Quantum Electronics
Prof. K. Thyagarajan
Department of Physics
Indian Institute of Technology, Delhi

Module No. # 05
Lecture No. # 33
Quantum States of EM Field (Contd.)

So, we continue with our discussion on single mode states; what we looked at was initially number state of a single mode field, n ket of flock states, and then we looked at coherent states.

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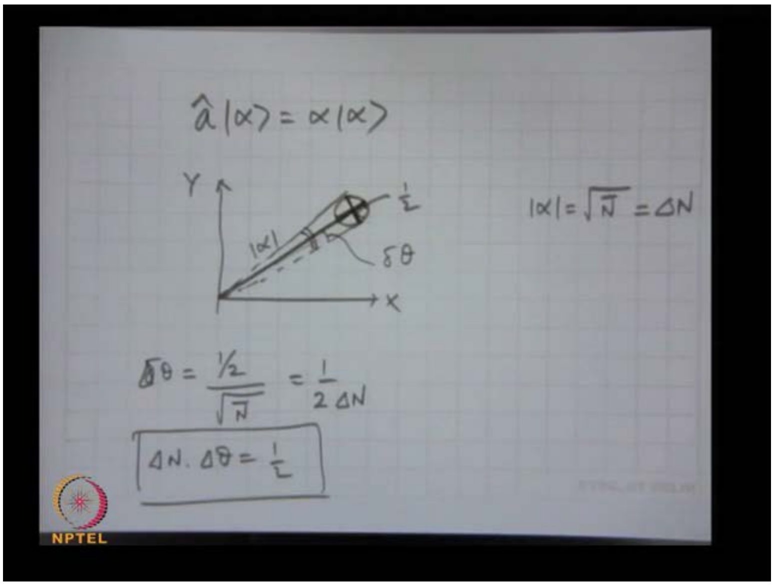

So, these coherent states when we recall are Eigen states of the a operator. We calculated the expectation value of the electric field of this state, and we found it is very similar to the classical electromagnetic wave; expectation value goes as sins omega t minus k z. You also calculated the variance in the electric field, and we found that a variance was equal to that in vacuum. So, this is the state, which is closest to the classical electromagnetic field; and the variance, the relative variance will decrease as you increase the amplitude of the electromagnetic field; as you increase mod alpha, which the electromagnetic field is proportional to mod alpha here; and as you increase the value of the mod alpha, the field amplitude keeps on increasing, and the noise remains the same as a vacuum.

So, the field will appear to be almost noise free at very high amplitudes; and hence, as a very good description of a classical field, where you can sort of forget about noise. We also found that the two quadrature operators, we had calculated the noise, and the expectation values of the two quadrature operators; and what we found is let me recall the diagonal that we had drawn, this is the X quadrature; this is the Y quadrature; the field is represented by a circle; this is mod alpha; and let me recall mod alpha is equal to square root of N bar, which is equal to delta N. The fluctuations and number of photons in that state is square root of N bar; N bar is the average number of photons in that state; in the state alpha, there is an average number of photons, and this is the fluctuation is square root of the average number.

And this is circle of radius half. So, in a way, we can actually interpret; this figure as this length, the square root this length is actually N bar, which is the average number of photons; the square of this length is N bar, which is equal to the average number of photons in this state. And we can also look at this angle, this angle is the angle submitted by this diagonal, by this diameter at the center. You can interpret this angle as something like an uncertainty in the phase of the electromagnetic field; and this along the length of this, as an uncertainty in the number of photons in that state.

Just like you can interpret delta X, delta Y as the quadrature operators, where the uncertainties in X and Y quadrature; I can also interpret this, as this being the approximately proportional to the uncertainty, the number of photons and this angle being the uncertainty in the phase of the electromagnetic field. So, you can actually write down at approximate relationship between these two; if I assume mod alpha to be large, then this angle suppose, I call this delta theta; how much is delta theta? This, this, by this distance; so half by square root of n bar; this half divided by this distance, which is mod alpha, which is square root of n bar, and which is equal to 1 by 2 times delta n.

So, this is something like delta n into delta… let me write this delta delta theta. Some kind of an uncertainty relationship connecting the number of photons in the state, there are fluctuations in the number of photons to the state to the fluctuations in phase. Note that this circle has the same diameter irrespective of the value of mod alpha.

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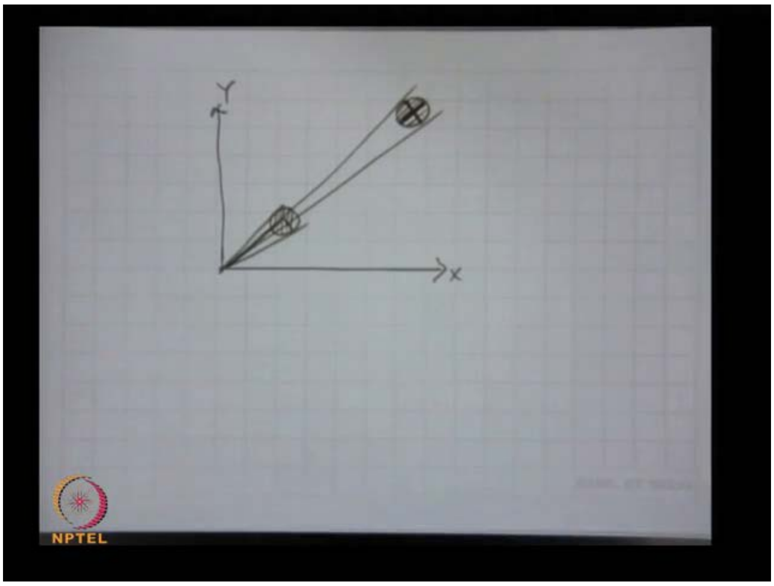

So, if you look at two states; one which is a small mod alpha, and one which is the large mod alpha; this phase uncertainty is much larger here, compared to this. The vacuum has the that is the same noise, vacuum noise all the time; wherever the mod mod alpha value is this particular state with smaller value mod alpha, as the larger phase uncertainty compare to this, because actually delta N is becoming smaller, as you come to closer numbers here, across the values and so, delta theta is actually increasing. So, this is also good picture to sort of have in mind that the number of photons in the state, the fluctuations in the number of photons in the fluctuation in phase are sort of related through a kind of an uncertainty relationship. Because this picture will become useful to us, when we go into the next state, what are called as squeezed states.

Which this state is…

I can interpret the this distance, square root of this distance is the average number of photons, because mod alpha square is N bar. I can interpret this angle subtended by the circle on the X, on this origin as safe it is an uncertainty in phase. So, if I look at two coherent states, one with small value of mod alpha, and one with large value of mod alpha, because the vacuum noise is the same in both cases, the angle subtended by this circle here is larger than the angle subtended by this circle.

So, this has more phase noise than this one that is because delta n for this is larger, because n bar for this is smaller sorry delta n delta n is smaller here; and so, delta theta is larger essentially, because these two relations approximately, relationship delta n delta theta is equal to half. So, I can solve a interpret the this along the radial direction as then uncertainty in the number of photons, and the this distance to be approximately proportional to the uncertainty in the phase of the electromagnetic field.

Sir, according to this it should be the same always, because the circle you know, in the same directions wherever we go. So, uncertainty in number of photons will also 1 and number of…

No.

Page you also you say, it is we can interpret this, you are tied with a along one of the axis we said can be interpret as uncertainty in phase, I mean that…

This this angle, this angle will decrease as you go to larger and larger value n bar; as we increase the number of photons in the state, in this coherent state, the phase uncertainty starts to reduce. And so, with large values of mod alpha, which is very close to classical states, the phase uncertainty is very little. So, you can very well defined a field has e 0 Cos omega t minus k z of sin omega t minus k z.

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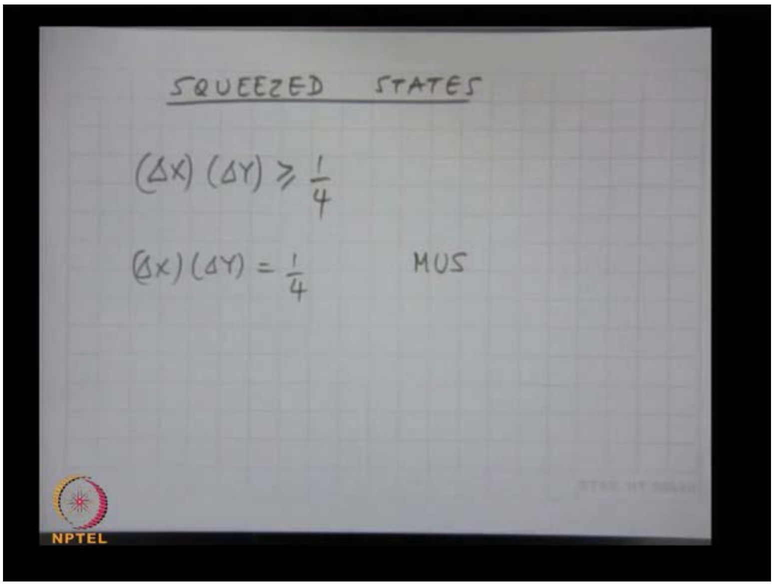

Now, we go to another class of states called squeezed states. We are still looking at single mod states; we are still looking at states, in which there is only one exit, one mode excitation, because after all this discussion, we will come to what are called as multimode states, where I will have fields in more than one mode; right now there is only one mode, which means one frequency, one propagation direction and one polarization. Now if you recall, we had this relationship between the uncertainty in two quadratures; X quadrature and Y quadrature have to satisfy the uncertainty relation. And when delta X delta Y becomes equal to 1 by 4, these are called the minimum uncertainty states, because this is the minimum value of uncertainty product that these states have.

And coherent states have equal uncertainties in both X and Y; delta X is equal to half, delta Y is equal to half, this is what we have derived. So, those are states, in which the uncertainty in X and Y are equal. But there is no requirement that delta X or delta Y need not be less than half, because all that I need is the product must be greater than equal to 1 by 4; and for all minimum uncertainty states, the product must be equal to 1 by 4. So, I can have states, in which one of the quadrature has less uncertainty than the other quadrature. And these are called squeezed states, because in the phasor diagram, there will not be circles any more, but there will be ellipses, squeezed in one direction compared to the other. So, if they are called squeezed states, and so, we will look at these squeeze states.

So, I can have reduced uncertainty in one of the quadrature, and this becomes very interesting for many applications of these kinds of non classical states of light. To introduce squeeze states, before this I forgot… Let me try to answer one question, and that is which was post at the end of the last class, just after the class was over that what is the picture we are using in discussing this coherent states; have we use Heisenberg picture or have we use Schrodinger picture? Yes…

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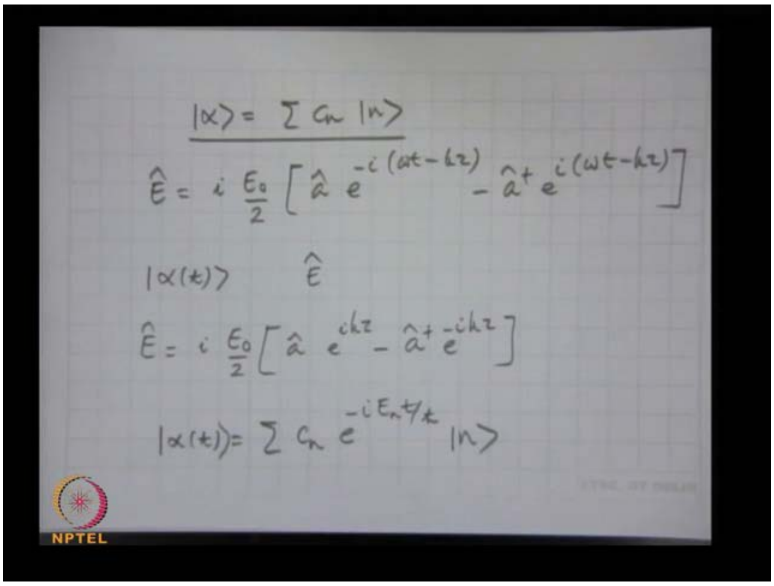

Please note, we have written alpha as sigma C n n and we have written E vector as i E naught by 2 a exponential minus i omega t minus k z minus a dagger exponential i omega t minus k z. We have this Heisenberg picture, because we have kept the alpha state to be independent time; and we have taken all the time dependence into the operator E. So, suppose I have to use the Schrodinger picture; how would I approach this problem? Because there are places, where you would like to use the Schrodinger picture, sometimes you would like to use Heisenberg picture. So, let me do an analysis now of the same problem in the Schrodinger picture and I should get the same expectation early of electric field, irrespective of which picture I use.

So, in the Schrodinger picture, alpha will become a function of time, and E will be independent of time. So, E operator electric field operator will be i E naught by 2 a exponential i k z minus a dagger exponential i k z a and a dagger are constants that is at t is equal to 0, the electric field of the two pictures match and alpha is a function of time. Now, how do I find out, how this alpha state varies with time? Can one answer what how do I find out how alpha state with vary with time? Here is here is alpha t is equal to 0. So, alpha t will be equal to C n exponential minus i E n e by h cross n each 1 of the flock states evolves in time as exponential minus i E n t by h cross, where e n is n plus half h cross omega h cross omega; is that clear? So, this is the state at t is equal to 0, as a function of time this is the evolution.

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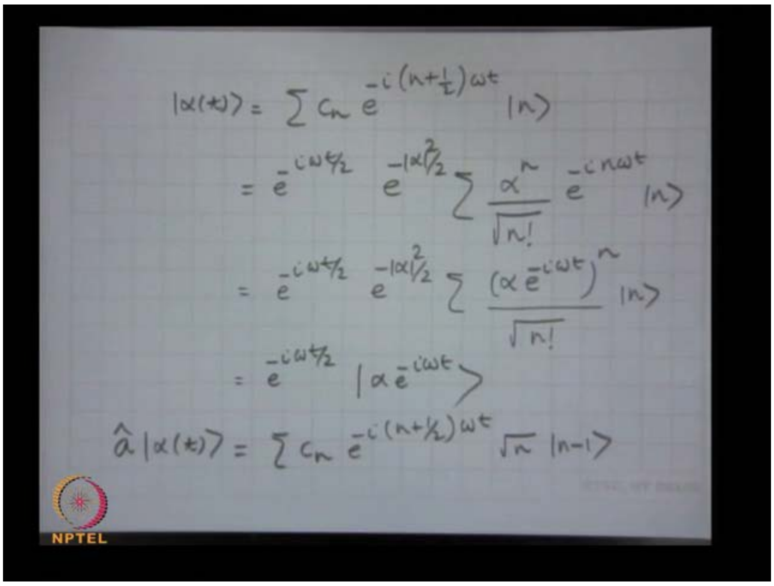

So, I can actually substitute for e n into this equation and get alpha of t is equal to sigma C n exponential minus i n plus half omega t n, which is equal to minus i omega t by 2; let me substitute for C n, so minus mod alpha square by 2 sigma alpha raise power n by square root of n factorial exponential minus i n omega t n. I have replaced e n by n plus half h cross omega, and I have just taken this out, and this is equal to minus i omega t by 2 minus mod alpha square by 2 sigma alpha exponential minus i omega t raise power n by square root of n factorial n. So, this is actually…

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Because if alpha is given by without this factor, alpha is just this without this factor, so if i alpha as changed to alpha into exponential minus i omega t, because the mod alpha square remains the same as before. So, what has what has happened is the coherent state evolves into new Eigen value alpha exponential minus i omega t. In fact, what is a of alpha t? So, alpha t is given by here. So, this is equal to sigma C n minus i n plus half omega t a times n.

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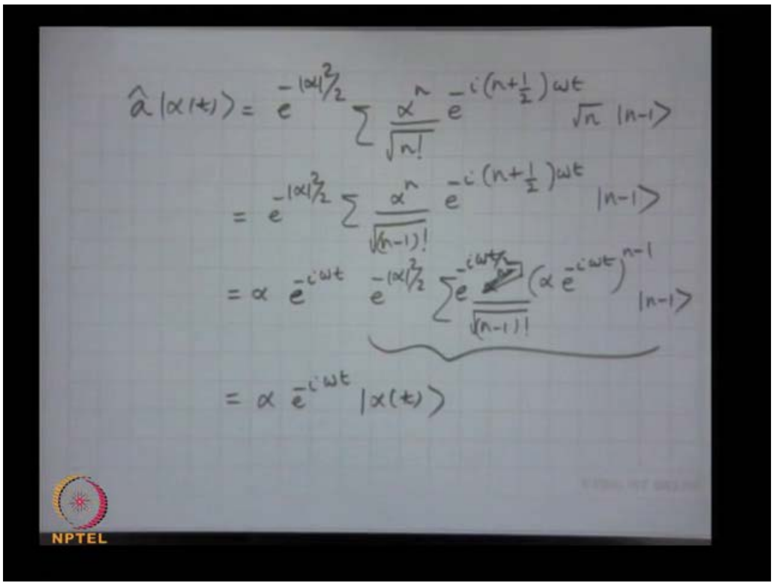

So, a alpha t minus mod alpha square by 2 sigma, let me just write exponential (( )) So, alpha raise power n by square root of n factorial exponential minus i n plus half omega t square root of n n minus 1, which is equal to minus mod alpha square by 2. So, this becomes alpha raise power n by square root of n minus 1 factorial with the square root of n here minus i n plus half omega t n minus 1. So, I can write this as alpha times exponential minus i omega t into minus mod alpha square by 2 sigma alpha raise power n minus 1 by square root of n minus 1 factorial. So, I write that is alpha exponential minus i omega t raise power n minus 1. I have taken out one alpha from here, and one exponential minus i omega t from here. So, I am left with actually into minus i omega t by 2.

No. So, what is this? This is nothing but alpha t is this, minus i omega t by 2 into this. So, this summation is still the same as before; so this become simply. If you look at this expression for alpha t, you have exponential minus i omega t by 2 into this ket, this ket has a sum over n or sum over n minus 1, it is the same. So, exponential minus i omega t by 2 minus alpha square by 2 alpha exponential minus i omega t raise power n minus 1 by n minus 1 factorial into n minus 1 ket and that is alpha t.

So, it is essentially the Eigen value of this as a function of time keeps on changing to alpha exponential i minus i omega t. So, the state is changing with time now, the field operators are constant with respect to time. Now, my objective is to calculate the expectation value electric field; and to show you that it is the same as we had obtained using the Heisenberg picture.

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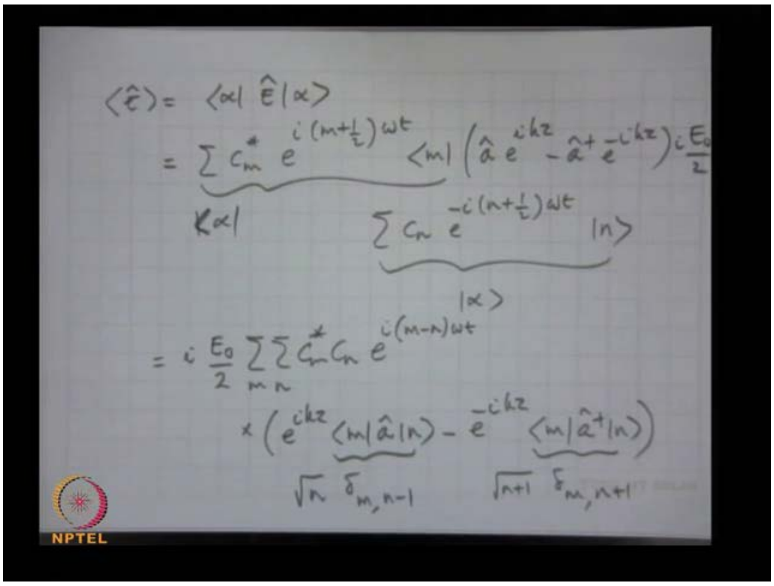

So, I am given alpha t, I know the electric field expression. So, let me calculate the expectation value; expectation value of electric field is equal to alpha E alpha. So, this is equal to sigma C m star exponential i m plus half omega t m, this is the bra alpha into a exponential i k z minus a dagger exponential minus i k z into sigma C n exponential minus i n plus half omega t ket n; this is ket alpha, this is bra alpha. Summation indices are different; this is ket alpha, this is bra alpha; and that is the electric field operator with sorry there is i e naught by 2.

So, this is equal to i E naught by 2 sigma m sigma n C m star C n exponential i m minus n omega t into now, exponential i k z m a n minus exponential minus i k z m a dagger n. So, this m, n, this is the m a n from here - first term, and m a dagger n from the second term. Now, can you tell me how much is this? A operating on n is square root of n n minus 1. So, m n minus 1 is, so this is square root of n delta m n minus 1, and this one is if you operate with a on n, you get square root of n n minus 1, m n minus 1 bracket is delta m n minus 1, a dagger on n is square root of n plus 1 n plus 1, m n plus 1 is delta n n plus 1. So, because of these two chronicle delta function, the double summation, but let us say single summation.

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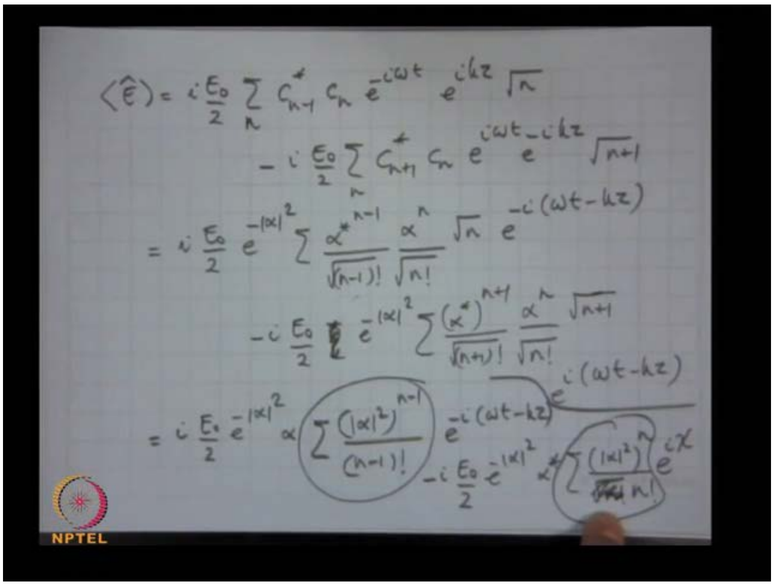

So, what will I get if I use this delta the chronicle delta functions? I will get expectation value of E is equal to i E naught by 2 sigma m. So, let me put in the in the first case, in the first term, I will have m is equal to n minus 1. So, C n minus 1 star C n m is n minus 1, so I get exponential minus i omega t, then I will have exponential i k z; and the second term will be minus i E naught by 2 sigma n C n plus 1 star C n exponential i omega t into exponential minus i k z. This is square root of n, and this is square root of n plus 1.

So, this is i E naught by 2. Now both this is contain exponential minus mod alpha square by 2. So, the product becomes minus mod alpha square sigma alpha star raise to the power n minus 1 by square root of n minus 1 factorial alpha raise to the power n divided by square root of n factorial square root of n exponential minus i omega t minus k z; i e naught by 2 C n minus 1 star is exponential minus alpha mod alpha square by 2, alpha star raise power n minus 1 by square root of n minus 1 factorial, C n is exponential minus mod alpha square by 2, alpha raise power n by n factorial square root.

And second term is minus i e naught by 2 sigma you will again get minus mod alpha square sigma alpha star raise to the power n plus 1 by square root of n plus 1 factorial alpha raise to the power n by square root of n factorial square root of n plus 1 into exponential i omega t minus k z; so, this is i e naught by 2 minus mod alpha square. So, let me take out alpha from here; alpha and I will be left with sigma mod alpha square raise to the power n minus 1 by n minus 1 factorial; the square root of n removes one of the square root of n from here, to makes it makes makes it square root of n minus 1 factorial into n minus 1 factorial square root is n minus 1 factorial; one alpha I have taken out; so alpha star raise power n minus into alpha raise power n minus 1 is mod alpha square raise power n minus 1, and I will have exponential minus i omega t minus k z.

And the second term will be minus i e naught by 2 exponential minus mod alpha square, now I take out alpha star, and I will get sigma mod alpha square raise power n by square root of n factorial sorry just n factorial into exponential this factor; exponential this is i chi actually. I have taken out alpha alpha common out here, to make to get mod alpha square is n minus 1; I have taken out alpha star common here to make it mod alpha square is for n for n factorial. What is this? And what is this? x raise power n minus 1 by n minus 1 factorial summation, e to the power x, this will also e to the power mod alpha square, this e to the power mod alpha mod alpha square, this is e to the power mod alpha square, and that goes off with this e to the power minus mod alpha square cancels off.

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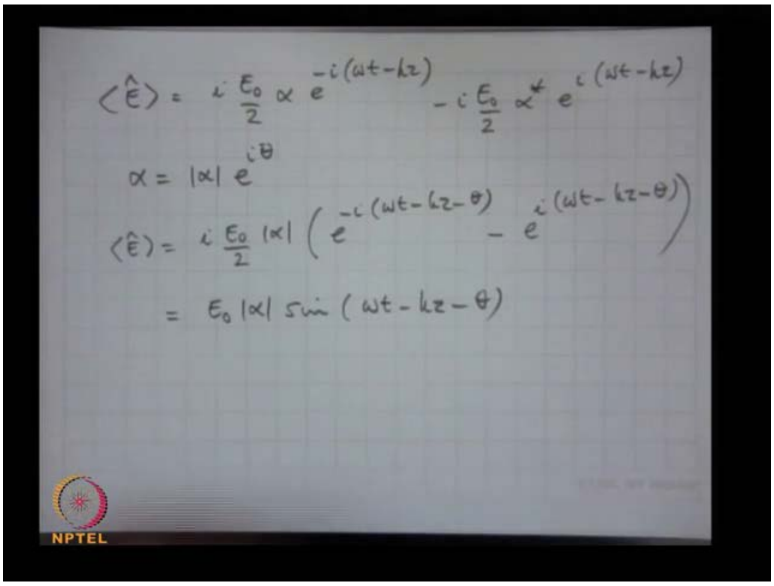

And I am left with much more simplified form i E naught by 2 alpha exponential minus i omega t minus k z minus i E naught by 2 alpha star exponential i omega t minus k z. The two summations give you exponential mod alpha square, which then cancels off with exponential minus mod alpha square. So, if as before I write alpha is equal to mod alpha exponential i theta; so expectation value of E becomes i E naught by 2 mod alpha into exponential minus i omega t minus k z minus theta minus exponential i omega t minus k z minus theta. This is alpha star. So, I get minus theta here, and that is this is nothing but 2 i sin with a minus sign minus 2 i sin omega t minus k z minus theta. So, this becomes e 0 mod alpha sin… exactly as before.

This was a little more involved derivation through the Schrodinger picture; the Heisenberg picture was very quick, equal to immediately calculate the expectation values through using the operators, which would time dependent. Here the states are time dependent, and we are to go through little bit of procedure, but the expectation values are the same. So, I can actually discuss the entire procedure either in terms of the Schrodinger picture or the Heisenberg picture.

In some situations may be one is easier; and so I will use that, but I thought I thought I will just show you that I could have a solve the coherent state problem, expectation values of electric field using the Schrodinger picture, and I will exactly get the same equations. All the expectation values will be exactly the same; the only difference is in one case the operators are time dependent, and in the other case that operators are time independent. So, I thought this is nice exercise to use the two pictures, and to compare the values which are of course, expected to be the same.

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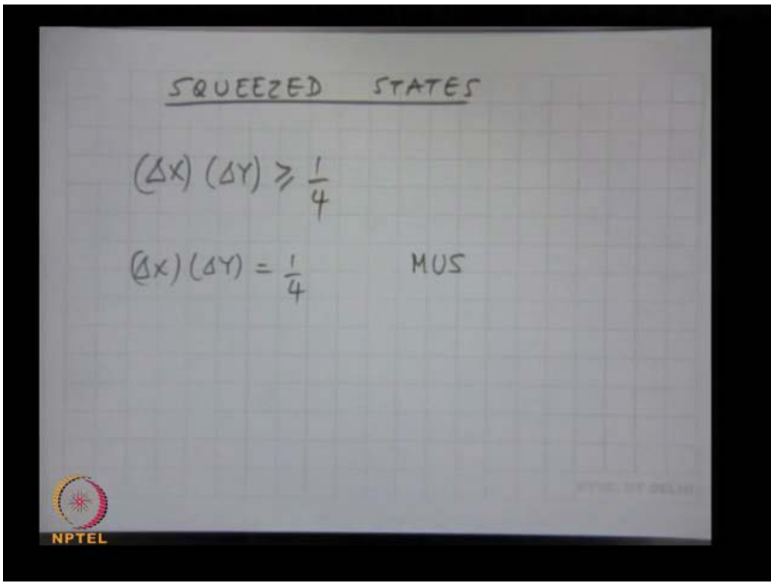

So now, we go back to squeezed states. So, as I was mentioning the uncertainty products between X and Y quadrature, minimum values 1 by 4, but there is no restriction on either delta X or delta Y. And squeezed states are states, in which the uncertainty in one of the quadratures is less; these are called quadrature squeezed states. For example, delta X could be less than half or delta Y could be less than half, and I will get the two quadrature squeezed states. So, let us try to discuss these states, and again in single mode, I still have only one frequency, one mode of propagation.

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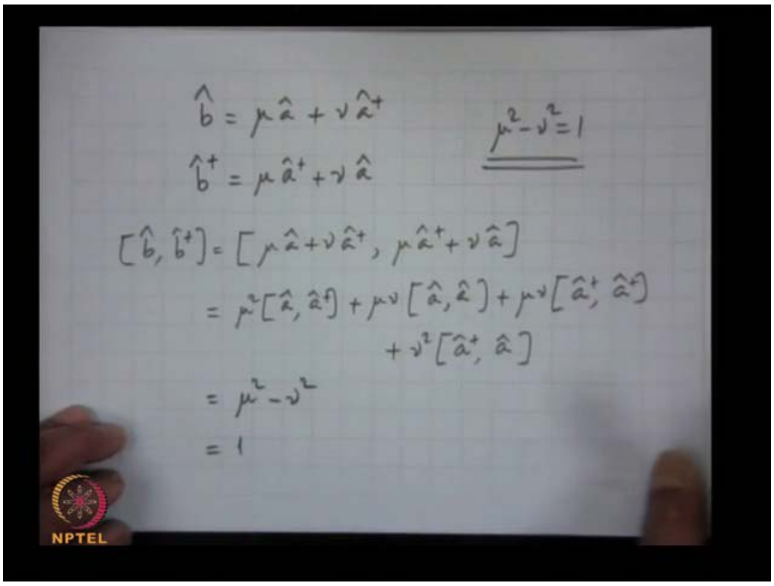

Now to discuss squeezed states, we will introduce another operator, we introduce a operator, which is b is equal to mu a plus nu a dagger; linear combination of the annulations creation operators; so, b dagger will be mu a dagger plus mu a. Now, in general, mu and nu are complex; but to have a simplified analysis, I will assume mu and nu are real. But in general, mu and nu are complex. So, I will assume mu and nu are real in the analysis that we are doing just to simplify the mathematical equations here. So, what is b b dagger? This is equal to mu a plus nu a dagger mu a dagger plus nu a, which is equal to mu square a a plus mu nu sorry a a dagger a a plus mu nu a dagger a plus nu square a dagger a a dagger a; mu square a a dagger plus mu nu a a plus mu nu a dagger a dagger plus nu square a dagger a.

So, what is a a dagger - is 1, and this is 0, this is 0, and this is minus 1. I also restrict my mu and nu to the fact that mu square minus nu square is equal to 1; and so, this becomes equal to 1; just like a a dagger is equal to 1 b b dagger is also equal to 1. So, I am looking at states, I am looking the operator b is equal to mu a plus nu a dagger with a condition mu square minus nu square is equal to 1. So, if mu and nu are real; I can represent this by a single condition for example, mu could be written as cos hyperbolic sigma and nu could be sin hyperbolic sigma; you satisfy this equation exactly.

So, later on I will put this numbers, this mu is equal to cos hyperbolic sigma, and nu is equal to sin hyperbolic sigma; so, that this condition is satisfied, I do not have to keep track; so, that I could have written a cos hyperbolic sigma plus a dagger sin hyperbolic sigma. Now please note, where did you get these hyperbolic function solutions in non-linear process? Parametric down conversion; this operator is actually will come there in the parametric down conversion process. In the quantum mechanical analysis of the parametric down conversion, we will bring in this operators, because they contain cos hyperbolic and sin hyperbolic functions, and they satisfy their condition. So, this is the b operator, you can actually invert these operators to write a and a dagger in terms of b and b dagger.

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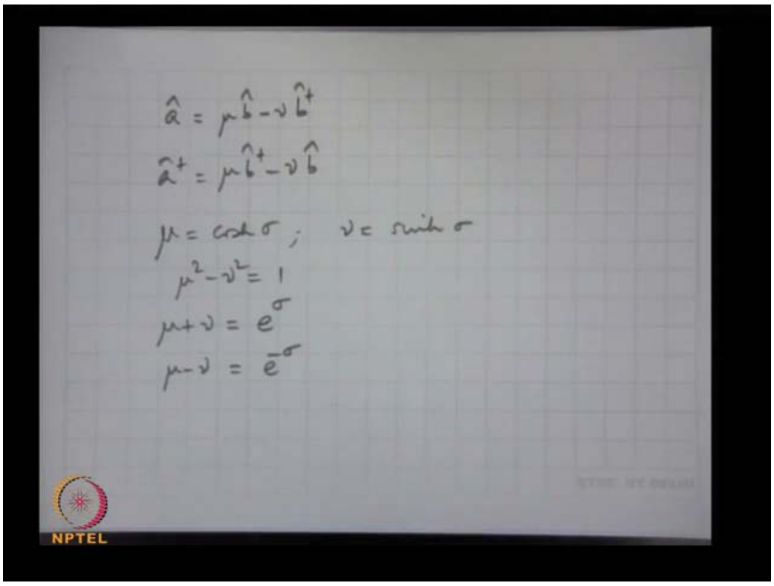

So, let me just give you the equations here; a is equal to mu b minus nu b dagger, and a dagger is equal to mu b dagger minus nu b. If mu and nu are complex, I need to keep track of the phase of mu and nu. So, I could later on I will write mu is equal to cos hyperbolic sigma and nu is equal to… So, this satisfy, and you also see it here mu plus nu is, how much is mu plus nu?

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Not i sigma, sigma; and mu minus nu, this is assuming… These relations for mu and nu. Now we defined, so b and b dagger operators satisfy similar equations as a and a dagger operators. Just like we defined the coherent state as an Eigen state of the a operator, we will define the squeezed state as the Eigen state of the b operator.

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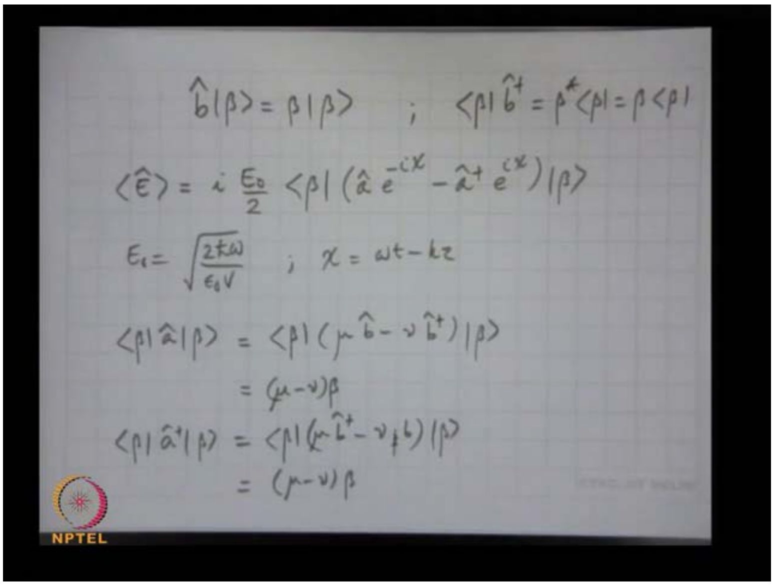

So, let me look at a state, which is called as beta states, so beta; beta is a Eigen value of the b operator. And beta in general could be complex, but again to keep the analysis simple; I am going to assume beta is real. The mathematics that we will do will be very general, except that if you if you would assume beta was not a complex, not real or mu and nu are not real, you have to be dealing with mu mu star nu nu star beta beta star etcetera. So, the just to keep the mathematics are little more compact; I am assuming mu and nu are real and beta is real.

So, these are states; so let me calculate the expectation value of electric field in the state. Suppose I have to generate a state, which happens to be an Eigen state of the b operator, for some value of mu and nu and with some Eigen value beta. So in this state, let me calculate now, remember E expectation E is given by i E naught by 2, so this is beta;

now we go back to Heisenberg picture; so, a exponential minus i chi minus a dagger exponential i chi. So, let me recall E naught was equal to square root of 2 h cross omega by epsilon 0 v and chi is omega t minus k z.

We are looking at a, a more propagating along this z direction at a frequency omega, and polarized - linearly polarized in some direction. So, we are forgetting about the vector nature of the electric field operator here. So, I need to calculate beta a beta and beta a dagger beta. So, beta a beta is beta. Now, let me substitute for a, a was shown to be mu b minus nu b dagger. So, because beta b beta is equal to beta beta, what is suppose a beta star beta and I am assuming beta to be real. So, I am assuming beta beta otherwise, I have to keep beta star there.

So, what is the value of this? And beta is normalized; beta beta is equal to 1 mu minus nu into beta right, because b beta is beta, beta dagger beta b beta is beta, beta still so. If it was not, I would have got mu beta minus nu beta star, what about beta a dagger beta? So, this is beta mu b dagger minus nu b, which is equal to same; because beta is real otherwise, I would have got here, mu beta star minus nu beta, which is different from the other one. So, I have got expectation value of a, and expectation value of a dagger in this state, so what is expectation of the electric field?

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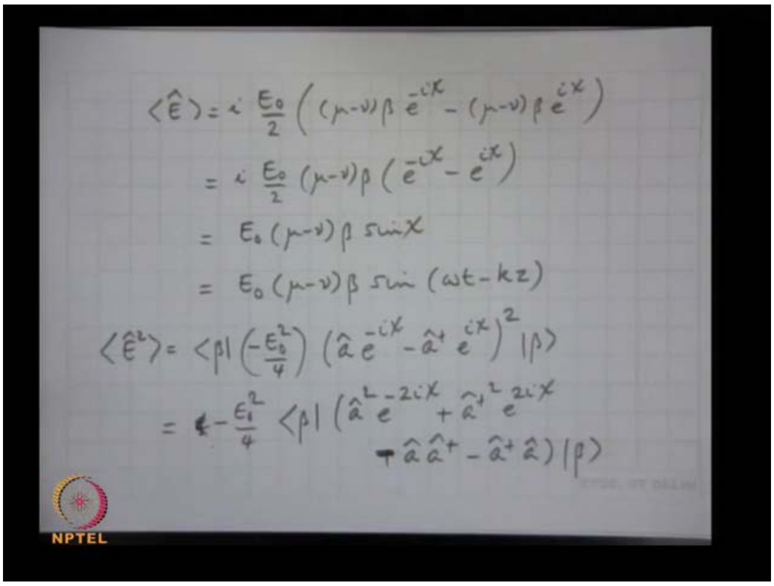

So, expectation value of electric field is equal to i E naught by 2. So, mu minus nu beta exponential minus i chi minus mu minus nu into beta exponential i chi; which is equal to… So, this is minus 2 i sin chi, so I get E naught mu minus nu beta, which explicitly in terms of time and space dependence is actually…

(No audio from 41:18 to 41:39)

So, the Eigen value beta appears here, and through does the mu and nu factors here, because the b operator is defined in terms of mu and nu, and the Eigen value is beta. And the expectation value of electric field goes exactly like a sin omega t minus k z; just like a classical field. But what would be interesting is to see, what is the variance in the electric field as a function of time? In the coherent state, how did the variance vary with time? The variance of the electric field in the coherent state, it was independent of time, it was constant; here I will show you that the variance depends on time, and there are times, in which the uncertainty in the electric field is less than what you are getting in the coherent state; at the expense of increased uncertainty, at other times in this state, because delta X delta Y has to be greater than equal to 1 by 4. So, I will not be able to I mean, I cannot reduce the uncertainty at all times, for or which means for both quadratures, I can only do so for one at the cost of the other. Now, I need to calculate the expectation value of e square. So, what I need to do is, this equation.

Sir,

Yes.

Sir, as we have delta x into delta is throughout by the (( )) as well; similarly, if there is a variance in the electric field, what should be the corresponding…

Conjugate variable?

Yes.

That is vector potential actually here. So, we are not looking at the corresponding conjugate variable of the electric field here, we are looking at the two quadrature operators of the electric field itself. But the variances between these two get distributed depending on the state of the electromagnetic wave.

In our case, when the variance of electric field becomes less than the vacuum, the treasure, as you are saying…

At given times.

At a given time.

Then the variance in the…

Conjugate variable conjugate variable will blast higher.

So, what would it mean?

No, but if I measuring the electric field or if I measuring something related to the electric field, and if the variance in electric field is less, I will get at less noise; photo detection process for example, depends on the electric field. And I will little later introduce the fact the concept of what is the meaning of photo detection.When light falls on a photo detector, what is happening? And how do I calculate quantum mechanically, this issue of detecting photons, when they arrive on a photo detector. So, it depends on the electric field. So, if the electric field has less variance, it will need to a certain effect on the photo detection process itself.

So, for example, normally you do not measure electric fields, you measure this quadrature electric field, but there are there are techniques homodyning etcetera, in which you can actually measure either of the quadrature. So, I will come to this little later. Vishwesh.

Sir, why the coherent state is called as an Eigen state of field?

No, I have defined a state.

Why Roy Glauber difference called the equivalent field?

No, this was introduced by Roy glauber in 60, in the early 60s. If you calculate the state generated by classical current, so detailed analysis, one can show it is a coherent state. Now I have started, I did not derive that, I will get a coherent state; I have started from the coherent state, I have looked that operate state with which happens to be the Eigen state of the operator, and then I calculate the properties of the state, I am not derived the coherent state. But if you calculate, if you try to obtain, what is the state generated by a classical current source, classical source, it is like a coherent state, so which we are not done in the class here now.

So what I need to calculate are these quantities beta. So, it becomes minus E naught square by 4. So, let me just write the equations, and we will discuss this later next class. So, a a exponential minus i chi minus a dagger exponential i chi whole square beta. So, I need to calculate all these quantities, so this is minus E naught square by 4 beta a square exponential minus 2 i chi plus a dagger square exponential 2 i chi plus a a minus a a dagger minus a dagger a. Now, so I need to calculate things like beta a a square beta, beta a dagger square beta, beta a a dagger beta, and beta a dagger a beta.

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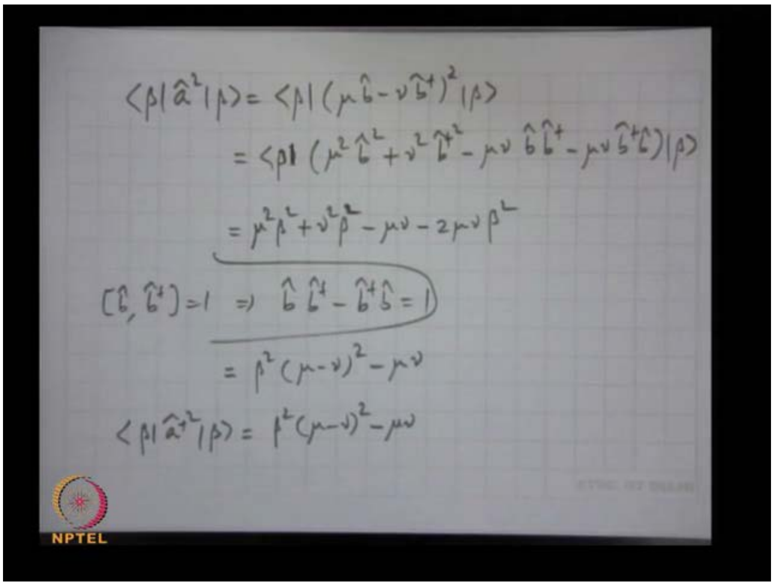

And I have to substitute the expressions for a in terms of b in those equation; so, for example, if I need to calculate beta a square beta. So, this will be beta, now a a mu b minus nu b dagger square beta. So, this is equal to beta mu square b dagger square plus nu square sorry b dagger square minus mu nu b b dagger minus mu nu b dagger b. So, what is beta b square beta? It simulate beta square, because b beta is beta beta, and one more b if you operate, you get another beta. So, you will get mu square beta square plus nu square beta square, I would have got beta star square, but I am just getting a beta square, because beta is assumed to be real minus mu nu. No, b dagger on beta is not beta beta. So, I have given the commutation relation between b and beta b and b dagger.

So, b b dagger is equal to 1, implying b b dagger minus b dagger b. So, this b b dagger is 1 plus b dagger b. So, I get minus mu nu minus there is another mu mu nu b dagger b. So, I get 2 mu nu beta square. Please note b beta is beta beta; beta bra b dagger is beta beta bra, b dagger beta is not equal to beta beta. So, this will give this gives me, so let me this is equal to beta square into mu square plus nu square minus 2 mu nu. Now, let me give you the expressions for the others; and I will leave it you to please go back and calculate. So, which is same as this; why not I will leave it; please calculate the other two terms that is beta a a dagger beta, and beta a dagger a beta.

And we will get finally, an expression for E square expectation value, from which we will be able to calculate the variance in the electric field. And then, I will discuss this variance, because which has interesting properties, the variance in the electric field is a function of time now, and in certain instance of time, the variance of electric field is less than, what you can obtain in vacuum state. So, this can lead to a state of light, in which at certain instance of time, the noise is less than what you can achieve with vacuum; and these are called the squeezed states.