Quantum Electronics

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Module No. # 01

Brief Review of Electromagnetic Waves;

Light Propagation through Anisotropic Media

Lecture No. # 03

Anisotropic Media (Contd.)

We continue with our discussion on anisotropic media. Do you have any questions?

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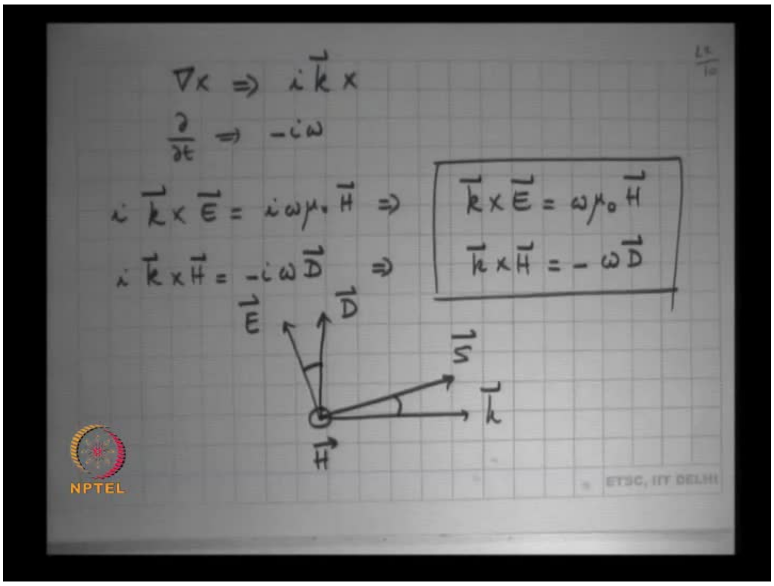

We started discussing anisotropic media last in the last class.

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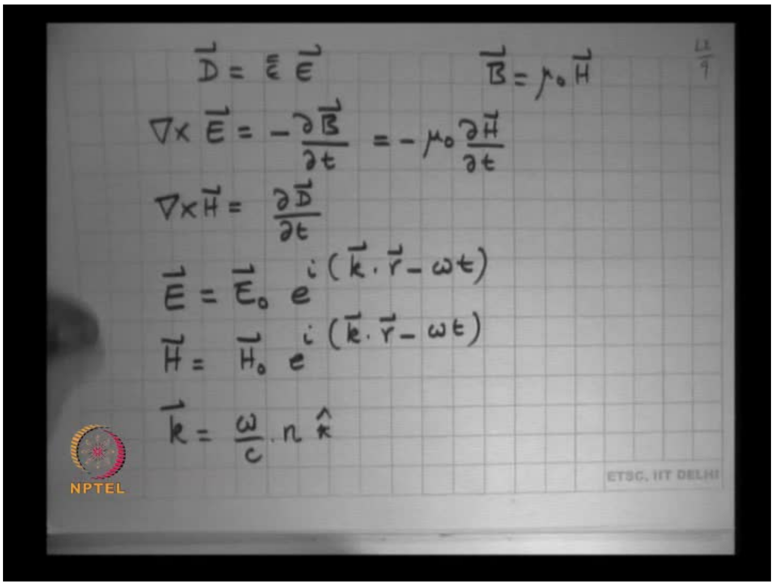

So, let me recall, we had written this equation D is equal to epsilon E; where, epsilon is now no more a scalar; it is a tensor. So, this will essentially imply that in general, D and E are not parallel to each other. These are the two Maxwell's equations we will use to study propagation of waves. And, what we try to look for is plane wave solutions of the type given by E is equal to E 0 exponential i k dot r minus omega t.

Now, as I was mentioning in the last class, if I assume E naught vector to be independent of position, it implies that we are looking for solutions, where the state of polarization represented by this E 0 vector is maintained as the wave propagates. We do not know what polarization states will lead us to the solution. So, E 0 vector is still an unknown. What is the direction of the E 0 vector? Whether it contains components, which are in phase, out of phase, etcetera? We still do not know. And, the corresponding H vector, we wrote as H naught e to the power of i k dot r minus omega t. And, here the propagation vector k is actually… Let me correct here – so, there has to be a unit vector here (Refer Slide Time: 02:04). So, this is omega by c times; some refractive index times a unit vector. This refractive index I still do not know, because the medium is characterized by three principal refractive indices, n x, n y and n z or three principal dielectric constants, k x, k y, k z.

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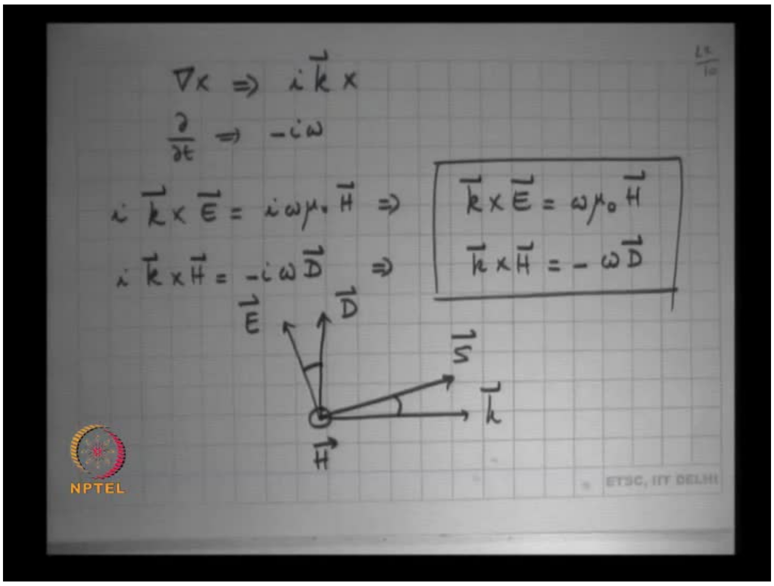

So, how is this n related to these? We do not know. So, we substitute this E and H in this equation, and also, this expression for D. And, as I mentioned before, we will assume B is equal to mu naught H and we land up with set of equations (Refer Slide Time: 02:40) – two equations – k cross E is omega mu naught H and k cross H is minus omega D. So, we discussed this last time and we saw that in general, because D and E are not parallel to each other, k and S are not parallel to each other. So, the wave front direction of propagation is represented by k vector; the energy propagation is directed by the S vector; and, they are in general, not equal, not the same. So, what you will have is a wave front like this, (Refer Slide Time: 03:11) facing forward, but going at an oblique angle. So, our objective is now to use these set of equations to find out what are the values of refractive indices that the wave will propagate with and with what polarization states. So, we use these two equations.

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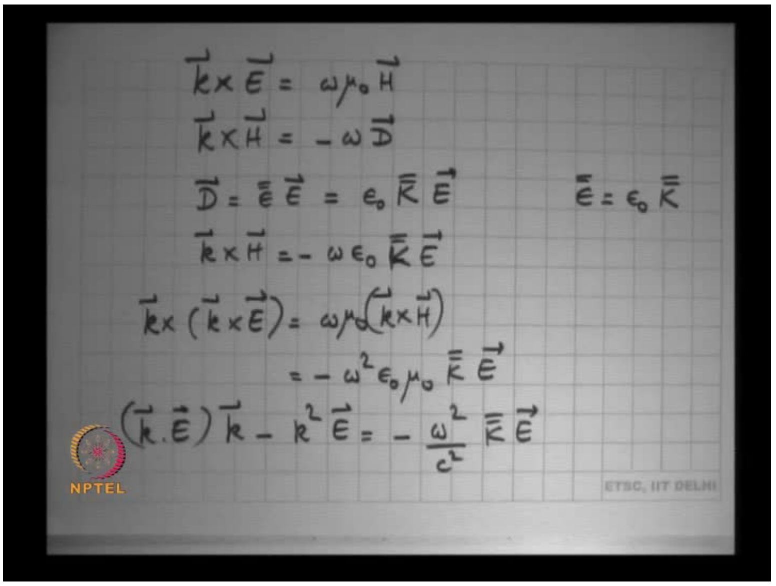

Let me write these equations again. So, k cross E is equal to omega mu naught H and k cross H is equal to minus omega D. Now, remember, we have written this equations D is equal to epsilon E. So, this equation actually becomes k cross H is equal to minus… Let me write this as epsilon 0 times dielectric constant tensor times E vector. The epsilon tenser is actually epsilon 0 times K tensor. So, in isotropic media, epsilon would be a scalar, K would be a scalar and K would represent the dielectric constant of the media. So, this is k cross H is minus omega epsilon K into E vector.

Now, let me take this equation and premultiply by k cross. So, k cross k cross E is equal to omega mu naught k cross H. I substitute for k cross H from here (Refer Slide Time: 04:59). So, this becomes minus omega square epsilon 0 mu 0 K times E. I can expand this a cross b cross c. So, I will have k dot E times k minus k dot k, which is k square times E vector, is equal to minus omega square by c square into K E. Epsilon 0 mu 0 is 1 by c square; and, I have just expanded this k cross k cross E in terms of k dot E dot k into k minus k square E. So, this equation is actually… This contains three equations, three Cartesian component equations. So, let me write each equation.

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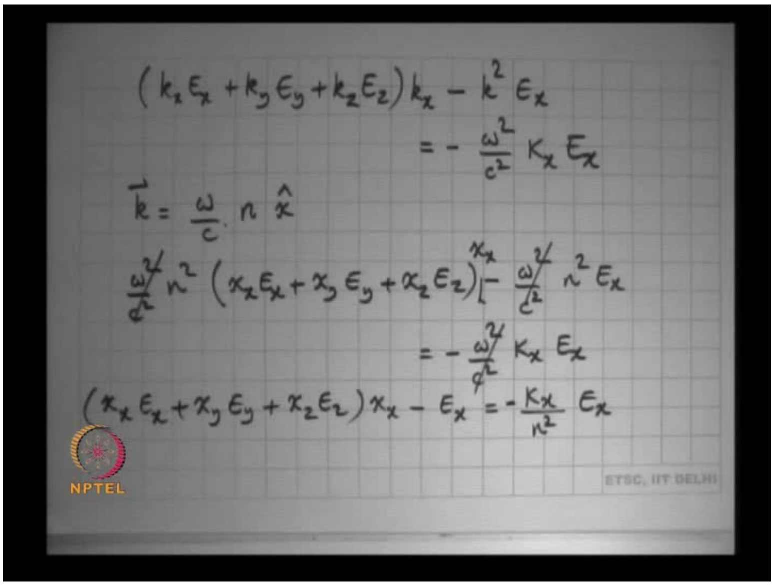

For example, the x component of this equation becomes k x E x plus k y E y plus k z E z – is a bracketed term times k x; that is, x component minus k square times E x is equal to minus omega square by c square. Now, remember, we are always using principal axis system. So, in the principal axis system, the K tensor is diagonal. So, what will be the x component of K times E? K x E x. So, I will have simply K x times E x.

Now, this k vector is omega by c times n times kappa cap; kappa cap is unit vector. So, let me substitute for the k x, k y, k z. So, what I will get here is omega by c times n is common in k x, k y and k z and this k x. So, I will get omega square by c square into n square into kappa x E x plus kappa y E y plus kappa z E z minus omega square by c square into n square into E x is equal to minus omega square by c square K x into E x. Now, I can cancel this omega square by c square in all the three terms and I can divide by n square the whole equation – there is a kappa x here sitting; this kappa x from here is sitting here (Refer Slide Time: 08:00). So, I divide by n square. So, I will get kappa x E x plus kappa y E y plus kappa z E z into kappa x minus E x is equal to minus K x by n square times E x.

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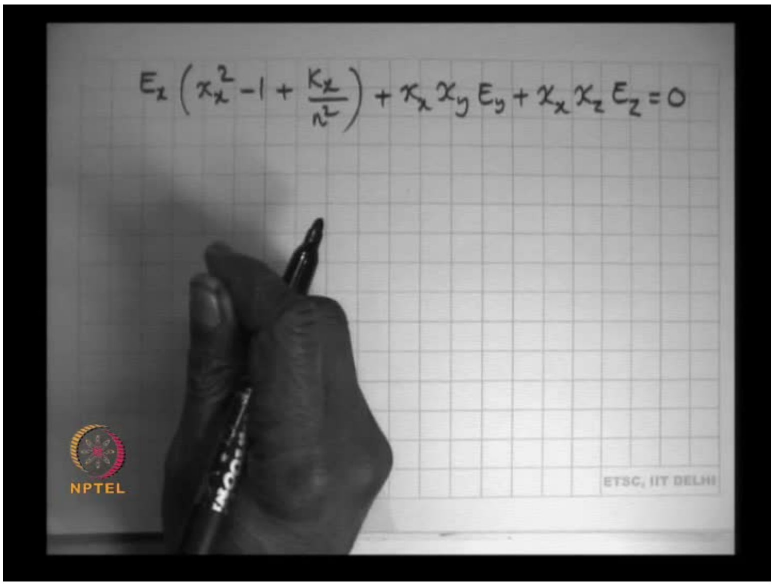

So, let me now write this equation after opening up the bracket; and, what I will get is – so, I will have E x into kappa x square minus 1 plus K x by n square. So, this is (Refer Slide Time: 08:51) kappa x square minus 1 from here and plus k x by n square into E x – plus kappa x kappa y E y plus kappa x kappa z E z (Refer Slide Time: 09:05) is equal to 0.

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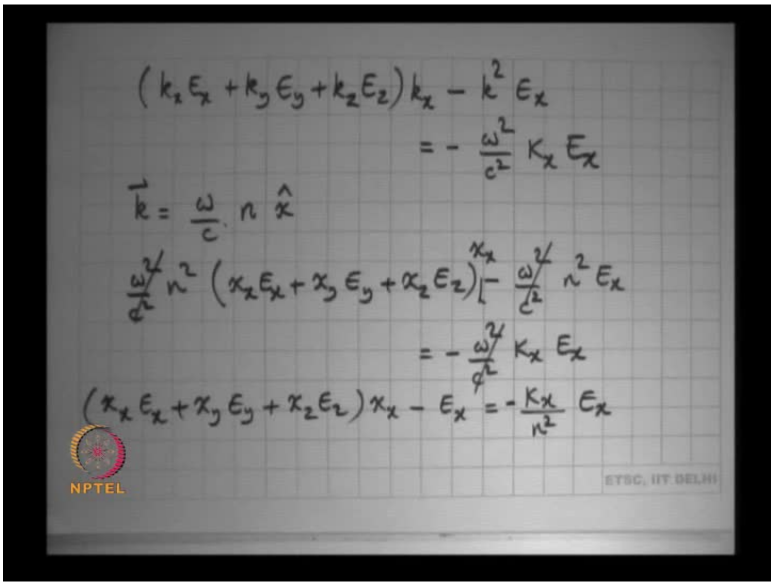

So, I have essentially (Refer Slide Time: 09:11) kappa x square E x minus 1 minus plus K x by n square into E x. And then, I have kappa x kappa y E y, kappa x kappa z E z.

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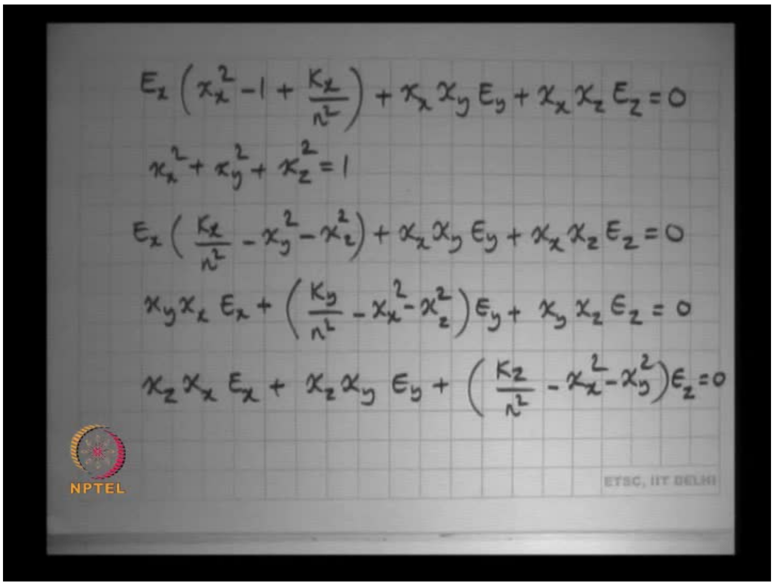

Now, remember, kappa is unit vector. So, kappa x square plus kappa y square plus kappa z square is equal to 1. So, kappa x square minus 1 is essentially minus kappa y square minus kappa z square. So, this equation can be written as E x into K x by n square minus kappa y square minus kappa z square plus kappa x kappa y E y plus kappa x kappa z E z is equal to 0. So, that is the x component of that vector equation. You can similarly, write out the y and z components of this equation (Refer Slide Time: 10:12) and you can actually use symmetry to write the remaining two equations. So, for example, the second equation you will get is kappa y kappa x E x plus K y by n square minus kappa x square minus kappa z square into E y plus kappa y kappa z E z is equal to 0; and, kappa z kappa x E x plus kappa z kappa y E y plus K z by n square minus kappa x square minus kappa y square into E z is equal to 0.

I have got a set of three linear equations in E x, E y, E z, in which, the unknowns are still n. I do not know the value of n. If I give you a medium, I know K x, K y, K z, the three principal dielectric constants. If I specify the direction of propagation, I know kappa x, kappa y, kappa z. So, I still do not know n, the refractive index and the corresponding E x, E y, E z values. So, the values of n will be the eigenvalues of this problem. And, electric field vector, which has components E x, E y, E z, will be the eigenvectors. Now, if you have a set of three linear equations in E x, E y, E z, what is the condition for a nontrivial solution? Determinant of coefficients must be 0. So, if I write out the determinant of these coefficients, you will get an equation in… So, first of all, you notice that it is all n squared; there is no n anywhere. So, you will have n squared terms, 1 by n squared terms; you will have 1 by n 4th terms, 1 by n 6th terms.

Now, what you can show is, when you expand this determinant, you will get a quadratic equation in n square, the 6th power; n to the power of 6 coefficient just vanishes. So, you will have a quadratic equation in n square. So, quadratic equation in n square means there will be four roots. And, you will find that out of these, only two are real roots of the problem. So, there will be two real eigenvalues of this problem, which will actually correspond to the two states of polarization or propagation, because light is a transfers electromagnetic wave and you will find two independent polarization states. So, let me leave it as a problem for you to show that opening the determinant will only give me a quadratic equation in n square; and, that quadratic equation determinant will give me the solution; and, I will have two real solutions.

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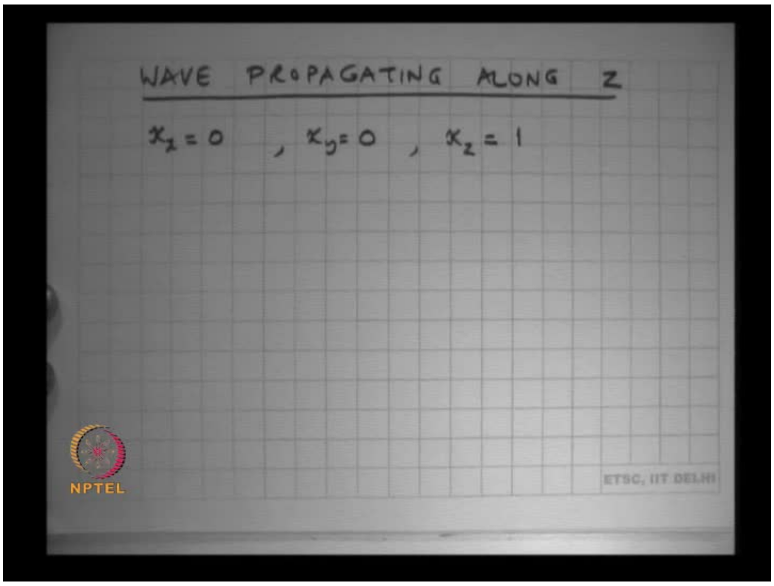

Now, instead of doing a general problem, let us look at some example. So, first example I want to see is a wave propagating along z. Please remember, this x, y, z is the principal axis system in the crystal; they are not arbitrary. When I say this z, this z is the z direction of the principal axis of the crystal. So, if I have a wave propagating along z, what is the value of kappa x and kappa y and kappa z? So, what should be kappa x? 0; kappa y – 0 and kappa z – 1; you have the wave propagating along the z direction. So, the unit vector has 0 components along x and y and has a component along z direction.

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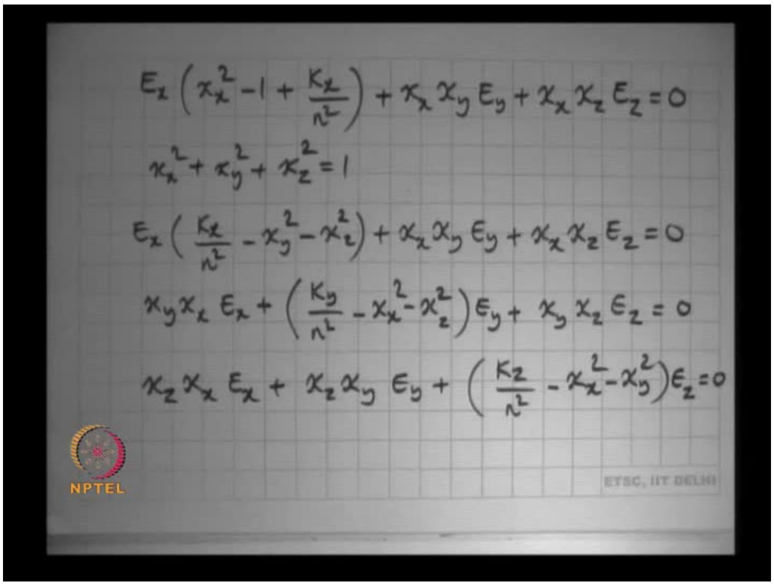

I substitute this into all the three equations (Refer Slide Time: 14:41). So, let me write down the three equations individually. So, I will have (Refer Slide Time: 14:46) E x into K x by n square – minus kappa y square is 0 – minus 1;

K x is 0?

No, K x – please, this is different; this is kappa x. K x is the one of the principal dielectric constants of the medium. So, K x, K y, K z – these three coefficients sitting here (Refer Slide Time: 15:11) K x, K y, K z are the three principle dielectric constants. This is kappa x, kappa y, kappa z. These are the components of unit vector. If I am given a medium, I am given K x, K y, K z.

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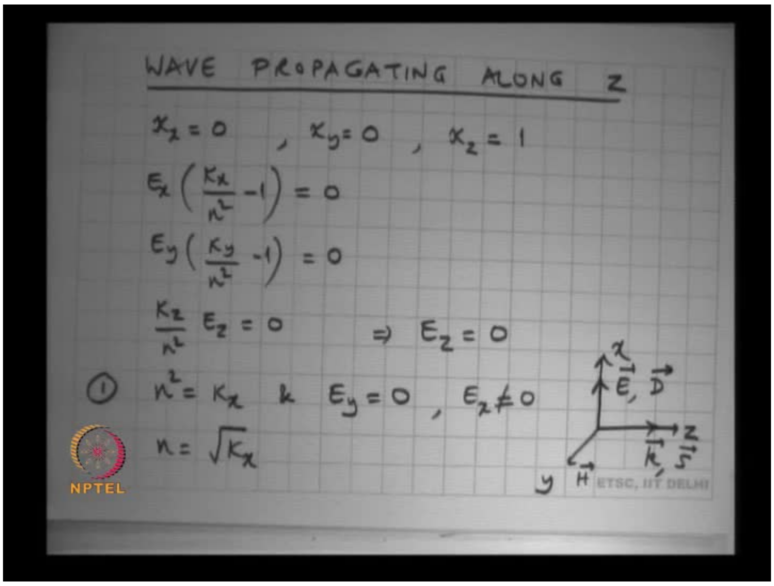

So, the second term and third term will be 0, because there is a kappa x, kappa y. So, this will be equal to 0 – first equation. Second equation will give me E y into K y by n square minus 1 is equal to 0. And, the third equation will give me K z by n square into E z will be 0. I substitute these values of kappa x, kappa y, kappa z in the three equations and the three equations simply reduced to these three equations; simply three simple equations.

Now, what is the first conclusion I can draw from here? E z is 0, because K z is a principal dielectric constant value; n is some refractive index. So, this implies E z is equal to 0; that is, for a wave propagating along z, the electric field has no component along the z direction. So, I am left with these two equations (Refer Slide Time: 16:32). So, I can satisfy these two equations under two conditions. The first solution is I can have n square is equal to K x, which will satisfy the first equation and E y is equal to 0, which will satisfy the second equation. So, if I choose n square is equal to K x as the solution, the first equation is satisfied; and, by choosing E y is equal to 0, second equation is satisfied. And of course, E x must not be equal to 0, because if E x is also 0, the electric field itself is 0. So, that is the first solution. So, the first solution corresponds to electric field along the x direction; and, the corresponding refractive index as seen by this wave is equal to square root of K x.

If I can draw the axis here, this is z, this is x, this is y. So, the electric field is along x direction; the propagation vector is chosen to be along the z direction. What is the direction of D vector along x? Because remember, if the electric field is along one of the principal axis direction, the corresponding D vector is also along the same principle axis direction. So, D is also here (Refer Slide Time: 18:00). And, what about S vector? S vector is parallel to k. This is the direction of H vector here. By using the relationship between the direction of vectors of k, E, D and H, you can calculate S vector. And, you see, for the solution, the wave is polarized along the x axis; propagates with a refractive index – square root of K x; and, propagates along the z direction; and, the pointing vector and the propagation vector are parallel to each other. So, this is the wave; this is just like any ordinary isotropic medium with a polarization now along the x direction.

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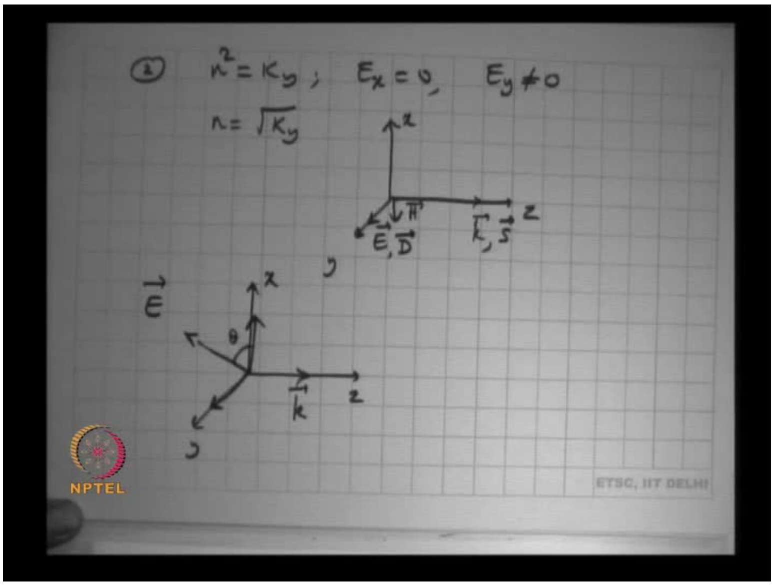

So, let me look at the second solution. The second solution is n square is equal to K y; E x is equal to 0; and, E y not equal to 0. So, the refractive index as seen by this wave will be square root of K y. And, if I draw the diagram again, this is z direction, this is x direction, this is the y direction. So, this is the k vector; the electric field is along this direction. What is the direction of H vector? Down – minus x. So, it will be like this. D vector parallel to E vector; and, k vector and S vector are parallel again. So, in this case, the electric field points along the y direction and the wave propagates along this z direction; and, the velocity of this wave is determined by this refractive index – square root of K y. So, if I have a wave propagating along the z direction, if I choose my electric field to be along x direction, if it propagate with a speed of c by square root of K x, if the wave… if I choose the electric field to be along the y direction, it will also propagates unchanged, but with a speed c by square root of K y.

Please remember, the solution we have got is a situation, where the electric field direction does not change with propagation. So, if you launch a wave along the z direction, but with its electric vector along x direction, it will not change at all; it will propagate x direction oriented completely. If their incident like is bipolarized with E vector on the y axis direction, it will also propagate along z unchanged, but with a different speed, c by square root of K y. If you choose an arbitrary direction of E vector – so, if x is like this; y like this; z like this; and, if my propagation is like this; and, if I happen to choose an E vector like this, what would I have to do? I have to break this up into this component and this component (Refer Slide Time: 21:17). So, if this angle is theta, I have E cos theta oriented along x cap direction E, sine theta oriented along y cap direction; the E cos theta component will travel with the speed c by square root of K x; the E sine theta component will travel with the speed c by square root of K y. And, if K x is not equal to K y, they will develop a phase difference. And, when you add two orthogonally linearly polarized light waves with a finite phase difference, you will find a different polarization state.

So, as light propagates along the z direction, please remember, the electric field, which is the x component, which has E cos theta component, has its S vector also along z; the E sine theta component along y also has its S vector along z. So, the net energy is propagating along z, but with the polarization state, which keeps on changing, because the two velocities are in general not equal. So, the state of polarization will change and these two solutions we have got are the two eigenmodes, because light transfers is electromagnetic; I will find two orthogonally polarized eigenmode solutions and these are the two solutions – one along x and one along y for propagation along z direction.

In the first case, when you say that the electric field would not change, you were talking about the direction of the electric field; magnitude still going to oscillate right?

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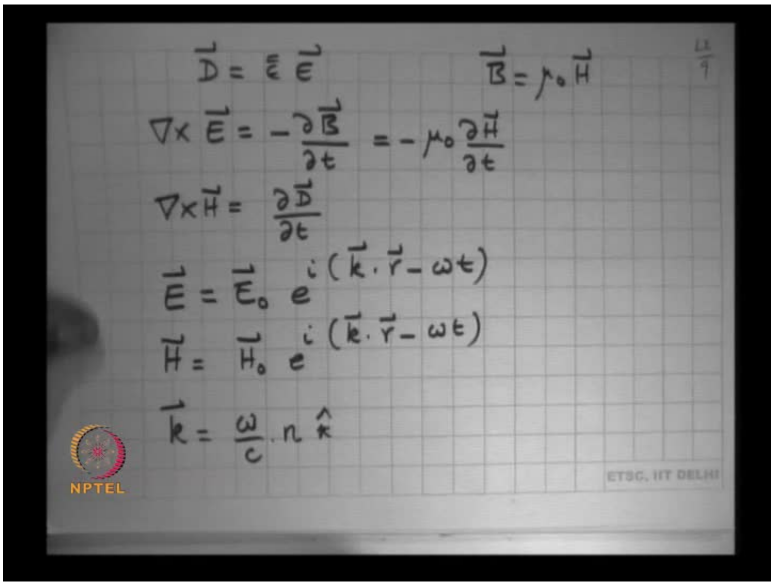

Yes, the magnitude oscillates. The magnitudes oscillates as per this equation – E to the power i k dot r minus omega t. What I am trying to say is, E 0 vector is not going to change at all with z. If the E 0 vector at the input was x directed, it will always remain x directed; if the E 0 vector was y directed, it will always remain y directed. But, if the E 0 vector is some arbitrary direction now, it will change, because that E 0 vector if I try to substitute into the Maxwell's equations, I will find that it does not remain constant. So, the way I solve it is that any given E 0 vector – I break it up into x component and y component; I know how x component propagates with distance; I know how y component propagates with distance. And then, I add up at any value of z. It is exactly like Fourier decomposition; it is in terms of two normal modes of propagation. So, I split the problem into the normal modes, whose solutions I know; how each normal mode propagates with time.

So, if I propagate along x direction, what will be the solutions I will get? It is again by symmetry. If I propagate along x, one will be along y; the other will be along z; the y component electric vector will see a refractive index, square root of K y; the z component will see a refractive index, square root of K z. So, if I propagate along any of the three principal axis directions, the two eigenmodes are the two other orthogonal principal axis directions; and, the corresponding refractive indices seen by the wave are c by the corresponding principal directive indices. So, for propagation along the principal axis, the solutions are very simple.

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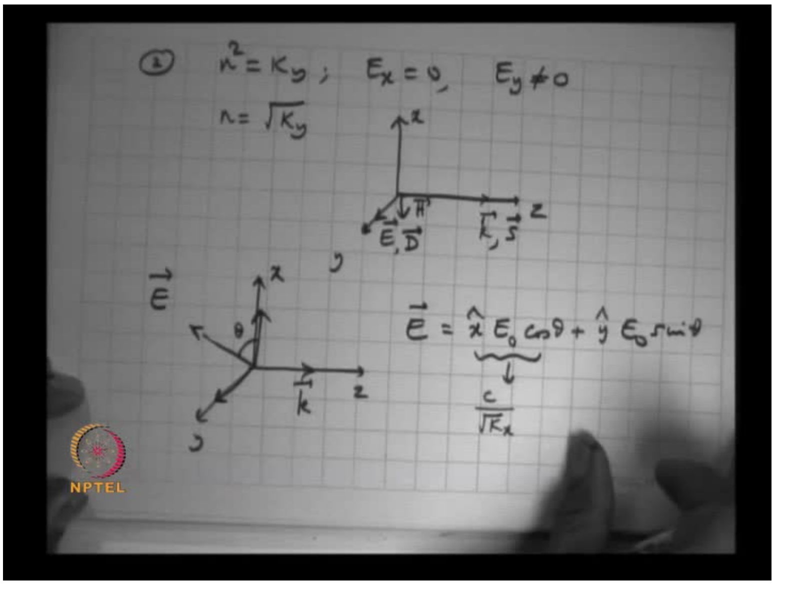

And, in this case, S vector and k vector are always parallel to each other; D vector and E vector are parallel to each other. And, the propagation is exactly like an isotropic medium; the only difference is the different components of polarization see different velocities. So, if you launch an arbitrary polarization state into this medium and still propagate along the principle axis, the polarization state will change in general.

Now let me go to little more general case.

This case is only possible when K x equal to K y?

Which case is only possible?

When the electric field has both x and y components.

No, suppose I take electric vector like this (Refer Slide Time: 25:32). So, E vector is equal to x cap. So, let me call it E 0 cos theta plus y cap E 0 sine theta. Now, what I am saying is, this component propagates with a velocity c by square root of K x. So, what is the k vector for this? k vector for this component…

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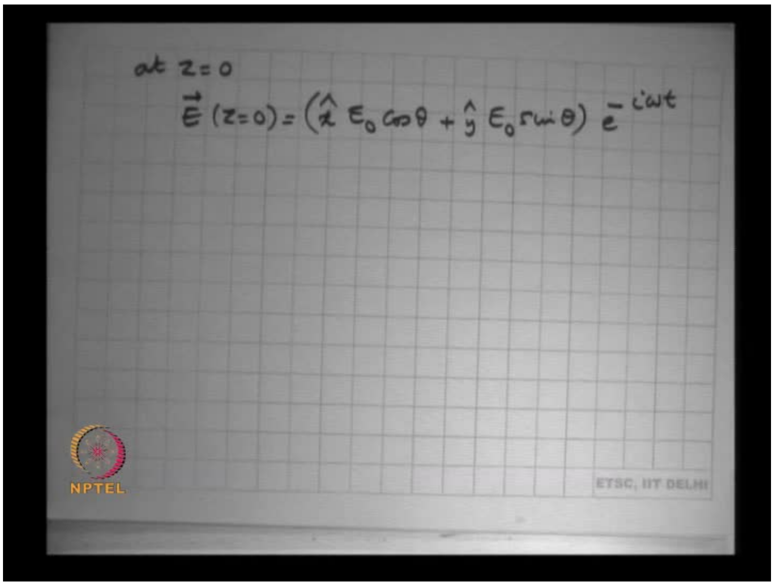

Let me write like this. At z is equal to 0, the equation is E vector – x cap E 0 cos theta plus y cap E 0 sin theta e to the power minus i omega t.

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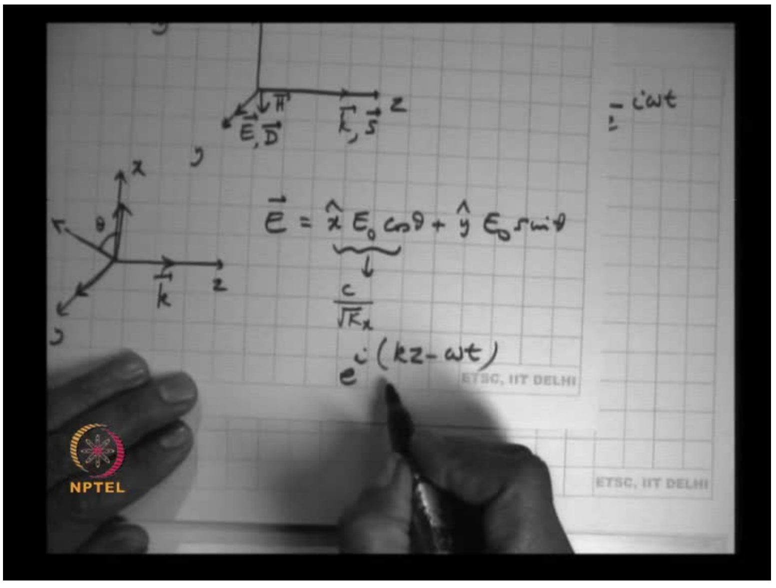

The propagation is along the z direction. So, the exponential factor would have been exponential i k z minus omega t, because propagation is along z; k vector is along z. And so, k dot r is k z. So, I have exponential i k z minus omega t.

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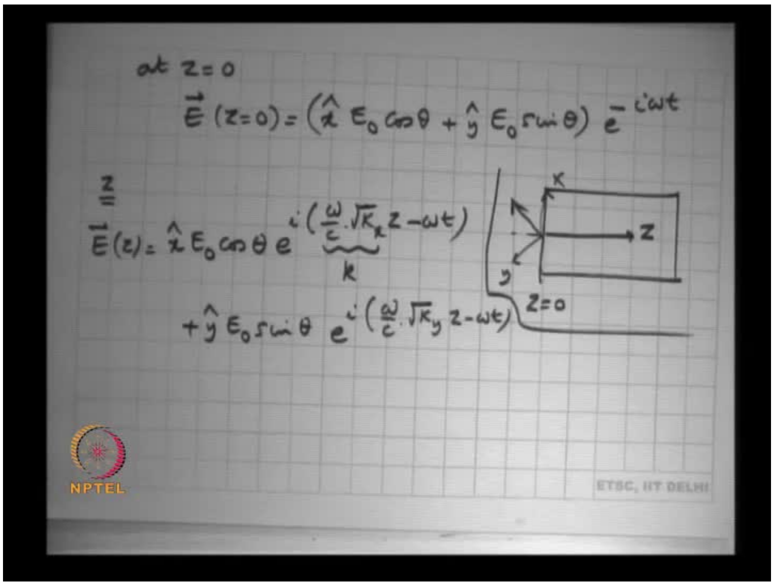

At z is equal to 0, this is the crystal. The crystal is here. This is z axis; this is z is equal to 0. I have launched the wave here with an electric vector like this, oriented at some angle to the x and y direction. Now, what will happen at z? At z, I will write this as… Now, look at this. When I have x component, the corresponding phase change will be i – what is the value of k? omega by c square root of K x into z minus omega t. This is the value of k for the x component of electric field.

For the y component, the k vector – the magnitude of the k vector is omega by c into the refractive index, which is square root of K y into z minus omega t. I cannot write a k vector for this x cap plus y cap component (Refer Slide Time: 28:14). I have got the solution. I have shown you that if electric vector is along x cap, it is a plane wave propagating with the velocity or with the propagation constant omega by c times square root of K x. If the electric vector is along y cap, it is a plane wave propagating with a propagation constant omega by c square root of K y. So, I have broken up the input electric vector into two components. And then, the corresponding phase changes, I have written as K times z.

Now if K x is not equal to K y, these two values are different. And so, when I add them up at any value of z at this output for example, the polarization state will not be linear anymore; it will be different.

But, if we go back to the two original equations?

Yes.

If both E x and E y are non-linear?

You mean where?

The two equations right; it may be non-linear.

These equations? (Refer Slide Time: 29:09)

No. E x into K x by n square.

Yes (Refer Slide Time: 29:15).

Now, if both E x and E y are non-zero, that would imply that, both K x and K y are equal and equal to n square, which would mean that they would…

That is not the solution of this problem, because n square cannot have two different values for the same solution; the two eigenvalues. The two eigenvalues are n is equal to square root of K x and n is equal to square root of K y. If n is square root of K x, E x is non-zero; if n is square root of K y, E y is non-zero. These are the two solutions. If you choose any other arbitrary E direction, I would have to write in terms of these eigenmodes, which are E x and E y components. E x component will propagate with its own phase constant; E y component will propagate with its own phase constants; and, they will develop a phase difference.

It is like, if I have a string, I have the modes of oscillation, the fundamental mode, the first excited mode. If at t is equal to 0, I excite the first and the fundamental of the first harmonic of the string, what will be the shape of the string after some time? You will say the fundamental has changed by so much, because the frequency is omega. The first harmonic has a frequency omega dash. And so, you will then find the shape of the string corresponding to the fundamental and the first charges excited after some time, and add them up. Exactly, similarly, I am doing here. So, what I am doing is, I am splitting the input electric vector in terms of the eigenmodes, because I know each eigenmode propagates as an independent solution. X component eigenmode will propagate with a certain propagation constant; the y component eigenmode will propagate with a different propagation constant; and they will propagate independently. See the eigenmodes are all independent, which means if I launch x component electric vector, it will always remain x component electric vector; if I launch y component electric vector, it will always remain y component electric vector.

If I launch an arbitrary direction, it will change its orientation. It is not a mode. For example, if the light was polarized at 45 degrees to the xy direction, after some distance, you will find it linearly polarized like this. At this position, the phase difference between this and this was 0 (Refer Slide Time: 31:38). After propagating through a certain distance, there will be a phase difference of pi. When the phase difference becomes pi, the linearly polarized light becomes the other way around. So, this polarization state is not remaining constant, because that is not a mode of the system. Whether you look at this or this, this (Refer Slide Time: 31:56) component and this component are remaining the same; there is no change in these components. The amplitudes of those two components remain E 0 cos theta, E 0 sine theta. The only thing that is happening is the change of phase. So, that is the beauty of eigenmode solutions. You break it up; you find the eigen solutions for any given problem, you break the problem into the eigen solutions and analyze the problem in terms of eigen modes.

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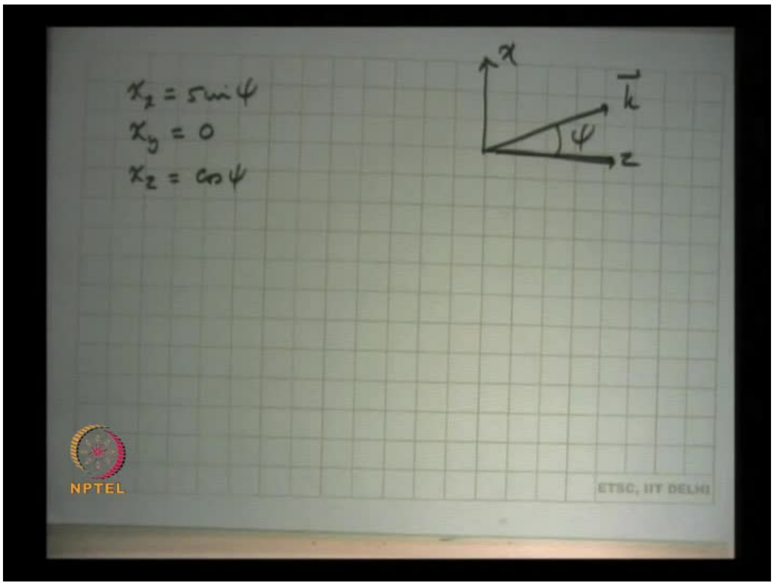

Now, let me look at a more general case. This is z; this is x. By propagation direction, is some direction psi with respect to z axis and in the xz plane. So, what is kappa x? Sine psi; kappa y? kappa y is 0; and, kappa z is cos psi.

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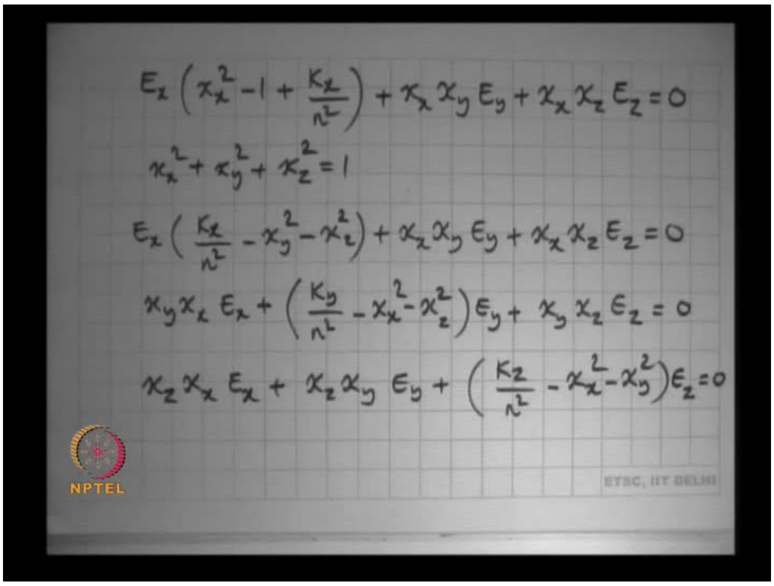

Now, let me look substitute these into those three. So, I will go back to these equations; substitute these values of kappa x, kappa y, kappa z, and let me write those equations.

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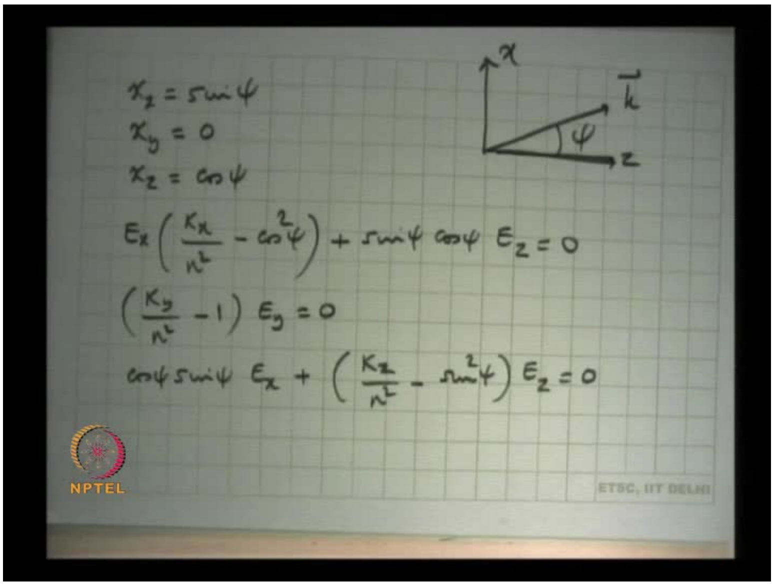

So, E x times k x by n square minus kappa z square, which is cos square psi. Second term is kappa x kappa y is 0 plus kappa x kappa z is sine psi cos psi E z is equal to 0. The second equation is K y by n square minus kappa x square minus kappa z square is 1 into E y is equal to 0. And, the last equation is cos psi sine psi E x plus K z by n square minus sine square psi into E z is equal to 0. So, I have just substituted the values of kappa x, kappa y and kappa z in those three equations; and, those three equations simplify to this (Refer Slide Time: 34:31) – this sort of three equations.

The first thing you notice is these three equations have split into two groups. In one group of equations, the first and the third only contain E x and E z. And, this intermediate equation has only E y. So, actually, in principle, what I need to do is to find the determinant of the coefficients; put it equal to 0; get the solutions. But, because of these special cases, I can find solution very quickly just by looking at these equations.

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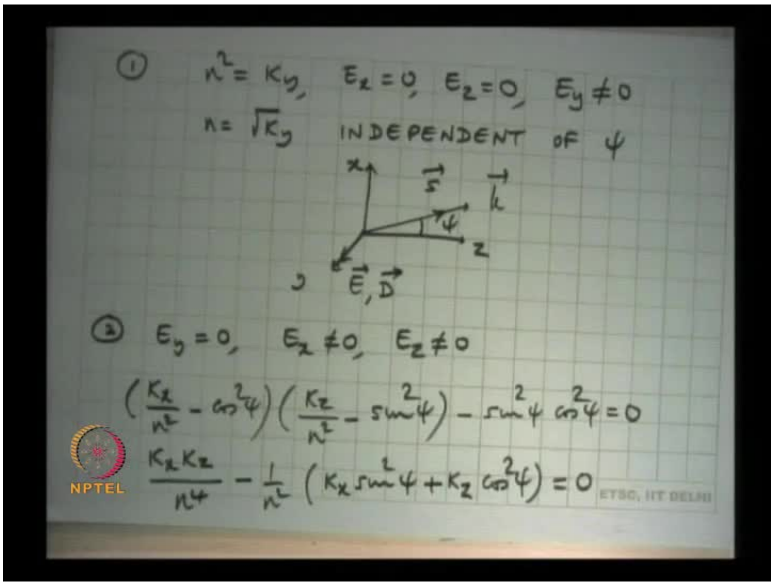

For example, one solution can be… If n square is equal to K y, this equation (Refer Slide Time: 35:13) gets satisfied. So, I can choose n square is equal to K y, E x is equal to 0, E z is equal to 0 and E y not equal to 0. If I choose n square is equal to K y, this equation is satisfied; and, by choosing the electric vectors, components E x and E z to be 0, I satisfied the first and the last equations (Refer Slide Time: 35:45). So, this gives me a refractive index, which is square root of K y, (Refer Slide Time: 35:51) independent of psi. What is psi? Psi is the angle between the k vector and the z axis. So, whatever psi you choose, one of those solutions will have its electric vector along the y direction. And so, if I again draw, this is y. So, this is my propagation direction k vector. So, E vector is here. What will be the D vector? If E is along the principle axis, D is also along the same principle axis. So, D is also here. And, what will be the S vector direction? Along k. So, this solution has its S vector parallel to k vector; the electric fields in the displacement vector are perpendicular (( )) plane and the velocity of this wave is independent of psi; this angle is psi here (Refer Slide Time: 37:14).

Second solution – Second solution is when I put E y is equal to 0, (Refer Slide Time: 37:24) and then, E x and E z have to satisfy these equations. So, let me put E y is equal to… So, the second solution is E y is equal to 0, E x is not equal to 0, and E z is not equal to 0. And, how do I find the value of n? From the first and the last equation, (Refer Slide Time: 37:43) the determiner of coefficient must be put equal to 0. So, this gives me (Refer Slide Time: 37:50) K x by n square minus cos square psi into K z by n square minus sine square psi minus sine square psi cos square psi is equal to 0. So, the determinant is essentially this into this; (Refer Slide Time: 38:13) minus this into must be equal to 0. And so, I open up the brackets and I get K x K z by n to the power 4 minus 1 by n square into K x sine square psi plus K z cos square psi is equal to 0. Sine square psi cos square psi terms cancels off.

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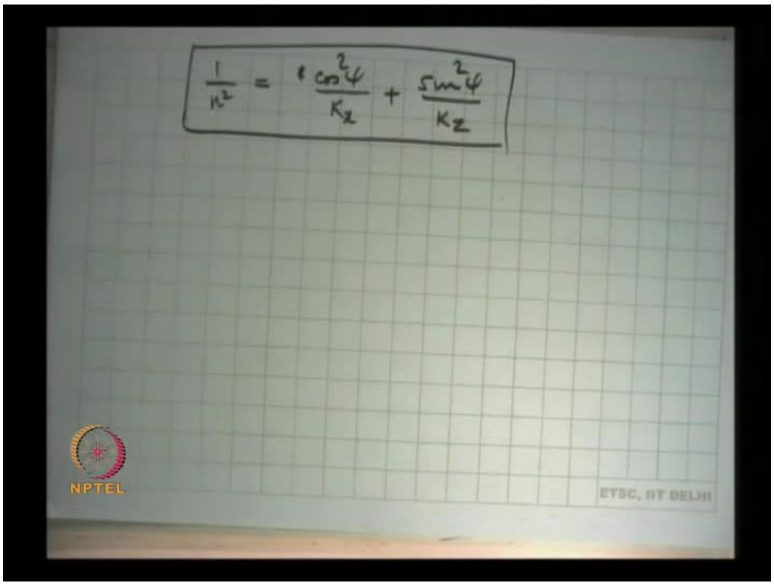

And, because 1 by n square is not 0, I take a (( )) by n square common from both sides and I will get an equation 1 by n square is equal to cos square psi by K x plus sine square psi by K z. In this equation, what I have done is, I have removed 1 by n square; it is not 0; and, I have divided by K x K z. So, I will have cos square psi by K x plus sine square psi by K z, is equal to 1 by n square.

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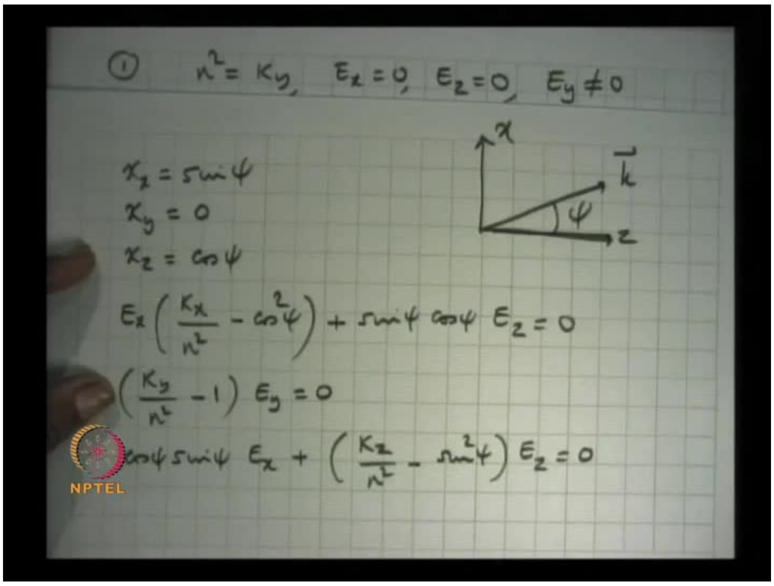

And, this has both E x and E z non-zero. So, let me look at the ratio of the E x to E z. How do I get that? I substitute the solution into one of these equations (Refer Slide Time: 39:44).

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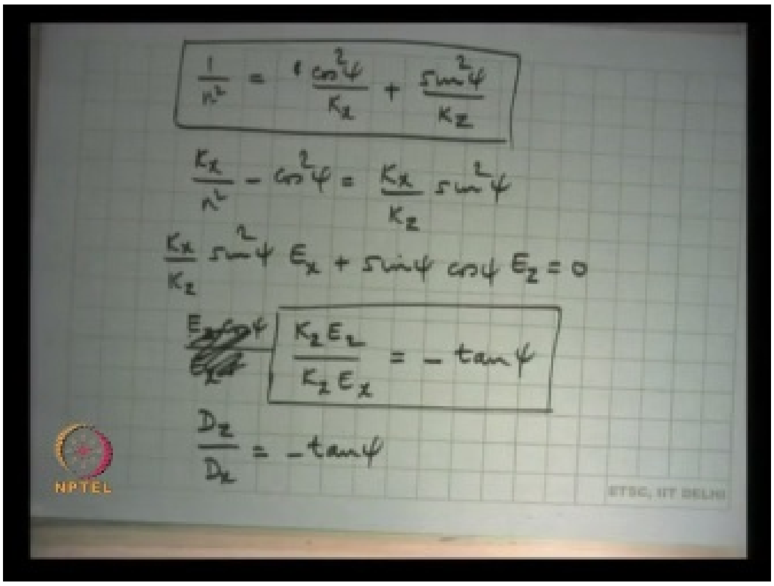

So, for example, K x by n square minus cos square psi from here – this implies K x by n square minus cos square psi is equal to K x by K z sine square psi. I just multiply by K x both sides. So, I get K x by n square minus cos square psi is equal to K x by K z sine square psi. So, I substitute for (Refer Slide Time: 40:10) K x by n square minus cos square psi into this equation. And, I will get K x by K z sine square psi E x plus sine psi cos psi E z is equal to 0. So, if psi is not equal to 0, then I can remove sine psi from here and I will get K z E z by K x E x is equal to minus tan psi. Now, what is K z E z? Please remember, xyz is the principle axis system. So, D x is K x E x, D y is K y E y, and D z is K z E z. So, this is nothing but D z by D x is equal to minus tan psi.

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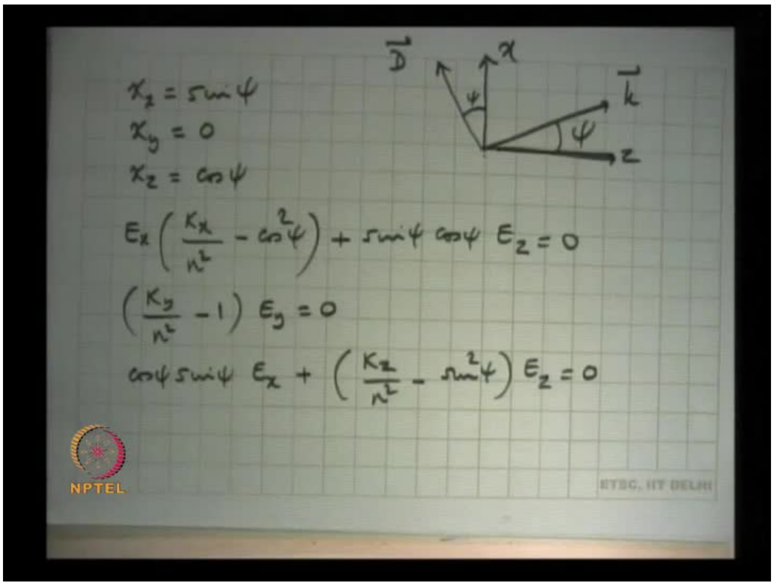

So, what is the meaning of the statement? If you go back to this figure, what is the vector, which has z component by x component, must be minus tan psi? It is a vector perpendicular to this k vector. And, this angle is psi. D – D z by D x is minus tan psi. This vector has a slope – tan psi; this vector has a slope – minus tan psi. So, D z by D x is minus tan psi and that is the direction of the D vector. And, it is expected, because D vector has to be perpendicular to the k vector. I need not have derived like this, but I know the D vector will always be perpendicular to k vector. So, this is the D vector of this propagation direction.

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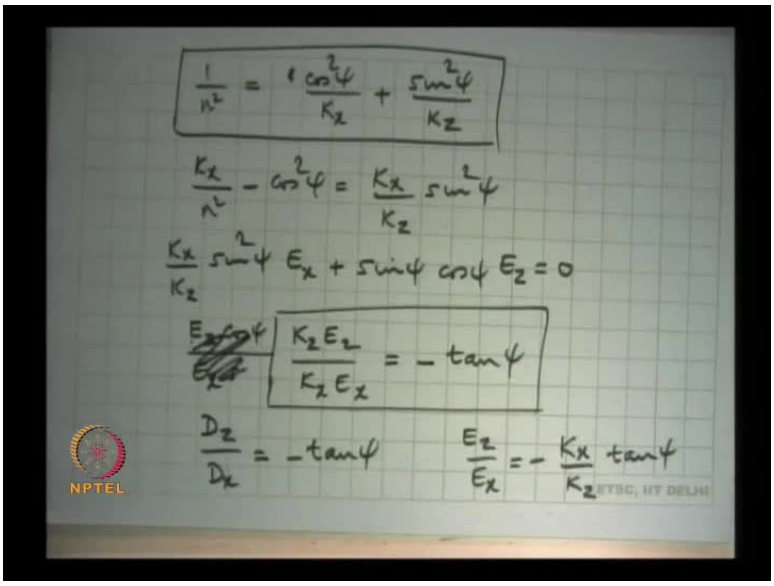

But, also note that this equation tells me that E z by E x is equal to minus K x by K z tan psi. Now, if K x is not equal to K z, the electric vector, z component by x component is not equal to minus tan psi. So, electric vector points in a different direction than the D vector. So, if K x is greater than K z, where should I draw E vector? If K x is greater than K z, then it will be to the left or to the right of D vector.

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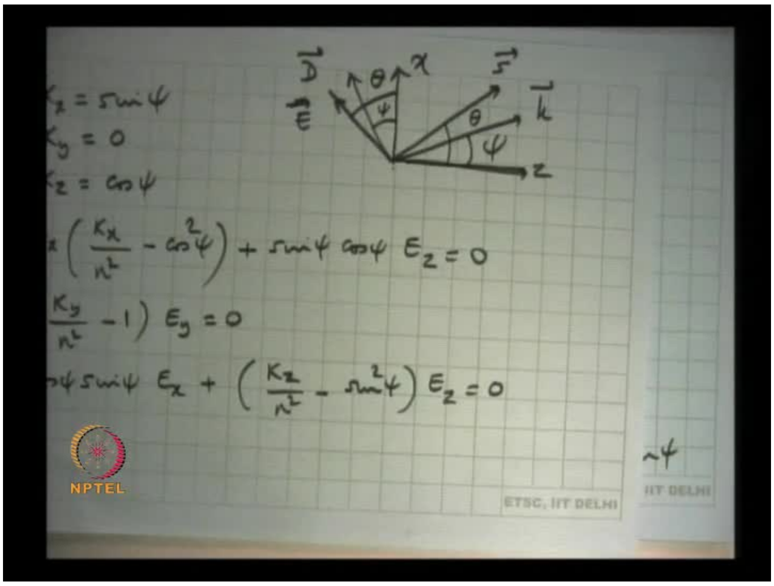

E z by E x will be bigger than D z by D x. So, E vector has to be here or here – between D vector and x vector or on the other side. D vectors, E vectors are here, because its z component by x component will be more than the z component by x component of the D vector. This is if K x is greater than K z. If K x is less than K z, it will be on the other side. So, for this mode, for this propagation, the D vector and E vector are not parallel. And, S vector now will be here (Refer Slide Time: 44:28). So, if I call this theta, this is also theta. So, please note that again, for propagation in the xz plane with k vector making an angle psi with the z axis, I have found two solutions.

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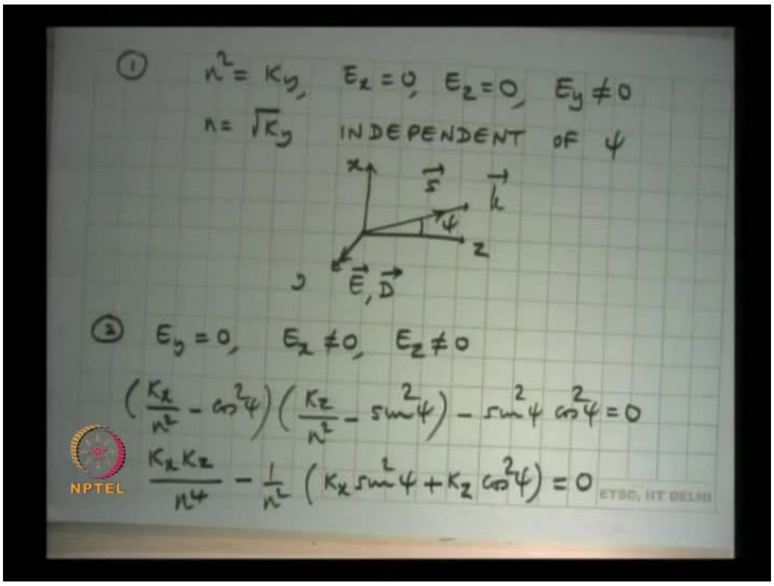

The first solution is a wave, which is with this electric vector along the y direction; and, S vector and k vector are parallel. The velocity of this wave is c by square root of K y, independent of psi.

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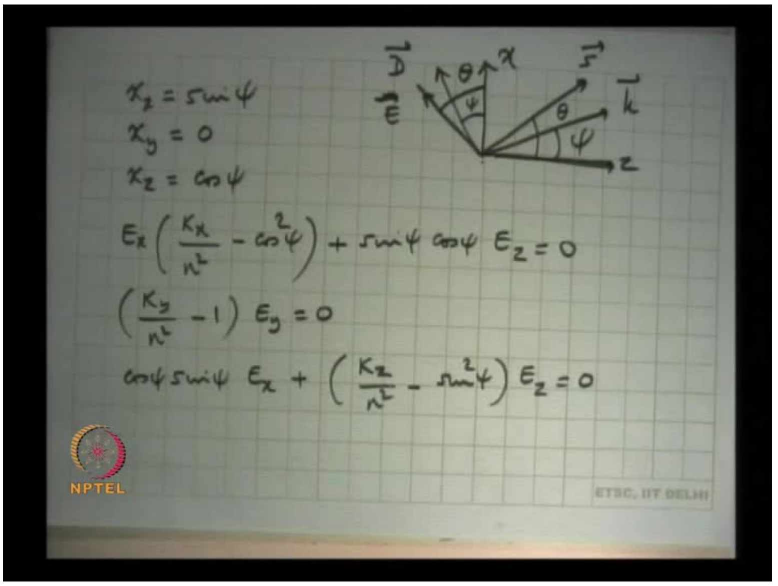

The second solution has its D vector perpendicular to k vector; and, E and D are not parallel to each other. So, S and k are not parallel to each other. But, the wave has the refractive index now, (Refer Slide Time: 45:31) which depends on the value of psi. Since you changed psi, the velocity of this wave will keep changing. If you propagate with psi is equal to 0, that means, if you propagate along the z axis, (Refer Slide Time: 45:44) then n will be square root of K x (Refer Slide Time: 45:49). If you propagate with psi is equal to pi by 2, that means along the x direction, n will be square root of K z. These are the same results we obtained in the special cases when you propagate along the principal axis direction.

But, in general, if you propagate in the xz plane at some arbitrary direction psi, then this particular polarization state, this D vector direction, which is perpendicular to this k vector direction, has its refractive index changing as you change the value of psi. And, that is the typical of anisotropic media that… Please remember, this solution has a psi dependent refractive index; the other solution has a refractive index, which does not depend on psi. So, in anisotropic media, again, for every propagation direction, there will be two orthogonal linearly polarized waves, which will propagate as modes of propagation. The refractive indices of these two modes will depend on K x, K y, K z and the angles of propagation.

Now, in the course that we will discuss, we will be most of the time looking at uniaxial media. Please recall, uniaxial means K x is equal to K y is not equal to K z. Because of the symmetry between x and y directions, if you give me an arbitrary direction of propagation, I will always orient the x-axis such that the propagation direction lies in the xz plane. Please remember, in uniaxial media, x and y are symmetric completely, because K x is equal to K y. So, if I choose another axis in the xy plane, which is oriented with original x and y-axis, there will be no change in dielectric constant tensor. So, if you give me a propagation direction, if this is the z-axis, then I will choose the x-axis such that the xz plane contains the propagation direction and the problem that we have solved is the more general situation. So, in uniaxial media, because there is only one special (Refer Slide Time: 48:04) direction z, K x and K y are equal; there is K z. So, if you give me a uniaxial medium with these three principle axis directions and if I propagate like this, and this is z axis, I will rotate the x-axis, so that the k vector, x-axis and z-axis lie in the same plane.

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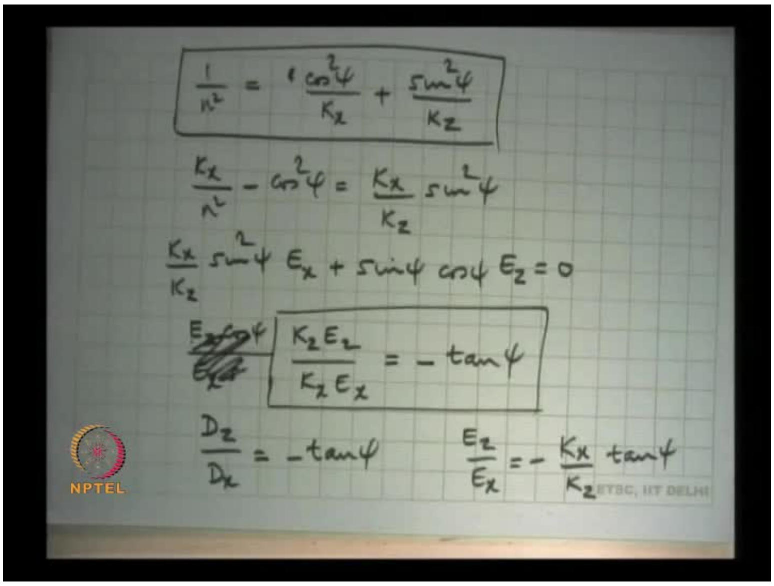

Once I do that, the solutions are one electric vector along y; the (Refer Slide Time: 48:28) corresponding refractive index given by this. The other electric vector in the xz plane, the D vector perpendicular to k vector (Refer Slide Time: 48:37); and, the corresponding refractive index is given by this equation (Refer Slide Time: 48:40). So, I need to know only the angle psi with the z-axis.

Now, what is the refractive index seen by this wave (Refer Slide Time: 48:48) when psi is equal to 0? Cos is 1, sine is 0. So, I get n is equal to square root of K x. And, what was the refractive index seen by the other wave? Square root of K y. And, because K x is equal to K y, the two refractive indices are equal. So, if I propagate in a uniaxial medium along the z direction, the two eigenmodes have the same speed, have the same velocity. And, this axis, z direction is called the optic axis of the crystal. And, there is only one such direction in uniaxial medium and that is why it is called uniaxial.

If you have K x not equal to K y not equal to K z, one can show there are two directions in which the two eigenmodes will have the same speed. And so, it is called biaxial. There are two optic axes in biaxial media; there is one optic axis in a uniaxial medium. And, in a uniaxial medium, by definition, the z axis is the optic axis. An optic axis means, if you propagate along the z direction, the two eigenmodes have the same speed. So, next class, what I will do is little more details, little more analysis or discussion of anisotropic media. We will discuss, look at the velocities of propagation and the polarization states. So, in the most non-linear interactions, we will see, we use the fact that you have now a medium, in which you can tailor the refractive index by choosing an orient propagation direction. In isotropic media, you are only given one refractive index. Here it is possible to choose a direction of propagation, so that you can vary the refractive index depending on the direction of propagation and this will become very helpful in non-linear interactions.

So, do you have any questions?

Sir, in uniaxial medium, K x is equal to K y; K z is 0?

No, in uniaxial medium, please differentiate K and kappa. K x is equal to K y is not equal to K z; all three are non-zero. The three diagonal elements of the dielectric constant tensor – K x, K y, K z are the three diagonal elements of the dielectric constant tensor. And, in the principle axis system, the matrix is diagonal. So, kappa x, kappa y, kappa z are the three unit vector components of the propagation direction. So, they are different. So, if you propagate along the z axis in a uniaxial medium, the two refractive indices, the two eigen polarization states, will have the same speed, same velocity.

Yes?

Sir, what will happen to the intensity of light?

What will happen to the intensity of light? Intensity of light does not change, because we are not looking at the absorption or reflection in the picture here. So, light will propagate as such. The polarization state will change; the two eigen components will remain the same; but, the phase difference will keep changing with propagation. If the phase difference changes, that does not change the intensity. If I take a string and make it vibrate in multiple modes and if there is no damping in the string, the total energy contained in the string remains constant with time. It is just the interference between the modes, which is creating the change of shape of the string; there is nothing else.