. They are highly non classical states. They are very interesting from the perspective of their applications in branches like quantum teleportation or quantum computing and so on. One of the standard methods to generate such entangled photon pairs is by using parametric down conversion. Remember, we had studied earlier spontaneous, we have studied parametric down conversion where, a frequency ω_p photon at frequency ω_p splits into one at ω_s and one at ω_i . Depending on the orientation of the pump depending on the crystal characteristics we can generate the ω_s and ω_i photons in orthogonal polarization states or same polarization states etcetera. This parametric down conversion process is one of the very widely used techniques to generate such states of light. Which are called entangled states.

What we have done today is, essentially discussed some interesting 2 mode states and finally, the entangled state is the 4 mode state. The first one is actually there are 2 photons in that state and 1 photon is occupying mode 1, the other photon is occupying mode two. The second state is 1 photon state but, linear superposition of being in mode 1 and mode two. Finally, this is an entangled state that we have discussed with some very interesting properties.

Do you have any questions in the 2 mode state that we have discussed ? In fact we can generalize this 2 mode states to multimode states. Where, instead of having only 2 modes we can consider a large number of modes. Actually, electromagnetic fields that you generate or not in a single mode state or a 2 mode state. usually they are a multimode state. Many times you can approximate them by these states and get some characteristics of the states.

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In the next class what we will do is, study the properties of beam splitters. We look at a state where I have light coming in the one arm of the of the beam splitter. You will look at a 50 percent of beam splitter, which classically reflects 50 percent of the light and transmits 50 percent of the light. Look at some quantum analysis of this beam splitter because, this can generate states which are superposition states. Also this is one basic element that is used in many quantum objects, experiments including balanced homodyning, which you will discuss much later.

Any questions? Thank you very much