Quantum Electronics

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Module No. # 05
Lecture No. # 30
Quantum States of EM Field

So, what I want to cover now is, so what we have done is we have quantized the electromagnetic field; we have shown that the total field can be considered to be a superposition of an infinite number of harmonic oscillators - independent harmonic oscillators - through a formal analogy of the Hamiltonian with the harmonic oscillator Hamiltonian.

So, each mode which means - a mode will be specified by the k x k y k z - which means the directions of propagation of k vector, which defines the frequency, also, because k x k y k z defines the frequency of the wave and depolarization state.

So, if I choose a specific set of numbers - n x n y n z that fixes k x k y k z, which fixes the frequency and I chose a polarization state; so a wave going in one particular direction, with a certain frequency in free space, with a certain polarization state, corresponds to mode.

If you have a wave at the same frequency propagating the same direction, but with an orthogonal polarization state, that is another mode. If I have the same direction of propagation, the same frequency, but sorry, if I have the same polarization, but the different frequency, that is a inner mode.

So, when you change the directional propagation or polarization state or the frequency, you will change the mode, you have a different mode. In free space, this is a continuum, all values of k x k y k z are allowed, which means all frequencies are allowed and for every core k vector direction, there are two orthogonal polarization states.

We have been able to discretize the problem by restricting our discussion to a volume of a cube of psi l and applying pediatric boundary conditions.

So, that gives me a discrete, but an infinite number of set of modes and each of these modes are independent of the others. So, as you wrote last time, the most general state can be written something like this.

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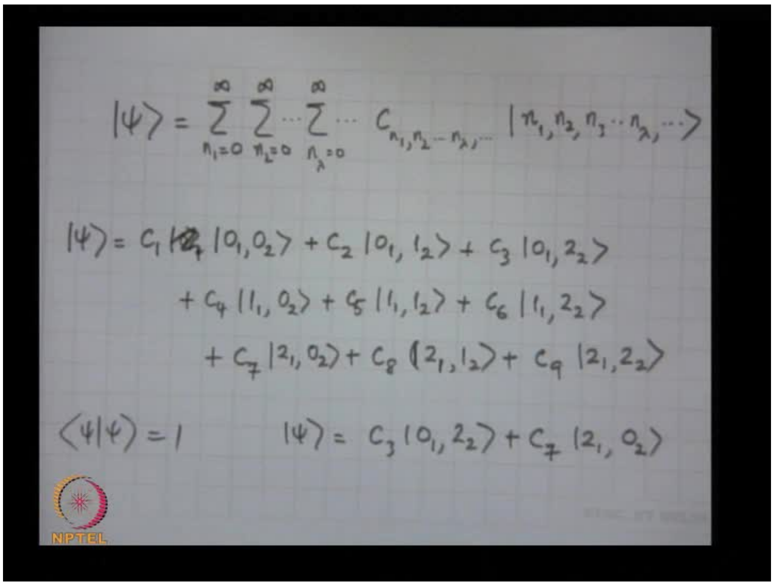

Sigma n 1 is equal to 0 to infinity, sigma n 2 is equal to 0 to infinity, etcetera, sigma n lambda is equal to 0 to infinity - infinite number of sums - c n 1 n 2 n lambda multiplied by n 1 n 2 n 3 n lambda. This implies that this is the number of photons in mode 1, which means, in mode 1 - mode 1 has an energy h cross omega 1 n 2 n 1 plus half.

There are n 2 photons in mode 2, which means energy in mode 2; which means, for example, this could be frequency, one frequency, this is another frequency; there are n 2 photons; that means, the energy of that mode is h cross omega 2 into n 2 plus half and similarly infinite number of modes. So, for one particular set of values, you have a certain constant and each of these numbers can go from 0 to infinity. So, I will take an example and show you what is the meaning of this.

That is the huge set of states; so, for example, let me look at a situation where I restrict the number of photons and the number of modes. So, suppose I say that I have two modes and I can have up-to two photons per mode, what will be the states that I can update? What will be this sum? So, I am restricting my problem now, just to understand that I have considered two modes; for example, I will say that I am propagating in a particular direction, say k x is equal to 0, k y is equal to 0, k z is equal to some value, which means a wave propagating in the z direction and that defines my frequency; if I define k z - the magnitude of k z - it defines my frequency and I say, I have vertical polarization or horizontal polarization; we have two different modes.

So, if a plane waves propagating along z direction, with vertical polarization is one mode; a plane wave propagating in the z direction at the same frequency, but with horizontal polarization is the another mode. So, I consider these two modes and I say that let me look at the possibilities, if each of these modes can be occupied by two photons, which means the energy can be at the most h cross omega into 2 plus half. So, the energy can be h cross omega into 0 plus half - no photon; h cross omega into 1 plus half - one photon in that mode; h cross omega into 2 plus half - two photons in that mode. So, what are the combinations I will get?

So, the most general state will be like this; so c 1 no photon in mode 1, let me write in the same this thing, no photon in mode 1 no photon in mode 2, I am not writing the others, there only two modes I am considering.

I am assuming that all the other modes do not a have no photons in them and so, they are not I am not writing them here.

I can have no photon in mode 1 1 photon in mode 2 plus I can have no photon in mode 1 2 photons in mode 2 plus I can have 1 photon in mode 1 no photon in mode 2 plus c 5 1 1 1 2 plus c 6 1 1 2 2 plus c 7 2 1 0 2 plus c 8 2 1 1 2 plus c 9 2 1 2 2 - nine possibilities; two photons per mode maximum. So, there is a possibility of 0 photon, 1 photon or 2 photons and there are two modes. So, there will be nine possibilities; so, these are the only possible states in which there are two modes and occupied by maximum of two photons. So, in this situation, you see already 1 2 3 4 5 6 7 8 9 different possibilities exist.

Classically, if I take two modes which means a horizontal polarization and a vertical polarization, I need only two complex numbers - the amplitude and phase of each of those modes; there are only two possible.

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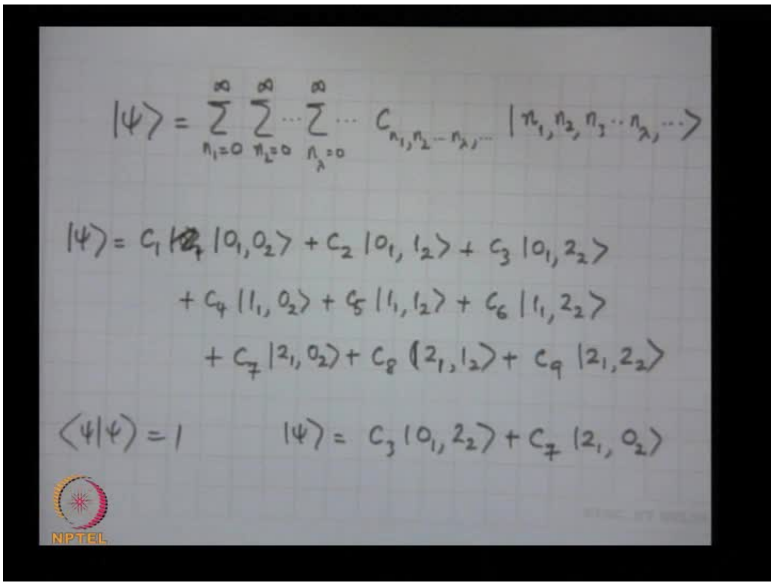

Here, there are so many different possibilities with please note c 1 c 2 c 3 that all arbitrary except for the condition that psi (()) must be equal to 1, c 1 all these are not independent there is this condition - normalization; so, that will give you mod c 1 square plus mod c 2 square plus mod c 3 square etcetera to mode c 9 square is equal to 1

For example, you could have a state, out of this combination I could have a state which looks like this. Let me pickup these two of them (Refer Slide Time: 08.43). So, I can have a state in which I have c 3 no photon in mode 1 and 2 photons in mode 2 plus c 7 2 photons in mode 1 and no photons in mode 2.

I assume all the other c's are 0, only 2 c's are finite; so, this is a state which corresponds to a linear combination of no photon in mode 1 and 2 photons in mode 2 or 2 photons in mode 1 and no photon in mode 2; that is one example for instance from this. So, there are actually, this is an, it is a huge set of states that is possible. So, the quantum states actually are very very huge in number and largely unexplored still.

We will look at certain types of states, that we people have studied and one can generate and work which are which have applications. And apart from them, there are still a large number of states which one can look at and may be still unexplored.

So, the space of quantum states of the radiation field is enormous. So, let us start with the simplest situation where I look at one mode of radiation - single mode radiation field.

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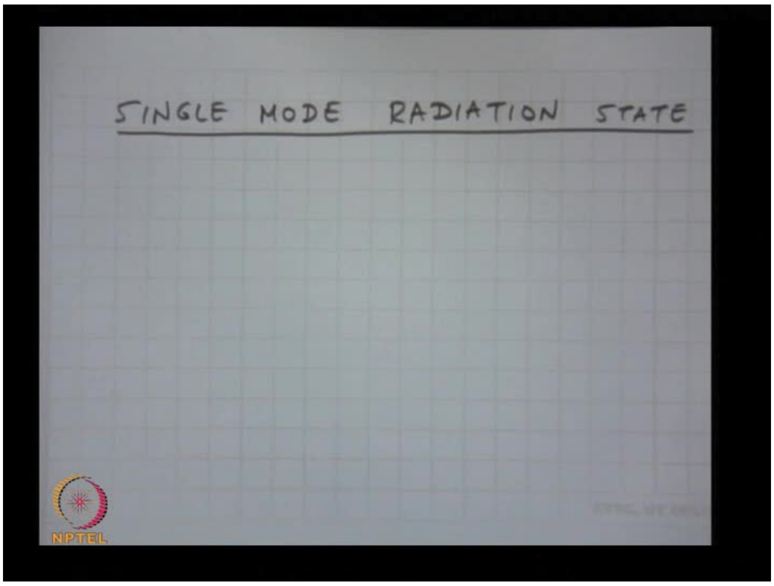

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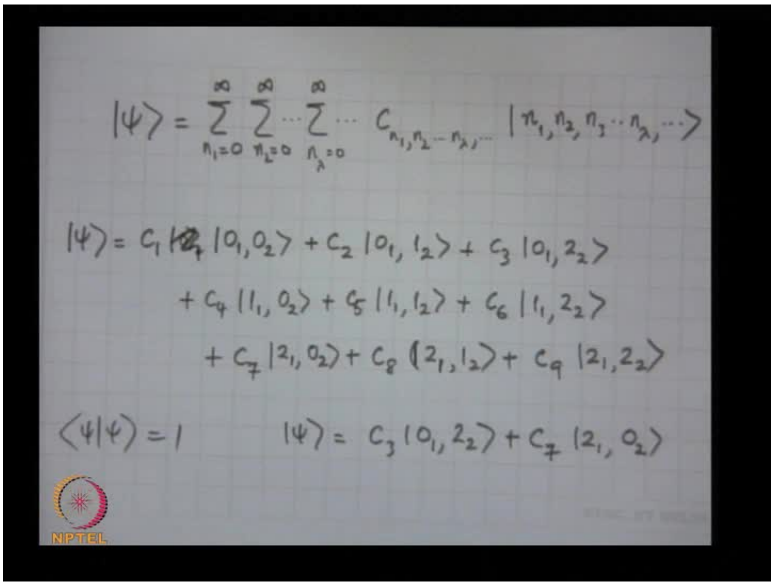

Single mode radiation state: by this sign, I mean that, my radiation is in a state in which all modes except one are empty; which means, this in the state, all n values except one of them is non-zero; that means, there is only one mode excited. It could be, for example, mode what I call as 3 here, so only n 3 is finite, n 1 is equal to 0, n 2 is equal to 0, n 4 is equal to 0 and etcetera, etcetera.

Please note, yesterday I have told you that if all the n values are 0, it is called the radiation vacuum state - complete vacuum - vacuum in terms of absence of any excitation of the radiation states, that is corresponding to each mode you are at the vacuum level half h cross omega.

We proceed and discuss things will become clearer. So, by single mode radiation field - so what I am looking at is a situation - for example, I have wave going in the z direction with a certain frequency and I defined polarization state, that is one mode; so, I have a vertically polarized slide going if here at a certain frequency in free space, I am looking at one mode.

I am assuming all other modes are of no concern to me or all empty, which means the corresponding horizontally polarized slide at the same frequency is not there; there is no photon in that state. That is, it is in the lowest possible state or same vertical polarization - all other frequencies are nonexistent or all waves going in all the kinds of direction - there all absent; absent, by that I mean that the corresponding n value of that mode - the excitation state of that mode - corresponds to n is equal to 0.

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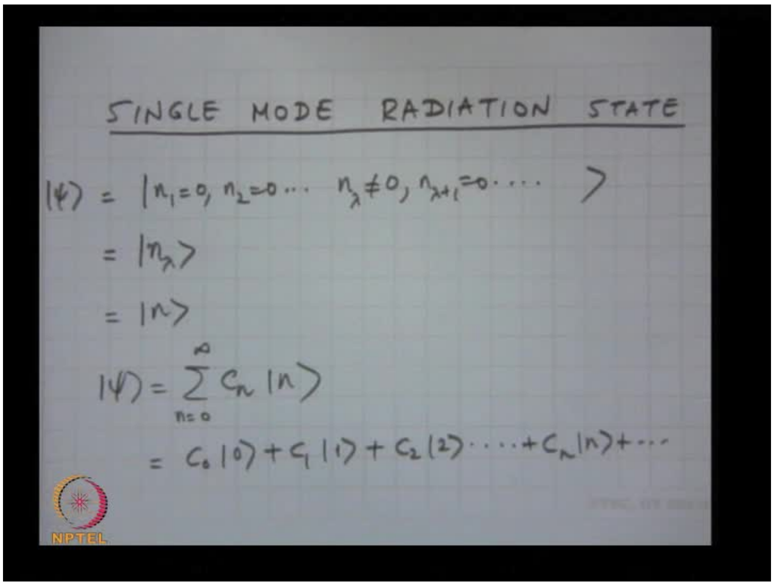

So, this would mean essentially a state in which I am looking at n 1 is equal to 0, n 2 is equal to 0, some n lambda is not equal to 0, n lambda plus 1 is equal to 0 (Refer Slide Time: 13.20). All states other than one particular state which I call lambda here,subscript lambda some number or (( )) this can also be 0, but that is complete vacuum state. So I am looking just at one state, so I will just, let me forget about all numbers and I just write this n lamda.

In fact, because I am looking at one mode, I can forget about the subscript; so, I just write as n. As long as I know that I am restricting myself to single mode field, I do not need to put a subscript.

So, what it implies is I am looking at a state - I am looking at those states - I am looking at those states where there is an excitation of only one mode. For example, this is, this is an example (Refer Slide Time: 14.14). In general, this is not the most general state; what is the most general state? This one with n lambda taking on any number, all other n values are 0 which means that these sums have only one term - n 1 is equal to 0 n 2 is equal to 0 term only, this sum is still not summed over, because you are having n lambda which can vary from 0 to infinity and all other terms are nonexistent (Refer Slide Time: 14.30). So, the most general state, single mode radiation state will be like this - c n n is equal to 0 to infinity.

Sir, this single mode radiation does not have a gap on the energy that it delta wave dependent.

No.

It just has that only one mode should be there, it can have any?

Because the energy is are n plus half h cross omega, so it can have 0, I mean half h cross omega to only thing is that it can have a only discrete states, but it can go to infinity.

It cannot have half h cross omega.

Vacuum state, when there are no photons,

Is that a single mode radiation of it I mean if,

No. if I am concentrating myself only on that frequency, on that propagation direction, on that polarization state and I find by some mechanism that the excitation corresponds to a 0 ket state, it is single mode state but with 0 ket - no photons incident.

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Corresponding to that single mode, it is the vacuum state of that single mode. So, that means I am looking only at one particular state, one particular mode of the radiation field, and in that mode I can have 0 photon, 1 photon, 2 photons or a combination that is what this is.

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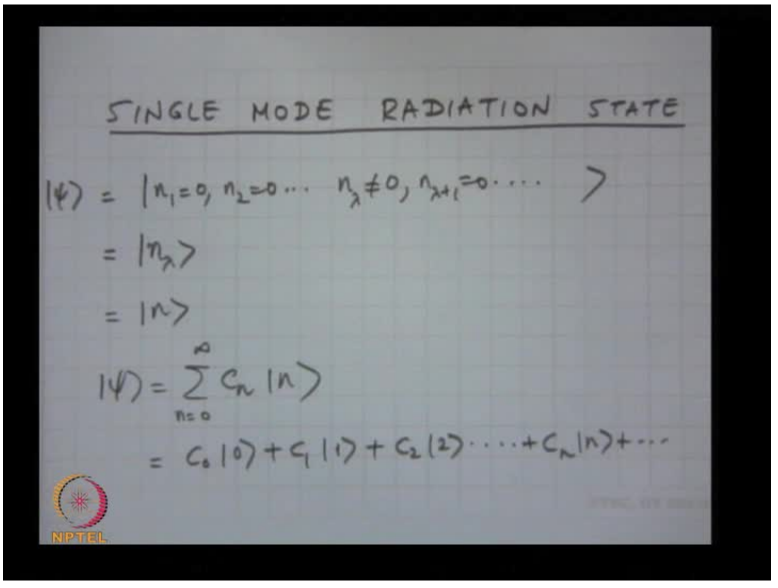

It is like one harmonic oscillator; a single harmonic oscillator will have n ket states and I can have linear combination of n ket states, because in electromagnetic field each mode is a harmonic oscillator. So, this means essentially I am concentrating on one of the radiation modes, which means one of the harmonic oscillators corresponding to the radiation fields.

As we go through, things will become clearer and clearer. So this, when I write n ket here, it only means that - now we are only discussing single mode radiation states - which means that there is only one mode which is under consideration. And this is the most general state; yes, you can have a linear combination of these states. So, this could be written in general as, which means, c 0 0 plus c 1 1 plus c 2 2 ecetera plus c n n plus ecetera; it is a linear superposition of vacuum state plus one photon state plus 2 photon states plus n photon state plus etcetera etcetera (Refer Slide Time: 17.19).

When I say a photon here, it only means that, in that mode I am talking about that mode, when I say this mode is occupied by n photons, I mean it has an energy, n plus half h cross omega. Later on, we will come to multimode states, but single mode states I am looking only at one of the one of the states of the radiation field and I am looking at a general state of that one mode.

So, this is vacuum state corresponding to that mode, this is 1 photon excited state, 2 photons excited state, etcetera etcetera (Refer Slide Time: 18.11).

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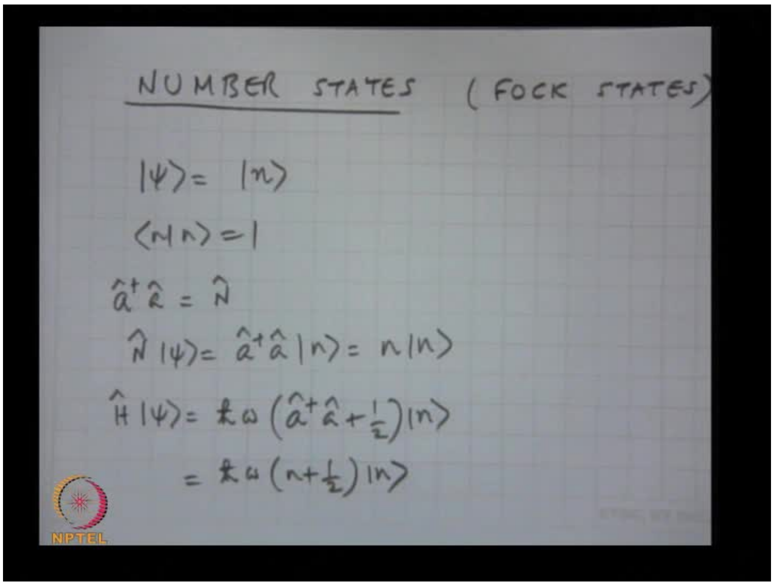

Now, what we will like to do is, the first thing that I want to do is, to look at what are called as number states of that single mode. This are also called as fock states; that means, I am assuming that only one of the coefficients is non-zero. In this expansion, only one of the coefficients is non-zero and all other coefficients are 0, and because n bracket is one normalized so that coefficient as to be 1, because the n ket states are normalized and so one of the c is non-zero, all other c's are 0. This is a special state of this radiation field, it is exactly like the harmonic oscillator being in one of the eigenstates of the Hamiltonian.

This is an eigenstate of the Hamiltonian for this mode (Refer Slide Time: 19.35). So, what it implies is a state in which in that mode, there are precisely n number of photons. I will come to this later and I will show you that, there is uncertainty in the number of photons in this state. Here, I precisely know because, why do I know? Because, what is the number operator? a dagger a, this is the number operator.

So, if I operate with a number operator on the state psi, which is a dagger a n which I get n n, because a dagger a n is n. And this, I am not writing any subscript, this is the creation and annihilation operators corresponding to that mode. In principle, I should write n lambda of psi is equal to a lambda dagger a lambda n lambda is equal to n lambda n lambda; that substitute I am removing, because I am looking at only one mode and I do not need to put a subscript.

The number of photons in this state is precisely known as n; the energy of the state is precisely known, because H operating on psi is h cross omega a dagger a plus half into n which is equal to h cross omega into n plus half into n, because a dagger a into n is n into n.

This number state of a single mode number state, please note that there are multiple number of modes - infinite number of modes, each mode can be occupied by an infinite number of sets of photons, I am picking up one mode of this infinite combinations and in that one mode, I am exciting it to a state where there are n photons precisely.

Now, let me tell you, to generate states like this is not very easy. Experimentally, people have generated states which are one photon states; that is, n ket - there is only one photon in the state. And usually, they are not single mode states, also they are multimode states; that means, just one photon actually exist not in one mode, but multiple modes.

So, I will get this clarified as we proceed into a little more arguments with these states, but to generate a radiation in a single mode, single number the in a number state is not very easy. That means, I state in which no other frequency is present, no other propagation direction is present, that are all in back, that is, they all are in the ground state, no other polarization ket is present, but except for one frequency, one propagation direction, one polarization which is occupied by a certain number of photons.

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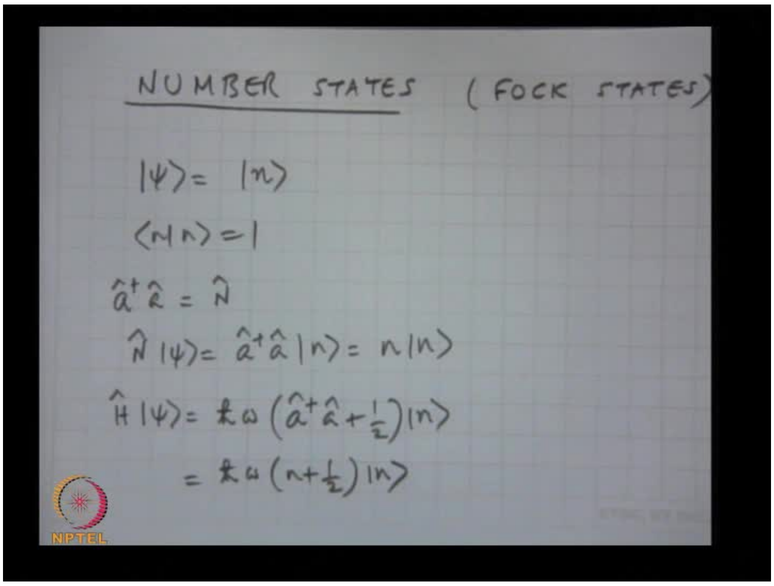

What is interesting for me to see is, suppose I had a state like this, what is the expectation value of electric field? Because that is what I can measure for example. So, I create a state like this and I want to measure the expectation value of the electric field.

Now please remember that, the expectation value of the electric field operator should resemble the classical electric field if it is a corresponding state. A classical electric field of a monochromatic wave will go as E naught sin omega t minus k z or whatever it is - k dot r minus omega t.

So, the correspondence is, if I calculate the expectation value of the electric field of my given state, that will give me the corresponding; if that corresponds to a classical state, if the quantum state corresponds to a classical state, then the expectation value of the electric field of my quantum state must vary as E naught cos omega t minus k z or something; with the correspondence has to be there, it is just like in a harmonic oscillator. I must satisfy the relation stating that the expectation values of the operators behave similarly to the quantum to the classical equations of motion.

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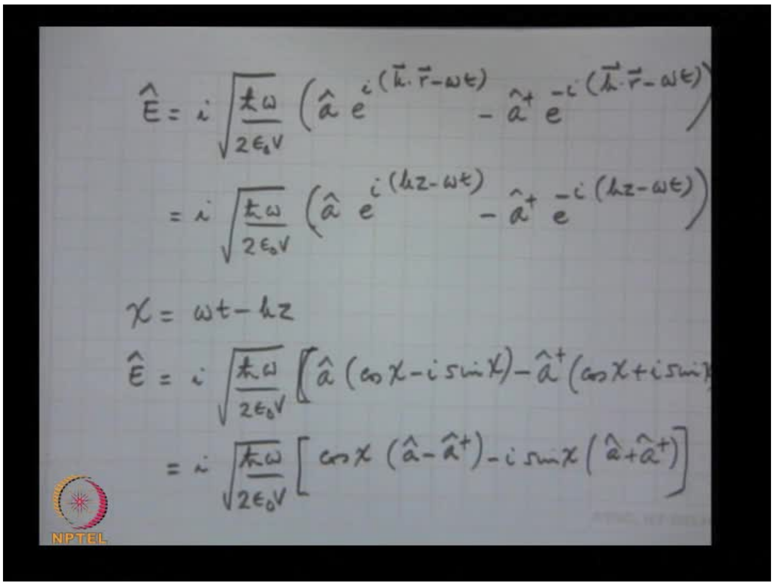

So here, let me calculate the expectation value of the electric field in this state. Now, for this I need the electric field operator, recall this, this is h cross omega by 2 epsilon 0 V a exponential i k dot r minus omega t minus a dagger exponential minus i k dot r minus omega t. This is for one mode which has a frequency omega, whose propagation direction is k and actually there has to be a polarization state, which I am assuming is some given polarizations (Refer Slide Time: 24.51); so, I can look at it as a scalar and not worry about the vector.

It may be horizontally polarized or vertically polarized. So, whatever polarization it is, actually I can simplified further assuming that the propagation direction is along z, in which case k vector is along z and this simply can be written as h cross omega by 2 epsilon 0 V a exponential i k z minus omega t minus a dagger exponential minus i k z minus omega t.

Now, the electric field is also can be written in a slightly different form, in terms of what are called as quadrature operators; so let me do that. So, let me define a chi which is omega t minus k z. I do not want to keep on writing k z minus omega t, all the time I just define chi, some quantity which is omega t minus k z; that is a phase. So, E cap is i times h cross omega by 2 epsilon 0 V, so let me write, so this is exponential minus i chi (Refer Slide Time: 26.24) which is cos chi minus i sin chi - chi is omega t minus k z, so this is exponential minus i chi - so cos chi minus i sin chi minus a dagger into cos chi plus i sin chi which is equal to i times square root of h cross omega by 2 epsilon 0 V into - so I will have a let me take the - cos chi into a minus a dagger minus i sin chi into a plus a dagger.

So, I am just expanding the exponential i chi and exponential minus i chi and exponential i chi in terms of cosine and sin and just coupling the terms. So, I will now define two operators which are (Refer Slide Time: 27.37); so, let me take the i inside first, if I take the i inside, I will get the following.

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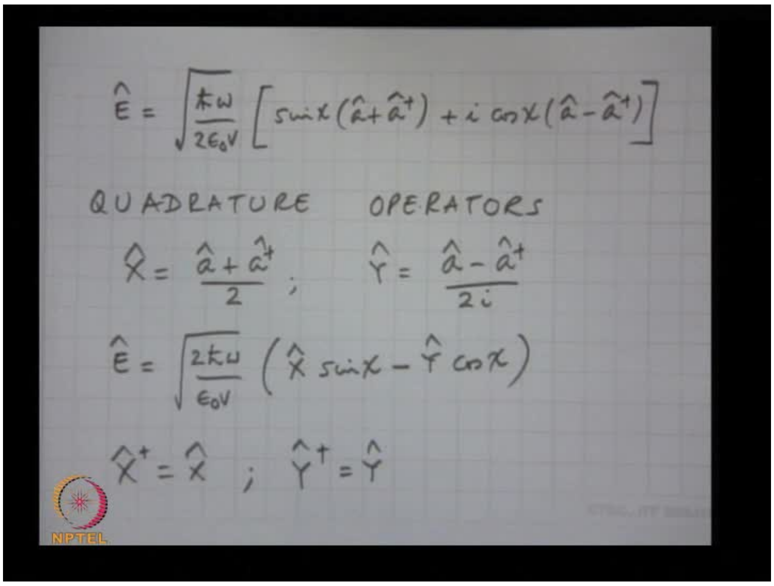

Square root of h cross omega by 2 epsilon 0 V into - so this i into minus i is plus 1, this i sits here (Refer Slide Time: 27.55) - so will get sin chi into a plus a dagger plus i cos chi into a minus a dagger.

So, I define what are called as quadrature operators: x is equal to a plus a dagger by 2 and y is equal to a minus a dagger by 2 i; so, this is electric field operator actually. So, I divide and multiply by 2 and that 2 which I take inside this square root, I get square root of 2 h cross omega by epsilon 0 V into x operator sin chi minus y operator cos chi. They are called quadrature operators, because one of them is the coefficient of sin chi term and the other one is a coefficient of cos chi term; sin and cosine are having a phase difference of pi by 2, they are in quadrature.

Is a Hermitian operator? It is not Hermitian, because a is not equal to a dagger. What about x and y? x and y are hermitian operator; x dagger is equal to x, because x dagger is a dagger plus a by 2 and y dagger is a dagger minus a by minus 2 I, which is a minus a dagger by 2 i. So, x dagger is equal to x and y dagger is equal to y; these are quadrature operators and they are observables, sorry, they are Hermitian and are observables.

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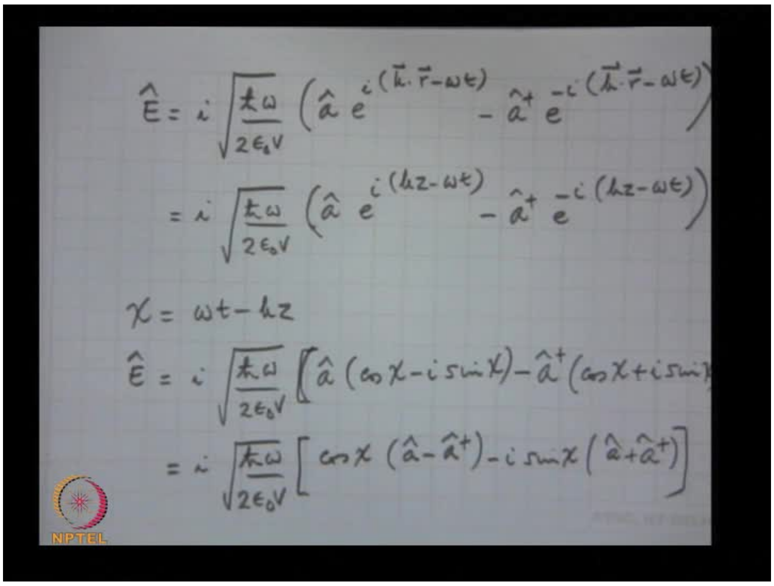

I cannot measure anything correspondent to a cap. a is an annihilation operator, a dagger is a creation operator, but x and y are Hermitian operators and observables. What it means is, in principle I can measure the sin quadrature of the electric field or the cosine quadrature of the electric field. I cannot measure this term or this term (Refer Slide Time: 30.56), because these are not observables.

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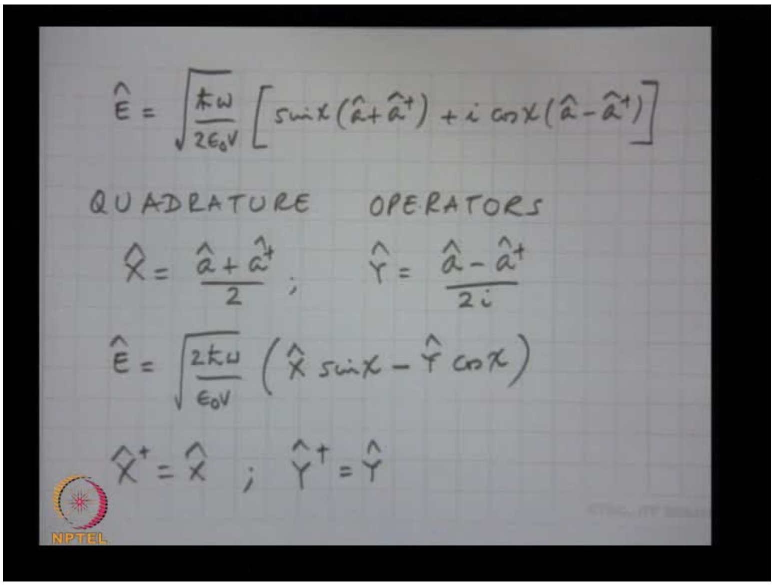

On the other hand, I can measure x cap operator, if I am measuring if I measure the value of this term here which corresponds to this term (Refer Slide Time: 31.09), I measuring the sin quadrature. Otherwise, if I measure the y cap observable, I am measuring the cosine quadrature.

All observables are 0 quantities could need to be Hermitian operators?

Yes, all observables.

Yes, but in the other way round also, all could, all Hermitian operators corresponds to some…

I think so, I think so, all Hermitian operators which have real eigenvalues should correspond to some observables.

All of Hermitian operators will have real eigenvalues?

All Hermitian operators will have real eigenvalues. Some of them may be difficult to observe may be, but they will correspond to observables.

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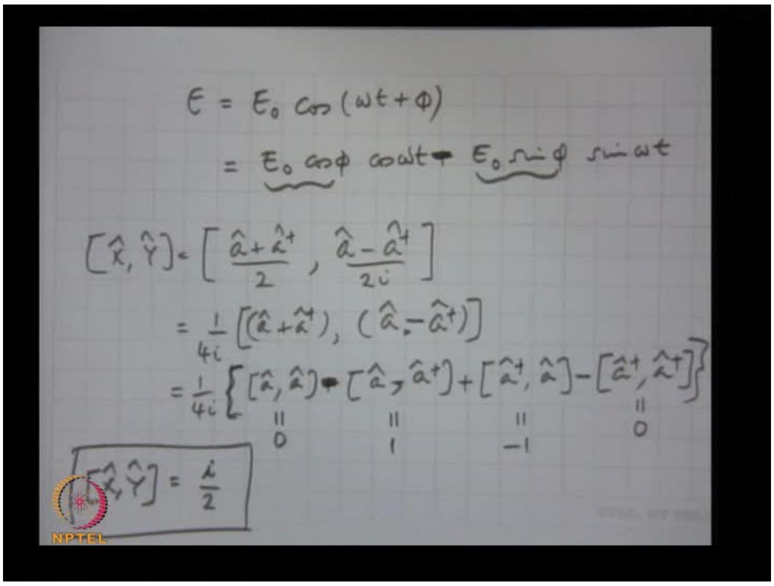

So, these are two new operators which we have defined - quadrature operators. This is like a, if you have a classical field like cos omega t plus phi, if I have classical field which looks like, suppose some classical field E is equal to E 0 cos omega t plus phi, this can be written as E 0 cos phi cos omega t minus E 0 sin phi sin omega t; this is one quadrature, this is the other quadrature (Refer Slide Time: 32.33). So, classically, either I measure E 0 and phi or I measure E 0 cos phi and E 0 sin phi, I get the same information.

So, there are techniques in engineering - electrical engineering - we chose a homodyning, heterodyning principles to measure the quadratures and we will look at some homodyning here later on. So, this is writing the field, this is a classical expression if we have given a function of time with some arbitrary phase (Refer Slide Time: 33.02), you can write this as some coefficient into cos omega t plus some coefficient into sin omega t; cos omega t and sin omega t are phi by 2 out of phase.

So, measuring the coefficient of cos omega t gives me E 0 cos phi, measuring the coefficient of sin omega t gives me E 0 sin phi and I can measure, then I can calculate E 0 and phi classically.

If I would lock in amplifier, what sum this principle. You tune the phase of the local oscillator and you can measure either of the quadrature components. I also want to see what is the commutator between x and y. So, let me look at this, x y is equal to a plus a dagger by 2 a minus a dagger by 2 i. So, these factors come out - 1 by 4 i, so this is a plus a dagger commutator with a minus a dagger which is 1 by 4 i commutator a a minus commutator a a dagger plus commutator a dagger a minus commutator a dagger a dagger, let me use another bracket here.

What is a commutator a? What is commutator a dagger a dagger? a a dagger? And a dagger a - which is 1 and which is minus 1? (Refer Slide Time: 34.53)

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This is minus 1 and this is (Refer Slide Time: 35.13); so, this is how much? This is minus 2 (Refer Slide Time: 35.24). So, that means, i by 2; they do not commute, what is the meaning of this statement? They do not commute which means...

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I cannot measure x the observables of x and y simultaneously; I cannot measure both the quadratures or the electric fields simultaneously; and there is an uncertainty relation now between the two quadratures.

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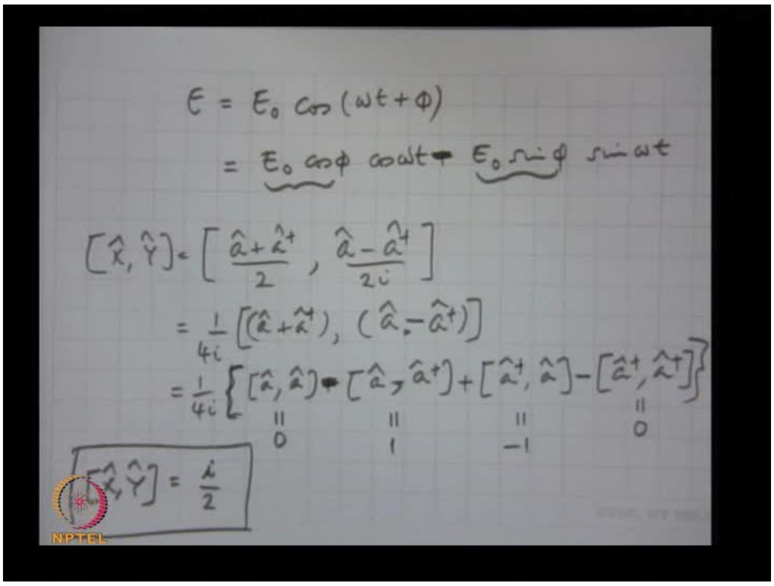

So, the two quadrature observable, the quadrature operators as essentially give me the sine and cosine terms in my electric field. And because they satisfy commutation relation x x comma y is equal to i by 2, they do not commute and they cannot be measured simultaneously, accurately, precisely. I can measure in simultaneously, but with certain uncertainties; I cannot measure both of them precisely.

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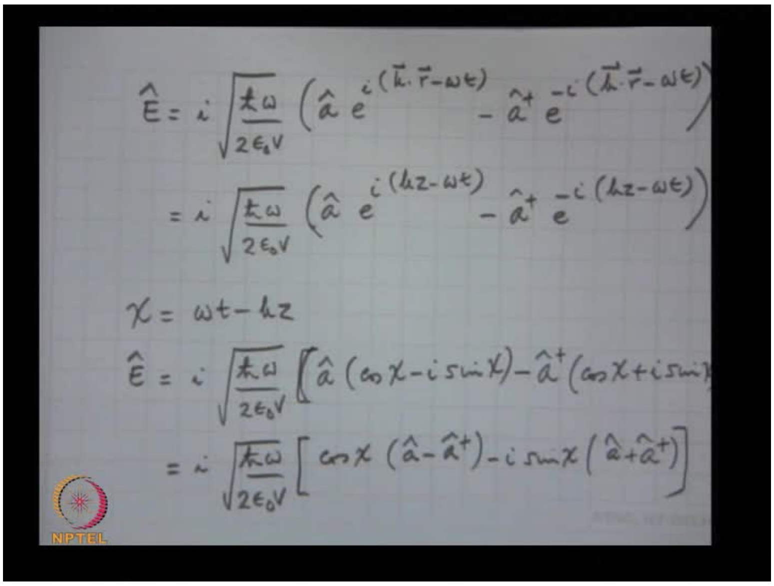

Now, let me look at the expectation values of electric field in my number state. So, this is electric field; I can use this one or this one, sorry, either the original expression for the electric field I can use, let me use this, because it is much simpler here (Refer Slide Time:36.40). So, let me use this equation in terms of chi.

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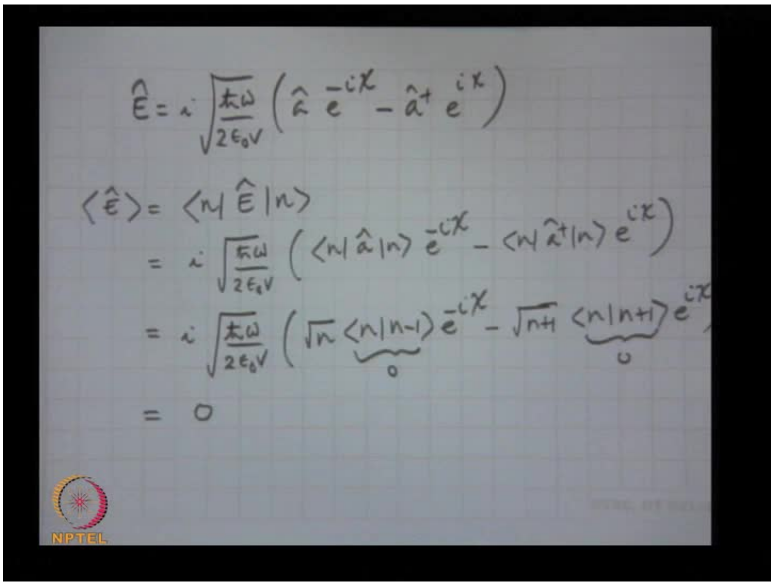

So, the electric field is E is equal to i under root h cross omega by 2 epsilon 0 V a exponential minus i chi minus a dagger exponential i chi. So, expectation value of E for this state, n E n which is equal to - can you tell me what is the value of this? So, I will have terms like i under root h cross omega by 2 epsilon 0 V into n a n exponential minus i chi minus n a dagger n exponential i chi which is equal to i square root of h cross omega by 2 epsilon 0 v - what is a n?

So, I will have square root of n n minus 1 minus i chi minus square root of n plus 1 n n plus 1 exponential i chi; this is 0 and this is 0 (Refer Slide Time: 38.25). a n is square root of n n minus 1, a dagger n is square root of n plus 1 n plus 1 ket; so, does it mean electric field 0? What kind how do I average? What is meant by average?

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Not time average, ensemble average.

This means I prepare a large number of identical quantum systems in exactly the same state, so I prepare a large number of n ket states, I measure the electric field at some instant of time. At one instant of time, there are so many people standing and everybody measure the electric field of the n ket state that he has. Everybody gets a value of electric field, but if you take an average, it becomes 0. This is because, the phase of this field is completely undefined.

The number of photons in this state is precisely defined (Refer slide Time: 39.44) and there is a kind of an uncertainty between the number of photons and the phase of the field. If you know precisely the number of photons in the field, you have no idea of the phase of the field. So, what is happening is one of the observer may be finding the electric field at its maximum value, another observer find it 0, another observer finds it a quarter of a cycle, third observer finds a negative. Please note that they are all identical states - identically prepared states - but the phase at which the value of phase that each person get is different. So, when you take an average of a large number of sinusoides which are not overlap - I will draw a figure little later - then you get 0 electric field; average electric field is 0.

Sir, how could be (( )) show that, there is some kind of uncertainty could be in the number photon phase.

I will come to it later. There is a problem, because there is a problem in defining the phase operator itself.

Chi operators?

Not chi operator, the phase of electric field. The electric field has a certain phase and amplitude; there is a uncertain. there is So, I have to define the phase operator and there are, it is not very well define, because there are some problems in defining the phase operator itself, but I will draw a picture later on.

And it will become slightly clearer that what the meaning of this statement is. So, expectation value of E is zero in this state (Refer Slide Time: 41.19). So, we are looking at one n ket state. Let me calculate the expectation value of E square

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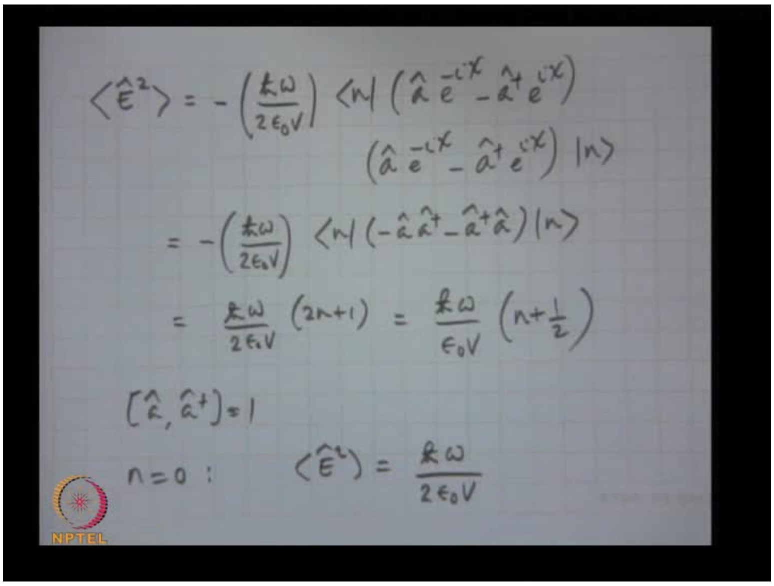

(( ))

No

Because they could be vary other cosine or sin so if I (( ))

No, classically, suppose I prepare 100 identical electromagnetic fields at frequency omega propagating in this direction z with a same polarization state - identical remember - everybody will find the same electric field like this.

No they have to be (( ))

Yes, when one person measures 0, everybody will measures 0; one person maximum, other; everybody measure the same; for all them, it is going like this. The problem is here, the moment I have defined, because of the uncertainty principle here; defining precisely the number of photons in the state removes all, I mean, makes the phase completely uncertain.

This is not a classical electromagnetic field (Refer Slide Time: 42.33); because classically, electromagnetic field should be such, if there is a quantum state which gives me a classical representation of the quantum representation of a classical field, the expectation value of electric field must vary sinusoidally with time and space, because classically I know the electric field goes as sin k z minus omega t or sin omega t minus k z.

So, a quantum state should give me expectation value of electric field in that state must be like sin omega t minus k z.

This n ket state is not a state which corresponds to a classical electromagnetic field; it is a completely non-classical state - the state cannot be explain classically.

Sir, is there any is there any can (( )) it is a general the theorem in quantum mechanics that the expectation values must satisfy the classical equations, the expectation values (( )) now when we are not be satisfy

This is not a classical state, there is no classical correspondence to this state. So, quantum states are much more richer much richer than classical states; classical states are a subset of the entire classic quantum states. So, I will tell you a quantum state, I will define a quantum state, whose expectation value will grow as E zero sin omega t minus k z, yes.

But what else exactly (( )) theorems states that, it is as I think that its states that for any observable the expectation value must satisfied the classical equations for according to that of the (( ))

But, you see I think that because quantum mechanics is the richer than classical mechanics, classical states, there are states in quantum which have no classical analogs. So, with what do I compare? Every classical state will have a quantum state I think, a quantum, a corresponding quantum state which will reduce to the classical state under certain conditions.

Please note here that if n becomes very large, we are still not the classical state. Sometime people think a large quantum number gives me - I get closer and closer to classical, no. Even if I n million or 10 power 20, expectation value of E still 0. There are other states called coherent states, whose expectation value will look like sin omega t minus k z, which will with its states which is closest to the classical state of electromagnetic fields.

What I want to do is to calculate E square expectation value. So, I will have this will be minus h cross omega by 2 epsilon 0 V n a exponential minus i chi minus a dagger exponential i chi into a exponential minus i chi minus a dagger exponential i chi n.

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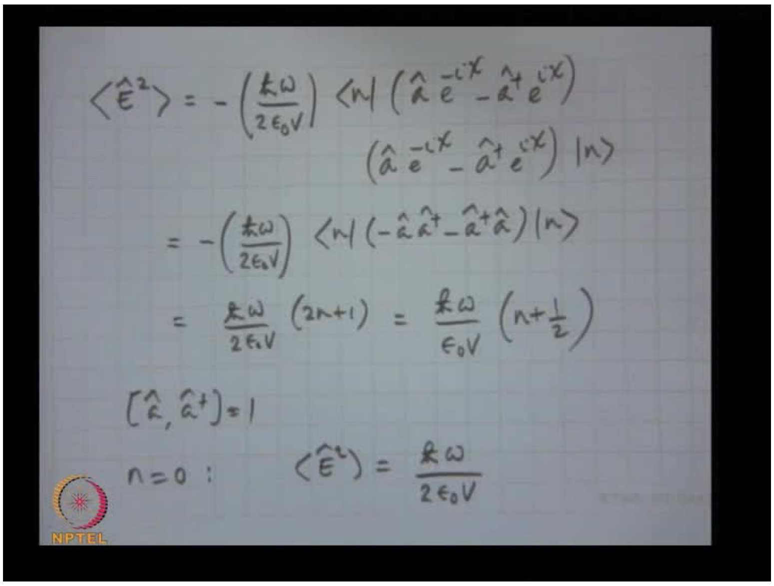

So, if you will have a square, a dagger square, a a dagger, and a dagger a. Which one will survive? a square and a dagger square will give you 0, because every time we apply the operator a to reduce the number. So, what you will be left with is the only non-zero states that the non-zero contribution will come from n minus a a dagger minus a dagger a n; the exponential i chi terms cancel off. a square and a dagger square will give you 0 because a times n is under root n n minus 1 and once more you apply n a, you get under root n into n minus 1 into n minus 2 and because n minus 2 ket is orthogonal to n ket, we get 0. So, what is this? This is equal to h cross omega by 2 epsilon 0 V into, how much is this?

(( ))

A dagger a on n gives me n, but a a dagger - I must use the commutation relation which is - a a dagger is a dagger a plus 1 so I will get (Refer Slide Time: 47.11) which is equal to (Refer Slide Time: 47.17)

So, expectation value of E square is not 0. So, there is field, there is an electric field whose average happens to be 0 - ensemble average is 0 - but the expectation value of E square is not 0. And please note, even if you have no photons in the state, expectation value of E square is still finite and the lowest which is, so, in the state n is equal to 0, the vacuum state E square expectation value is h cross omega by (Refer Slide Time: 48.01) So, these are called the zero point fluctuations. Even if you have a state which has no hesitation, there are electric field fluctuations in that state.

We will stop here. So, I will continue with this discussion in the next class to look at the quadrature operators and look at try to do a pictorial representation - an approximate pictorial representation to understand - the result that we are getting mathematically.

Any questions?

Thank you.