Quantum Electronics
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Module No. # 05
Lecture No. # 34
Quantum States of EM Field (Contd.)

So, we continue with our discussion on squeezed states. Do you have any questions? So, let us recall, we are looking at one special state called the squeezed state and within this squeezed states we are looking at a single mode squeezed state - one mode - which means, only one mode is excited and that excitation is in a state which satisfies this condition - where b is an operator, which is defined by mu a plus nu a dagger and the condition that mod mu square minus mod nu square is equal to 1. States which satisfies this equation or are called squeezed states, because as we will see these states have less noise than vacuum, in some of the quadratures.

So, remember, we had shown that if you take the two quadratures delta X delta Y must be greater than equal to 1 by 4. The minimum uncertainty states corresponds to delta X delta Y is equal to 1 by 4 and in vacuum delta X and delta Y are equal and equal to half. If I have a state, I could have a state in which delta X or delta Y is less than half and that would correspond to what has called as quadrature squeezed states, which means in one of the quadratures, the noise is less than what is there in vacuum. And I am going to show you that this state which satisfies this equation, satisfies this condition of squeezing.

Now, as I told you to keep the analysis simple, we will assume mu nu and beta to be real, but in general mu and nu and beta are all complex numbers. So, if mu and nu are real, I can actually write mu is equal to cos hyperbolic sigma and nu is equal to sin hyperbolic sigma. So, that mu plus nu becomes equal to exponential sigma and mu minus nu becomes exponential minus sigma. Please note that sigma can be positive or negative; (Refer Slide Time: 03:40) if sigma is positive mu plus nu is exponential plus sigma and this is exponential minus sigma; if sigma is negative then mu plus nu becomes less than mu minus nu mu plus nu becomes exponential minus mod sigma mu minus nu becomes exponential plus mod sigma if sigma is negative.

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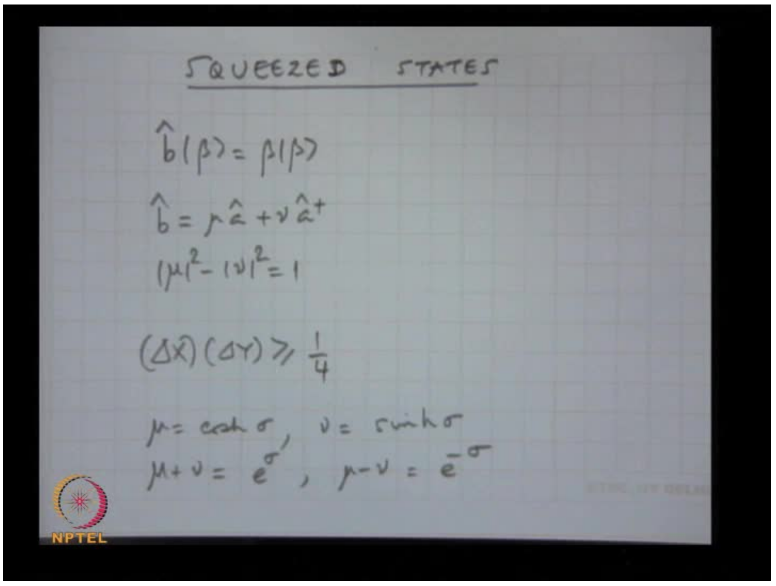

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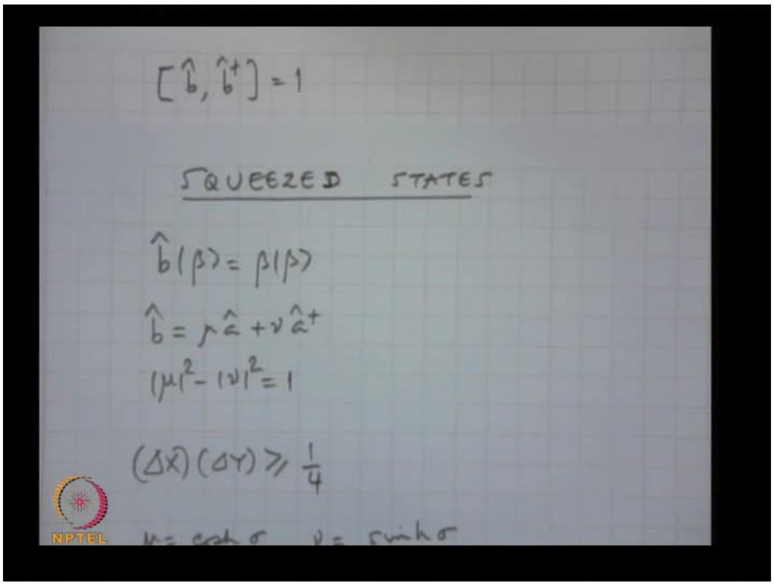

So, as you will see, the quadrature which gets squeezed depends on the sign of sigma. So, we are looking at special kind of states call squeezed states in which the squeezed states, beta satisfies this equation b beta is equal to beta beta; where b is an operator define in terms of the a and a dagger operators, the creation and annihilation operators. Actually, as I told you last time, you can substitute the commutation relations for a and a dagger and show that b b dagger is equal to 1.

So, they are similar to a and a dagger operators, b b dagger operators are similar to a a dagger operators and this equation is something like what we defines to the coherent state a alpha is equal to alpha alpha.

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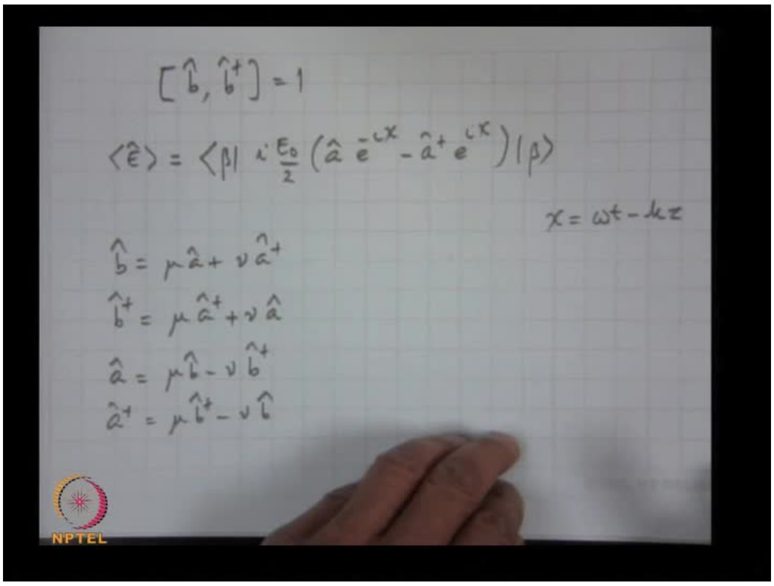

So, now, we have defined another state with respect to b operators as b beta is equal to beta beta. So, what I first calculate is the expectation value of electric field and I want to calculate the variance in the electrical field as a function of time and space.

So, last time we did this expectation value of E is equal to beta i E naught by 2 a exponential minus i chi minus a dagger exponential i chi beta; where, let me recall chi is omega t minus k z, I am assuming propagation direction to be along z and so I have to invert this equation. So, actually I am given b is equal to mu a plus nu a dagger. So, b dagger is equal to mu a dagger plus nu a. This is for mu and nu to be real. So, we can invert these equations and write, a is equal to mu b minus nu b dagger and a dagger is equal to mu b dagger minus nu b. If mu and nu is complex, I need keep here mu star nu star and similarly here mu star nu star. S,o I have to be careful when I am having the complex values of mu and nu. But in this right now, we are assuming mu and nu are real. So, that we can simply write a and a dagger.

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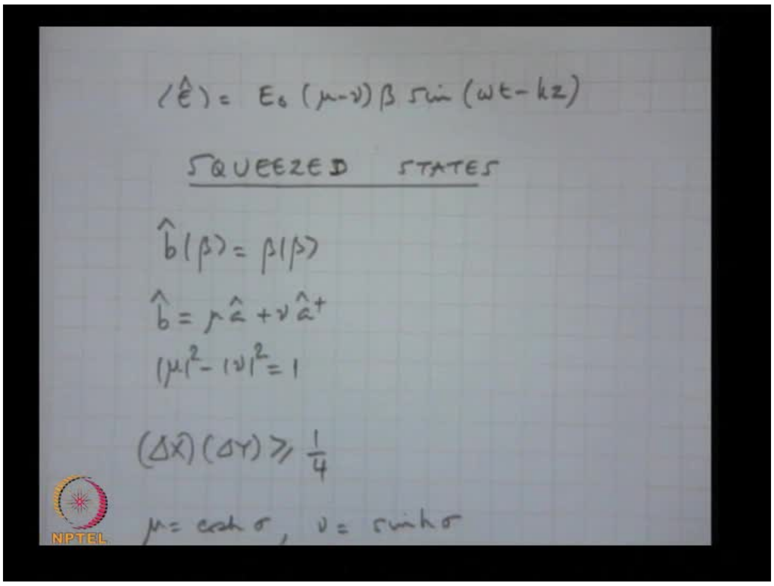

So, I need to substitute this a and a dagger here and calculate the expectation value of the electrical field, which we did last class and what we obtained was E. E expectation value is equal to E naught times mu minus nu into beta sin omega t minus k z.

Also, note that if I put nu is equal to 0, what will the state beta correspond to? It becomes a coherent states. Because if nu is 0 b becomes a operator, so b beta is equal to beta beta becomes a beta is equal to beta beta.

So, effectively the state becomes a coherent state if nu is equal to 0. So, here if you put nu is equal to 0, you will get epsilon 0 mu must be equal to 1, because of this condition if you put nu is equal to 0 mu is 1 and you get e 0 beta sin omega t minus k z which is exactly the same as what we had got for this coherent state.

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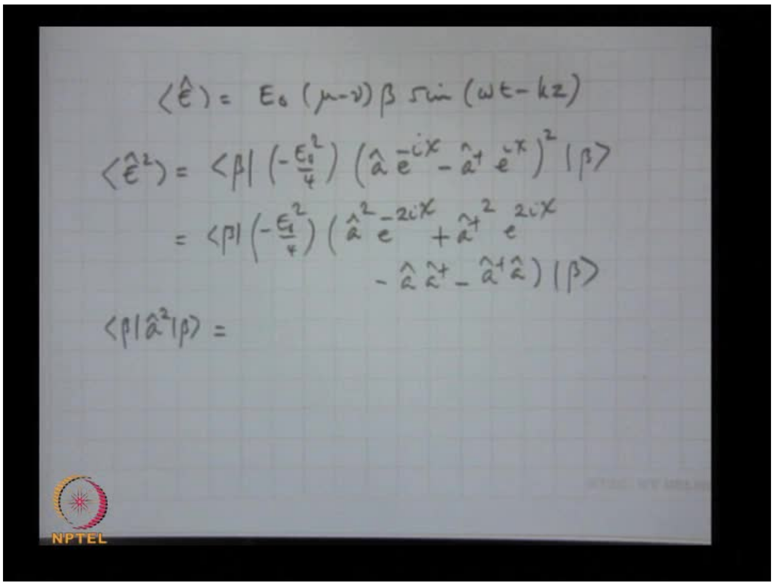

Now, I need to calculate E square expectation value and we started doing it last time. This is beta into minus E naught square by 4 a exponential minus i chi minus a dagger exponential i chi whole square beta.

When you expand this square root square here you will get minus E naught square by 4 a square exponential minus 2 i chi plus a dagger square exponential 2 i chi minus a a dagger minus a dagger a.

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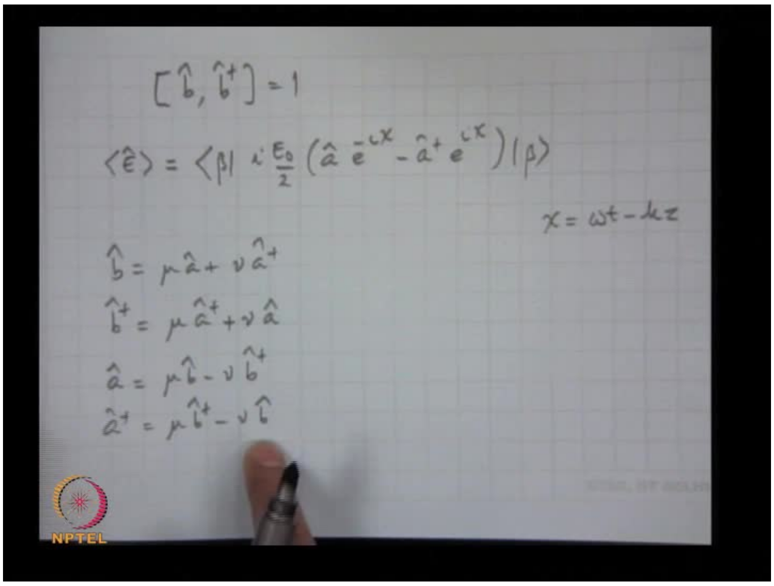

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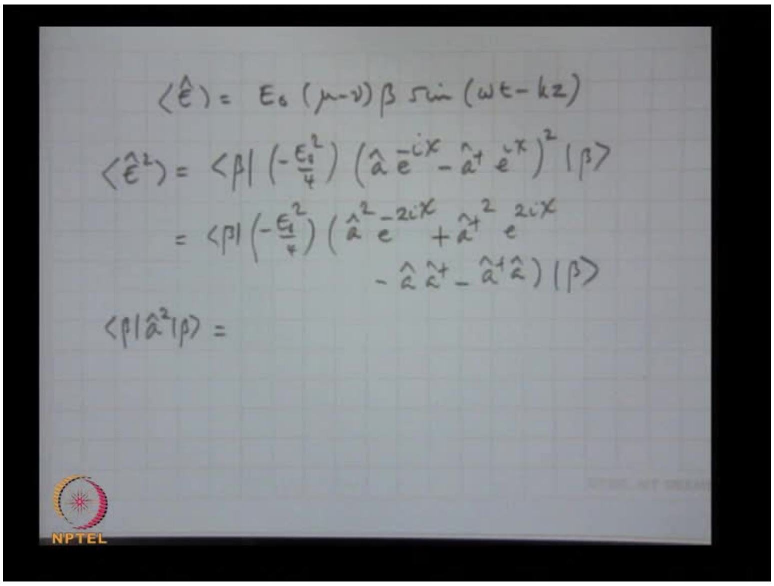

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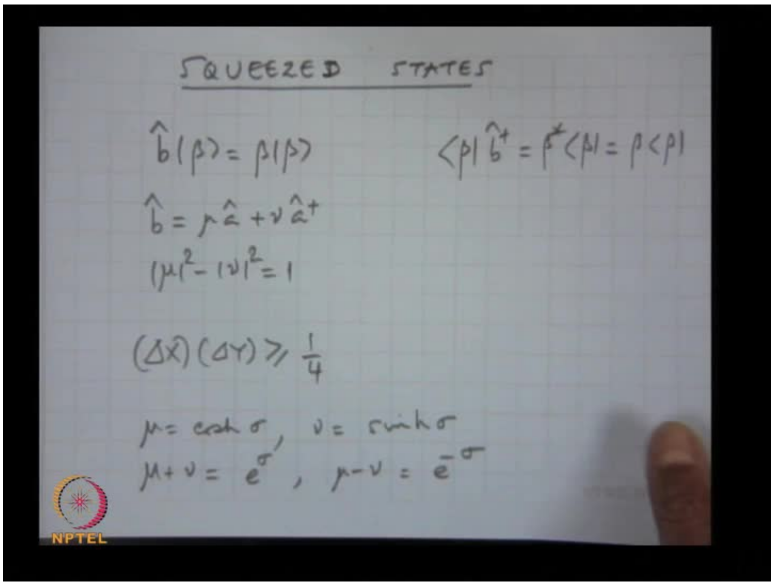

So, I need things like expectation value of a square; expectation value of a dagger square; expectation value of a a dagger and expectation value of a dagger a. So, I have to substitute for a in terms of b, which I have got here; I have to replace a by b and b dagger and use the condition that b beta is equal to beta beta and what is the corresponding conjugate expression for this beta b dagger? Actually beta star beta, if I assume beta to be real this is beta beta.

Please note b dagger beta is not equal to beta star beta, if b dagger if it operator on ket beta does not give me beta star. It is b dagger operating on bra beta which gives me beta star beta or beta times beta if beta is real.

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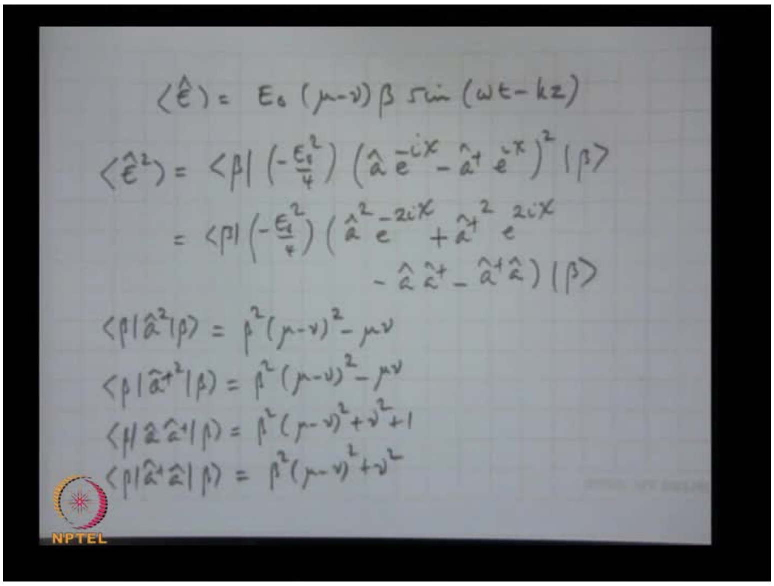

So, I need to use this equation, this equation and after substituting the expression for a and a dagger. This is a bit of algebra, very simple algebra and I will just give you the expressions. Please verify yourself. So, this is beta square into mu minus nu whole square minus mu nu beta a dagger square beta is equal to beta square mu minus nu whole square minus mu nu beta a a dagger beta is equal to beta square into mu minus nu whole square plus nu square plus 1 and beta a dagger a beta will be beta square into mu minus nu whole square plus nu square. Because a a dagger is equal to a dagger a plus 1, there is a difference of 1 between expectation value of a a dagger and a dagger a.

So, I need to substitute the expressions for a and a dagger in terms of b and b dagger, use the condition that b beta is equal to beta beta and similarly for the b dagger operator and a little bit of algebra will give you these expressions for a square expectation value and a dagger square expectation value and expectation value of a a dagger and a dagger a.

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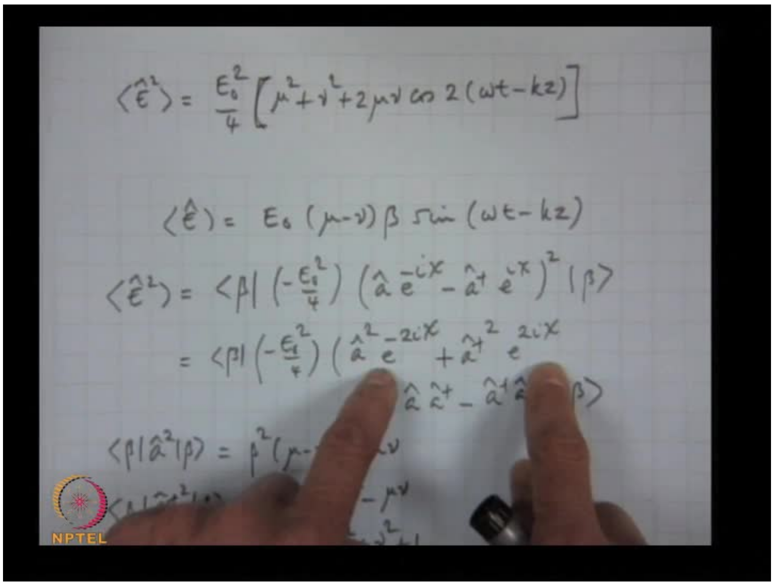

So, I now substitute all these into this equation and here is what I will get. Expectation value of E square is equal to E naught square by 4 mu square plus nu square plus 2 mu nu cos 2 times omega t minus k z.

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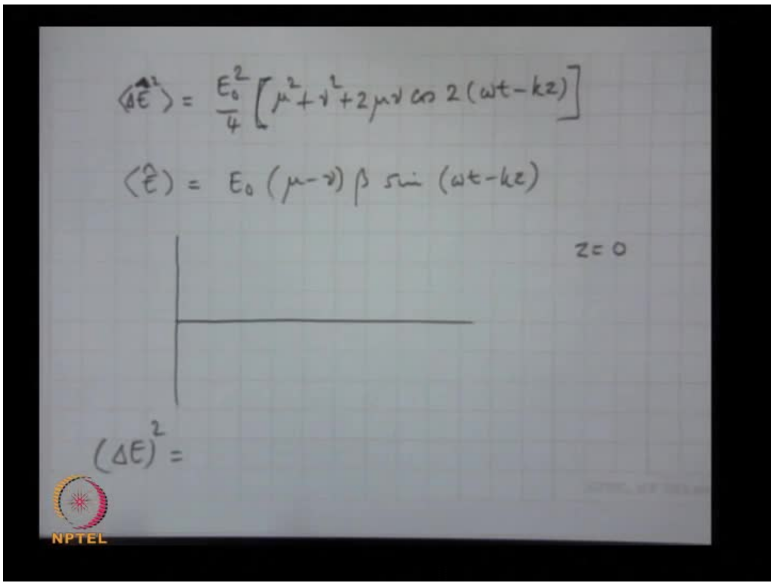

If you calculate all the expectation values substituted into this, this cos 2 times omega t minus k z is coming from this term. See, a square expectation value and a dagger square expectation value are equal. So, essentially you will get a cos 2 chi coming from here and I have the expectation value of E, which I have derived earlier is equal to E naught mu minus nu into beta sin omega t minus k z.

For the coherent state, I can obtain by putting nu is equal to and mu is equal to 1 and if you put nu is equal to 0, look here this expression disappears and the expectation value of E square becomes independent of time for a coherent state which is what we had seen.

(( )) Not audible form Time: 13:20 to 13:27E square no, if you put nu is equal to 0, you get the coherent state and the expectation value of E square in the coherent square is independent of time. Here it is a function of time.

So, let me try to plot the electric field variation. So, let me plot at z is equal to 0, I chose a point at z is equal to 0 and plot E as a function of time expectation value of E as a function of time and around that value expectation value of E square so what is delta E square? The variance in E is given by E square expectation value minus E expectation value square this is delta E square. Let me, let me, write the expression for E square, - 1 second.

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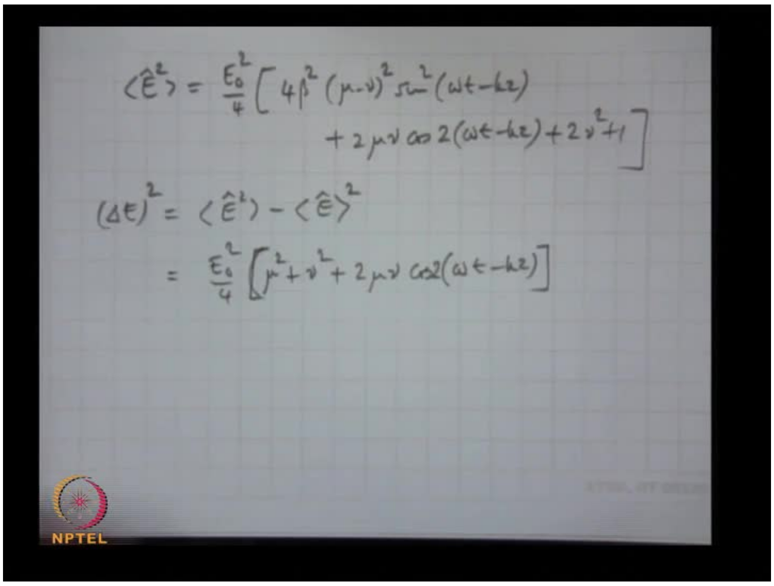

So, E square expectation value please correct this E naught square by 4 4 beta square mu minus nu whole square sin square omega t minus k z, you are right there is a beta here, plus 2 mu nu cos 2 times omega t minus k z plus 2 nu square plus 1.

Please correct this is the expectation value of E square and it depends on beta. So to calculate the variance I need to do E square expectation value minus E expectation value square.

So, the first term here is E naught square beta square mu minus nu whole square sin square omega t minus k z is nothing but E expectation value square E naught square beta square mu minus nu whole square sin square omega t minus k z is nothing but expectation value of E whole square. So, that cancels off and I am left with E naught squares by 4.

Now, 2 nu square plus 1 1 is mu square minus nu square. So 2 nu square plus 1 is mu square plus nu square. So, mu square plus nu square plus 2 mu nu cos 2 times omega t minus k z.

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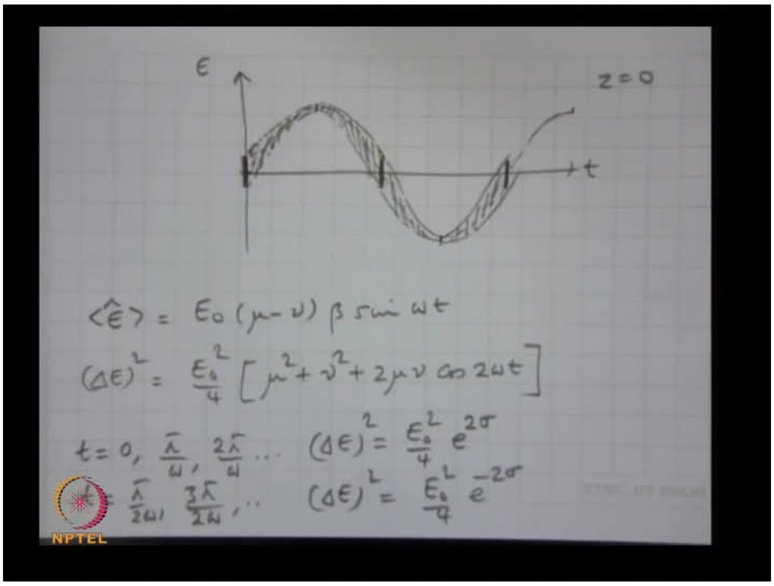

So, the variance in electric field is depends on time and varies in this fashion. So, let me try to plot how the electric field will look like? If I have to measure as a function of time and I position myself at z is equal to 0. So, I have expectation value of E is equal to E naught mu minus nu beta sin omega t and delta E square is equal to E naught square by 4 mu square plus nu square plus 2 mu nu cos 2 omega t.

So, depending on the values of mu nu and beta, the expectation values vary sinusoidally and let me the p plot dash line here this is the expectation value it goes like this. At the frequency omega which means on an average if you what measure the ensemble average of the electric field you will get this dependence, it is sinusoidal dependence but now if you look at this equation here the variance in the electric field depends on time.

So, at t is equal to 0 this becomes mu square plus nu square plus 2 mu nu which is mu plus nu whole square which if nu is positive . What is the value? So, at t is equal to 0 , and then pi by omega, 2 pi by omega ,etcetera; delta E square becomes E naught square by 4 into mu plus nu whole square which is exponential 2 sigma.

Assuming mu is equal to cos hyperbolic sigma and nu is equal to sin hyperbolic sigma and at t is equal to pi by 2 omega, 3 pi by 2 omega, etcetera; delta E square becomes E naught square by 4 exponential minus 2 sigma.

What is delta E square for the coherent state? E naught square by 4, nu is equal to 0 mu is equal to 1. So, at certain times the variance in the electric field is more than the coherent state and at other intervals of time it is less than the coherent state.

So, if I have to plot this I will have for example, this will be the variance here at t is equal to 0, t is equal to pi by omega, t is equal to pi by 2 omega, etcetera. Here the variance is much smaller. So, which means if I draw the region of electric field, which you will measure it will look like this.

The variance depends on time at certain times, the variance here is less than what you will achieve in coherent state or in vacuum, please remember the variance of the electrical filed in the coherent state or the is equal to that is the vacuum.

So, here the variance is less than in vacuum but correspondingly there are other times where is much bigger than vacuum, you cannot have variance at all times less than vacuum. This is related to delta X, delta Y greater than equal to 1 by 4. So what is actually happening is at these intervals of time, you are having larger variance than vacuum at other intervals of time you are having less variance than vacuum.

So, what is happening? The point of intersection with the axis is ill defined, the phase is ill defined. But the amplitude is more precisely defined than in the coherent state, if I have to measure the electric fields at this intervals of time, I will get an electric field value which is better than or less noisy than I would have got if I had vacuum or if I had coherent state.

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Yes, so I need to do a measurement which is called homodyning, which will measure one of the quadratures. when I will discuss the quadrature it will become clearer I must measure one of the quadratures of the electric field which has less noise than the other quadratures. So what is actually happening is the amplitude with the electric field is more precisely defined than phase here, much more precisely defined.

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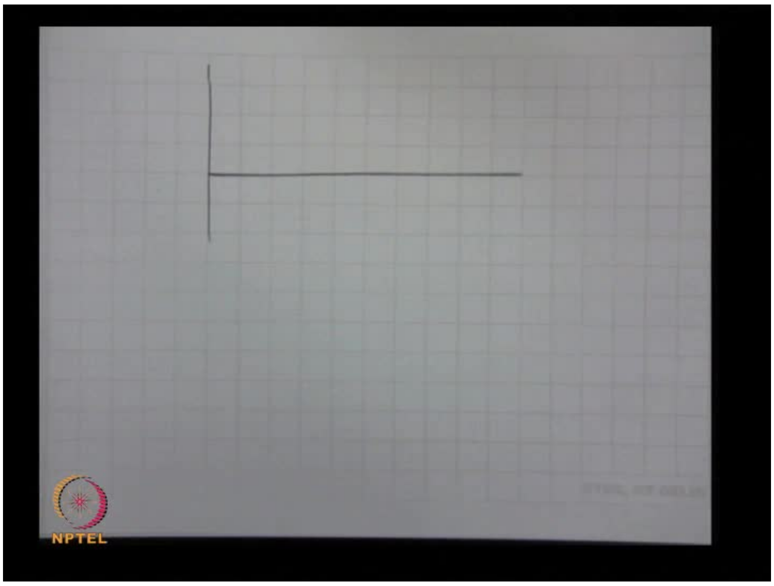

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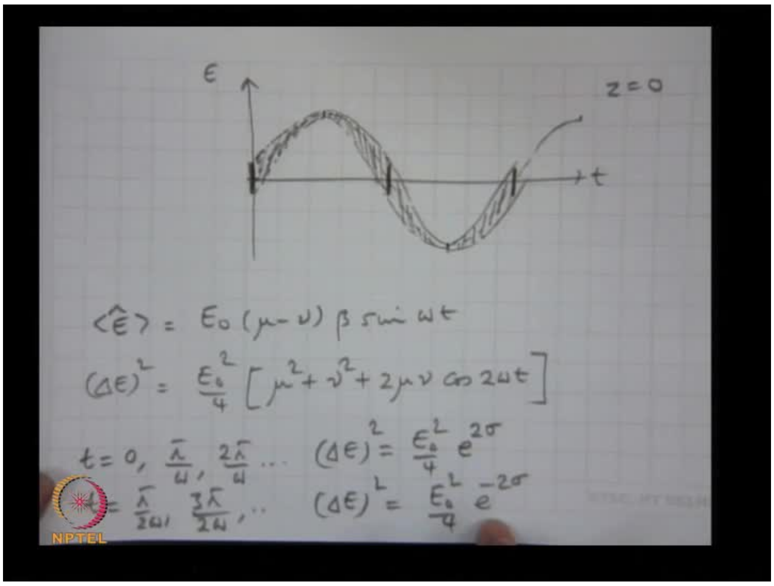

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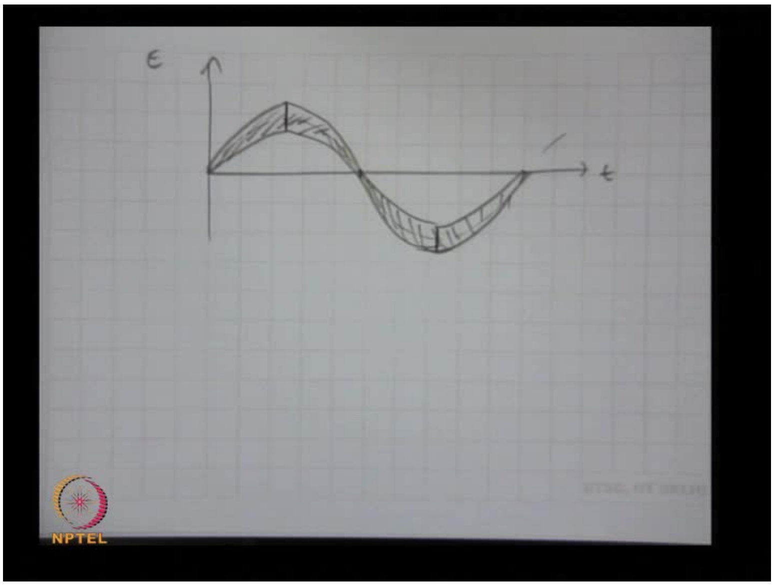

If mu nu is equal to 0, you get the coherent state and the variance is independent of time; it is a equal thickness curve which are plotted last time. So, what I am saying here is, this is characteristic by squeezed state. In fact, if sigma is negative, nu is negative; in that case I would have got another state which is different. I would have got a state like this, if sigma is negative then at these intervals of time, the noise is less than vacuum at these intervals of time; the noise is more than vacuum. So, I would have got suppose this is my expectation value - here the noise is less, here the noise is more. so I would have got like this. The phase is more precisely define here at the cost of a greater than noise or uncertainty in the amplitude the electric field.

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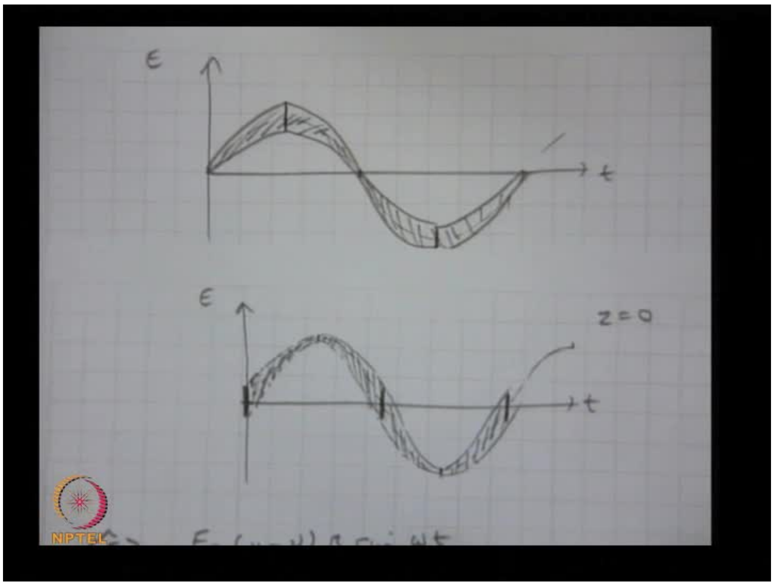

So, depending on the sin of sigma I can have either squeezing or reduce noise in the amplitude quadrature here or the phase quadrature here. We will now calculate the noise in x and y quadrature’s and that will give us a better picture and we can then draw the phasor diagram and see how the phasor diagram looks for a squeezed state, is this clear?

So, these are some special state, the problem is how do I produce these states? We will come to it later but these are some special states in which I can have reduce noise in one at the cost of increase noise in the other.

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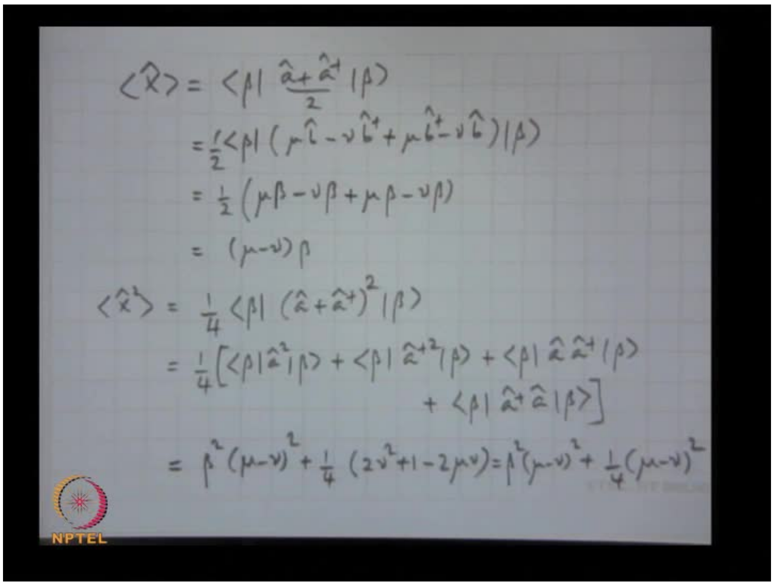

Now, let me calculate. So, for the phasor diagram I need to calculate expectation values of X Y and then variance in X and variance in y Y. So, I need to calculate expectation value of X, which remember this is defined as beta a plus a dagger by 2 beta.

Now, I will just do a calculation for 1 or 2 then I leave the remaining to you. So, this is beta half of beta, so this is mu b minus nu b dagger plus mu b dagger minus nu b beta. Half of now b on beta is beta beta and beta into beta is 1 normalized, so I get mu beta. The second term b dagger on beta bra is beta beta because beta is real. So, I will get minus nu beta. Again, this b dagger acting on beta bra gives me beta into beta bra beta bra in to beta ket is 1. So, I get plus mu beta and then minus nu beta.

I am assuming beta to be real. Otherwise, I would have got beta dagger operating on bra beta would have given me beta star into beta bra. So, this is equal to mu minus nu beta X square expectation value 1 by 4 beta a plus a dagger square beta, which is 1 by 4 beta a square beta plus beta a dagger square beta plus beta a a dagger beta plus beta a dagger a beta. It is just algebra a little sometime becomes tedious to simplified to calculate etcetera. But let me, give you the expression for this again; you need to calculate, substitute the values of a in terms of b and b dagger and use the equations and what you get is the following. Beta square into mu minus nu whole square plus 1 by 4.

Actually the second term is (( )).

Yes, so this is beta square into mu minus nu whole square plus, this is 1 by 4 of can we estimate expectation value of Y.

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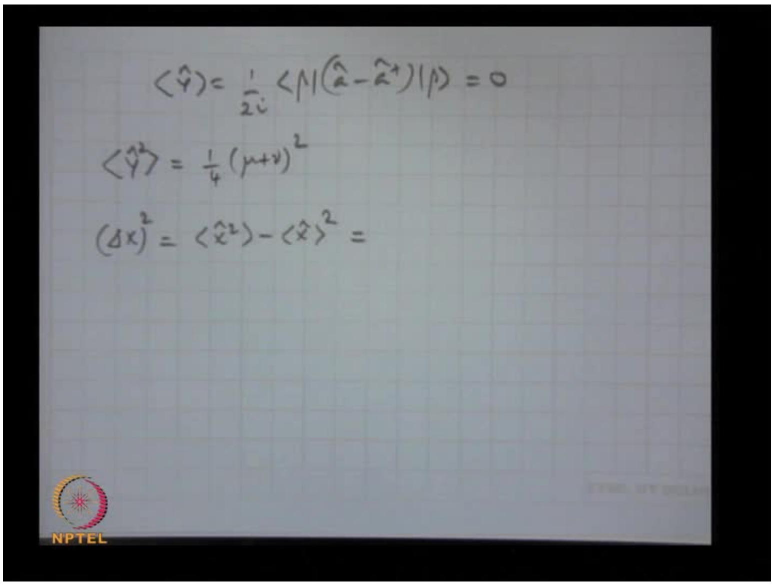

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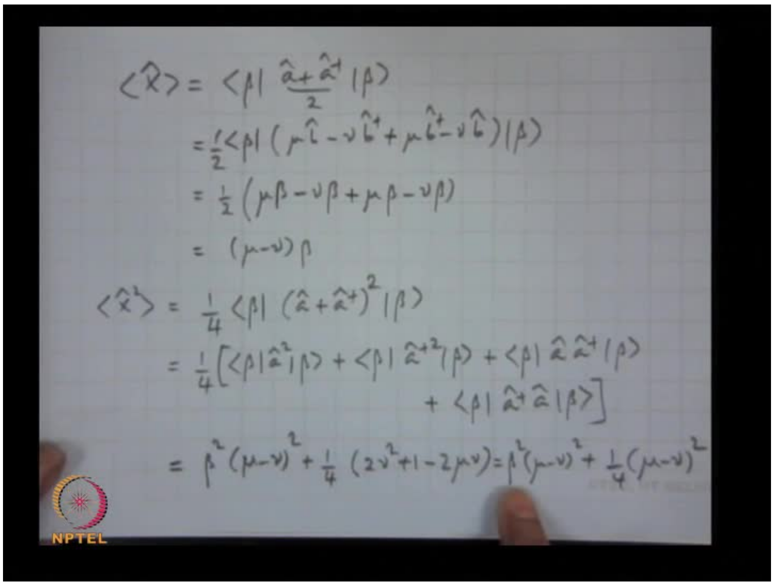

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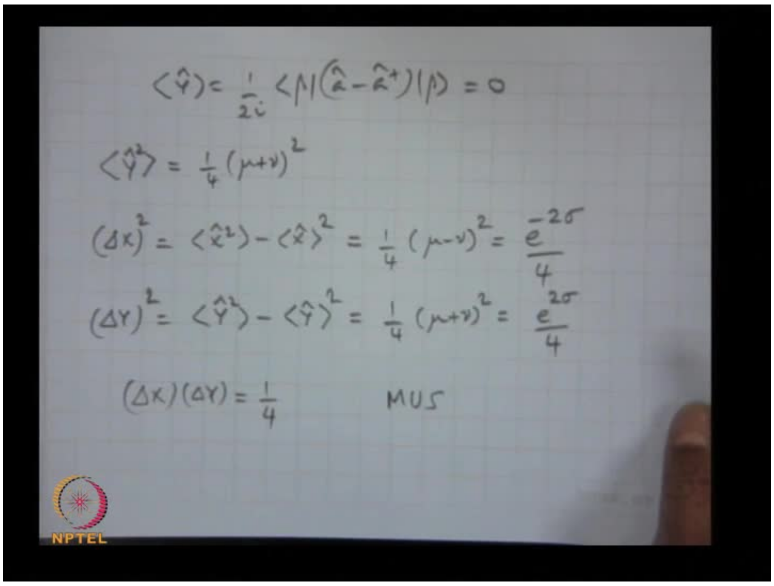

Yes, 0 yes, expectation value of Y is 1 by 2 i beta a minus a dagger beta its 0. So, I will leave this you to to you, show that expectation value of Y square is equal to 1 by 4. So, variance in X is X square average minus X average square, which is equal to (()). Now, this first term beta square mu minus nu whole square is exactly equal to X average square that cancels off and I am left with 1 by 4 mu minus nu whole square; which is mu minus nu is exponential minus sigma and delta Y square is Y square average minus Y average square which is equal to 1 by 4 mu plus nu whole square. Because Y expectation value is 0 and I get exponential 2 sigma by 4 and delta X delta Y is equal to second a minimum uncertainty state. It is a minimum uncertainty state because the product of the uncertainty in X and Y is 1 by 4 but delta X and delta Y are not equal and if sigma is positive, remember in coherent state delta X square was 1 by 4.

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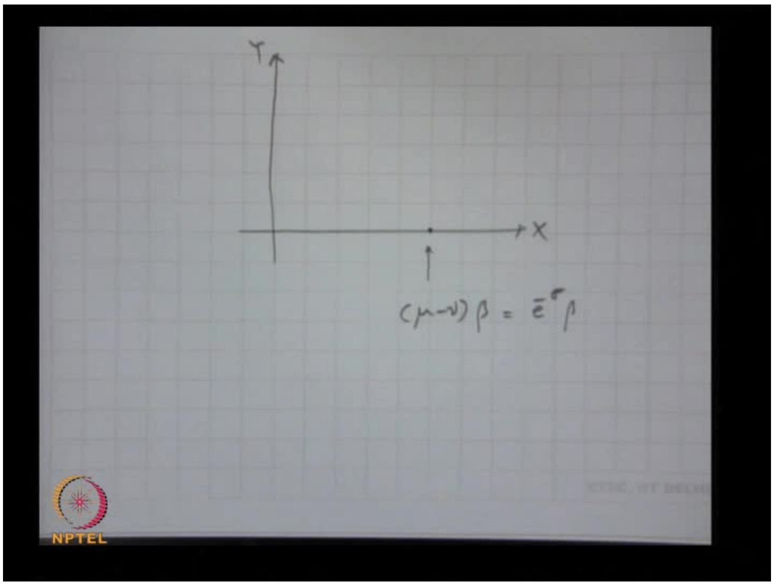

So, if sigma is positive delta X square is less than in coherent state by delta y square is correspondingly more in the than coherent state. Similarly, for sigma negative delta X squares is more than coherent state and delta Y square is less than for coherent state. So, I can actually draw a phasor diagram. Now, knowing these numbers expectation value of X expectation value of Y delta X square and delta Y square and here is the phasor diagram.

So, where will the spot appear? Expectation value of X, expectation value of Y, it some point where? On the X axis, because expectation value of Y is 0. So, it will be somewhere here, whose value is mu minus nu into beta; with this point is actually which is exponential minus sigma beta.

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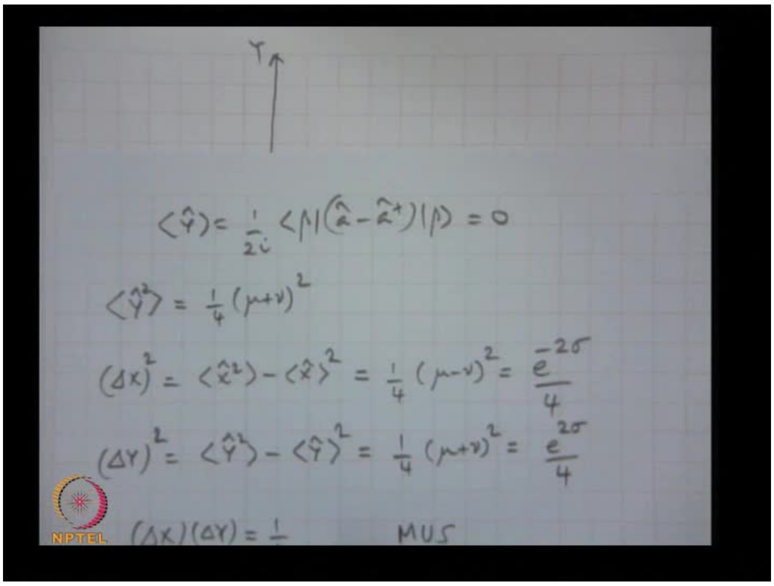

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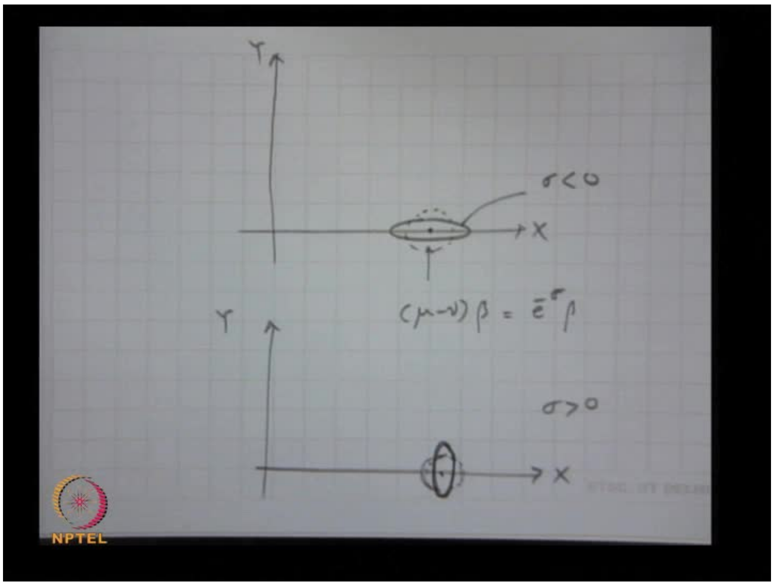

Now, how will I draw the area for corresponding to delta X and delta Y? For the coherent state remembers delta X is equal to delta Y and you get a circle. Here if sigma is positive, delta X is less than delta Y, delta X is less than for coherent state and delta Y is more than for coherent state. So, if the coherent state was something like this, this will correspond to figure like this delta X. This is for this is for sigma less than 0, let me draw for sigma greater than 0.

The circle in the case of coherent state which corresponds to equal uncertainties in both the quadratures gets squeezed into an ellipse. and In this case, the squeezing is in one direction. In this case, this squeezing it is in the opposite direction. This particular state here has less noise in the X quadrature than the coherent state but it has more noise in the Y quadrature. This state has less noise than the coherent state in the Y quadrature, while it has increased noise in the X quadrature.

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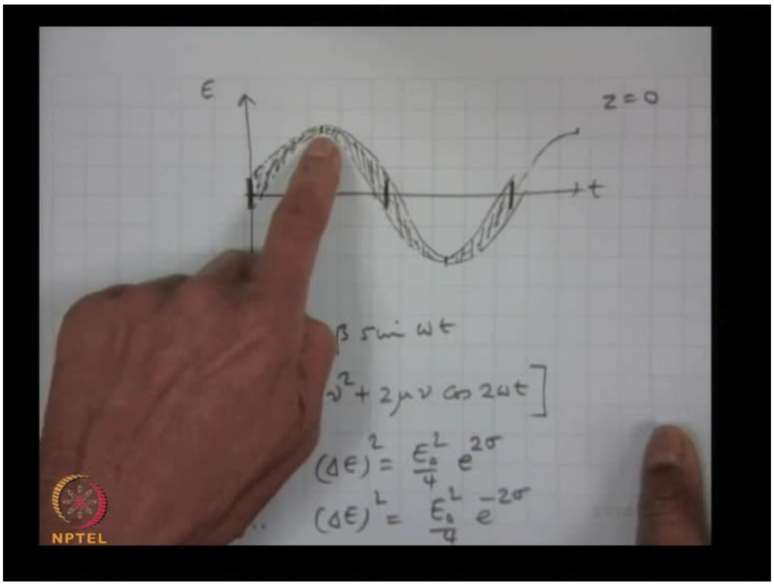

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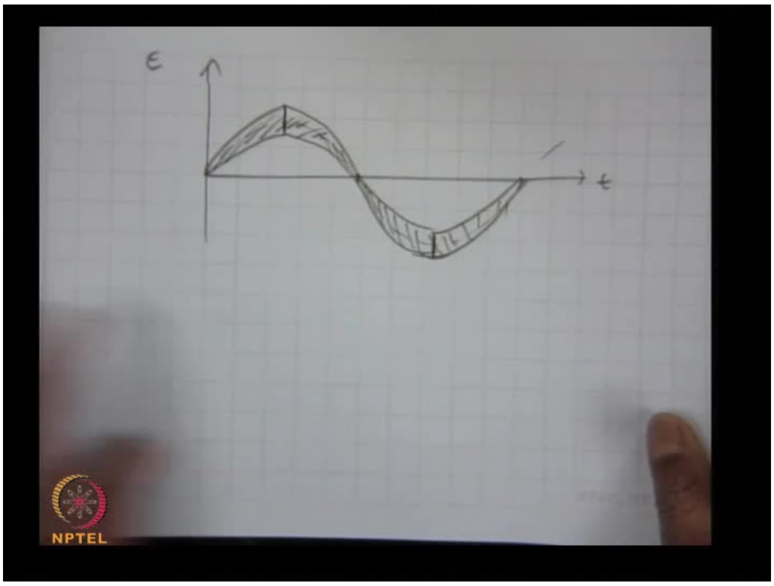

So, these are called quadrature squeezed states because this squeezing is in one of the quadratures and what it implies is that, please remember i had told you that this distance is something like the amplitude of the electric field. So, there is less noise as you can see here for sigma greater than 0. As you can see, from this plot that we had here hear that there is less noise for sigma positive. As you can see, here this less noise in the amplitude and more noise in the phase for sigma less than 0, there is more noise in the amplitude and less noise in the phase, here.

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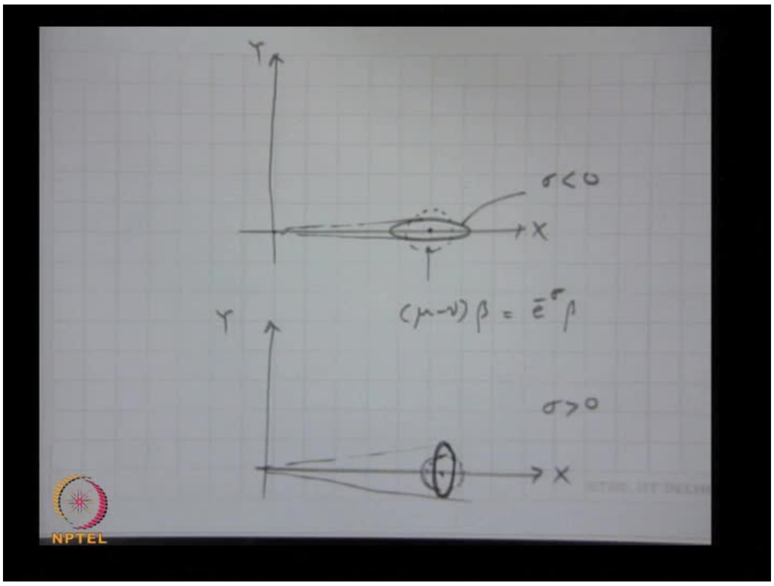

This angle, as I was telling you, is something like the angle. The phase uncertainty in phase is something related to this particular state has more uncertainty in phase, than the coherent state, but it has less uncertainty in the amplitude than the coherent state, less fluctuation in the electric field amplitude.

Here, the fluctuations are in the other quadrature's.

yes Mohit

Sir (( ))

Yes

(( )) (( )) that p corresponds to the amplitude and the points where they intersect?

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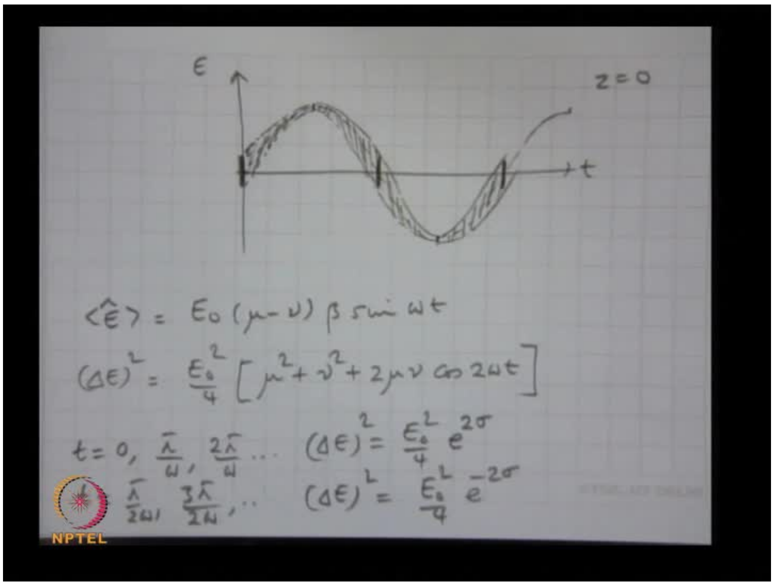

No, I am just by that I mean, the amplitude of the electric field which is proportional to something either the number of photons in the state has less uncertainty. Because the amplitude is most certain; here I know the amplitude more precisely than I knew for a coherent state.

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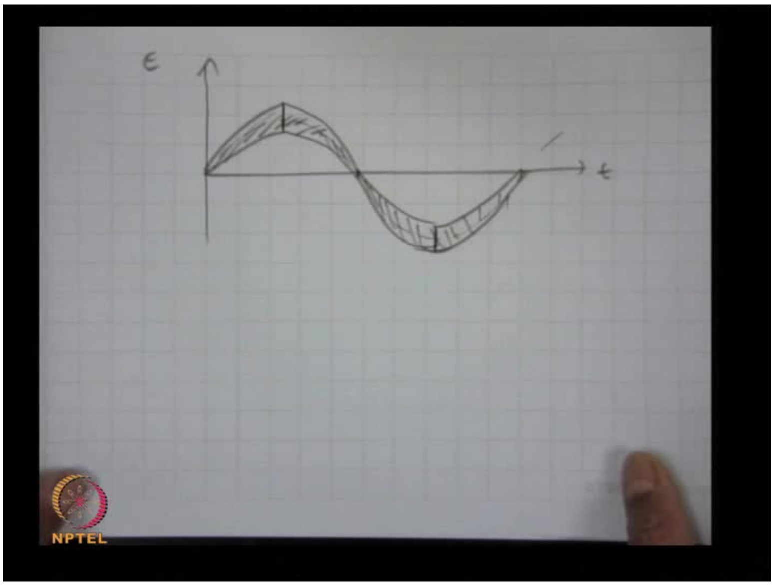

If I were to measure precisely the electric field at this instant of time, I will get a value of electric field more precise than what I would have got, if I had a coherent state. So, this state has less noise in amplitude than a coherent state and the coherent state it is on one that is produce by a laser. So, this particular state has less noise in amplitude quadrature or phase quadrature, depending on the sign of sigma that I have choose have I choose.

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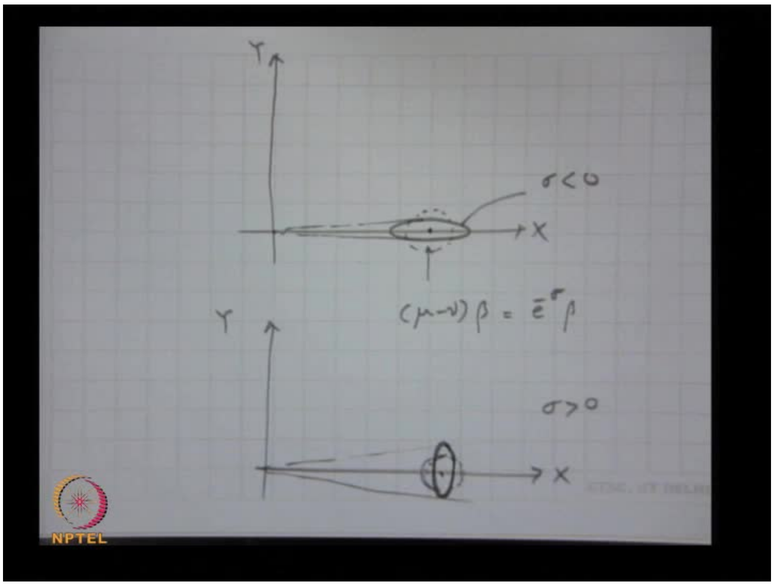

Later, I will tell you, how to measure one of the quadratures, how to measure the quadrature correspond to X or Y in an actual experiment? So, that I can actually use this less noise. The presence of less noise to increasing my detection sensitivity, and these are the type of squeezed states that are used for gravitational wave detection; because these states can have less noise than what you can get with simply by laser beams, in that you can reduce the noise.

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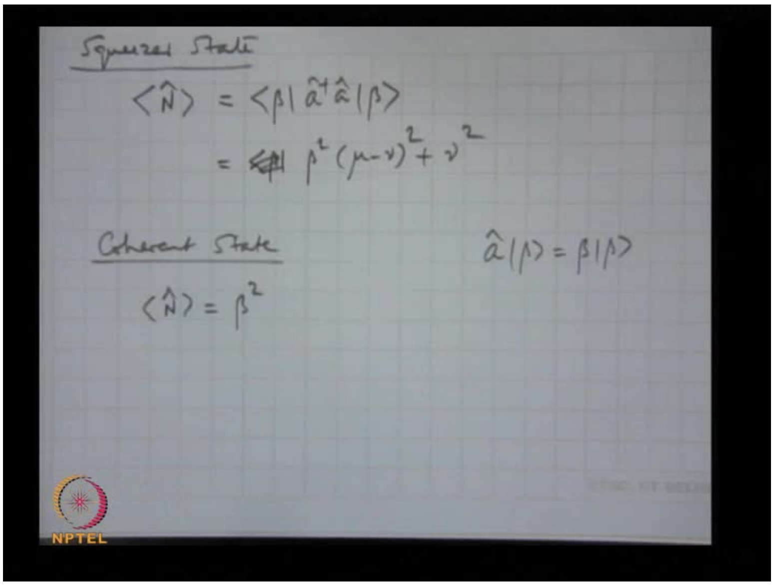

Now, there is something interesting that I want to check and that is let me calculate the average in number of photons in this state. What I have to calculate? Expectation value N, which is beta a dagger a beta.

So, this is beta. Now, a dagger in the I must substitute, whether we have calculate a dagger a expectation value? Yes, we have ,we have, calculated, so what is the value of this?

(( )) Not audible time from 37:15 to 37:30

beta square

Mu minus nu plus (( )) Not audible time from 37:40 to 37:47

Yes, so this for the squeezed state. What I will get for the coherent state? Assuming that a beta becomes equal to beta beta. So, I just put alpha is equal to beta in this case. So, what do I get? Which is mod alpha square we got, remember? Expectation value of N was mod alpha square average number of photons in the state was mod alpha square, I am just replacing alpha by beta and assuming beta to be real. So, I just get instead of mod alpha square which we did, I just get beta square.s

Now, here is an interesting state called vacuum state. What is the vacuum state? Vacuum in is the sense of radiation a state in which none of the modes are excited. So, if I look at the single mode, if I in the vacuum state of the single mode, what is the equation satisfied with the vacuum state? A normal vacuum state.

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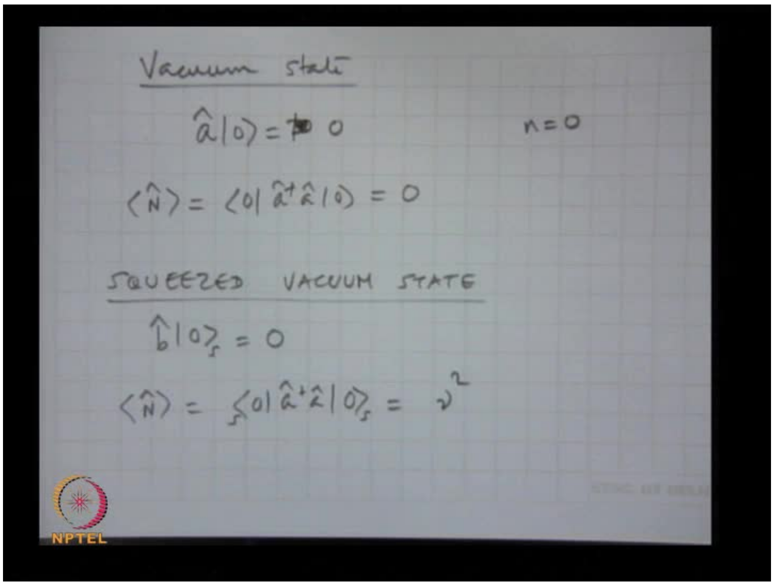

What is equation? What is the state, by which is the limit into which I reach as I keep on removing photon after photon from a [fock/flock] state. (39:38)

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a on 0 is equal to 0. This is called the vacuum state. Because if I start with any n ket state and I keep on applying a operator, I keep on destroying 1 by 1 photon and finally I will reach a state, which is the lowest energy state; which is called the vacuum state; where this corresponds to n is equal to 0.

So, what is the expectation value of n in this state? 0 a dagger a 0, there are expectation value is 0.

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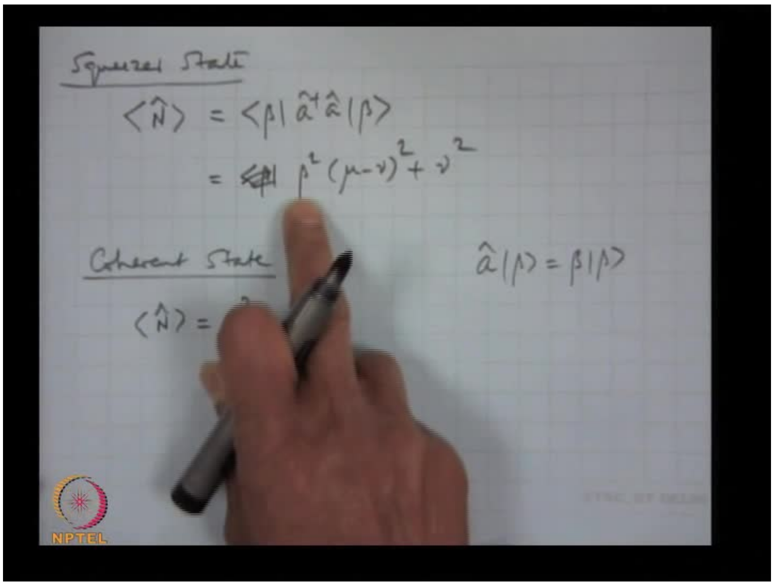

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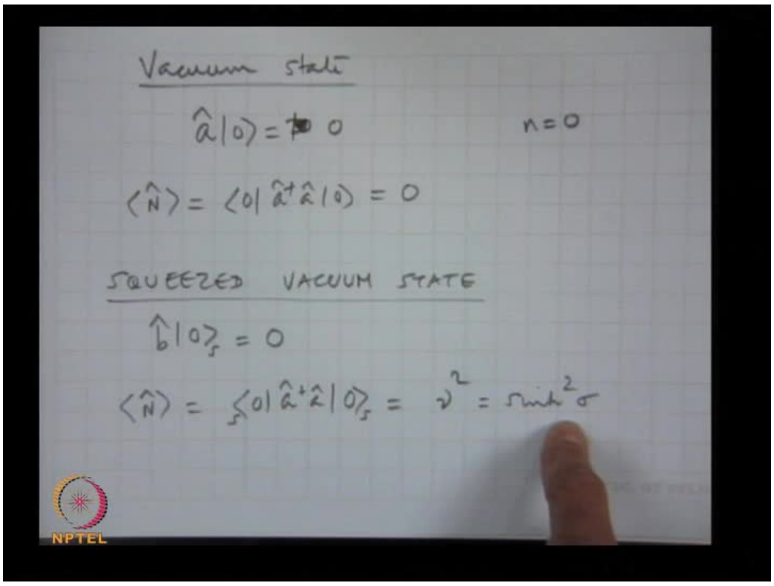

Now, we defined what is called as a squeezed vacuum state. Similar to this equation by saying b 0, let me put s here, just to tell you it is a state of the lowest Eigen value beta is equal to 0 Eigen value just like a 0 is equal to 0, I call as a vacuum state, I am calling b 0 is equal to 0 as the squeezed vacuum state and what is the expectation value of N. In this squeezed vacuum state is equal to because I can use this equation. Which I just now wrote beta is equal to 0. So, I can nu square which is sin hyperbolic square.

So, a squeezed vacuum state has photons in it. Expectation value of measuring photon number in that squeezed state is finite, it not 0 because I get sin hyperbolic square sigma it is called vacuum state. Because this is the lowest state corresponding to the b operator but it is called as squeezed vacuum state, because that is a state in which b 0 is equal to 0 and so this squeezed vacuum state, actually has photons in it and if I were to draw the vacuum and this squeezed vacuum state in a phasor diagram, how will I draw? So, please note what is delta n square? We had expression for (()) no, we did not calculate delta n square.

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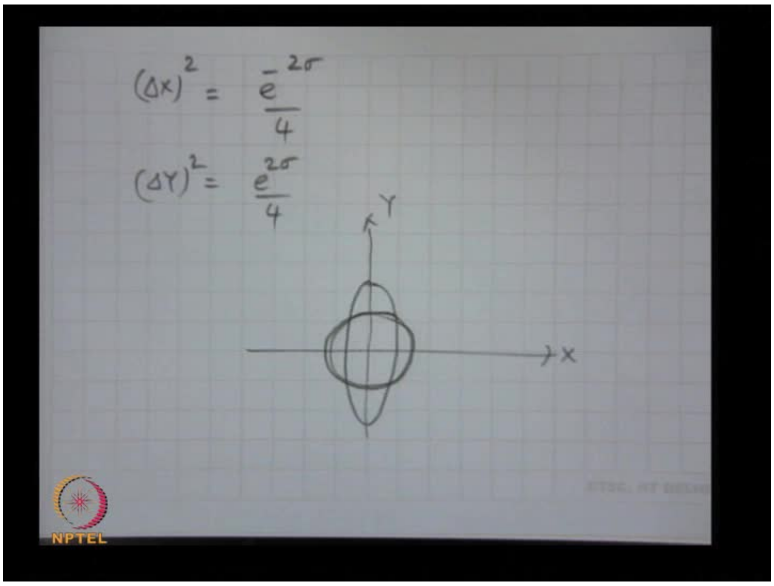

Let me, let me, calculate delta. What is delta X square for the squeezed vacuum state? We had an expression for delta X square, I put beta is equal to 0 it is actually independent of beta exponential minus 2 sigma by 4 and delta Y square is equal to exponential 2 sigma by 4.

And what will be the delta X, delta Y for coherent vacuum state? 1 by 4 and 1 by 4. So, if I draw the phasor diagram a coherent vacuum state will look like this, expectation value of X is 0, expectation value of Y is 0, delta X square is 1 by 4, delta Y square is 1 by 4.

This one a squeezed vacuum state with sigma positive will be like this, expectation value of X is again 0, expectation value of Y will be again 0; if you go back and look at the expectation values of X and Y for the squeezed vacuum state, if you put beta is equal to 0 then expectation value of X becomes 0; expectation value of Y is 0; delta X square delta Y square are given like this.

So, this is one squeezed state and the other one will correspond to squeezing in the other direction such interesting because even when I have nothing, I have this noise; if it was a coherent vacuum state which means I come to a state which is the 0 state corresponding to n is equal to 0 number of photons, in that mode there are is no photons.

There is a fluctuation in X quadrature, there is a fluctuation in the Y quadrature. They are equal delta X is equal to delta Y. Which means if I got the measure the electric field X quadrature or electric field Y quadrature, I will find noise. If I could prepare a state which I call as the squeezed vacuum state in which b 0 is equal to 0.

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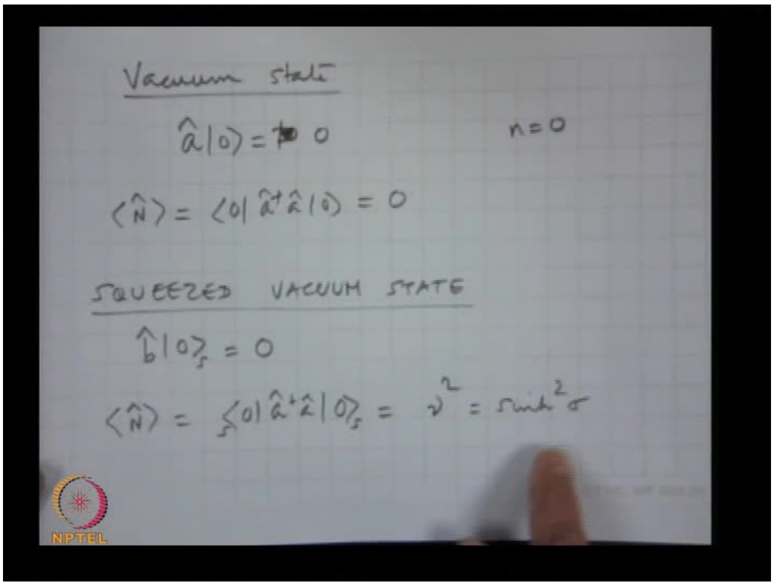

Then, with sigma positive for example, I will have less noise then I would have got even when there was nothing. This is a state in which there are finite number of photons. Please remember now, this is not a vacuum. There are now some photons in this state because expectation value of N is not 0, it is sin square sin hyperbolic square sigma

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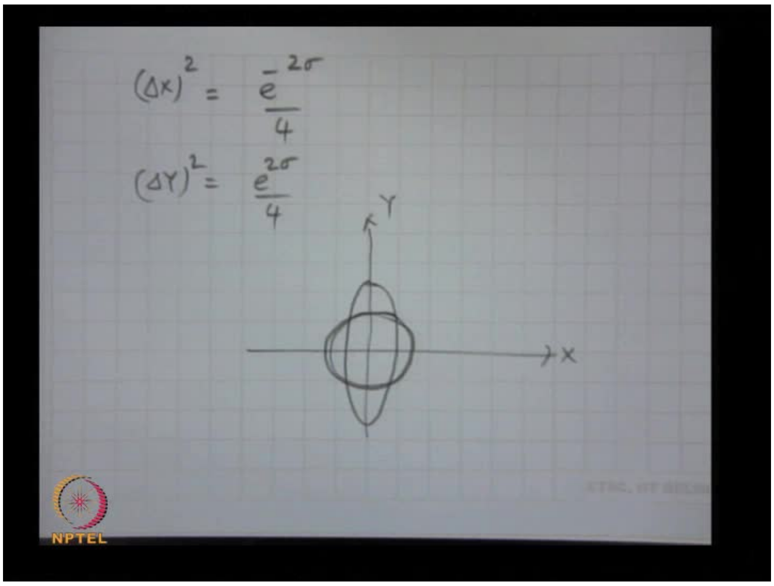

So, there are photons in the state now, but what is happening is the uncertainty in the X quadrature is less than, what you would have got had? You had no photons. This is called squeezed vacuum. This quadrature you can squeezed the vacuum in the Y quadrature by taking a sigma which is negative.

Of course, you pay price of the increasing the noise in the other quadrature. So, you have to do a measurement, which is phase sensitive because you need to measure the X quadrature or the Y quadrature, you need to do a phase sensitive measurement and that is typical in homodyning.

So, I will bring up this concept of homodyning a little later. But it is very interesting to note that if I could choose states for which satisfies the condition of the squeezed state condition. Then I can go to a state where which is the lowest state of the b operator in which case I have less noise than vacuum in one of the quadrature at the expense of increase noise in the other quadrature.

Yes

(( ))

(( )) x and y operator (( ))

Yes, remember the electric field has x sin omega t minus k z minus y cos omega t minus k z. So, the sin term sin and cosine are pi by 2 out of phase. So, they are the 2 quadrature, if I could measure the amplitude correspond to the sin term, I will get the expectation value of x, if I could measure the amplitude correspond to the y term, the cosine term I would have got the expectation value of y operator.

So, I need to be phase sensitive. Now I need to be sensitive to only pick up the sin part of the electric field to get expectation value of X and that is possible through because phase sensitive means I must have another reference phase with respect to which I match and measure and that is what is homodyning essentially.

Homodyne is a very general principle, which is used in engineering for signal detraction and signal processing but here, we will use optical homodyning I will explain to you, how it can lead to very interesting situations and let me tell you this decreased noise in vacuum is an experimentally measured quality and a parametric down conversion process which can lead me to these kinds of squeezed states.

So, yes, do you have questions on this? So, this, this, squeezed states are very interesting because you can have reduced noise in one quadrature, at the expense of the other. Here, we have assumed mu nu and beta to be real but in general, please note there I could have a state which should be squeezed may look like this.

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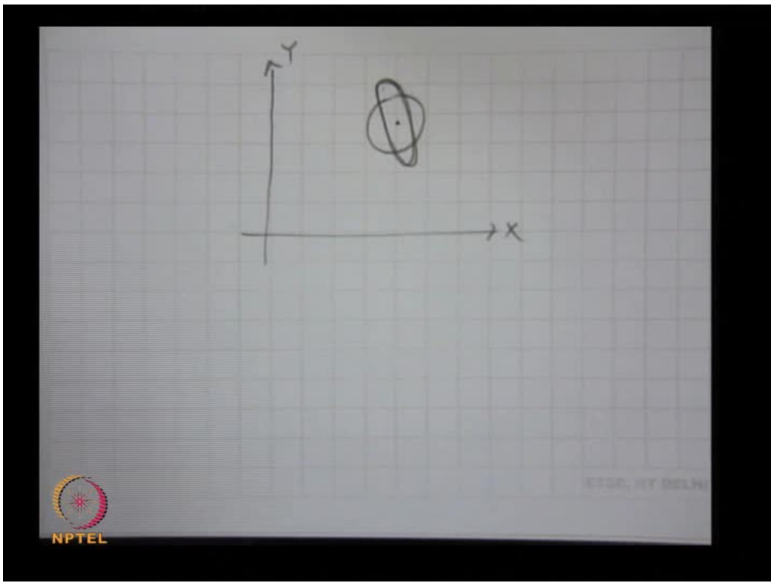

So, a coherent state may be like this is squeezed state. Squeezed in some arbitrary direction, here I am showing either horizontal or vertical ellipses but in general I could have an expectation value of X and expectation value of Y, which are nonzero both nonzero and the squeezing is not in the X and Y quadrature but in the another pair of quadratures.

So, this squeezing is taking place from a circle to an ellipse in some orientation and that is a more general squeezed state. This kind of states you will obtain if you were to choose complex values of mu nu and beta.

I think we will stop here. So, what I want to do next time is may be go on to what we call as multimode states, it still now we are talking of states in which only one mode is excite; light is present in only one mode, 1 frequency, 1 propagation direction, 1 polarization state.

I want to extend this into a state where I can have at least 2 modes. Which means same direction of propagation but 2 different frequencies or the same frequency but 2 different propagation directions and that is where I will start to see interference effects because with single frequency I do not see any interference.

The interfere I must have either 2 frequencies or 2 waves going in 2 different directions and that will be generated at least by a state which contains 2 modes.

So, we will start to look at 2 mode states and then I will ah try to calculate what do we actually measure? When we do an interference experiments? and This mathematical structure will explain, what we are started within the beginning. That in a beam splitter with single photons coming in only one retractor clits but if I make a Mach-Zehnder interferometer there is interference taking place; both these experiments will be explained by this mathematical structure.