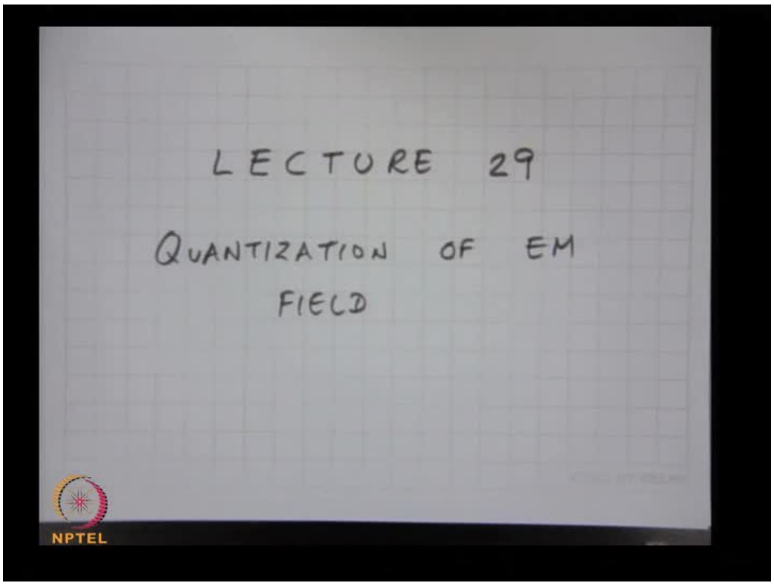
Quantum Electronics
Prof. K. Thyagarajan
Department of Physics
Indian Institute of Technology, Delhi

Module No. # 05
Lecture No. # 29
Quantization of EM Field (Contd.)

We will continue with our discussion on quantization of electromagnetic fields. So, do you have any questions of the derivation and discuss that we had last class.

(Refer Slide Time: 00:43)

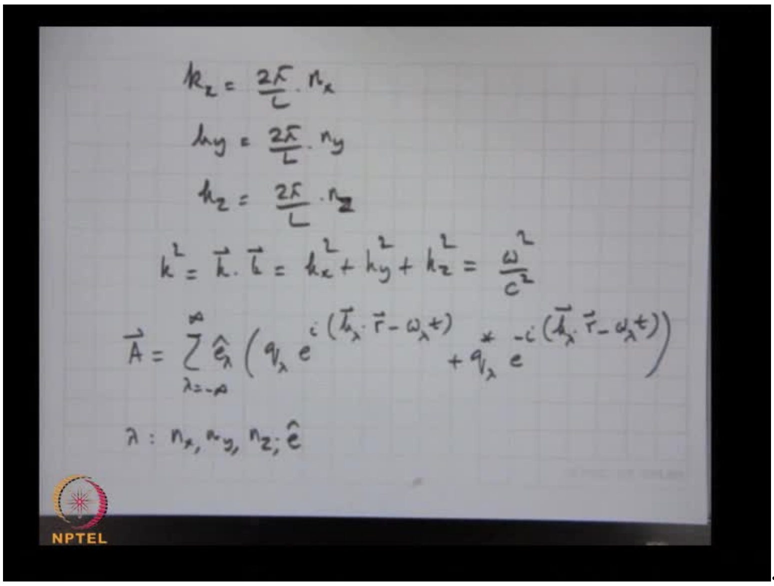


Let me recall, what we are trying to do is starting from Maxwell's equations deriving an expression for the Hamiltonian, the total energy of the electromagnetic field and what we did was we wrote the equation in terms of vector potential. The wave equation in terms of vector potential because knowing the vector potential I can calculate the electric field which is minus del i by del t and the magnetic field which is del cross k. So, if I get the solution in terms of vector potential, I can calculate the electric field, magnetic field, pointing vector, energy density everything. We wrote on the wave equation for electro of the vector potential and we use separation of variables technique to get solutions in terms of plane waves. Exponential i k dot r minus omega t, now in normal free space all frequencies are allowed, all direction of propagations are allowed. So, the total electromagnetic field would be an integral over all possible solutions. Now, instead of dealing with integrals it is much easier to deal with summations and at the end we will lead the space got infinity, what we did was we said that let me consider a cubic volume of side l and we apply what are called as periodic boundary conditions. That means, I say that the field on the plane x is equal to 0 must be equal to the field on the plane x equal to l, it does not tell me that the field is 0 its whatever the field at x is equal to l it is the same as field that x equal to 0. Similarly, for the y coordinates and similarly, for the z coordinates.

I have a volume of space of cube l which is such that it repeats again and again. So, the field on the plane x equal to l and x equal to 0 are the exactly the same and similarly, for the y and z coordinates, and this is called periodic boundary condition and this does not restrict the field to be within this volume. The field is going outside, also the fields are travelling waves all I am requiring it the field here and must be the same. This is difference from a condition which people can use in which I say that I consider an enclosed cavity of side l and I say that the field is completely confined within this cavity. Then I applied boundary conditions saying that the field here has to be 0 will give me another set of modes called standing wave modes and these are modes which called propagating wave modes. Because, these are solutions in terms of propagating waves and the solution, if you have a box which has perfect reflecting walls that will give you standing wave solutions within the cavity.

We are using the propagating wave solutions and we say that the field at x is equal to 0 and x is equal to l must be the same. Similarly, for y and z and what we find is because of this condition the frequencies get discretized the k x, k y, k z which are the components of the k vector gets discretized and we get a discrete spectrum rather than a continuous spectrum. So, remember we found that we had the k x must be equal to 2 pi by l into n x.

(Refer Slide Time: 03:51)

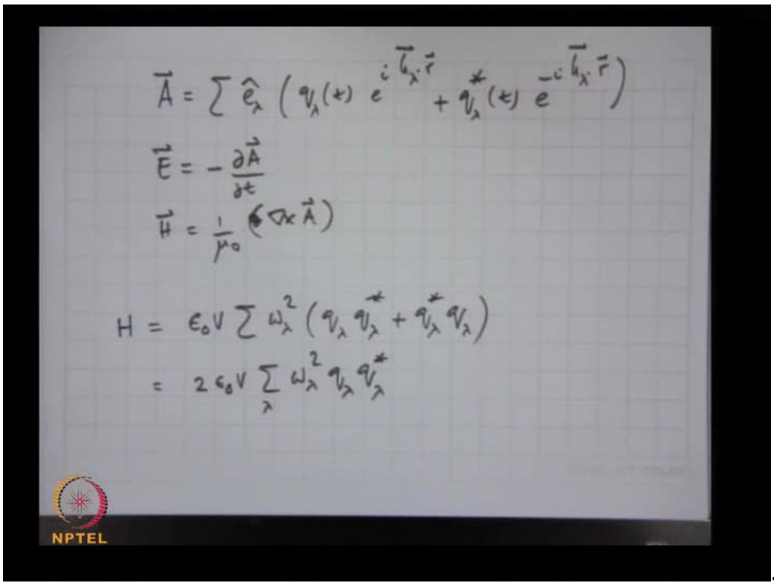


The propagation constant is k dot k which is equal to k x square plus k y square plus k z square is equal to omega square by c square and because k x, k y, k z are discretized, omega is also discretized not all frequencies are possible only those frequencies which are satisfying these conditions are allowed.

The modes now consist, these are different modes of the electromagnetic field with these boundary conditions and the total solution is the sum of all the possible modes. We wrote for example, the vector potential as sigma e lambda. Lambda is equal to minus infinity to plus infinity into q lambda, exponential i k lambda dot r minus omega lambda t plus q lambda star exponential minus i k lambda r dot r minus omega lambda t and lambda is a subscript which is a short form for all these n x, n y, n z and also the polarization states. One mod is characterized by one value of n x, n y, n z and one polarization state because divergence a has to be 0 k dot a is 0. So, for a given vector direction k lambda I can have two independent polarization states which we can choose to linearly polarized states or to circularly polarized states.

That is in my choice here for example, I can choose linearly polarized states. So, if my propagation direction is like this I can have one polarization like this another polarization like this. These are 2 independent modes. So, for every value of a set n x, n y, n z and e i have a mod, these modes are orthonormal to each other. They are independent, they are orthonormal, they are orthogonal that means they are, was I showed you then integral between 2 different values of lambda corresponding to a vector are 0.

(Refer Slide Time: 06:44)



Actually, I can also write we will later on also be able to write like this, in this form in which i absorbs the time dependence within the cube. So, i will, i can also write like this e lambda q lambda of time exponential i k lambda dot r plus q lambda star of time exponential minus i k lambda dot r. Where, i have absorbed the time dependence a time dependence is contained in this coefficient q lambda it has amplitude and the time dependence the amplitude is just the q lambda and there is a time dependence which is classically exponential minus i omega lambda t and exponential i omega lambda t.

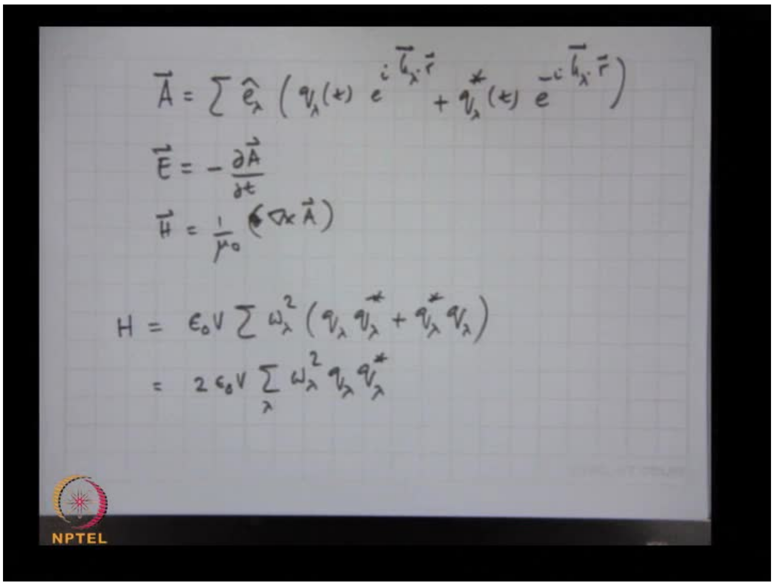
Because, later on we will see that in Heisenberg picture the operators will be time dependent and these actually become operators, and they will carry time dependence. The Schrodinger picture will correspond to by just putting t is equal to 0 and time independent operator they will becomes, I will make this clear as we proceed. Once you have got a vector we can actually calculate e vector minus del a by del t and h vector 1 by mu 0, curl a.

We did this calculation last time and finally, we obtain an expression for the Hamiltonian, the total energy please note this is an H without the vector sign this is the magnetic field here. This is the total energy Hamiltonian which we remember we had got this expression omega lambda square, q lambda, q lambda star plus q lambda star, q lambda. Which is actually in classical, there is no problem in classical. In the classical picture I can write this as omega lambda square q lambda q lambda star.

Remember what we did last time we calculated the electric field, calculated the magnetic field then calculated the total energy which is integral over the volume of half epsilon 0 e dot e plus half mu 0 H dot H. I left those simplifications to you, I have to use the orthogonal condition between different exponential factors here, and you will find that finally, the expression for the Hamiltonian reduces to this quadrant.

Now, what I want to do is, this is still classical please remember this discrete values of omega. I am getting, is a classical because of a classical boundary conditions

(Refer Slide Time: 06:44)



There is no quantization here. Yeah, we are still in completely classical picture. So, if I make the length go to infinity the spectrum will become continuous because remember these are different values of k x differ by 2 pi by l and as i have tends to infinity these will become continue all value of k x, k y, k z will be allowed all values of frequencies are allowed etcetera. These different values of lambda, this summation is coming because of the boundary condition periodic. Boundary conditions we have applied and this is a sum of the terms like omega lambda square q lambda q lambda star. Now, I want to quantize this problem for that I need to write the Hamiltonian in terms of canonical conjugates conjugate variables.

Just like we did for the harmonic oscillator you have to write it in the form which satisfy in which I find pairs of canonically conjugate variables which satisfy the Hamilton's equations of motion.

Yeah

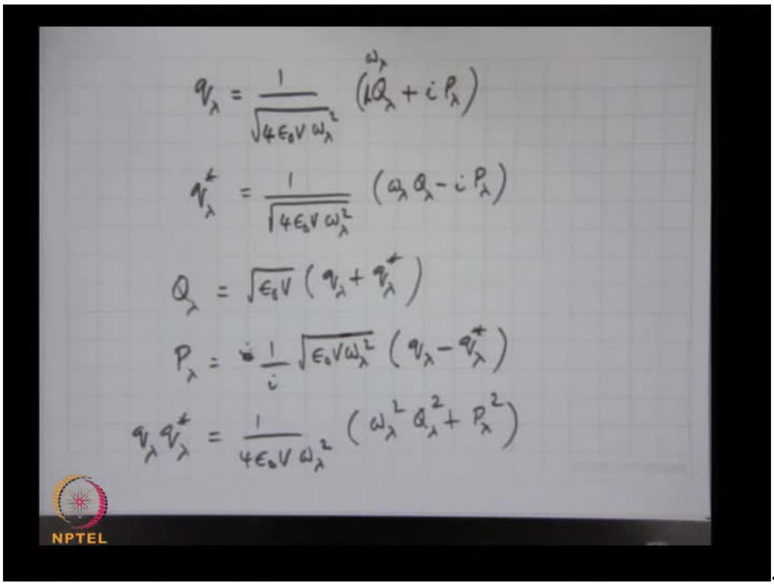
Sir, what are exactly the conjugate variables I mean whose commutation is non zero, no it satisfies the Hamilton's equations of motion which satisfies the Hamilton's equations of motion that is a classical picture, classical mechanics and the poser bracket I can calculate the poser bracket in classical mechanics and I replace the poser bracket by commutator bracket to quantize.

(( )) i H cross

Yes. So, this again will become clear because I want to put this in a form which looks like a sum of harmonic oscillator kind of value energy Hamiltonians

Now, there is a little bit of algebra very simple algebra because to convert this into that form I need to introduce some change of variables and these are simply just classically equation term. So, I want to write instead of q lambda i, I introduce 2 new variables.

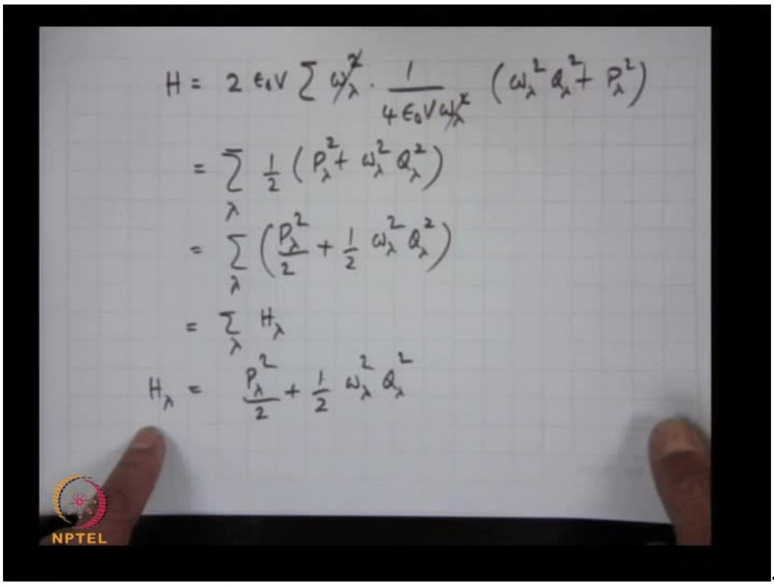
(Refer Slide Time: 11:48)



Q lambda is equal to 1 by square root of 4 epsilon 0 v omega lambda square q lambda plus i p lambda. Sorry, omega lambda, omega lambda, omega lambda please note its looks like n omega cube plus i p which we are done for the harmonic oscillator case. So, q lambda star is equal to 1 by square root of 4 epsilon 0 v omega lambda square omega lambda q lambda minus i p lambda. I am assuming q lambda and p lambda are real quantities. So, i am writing q lambda as a sum of real and imaginary parts with some multiplicative constants which will actually simplify my expression later on and bring it to a form which looks like that for a harmonic oscillator.

So, if I now multiply these 2 actually I can invert these 2 equations and calculate q lambda is equal to square root of epsilon 0 v into q lambda plus q lambda star and p lambda is equal to i sorry, 1 by i times square root of epsilon 0 v omega lambda square into q lambda minus q lambda star. Just inverting these equations I can get the capital q lambda and capital p lambda in terms of q lambda and q lambda star and these are as you can see here these are real because these are real imaginary parts of q lambda here. What is q lambda, q lambda star, q lambda, q lambda star is equal to 1 by 4 epsilon 0 v omega lambda square into omega lambda square q lambda square plus p lambda square. So, you just multiply q lambda, q lambda star I get 1 by 4, 4 epsilon 0 v omega lambda square omega lambda q lambda whole square plus p lambda square.

(Refer Slide Time: 14:17)

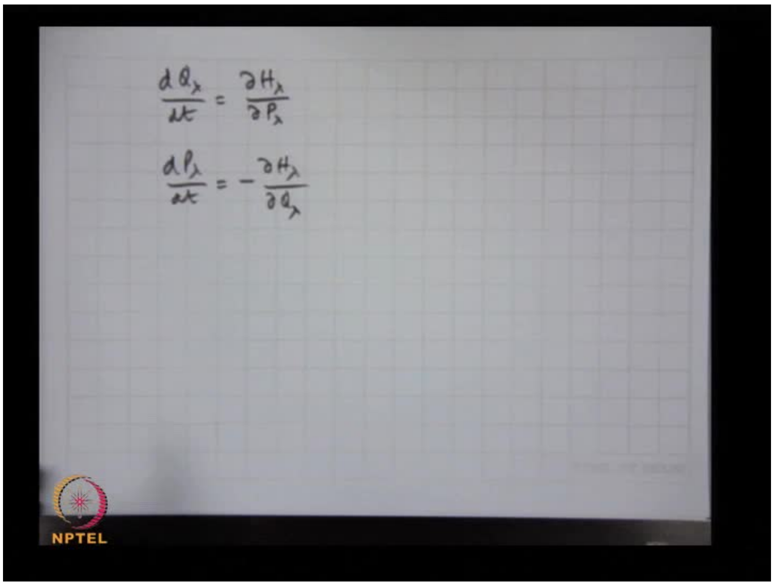


I can write the Hamiltonian, now replace q lambda, q lambda star by this and the Hamiltonian becomes H is equal to 2 epsilon 0 v sigma omega lambda square into 1 by 4 epsilon 0 v omega lambda square into omega lambda square Q lambda square plus P lambda square which is equal to sigma. So, omega lambda square goes off, I get half P lambda square plus. This looks like, this is like this now P lambda square by 2 plus half what is this exactly like harmonic oscillator Hamiltonian with m is equal to 1 p square by 2 m plus half m omega square x square.

By defining 2 new quantities Q lambda and P lambda what I have done is I have expressed the total energy of the electromagnetic field in that volume by in terms of an expression which looks like the harmonic oscillator Hamiltonian. So, in this picture I can consider the electromagnetic field to be made up of an infinite number of harmonic oscillators, each harmonic oscillator corresponds to one particular mod of propagation of that state plane wave.

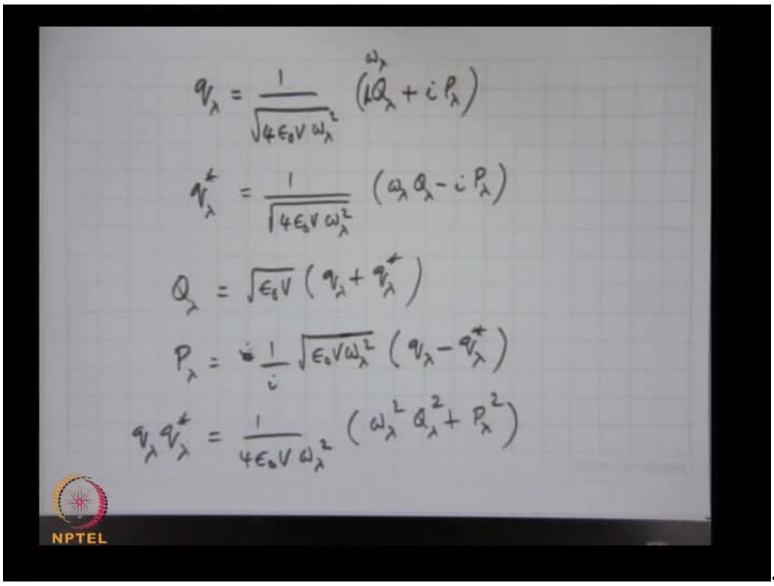
For every value of lambda I have a harmonic oscillator each mod for each value of lambda corresponds to different mod. So, each mod acts as if it is the harmonic oscillator there is nothing oscillating, there is no oscillation, there is no mechanical oscillator but, by analogy formal analogy its looks like that the electromagnetic field can be considered to be a superposition of an infinite number of harmonic oscillators. And the energy of each harmonic oscillator is half P lambda square plus half Q lambda square omega lambda square. So, I can write this as actually H lambda where H lambda is the Hamiltonian for the lambda mod. Now, I must make sure I would not, I will show you that Q lambda and P lambda are canonically conjugate variables and how do I show that I check whether this H lambda satisfies the Hamilton's equation of motion with Q lambda and P lambda as canonically conjugate variables.

(Refer Slide Time: 17:23)



So, what do I do, I find out I substitute this H lambda into the Hamilton's equation. So, what is the Hamilton's equation d Q lambda by d t is equal to del H lambda by del P lambda d Q by d t is equal to del H by del P and the other equation is d P lambda by d t your minus del H lambda by del Q lambda.

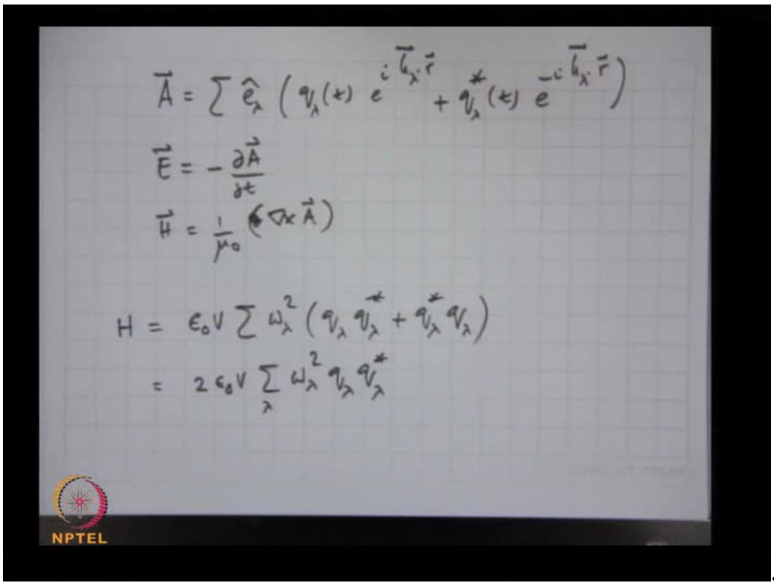
(Refer Slide Time: 17:54)



.

Let me check the first equation. So, i have here is the expression for q lambda now I am assuming these operators are time dependent I am not written the time dependence part here these operators are all time dependent in the Heisenberg picture.

(Refer Slide Time: 18:20)

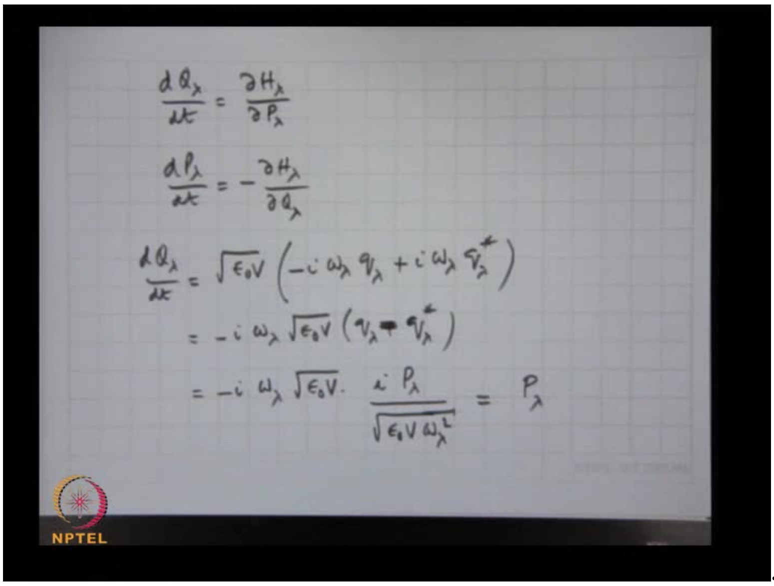


.

The Hamiltonian is independent of time because it is q lambda q lambda star if you would write the time dependence here exponential minus i omega t and the exponential plus i omega t it is time independent. So, these operators are time dependent in the

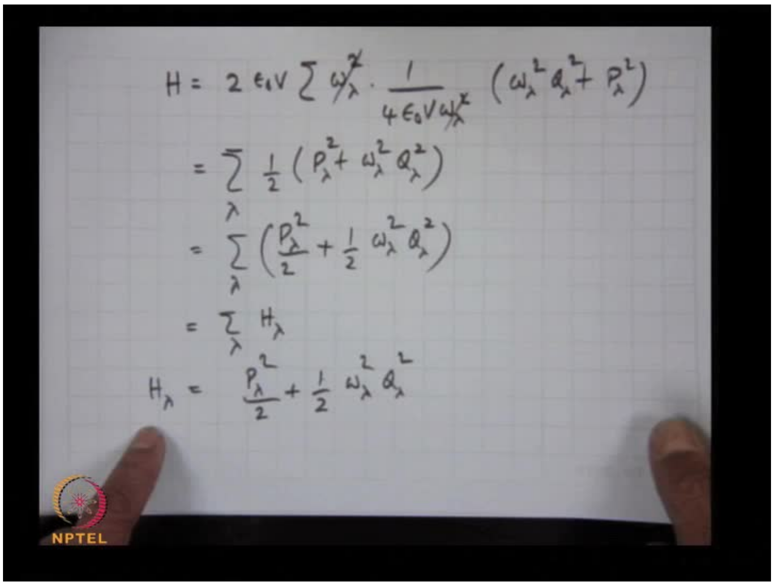
Schrodinger picture all these operators are time independent in the Heisenberg picture. All these operators are time dependent. Well, this is they will become operators right now they are just classical variables but, q lambda will contain the time dependence. So, what is d Q lambda by d t if I use this from here this Q lambda I will have d Q lambda by d t is equal to under root of epsilon 0 V.

(Refer Slide Time: 18:50)



I have d Q lambda by d t plus d Q lambda star by d t. So, d Q lambda by d t is minus i omega lambda Q lambda plus i omega lambda Q lambda star this is d Q lambda by d t plus d Q lambda star by d t. So, this is equal to minus i omega lambda square root of epsilon 0 V q lambda plus minus q lambda star. And q lambda minus q lambda star can be written in terms of p lambda. So, if I substitute for q lambda minus q lambda star this is minus i omega lambda epsilon 0 V into i p lambda divided by epsilon 0 V omega lambda square.

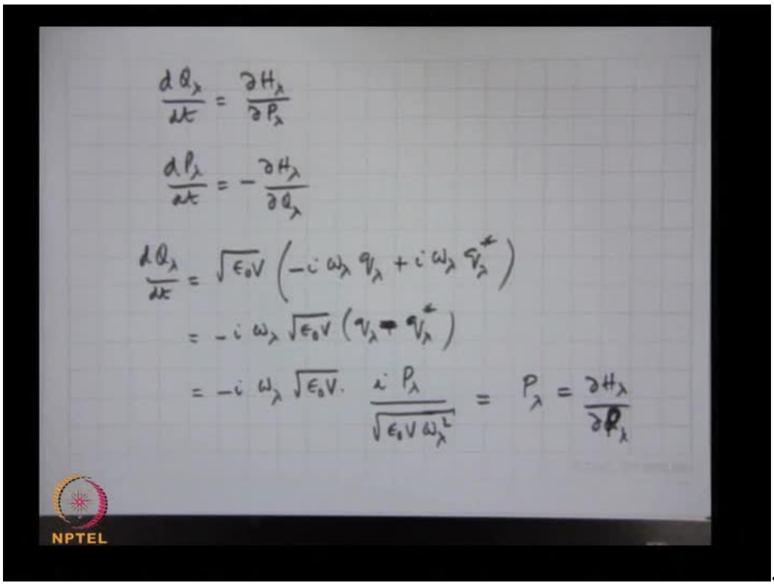
(Refer Slide Time: 20:24)



.

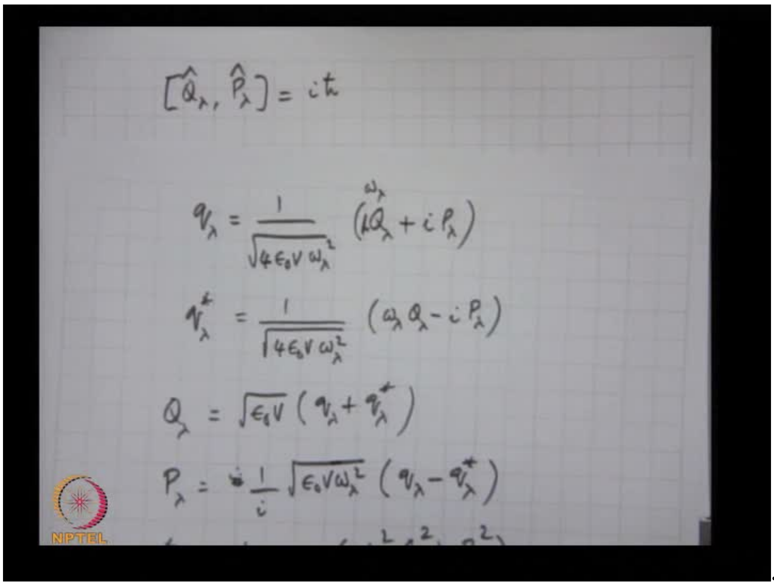
This is equal to minus i into i is plus 1 omega lambda cancels off, square root of epsilon 0 v cancels. Now I get p lambda, d Q lambda by d t is p lambda and what is del H lambda by del p lambda, with this lambda it is here is expression for del H del H lambda by del P lambda is P lambda. So, this is equal to del P lambda.

(Refer Slide Time: 20:32)



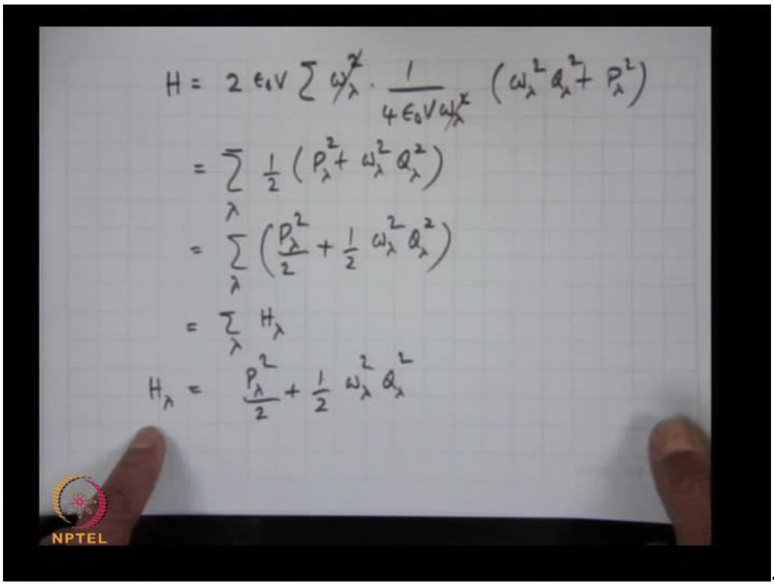
That is the first equation. Similarly, I leave it to you to show that this H lambda expressed in terms of Q lambda and P lambda satisfy the second Hamilton's equation implying that Q lambda and P lambda are canonically conjugate variables.

(Refer Slide Time: 21:25)



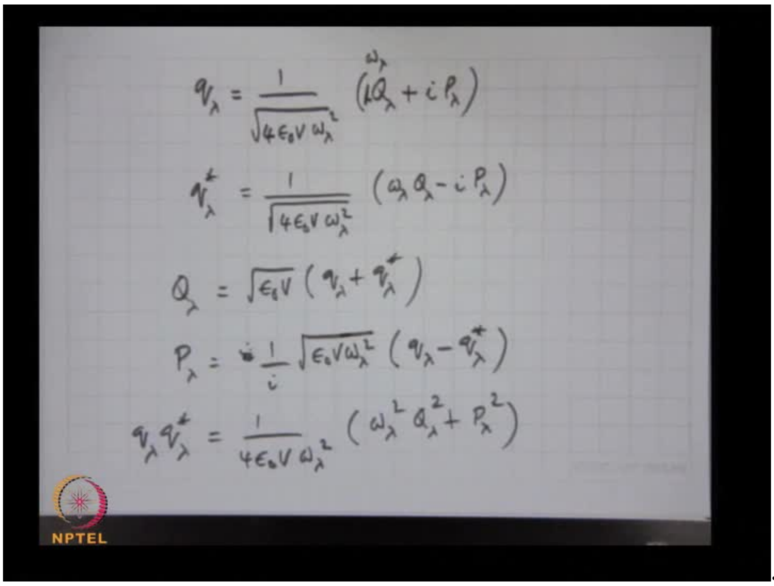
I have expressed this in terms of canonically conjugate variables and now I can quantize, and quantization means that I will now write a commutation relation between Q lambda and P lambda. So, I will say the commutator Q lambda P lambda is equal to i H cross and these are now operators. Q lambda P lambda where some classical functions having amplitude and time dependence with those phase dependence is contains an exponential minus i k dot r and i k dot r. There is a time dependence and amplitude here. So, till now there was no operator is all classical functions now I say the quantization means having we expressed the total Hamiltonian in terms of canonically conjugate variables.

(Refer Slide Time: 22:11)



That will show that the total Hamiltonian of the electromagnetic field consists of a sum of infinite number of independent harmonic oscillators each lambda corresponds to one harmonic oscillator. So, it is an infinite sum of harmonic oscillators of different modes The Hamiltonian of each mod is given by this and classically this is the equation I have shown you that Q lambda and P lambda are conjugate variables. So, I can quantize by expressing a commutation relation between Q lambda and P lambda, and I say Q lambda is an operator capital P lambda is an operator small q lambda is an operator small q lambda star everything is an operator. Now, it is no more star, it is a dagger operator.

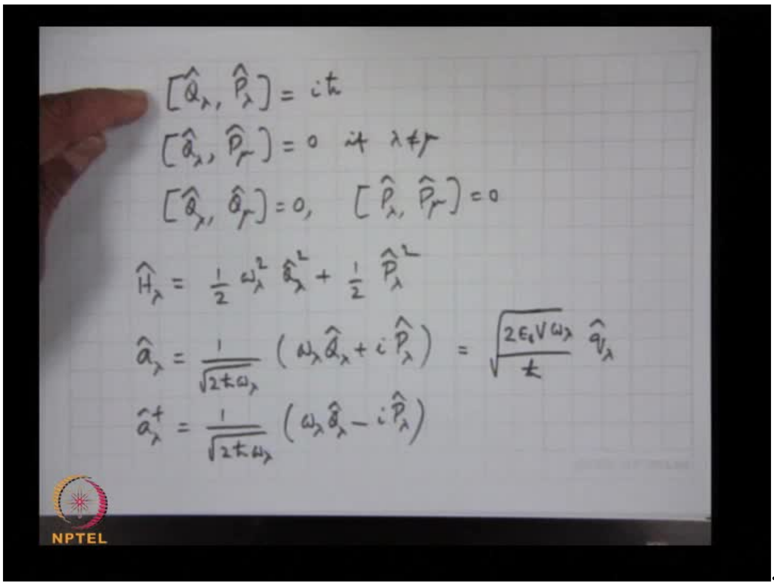
(Refer Slide Time: 23:08)



.

Hamiltonian becomes an operator vector potential becomes an operator because remember Q lambda is in terms of small q lambda and small q lambda if these 2 are operators this is an operator if this is an operator all these are operators and a vector becomes an operator.

(Refer Slide Time: 23:36)



.

Because, a vector is written in terms of Q lambda and Q lambda star. So, the vector potential becomes an operator electric field becomes an operator magnetic field becomes an operator all are operators. Now, all observables becomes operators and when I write this equation I have actually quantized and because I know that these modes are independent Q lambda P mu is equal to 0 if lambda is not equal to mu take a mu the Q operator and P operator corresponding to 2 different modes commute with each other and of course, you have Q lambda Q mu is equal to 0. And Q lambda Q lambda is also 0 right there is the commute in the standard harmonic oscillator problem you studied one harmonic oscillator which is defined by position momentum coordinates. So, a position operator, momentum operator, Hamiltonian operator etcetera

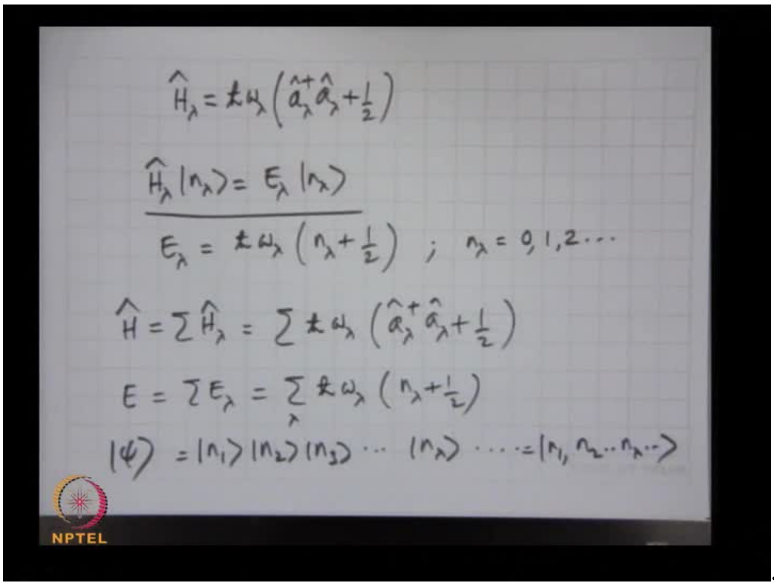
Here, we have now an infinite number of harmonic oscillators, decoupled harmonic oscillators they are all independent harmonic oscillators, each mod is a harmonic each has like a harmonic oscillator splits hormonology between harmonic oscillators and the electromagnetic field and the operators representing the harmonic oscillators are Q lambda and P lambda. Which are which will give me what is the corresponding Hamiltonian, which will give me the corresponding electric field operator? All these operators will depend on Q lambda and P lambda operators. So, the lambda at mod is now an operator. This is given by half omega lambda square q lambda square plus half.

Now, what it I do for harmonic oscillator, we calculated the eigen values of the hermitian of the Hamiltonian. We solve the equation the eigen value equation for the Hamiltonian and found out the various energy eigen values and eigen states. We can do exactly the same thing now, for every mod I can find out the eigen values and eigen states. For example, just like we introduce 2 new operators a and a dagger we can do the same thing here. So, I will introduce a lambda operator is 1 by under root 2 H cross omega lambda into omega lambda Q lambda plus i P lambda.

And, a lambda dagger is equal to 1 by square root of 2 H cross omega lambda into omega lambda Q lambda minus i P lambda Q lambda and P lambda are hermitian operators just like position, and momentum operators for the harmonic oscillator and a lambda and a lambda dagger they are not hermitian because a lambda dagger is not equal to a lambda. They are different operators and just like we did for the normal harmonic oscillator problem. I can introduce 2 new operators in fact these are related to a circular fashion back to Q lambda because see Q lambda is also written as omega lambda plus i P lambda classically.

This is also omega lambda Q lambda plus i P lambda. So, if you substitute you can actually show this is same as square root of 2 epsilon 0 v omega lambda by H bar into Q lambda where now Q lambda has become operators q lambda is related to Q lambda and P lambda. These 2 are operators, this is also an operator and because of the way I have defined I am introduce some extra factors here just like we did for the harmonic oscillator to get an expression for the Hamiltonian in terms of H cross omega.

(Refer Slide Time: 28:35)



We have the 2 operators the annihilation and the creation operators, and they are defined like this. The Hamiltonian if I substitute this, if you substitute this and use the commutation relation what you will get is essentially what will be the operator a dagger a plus half into the Hamiltonian operator there where H cross omega a dagger a plus half and to get this expression you need to use this commutation relation.

In fact the procedure is exactly the same only thing is now you have a subscript lambda here because you are looking at the lambda th mod of electromagnetic wave of the electromagnetic field. So, you can actually solve the eigen value equation for H lambda is equal to and E lambda will be H cross omega lambda into n lambda plus half. E lambdas are the energies of the lambda th mod. This is now quantized it is now coming because of the quantization of the problem what it tells me is the lambda th mod of the electromagnetic wave cannot have an arbitrary energy its energy has to be n plus half into H cross omega of that mode. So, if I take an electromagnetic wave at certain frequencies say 10 to power 14 hertz that electromagnetic wave cannot have an arbitrary energy its energy has to be H cross into omega of that frequency multiply by n plus half it gets discretized now and this is coming because of my quantization of the problem. Yes, mohith.

Sir, (( )) us towards the quantization of energy values. So,

Of the lambda th mod here.

Yeah

Yeah

(( )) So, can this also treated as quantization of the omega.

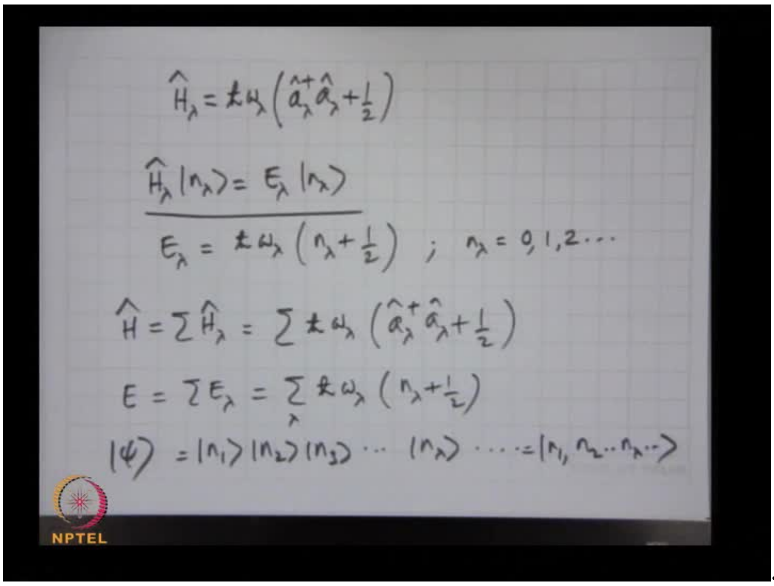
No

This

Omega is not quantized because the omega is discretized because we have to chosen periodic boundary conditions. If I extend the space to infinite then omega becomes continuous, all frequencies are allowed if there is no actual physical cavity if I really consider a laser cavity for example, then omega gets discretized also because of a classical boundary condition it is not quantized. If I consider laser cavity, what is actually happening is that this cavity will have certain allowed modes if you take one of those modes classically that mod can have any energy but, that mod can only have in multiples of H cross omega odd multiples of H cross omega because of n lambda plus half.

This is where the quantization comes in see if i take 1, i take a cavity or I will take electromagnetic wave at one frequency classically that electromagnetic wave can have an arbitrary energy. This tells me that no the energies can only be n plus half H cross omega of that mod it also tells me that you can never have 0 energy of that mode.

(Refer Slide Time: 28:35)



That mod is always there that there is energy in that mod always which is the 0 point energy like an harmonic oscillator the harmonic oscillator can never come to rest because if it comes to rest x 0 p 0 you know x and p precisely and you do not satisfy the uncertainty relation similarly, here what is happening is because of this quantization you cannot have a situation where the energy of any particular mod is 0 the minimum energy is half h cross omega. It looks like that there are quantum, the energy can only change energy of that electromagnetic field of that mod can only change in packets of H cross omega which I call photons, please note that photon is not at one point it is everywhere because this field is supposed to be everywhere it is a plane wave propagating. And that the way I have defined this, I have just given a name instead of saying that energy can only be n plus half H cross omega, I say that there are n photons in that mod when I say there are n photons in that mod all I mean is the energy of the mod is n plus half H cross omega.

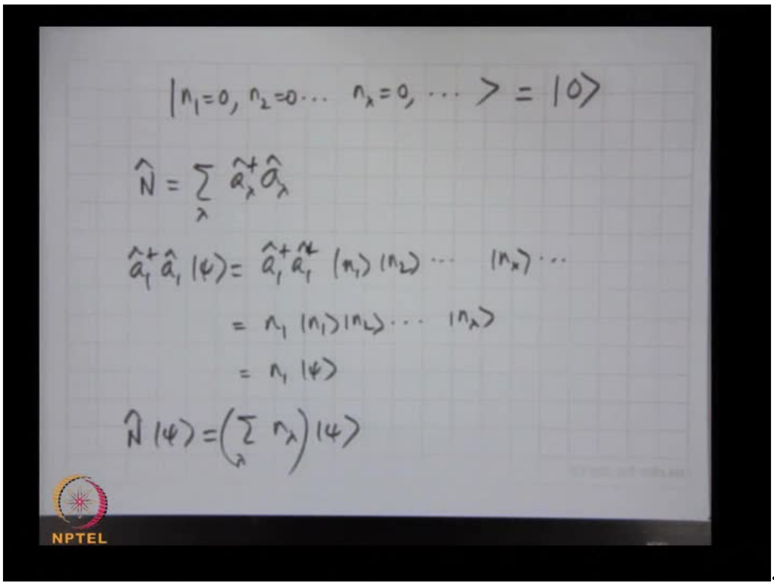
And that mod that energy of that photon is everywhere, a single this is a photon corresponding to a single frequency that I have taken because I have come to more interesting aspects of the photon picture. But, right now what I have shown is that each mod of the electromagnetic wave can only have energies given by H cross omega lambda n lambda plus half. The total Hamiltonian will be sigma H lambda which is actually sigma H cross omega lambda a lambda dagger a lambda plus half and what will be the total energy sigma E lambda this is sum over lambda. Lambda goes to 0 1 2 3 4 minus also there are all possible each lambda corresponds to a different mod and there are infinite number of modes each mod has an energy H cross omega lambda into n lambda plus half.

I will say that this electromagnetic field which has this energy has n 1 photons in omega 1 frequency mod n 2 photons in omega 2 frequency mod n 3 photons in omega 3 frequency mod etcetera. The total energy is given by this and the each of those modes is represented by a different number ket this n lambda which we obtain here. This eigen value equation which I wrote for the Hamiltonian operator where did it go. What does the equation satisfy here it is, yeah this equation. So, for each lambda mode there is an eigen ket which is called the number ket because it gives you the number of photons or number of the excitation state of that mode.

So, this state corresponds to a ket which is n 1 photons in mode omega 1 n 2 photons in mode 2 n 3 photons in mode 3 like this. So, what it implies is H 1 into this will give me n 1 times this ket that means this ket corresponds to a state in which there are the energy of the state is n 1 plus half H cross omega 1 plus n 2 plus half H cross omega 2 plus n 3 plus half H cross omega 3 plus n lambda plus half H cross omega lambda plus etcetera, infinite.

There are n 1 photons in mode 1, n 2 photons in mode 2, n 3 photons in mode 3 by photons I mean the number the excitation state of that electromagnetic field. I could have a situation for example, that there are no photons in any electromagnetic field in any electromagnetic mode. What will that corresponds to that will corresponds to a state. So, this is usually written as actually this is shorter form of this essentially stating that this is a product of these kets corresponding to mod 1, mod 2, mod 3, mod 4 etcetera.

(Refer Slide Time: 38:43)



If what is the lowest energy state, what is the lowest state possible when all n lambdas are 0 that is the lowest energy state and that corresponds to a state which will look like this n 1 is equal to 0, n 2 is equal to 0, n lambda is equal to 0 which is usually written as 0 ket. It is like the 0 ket for the harmonic oscillator where the excitation state was it was a ground state of the harmonic oscillator. This is the ground state of the total electromagnetic field that for all the modes in the lowest energy state that is possible. Yes mohith

(( ))

No. Number operator, you have to be what is the number operator now

A dagger lambda

Yeah. This is only the number operator for the lambda th mod. So, I have sum over all the lambdas.

So, for example, if I operate for example, if I operate on this state let me operate a 1 dagger a 1 into psi is equal to a 1 dagger a 1 sorry into n 1, n 2, n lambda. So, this operates only on the n 1 ket because this is not operates on the other kets, i will get n 1. So, operating with a dagger a 1 gives me the number of photons or the number of excitation state of the mod 1.

There will be equal operator of the (( ))

Correct. So, what will be the total? Right? Because, a 1 dagger a 1 be the operator of n 1 ket a 2 dagger a 2 will operator on n 2 ket a 3 dagger a 3 will be operator on n 3 ket.

Each of this is an operator number, operator corresponding to mod 1 that number operator operates only on the ket corresponding to mod 1 and that gives me the number of photons or a number of the excitation state of mod 1.

This n contains a sum of a 1 dagger a 1 plus a 2 dagger a 2 plus etcetera infinite number. Each one of those numbers operators operates on corresponding eigen ket and gives me a certain number n lambda. So, if you substitute here and use this fact that this particular operator operates only on the corresponding ket what you will see is the total number of photon that is present in the state the total excitation state of this state of that state.

This state which is the lowest possible energy state is called radiation vacuum. It is not vacuum in terms of no particles present it is a state which is the lowest energy state of the electromagnetic field which means that it mean, it classically there is no electromagnetic field there. But, quantum mechanically please note this has infinite energy because the energy of each mod is half H cross lambda as half H cross omega half H cross omega 1 plus half H cross omega 2 plus half H cross omega 3 etcetera. So, this gives you infinite 0 point energy actually there is a problem it may look as if i can shift my scale and get rid of this so most of the time we can do it but, this 0 point energy has real effects.

The fluctuations in the 0 point energy are responsible for spontaneous emission light from this lamp the fluctuation of this are responsible for spontaneous parametric down conversion. There are more interesting effects, there is an effect called Casimir’s effect. If you take 2 neutral surfaces they attract each other because of this vacuum field. This is measured compared with quantum mechanical predictions perfect match. This is not an artifact it is a real 0 point energy of fluctuations and we will see that this is responsible for the spontaneous parametric down conversion. That we have actually discussed little earlier without really from the classical equation. We could not predict the existence of spontaneous emission, spontaneous parametric down conversion. So, this is called radiation vacuum and there are no all the modes of the electromagnetic field are in there ground states. Yes.

(( ))

There is no 0 point energy in classical. No.

(( ))

No. This is quantum mechanical.

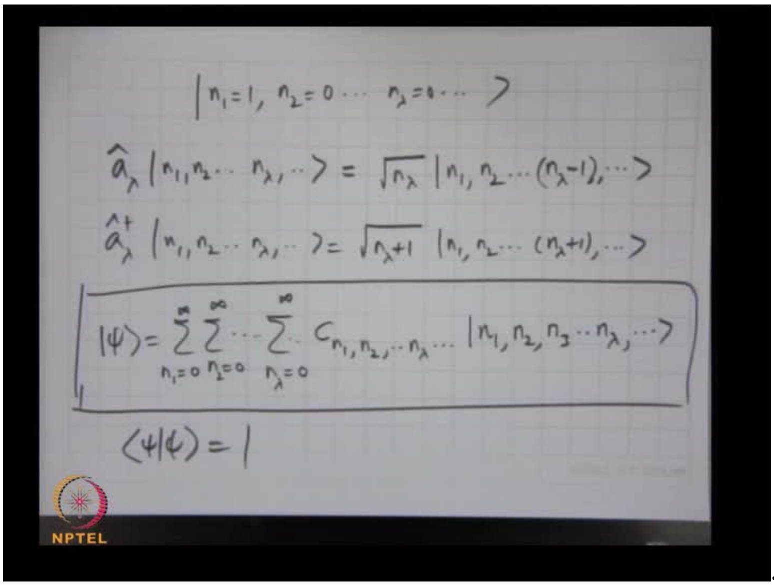
Most of (( )) energy is infinite.

0 point energy but, I cannot draw this energy because I cannot go below half H cross omega. So, I cannot use this energy if there was 3 by 2 H cross omega I can take out H cross omega from there.

But, if it is half H cross omega energy I cannot do anything that is the lowest possible state. So, I cannot draw energy from here but, does not mean in it, does not exist because if I create a field in that state I will show you that electric field there is a fluctuation electric field you can see the fluctuations.

So, if you do measurement you will see a measured value. So, this radiation vacuum is the lowest possible energy that you can go to when you have that means it is a state of electromagnetic field where there is no none of the electromagnetic wave modes has any classically excite you are not excited any of the modes. Vacuum in the sense that it is called the vacuum state; in the sense that there is no radiation there you have not excited the radiation you are in the lowest possible energy state of each of the modes of the radiation field when you are in this state.

(Refer Slide Time: 45:42)



You can for example, put 1 fourth one can go to a 3 by 2 H cross omega 1 and you will come to a state for example, if you can come to a state which is n 1 is equal to 1. This will be a state which has energy of 3 by 2 H cross omega 1 plus or actually H cross omega 1 plus the infinity.

So, in the sense every state is infinite.

Every state has 0 point energy is infinite. So, always there all the times sitting there

After

Yes. So, what you can do is you can go from one state to another state by every time adding or subtracting H cross omega corresponding to that mod. Let me write down some equations which will help us to work out further. For example, if I have a lambda operating on n 1, n 2, n lambda ket what do i get?

Please note this will only operate on the n lambda part. So, I will get square root of n lambda, n 1, n 2, n lambda minus 1. Yeah, it is the fluctuations because you cannot have a state quantum mechanically where certain pairs of conjugate variables are precisely known, you cannot have a state where Q and P which are becoming exactly measurable simultaneously. So, it is a result of uncertainty principle. They are all connected to each other in a standard normal harmonic oscillator. You cannot have 0 energy because if it has 0 energy the particle is that rest at 1 point which means you know its position it has a finite, it has a precise position or precise momentum and that is not allowed by quantum mechanics.

So, there have been fluctuations and that fluctuation is essentially contained here because each mod is like a harmonic oscillator that means there you can in principal have a state where there is no electromagnetic wave. You have not sent any wave at all you are not excited any wave but, still there are fluctuating electric fields corresponding to all frequencies all the time.

The vacuum is boiling all the time either whether we can look at the origin of those fluctuations this fluctuation is being explained quantum mechanic the mathematical structures explains to me that this must be there and that is experimentally verified.

(( ))

Sorry. Yeah.

Sir,

Yes

We can say 0 point energy is because of our observation because, by observing this we are (( ))

No

That's why (( ))

Whether you observe or do not observe there is 0 point fluctuation. See i am not observing anything I am trying to find out what are the energy states of my field of one particular mod I find that the minimum value is half H cross omega lambda. Whether I do a measurement or not it is half H cross omega i cannot this state this field cannot have an energy less than half H cross omega when it is in that particular mode.

For a harmonic oscillator the minimum energy is half H cross omega. You are trying to express this by stating that if I tried to measure the position and momentum simultaneously. They have to satisfy certain uncertainty principle and because of that is an explanation but, this contains the quantum mechanical structure contains the fact. The moment I write commutation relation I have already introduced uncertainty principle because 2 non commutative observables will satisfy an uncertainty principle delta Q delta P get (( )) equal to H cross by 2.

(( )) 0 point energy is generated because of (( )) initially (( )) exactly 0 energy (( ))

How do you, how do you say that?

(( ))

This is what I am asking, I am saying that whether you observe or not it is half H cross omega I am trying to explain by another way by, through the uncertainty principle but, it does not mean that if I do not measured energy is 0.

The energy is minimized half H cross omega well we will discuss further. So, let me write down the other equations a lambda dagger n 1, n 2, n lambda is equal to square root of n lambda plus 1, n 1, n 2, n lambda plus 1. Apart from that you had a lambda dagger a lambda operating and this gives you n lambda etcetera. So, let me write down the most general state of the electromagnetic field, and then we will stop. This is equal to of course, normalized.

These are all complex coefficients. See, this is a huge state what it tells me is they are actually infinite summations and each summation is infinite that means I can have state in which the photon number here in one state there is one photon one state there are 2 photon, one state there is 3 photons etcetera. Plus there could be another state in which this n 2 can go from 0 to infinity n lambda cross goes 0 to infinity and this summation itself goes to infinite. There are infinite numbers because there are infinite numbers of modes.

The space occupied by the quantum states is huge and much of it is still unexploded because we will go through in the course we will discuss some very interesting case states one are called as single photon states, coherent states, free states, entangle states etcetera. So, these are all essentially combination of this kind of combinations and these are some special interesting properties.

We will stop here, do you have any questions? Yeah, what we will do is we will calculate, i will take a film which is at the lowest possible state lambda th mod for example, the 0 lambda th state that means the lambda th mod there are no excitations.

So, I will calculate what is the average value of electric field in that state I will get 0. The average of e square will not be 0. So, if I have average 0 and e square not 0 that is variance there is fluctuation that is what i mean. There is a constant fluctuation in the electric fields the average electric field is 0 but, there is fluctuations the area of a fluctuating electric field there are fluctuating magnetic fields which is giving me finally, the energy which i am measuring has half H cross omega.

Thus fluctuations we will discuss when I come to, I will calculate the average of the electric field the average of the square of electric field and if all the terms are 0 then there is no electric field but, square of the field has a finite average that means there are fluctuations.

Thank you.