Quantum Electronics
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Module No. # 05
Lecture No. # 28
Quantization of EM Field (Contd.)

We will continue with the discussion on quantization of electromagnetic fields. Do you have any questions?

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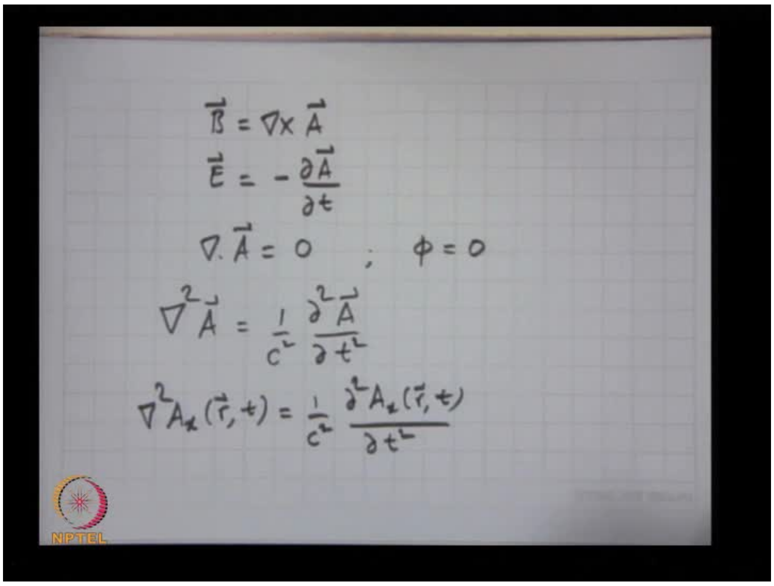

What we started doing was, we wrote the Maxwell's equations and then using Maxwell's equations we found the wave equation satisfied with electro potential, so remember we wrote B is equal to curl A and E is equal to minus del a by del t and the vector potential is chosen to satisfy this condition, it is called the coulomb gauge and we also assumed that the scalar potential is 0.

From Maxwell's equations from divergence B is equal to 0, we wrote B is equal to curl A then use the Maxwell's equations and impose the condition of divergence A is equal to 0, which is the coulomb gauge condition and we got an equation satisfied by the vector potential, the wave equation.

If i solve this equation and get the vector potential as a function of spatial and time coordinates i can use these first two equations to calculate the magnetic fields and the electric field corresponding to that vector potential. For example, we started looking at one of the Cartesian components of this equation, if you look at the x component this is the function of r and time is 1 by c square del square A x of r t by del t square and this is a partial differential equation and we can solve this equation by using separation variables technique.

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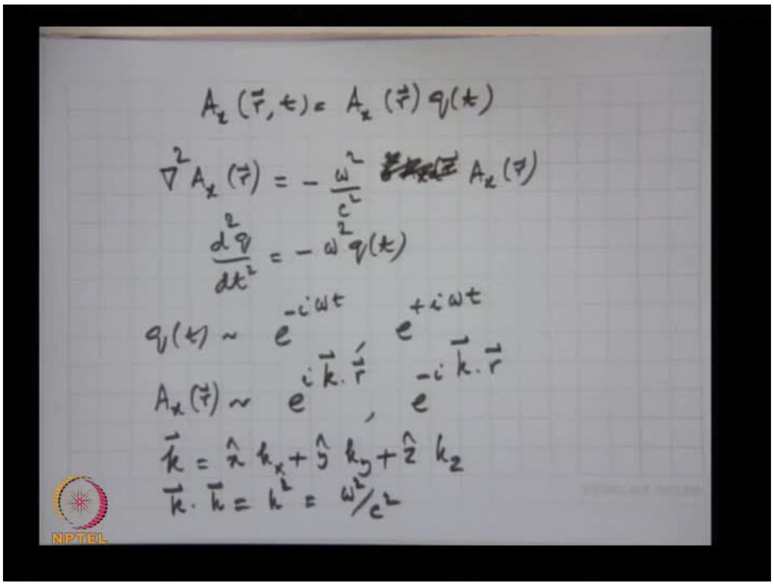

We wrote A x of r t as a product of a function of space coordinate and a function of time, we have A x of r t is equal to A x of r and some q of time, so we got these two equations z square a x of r is equal to minus omega into A x of r and d square q by d t square is equal to minus omega square q of t right?.

We substitute this use separation variables techniques the time and the space parts gets separated out, the time dependence solutions look like this exponential minus i omega t and exponential plus i omega t. This partial differential equation can again be solved by separation variables technique and we will get for example, plane wave solutions, so A x of r has functional dependence of the form exponential i k dot r and exponential minus i k dot r. Where k vector is equal to x cap k x plus y cap k y plus z cap k z. The magnitude of k is omega by c. This is free space we are looking at so k dot k, k dot k is equal to k square is equal to omega square by c square.

So, what we have essentially got are plane wave solutions to electromagnetic wave propagation in free space. i can similarly solve the equation for A y and A z the three components of the A vector and all of them will have the same form exponential i k dot r and exponential minus i k, i k dot r.

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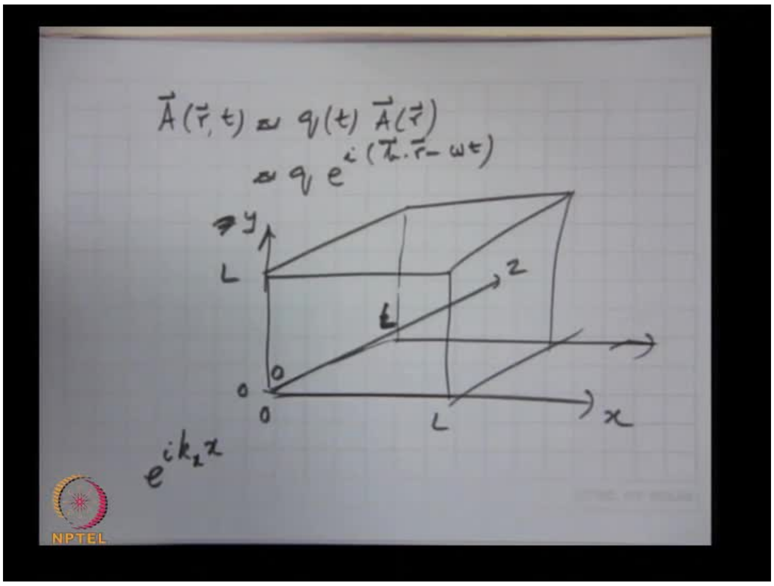

So, i can write the complete solution for example, will be of the form a of r t will be of the form q of t times A of r, which will be of the type some complex constant multiplied by exponential solutions of this form, let me write this thing exponential i k dot r minus omega t chi k dot r minus omega t, i will have k dot r plus omega t, i will have minus i k dot r minus omega t, i have all kinds of combinations of the all solutions.

Now, as i was mentioning yesterday, each plane wave is a mode of propagation in free space what does it mean? it means that these waves can propagate independently of all the other waves and they form a complete set that means any wave propagating in free space can be written as the superposition of an infinite number of plane waves. In fact, this expansion is what leads to diffraction, if you are given a beam with a certain width on a plane z is equal to 0, you can express this beam as a super position of plane waves going in different directions and if you calculate the field at any other plane z because different plane waves have suffered different phase changes in arriving at this plane, the field distribution here is different from the field distribution here, it is very similar to a situation where i pluck a string in some arbitrary shape and the shape of the string will keep changing with time, because that arbitrary shape of the string is a linear combination of the modes of oscillation of the string and because different modes of oscillation have different frequencies, their phase changes with time or change are different and after some interval of time, they will add in their appropriate phases and you will get a different shape in general.

So similarly, i can write any field here, as a superposition of plane waves because plane waves are modes of propagation which means that a plane wave propagates independently of all other waves, If you start with a plane wave going like this it will keep going like this and if i express the field distribution here as a superposition of plane waves, each plane wave propagates independently, i can calculate the total phase what happens to each plane wave as it propagates over a distance z add them up with appropriate phases and i get the field distribution here, so you can study diffraction exactly like this, this is the Fourier transform relationship actually.

So, what we have done is? we have broken up into any plane waves now, the problem with free space is as you can see here any omega is allowed and as long as k square is omega square b c square i can have any combination of k x, k y, and k z which means i can have plane waves propagating in all different directions at all kinds of frequencies from 0 to infinity.

So, if i want write the total field at any point, i have to write as an integral. Now, it is much more convenient for me to work with summation sums rather than integrals, what normally we do is? we put some virtual boundaries. For example, i will consider a cavity of a cubic cavity of size L oriented along x y z directions and then i apply what is called as periodic boundary conditions. i will say that the field on the plane x is equal to 0 is equal to the field on the plane x is equal to L the field on the plane, so i will have for example, i can consider a cavity like this a sort of space like this so, this is y and this is z.
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So, the fields on the plane x is equal to 0 must be equal to the field on the plane x is equal to L, so this is 0 and this is L. Similarly, the field on the plane y is equal to 0 must be equal to the field on the plane y is equal to L and similarly, for z the field on the plane z is equal to 0, which is this plane must be equal to the field on the plane z is equal to L, this is called periodic boundary condition.

There are different kinds of boundary conditions, somebody can say that i will assume that these are perfectly conducting surfaces and the field inside is the standing wave, it is like a cavity.

So, those will give me solutions in terms of standing waves these give me solutions in terms of propagating waves because all i am saying is the field here and here must be the same any propagating wave, which satisfies this will be a solution to this problem. But, putting this boundary condition immediately discretizes my frequency because as you can see for example, if i say that the field at x is equal to 0 must be the equal to the field at x is equal to L, what is the condition i will have? And what is the x dependence of the field? x dependence of the field is of the form exponential i k x, x look at the solution we have a solution of the type k dot r, so k x x plus k y y plus k z z. (Refer Slide Time: 02:41).

Spatial dependence of the field is of the form exponential i, k x x plus k y y plus k z z, so if i say that the field on the plane x is equal to 0 must be equal to the field of the plane x is equal to L, what is the condition? This must so k x into L must be equal to integral multiple of pi.

Integral multiple of pi, so let me call this n x into two pi.

Yeah.

Sir why would we say that it should be same at 0 and L what is the idea behind sir?

No, this is a method to solve the problem to discretize the problem and at the end i will tend to infinity.

My solutions will become independent of L. Now in other situation there could be a real cavity for example, i could be thinking of light inside a laser resonator for example, laser cavity, so this there is a actual cavity there are actual mirrors then i will have to put actual conditions or boundary conditions, but these are a set of boundary conditions which i am putting to discretize the problem, so that the total field will be a sum of various turns rather than integral over various turns, this is just for mathematical simplification.

Are we using the generality even when we are extending this box to infinity because we have said that time (( )) will be same are we using the generality?

Yes, i infinitely extend the plane wave, so it extends from x is equal to minus infinity or plus infinity, y is equal to minus infinity or plus infinity, and z is equal to minus infinity or plus infinity that is the entire space right?, so all i am saying is the volume of consideration is much smaller compared my actual this L for example, so if i am looking at a certain region of space this L will be much bigger than that.

So, my solution will become independent of L, some way later on, i will show you that i can actually take care of this L, so this is just helping me to look at a discrete set infinite number of discrete set of functions rather than a continuous set of infinite number of functions right? (Refer Slide Time: 05:06).

So, i will get k x L is equal to 2 pi into n x similarly, i will have conditions on k y and k z.

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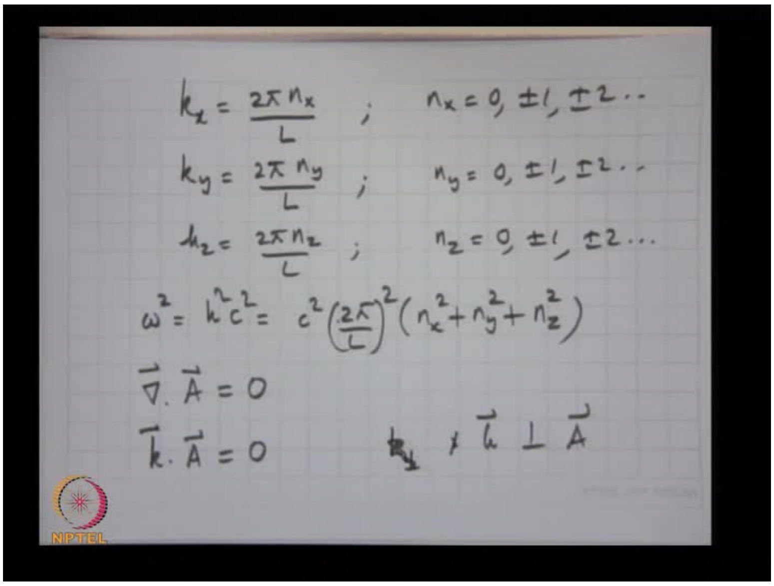

So, i have the three conditions on k x, k y, and k z for example, my k x can be 2 pi n x by L, n x is equal to 0 plus minus 1 plus minus 2 etcetera. Similarly, k y is equal to 2. pi n y by L, n y is equal to 0 plus minus 1 plus minus 2 etcetera and k z is equal to 2 pi n z by L, n z is equal to 0 plus minus 1 plus minus 2 etcetera.

The corresponding frequency will be omega square is equal to k square c square, which is equal to c square times you will have 2 pi by L whole square into n x square plus n y square plus n z square. This is a discrete spectrum of frequencies now. Discrete spectrum of frequencies and a discrete spectrum of propagation directions all values of k x are not allowed, all values of k y are not allowed, so by imposing these boundary conditions what i have ensured is, now the field consists of the solutions consist of a discrete number a discrete sequence or infinite number of possible solution, i have discretize the problem and the total solution will be a sum of all possible modes.

So, let me write the total sum. i want because there are three quantities n x, n y, and n z. i will have to use as an index which is the contracted form that equation will not become huge. There is also another condition i need to worry about and that is please remember that we had seen we had imposed this condition what does that imply? A is vector, so what does this condition imply? Remember the solution A is of the form exponential i k dot r minus omega t or plus omega t etcetera, so if i substitute this into this divergence a is equal to 0 what is i get? k dot a is equal to 0. (Refer Slide Time: 05:06)

So, i cannot choose any arbitrary k, there is a restriction, so this means k is perpendicular to a vector sorry k is perpendicular to a vector. So, for every k vector direction, the solution a vector should be perpendicular to this k vector direction and how many independent vectors can i write in terms which is perpendicular to a given vector? 2 you choose this vectors, i have one vector here and one vector here, two independent states and these are the two independent states of polarization, because light is a transverse electromagnetic wave the two solutions if i chose at certain propagation direction k vector, i will have two values of a vector directions which will be solutions to the problem any other vector A can be written as a super position of these two vectors just like these each are the two states of polarization, so there are two possible polarization states for every k vector, there is a particular frequency and for every set of frequency and k x, k y, and k z there are two possible solutions.

Both the solutions have the same frequency and have the same k vector right?, so for example, if i choose one set n x, n y, and n z which will define my frequency and k vector, i shall have two solutions which will be perpendicular to this k vector direction. Suppose i choose n x is equal to 1, n y is equal to 1 and n z is equal to 3, so i have certain k vector magnitude a certain frequency, certain k vector direction, and two possible solutions perpendicular to this k vector direction. Now, as i said to simplify notation what i am going to do is the following; i will contract all these n x, n y, n z and the polarization state into one single quantity.

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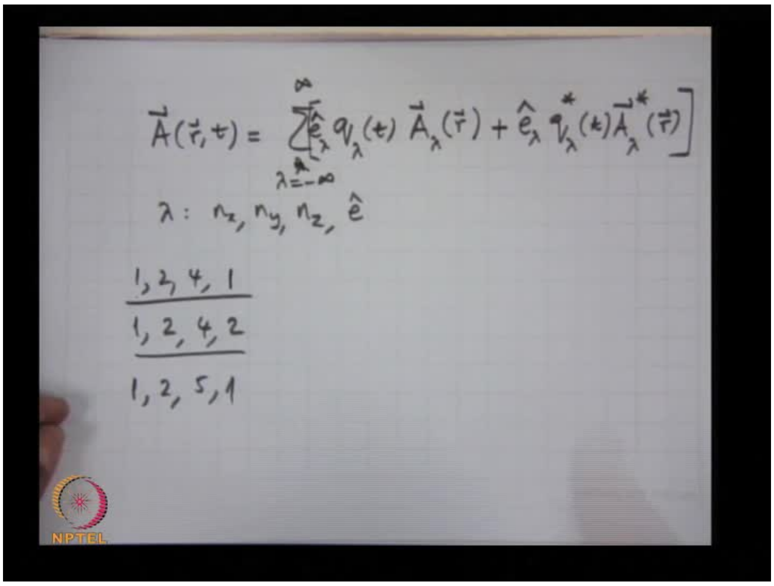

So, let me write the solution like this. So, the general solution A vector sigma lambda okay this lambda is not wavelength, this is a subscript, so i will have q lambda of time A lambda of r, let me write E lambda plus remember our fields are all real, so lambda stands for a set of n x, n y, n z, and E direction, so E could be 1 or 2 for example, there are two orthogonal polarization states, if you choose one set n x, n y, and n z, E can be 1 for one polarization state and E can be two for another polarization state.

So, for a given set n x, n y, and n z there are two possible solutions. All this everything is actually into this, so as lambda varies i will have to use in a particular sequence every mode is represented by one value of lambda. So, i will differentiate for example, 1, 1, 1, 1 and 1, 1, 1, 2 they are two different solutions, 1 will correspond to a particular polarization state suppose i choose the set of numbers 2, 2, 4, and 1, so this n x, n y, and n z polarization and if i have another mode, which is this, so this mode and this mode have the frequent or propagating in the same direction, but have two polarization states which are different, because in this case i just represent by 1 and 2 the two orthogonal polarization states.

So, this is one mode and this is another mode, i could have another mode, which is 1, 2, 5, 1, so this mode is different from this mode, because one of the numbers is different. So, if i change any one of the values n x, n y, n z, or E, i get a different mode and there is a degeneracy because all modes having the same n x, n y, and n z, but two different polarization states have the same frequency and the same propagation direction.

(Refer Slide Time: 17:53). Now, by writing like this actually, i have made the field real and actually lambda goes from minus infinity to plus infinity.

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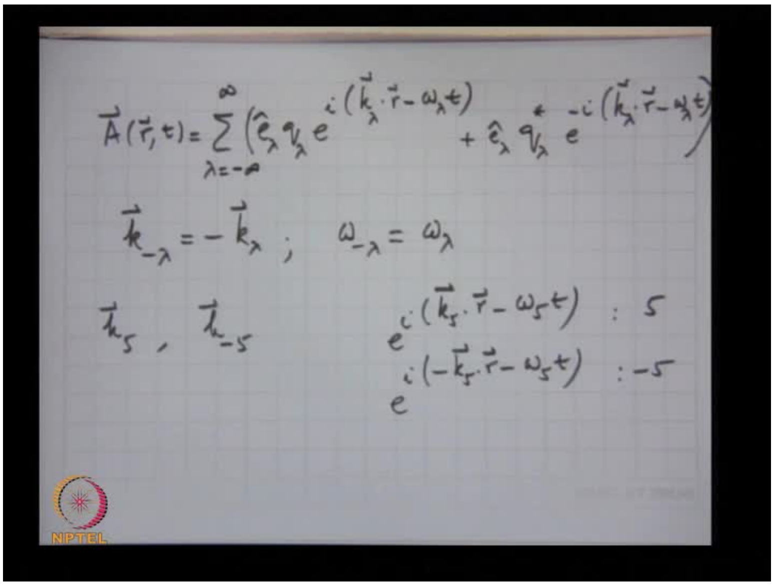

Now, let me try to expand this bracket and write it in an explicit form, which will become clearer to you okay. So, let me write the time and space dependence explicitly, so A of r t sigma lambda, i have E lambda, q lambda exponential i, k lambda dot r minus omega lambda plus this unit vector, E lambda, q lambda star exponential minus i, k lambda dot r minus omega lambda t. This q lambda and q lambda star are now time independent because time dependence has been taken into exponential, i have written explicitly the time dependence and explicitly the space dependence, so the amplitude is contained in q lambda this is the solution this is the solution sum of the these two is a solution and this makes it real, which direction is this propagating this wave k lambda vector both of them are propagating the same direction k lambda vector this complex conjugate is added make it real.

Now, i will also impose the condition that k minus lambda is equal to minus k lambda and omega minus lambda is equal to omega lambda. So, if i have two modes differing in the sign of lambda, they will differ only in the values of n x, n y, and n z, so 1, 2, 4, mode it will be lambda, 1, 2 ,4, 1 will be lambda, minus 1, minus 2, minus 4, 1 will be minus lambda. When i change the sign of lambda, i change the sign of k vector and not the sign of omega lambda, because omega lambda is the frequency, it is the same frequency, so what will be the corresponding solution for minus lambda, if i look for example, a field which corresponds to k 5 vector and k minus 5 vector what will be the difference between these two solutions?

Yes any one?

Yeah, to why? To where?

Exactly reversed, because this function in one of the cases it will be exponential i, k 6 dot r minus omega 5 t and the other case will be exponential i minus k 5 dot r minus omega 5 t, this corresponds to this corresponds to 5 and this corresponds to minus 5.

Yeah?

Sir k 5 notation what does it say about n x, n y, and n z?

This is (( )), so i am saying that i will have to number these modes these infinite number of modes in a particular method each value of lambda corresponds to one set of n x, n y, n z and polarization state, so i am not defining what is 5, 5 will be one combination of n x, n y and E, i do not know what it is and i do not need to, because what i need to know is that, if lambda is different the modes are different they may be different because the polarization states are different or they are different because they correspond to another set of n x, n y, and n z right?. So, all i need to know is this because when i solve the problem, i am witting in terms of a sum of all the modes and that is what i need to know.

So, the values they corresponding to negative values of lambda are actually modes which are propagating in the reverse direction of the mode solution corresponding to lambda positive. So, this contains all solutions wave is going in the plus z direction and minus z direction everything okay.

So, with these conditions this is the general solution to the problem. Please note we have not yet quantized anything. This is still a classical picture the quantization of frequencies has coming because we have put in boundary conditions, classical boundary conditions this is still a completely classical picture. (Refer Slide Time: 20:54).

These modes exist even in classical picture because we have put boundary condition simply. So, this discretization has nothing to do with quantization yet. So, this discrete frequency is the representation we have solved represented in the continuous frequency spectrum in terms of discrete spectrum by applying some boundary conditions.

Sir.

Yeah,

But, is it not the discrete frequency that i am not lying saying that in a finite portion of phase only discrete energy levels exist.

No, i am not calculated yet energy.

I will calculate the energy corresponding to each frequency okay. Now, this electromagnetic field will consist of s large number of frequencies, the general solution, i could for example, pick up one of the solutions and look at that solution that will be one mode, because one mode means one plane wave propagating in a one particular direction with one polarization state, if i take a wave infinitely extended a plane wave propagating in the z direction like this polarized vertically, for example that is one mode that is one of the solutions here (Refer Slide Time: 20:54) because k lambda will be z cap into k and corresponding frequency, i will get and polarization state is y cap or whatever it is.

Now, classically this wave can have any energy this wave can have any electric field or magnetic field i can measure the magnetic field and electric field perfectly accurately of this wave that is classical.

I will show you that when we proceed further ahead, quantization will come when i go into the next level and find out that this wave this mode cannot have arbitrary energy it will have energies only if as n plus half h plus omega, where omega is the frequency of this mode right?. Right now, there is no quantization yet we have just looked at a classical problem of a plane wave expansion and we have written the total vector potential in terms of the plane wave expansion, so this is one expansion in terms of plane waves.

This plane waves you have got by solving remember this equation this equation (Refer Slide Time: 02:41) and the plane waves are not the only solutions to this equation, there can be other solutions but, this is this is a del square of a x is equal to minus omega square by c square a x. In Cartesian system, i am getting a solution i could have another set of infinite set of modes of solution, so you see this modes, i am using a plane wave modes as solutions somebody else can use another set of modes as solutions, because this will become little more sort of intriguing when we go into next step and when we look at actually when we try to introduce the concept of photons.

Sir.

Yeah?

(( ))

As a sum rather than integral to simplify my problem because later on i will find out that l will never appears anywhere finally. It will be in equation, but it will be it will disappear when i try to do calculations of detection probabilities or whatever it is, i will find that what is the what is the intensity following on a photon detector? If i try to calculate L will disappear, it should not appear finally because it should not depend. i have taken a cubic structure, i could have taken a spherical one then i will not get plane, i will get another set of Bessel function solutions or something like that i will get okay, cylindrical cavity if i had taken, so it is just one mode of mode one set of modes and these are plane wave modes this is simplest modes. (Refer Slide Time: 20:54). Now, so there is a spatial part, there is a temporal part and there is an amplitude here q lambda say.

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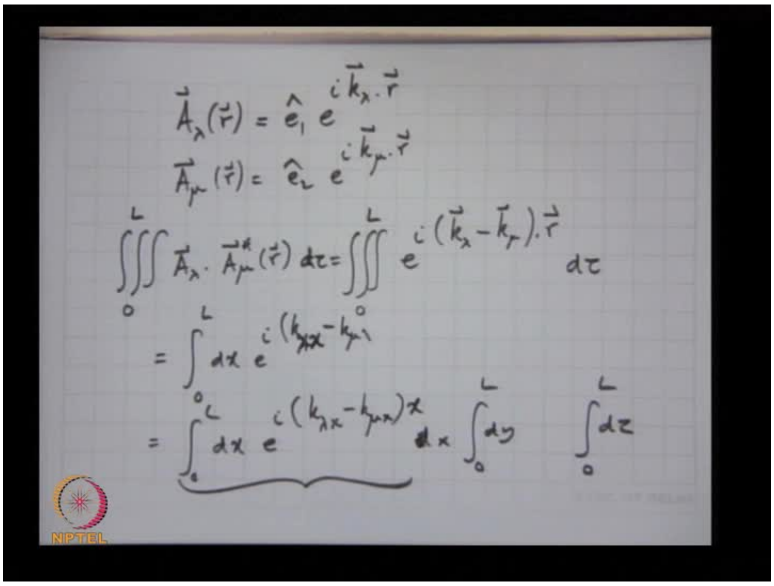

Now, the spatial part for example, if i look at one of the spatial parts A lambda of r, which is equal to sum polarization into exponential i, let me look at one of the one of the terms, so this lambda actually contains the polarization state also, when i when i change the lambda this polarization state will go from one polarization into another polarization.

Now, if i take another mode, let me call this E 1 and we call this E 2. Suppose, i want to calculate this value the integral of this from 0 to L over the volume of the cavity where i have quantized. If they are orthogonally polarized obviously, it is 0 because E 1 dot E 2 is 0, so that is that means even E 1 and E 2 are different obviously lambda and mu are different that means they are different modes, so that is gone.

So, let me take even if they are at the same polarization state what will i get? i will get to put integral 0 to d tau over volume d tau, 0 to L exponential i, let me take a star here a mu star, k lambda minus k mu dot r d tau. i want to show that modes are orthonormal orthogonal to each other.

So, this will split into three integrals. This will be equal to integral 0 to L, d x exponential i, k x lambda minus k mu sorry k lambda x k sorry let me write again is equal to 0 to L d x exponential i, k lambda x minus k mu x, x d x into the integral over y and integral over z, only difference is instead of k lambda x minus k mu x into x, you will have k lambda y minus k mu y into y and the third one will have k lambda z minus k mu z into z.

What is the value of this integral? Sorry, not delta for its zero to L.

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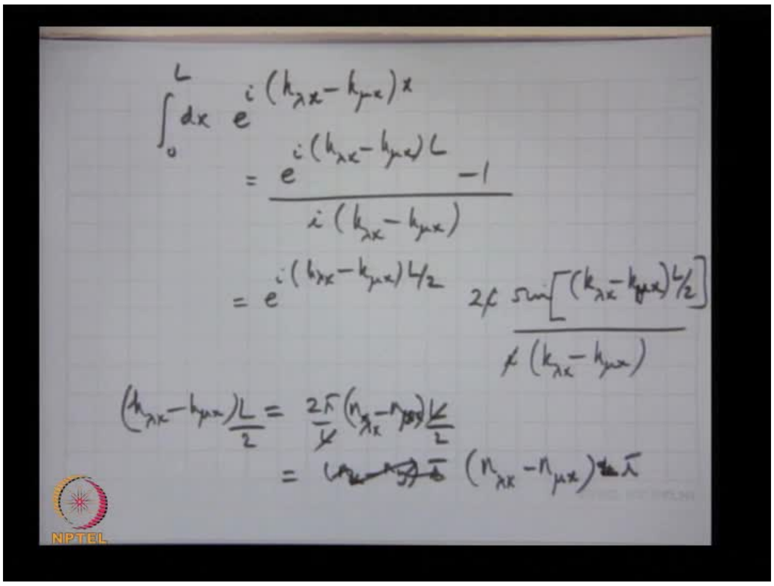

So, let me look at this integral 0 to L, d x exponential i, k lambda x minus k mu x, so this is equal to exponential i, k lambda x minus k mu x into L minus 1 divided by i times k lambda x minus k mu x, so this is equal to exponential now i, k lambda x minus k mu x into L by 2 into 2 i sign k lambda x minus k mu x into L by 2 divided by i times k lambda x minus k mu x.

So, what is k lambda x minus k mu x into L? Two pi by lambda, 2 pi by L n x minus n y into L, 2 pi into n x minus n y, so divided by 2 let me do so by 2, so this is equal to n x minus n y into pi, so can you tell me what happens to this?

(( ))

If n x is not equal to n y, this is a multiple of pi the numerator is 0 denominator is not 0 and the function is 0 can it be finite?

(( ))

If n x is equal to n y, then this is how much? So this is 1 any way and then how much is this value? i goes off sign square how L square? (( )), L by into 2, so if n x is equal to n y this is L otherwise it is 0. Similarly, the y integral will be 0, if n y is not equal to n lambda ah n lambda x is sorry n yeah not a not a n x n y sorry this should be n lambda x and n mu x sorry please correct this n lambda x minus n mu x into L into pi. This is k lambda x is 2 pi L into n lambda x, k mu x is 2 pi by L into n mu x that is the n x value corresponding to k mu the math solution.

So, if n lambda x is equal to n, n mu x the first integral is finite n equal to L similarly, the second one and third one, (Refer Slide Time: 29:44).

So, all this integral will be finite only if lambda is equal to mu that means each of the numbers n x, n y, and n z of the lambda mode and the mu mode must be the same and the polarization must be the same, so unless they correspond to the same mode this integral is 0. If they correspond to the same mode, the value of this is L q because each integral is L, L, L.

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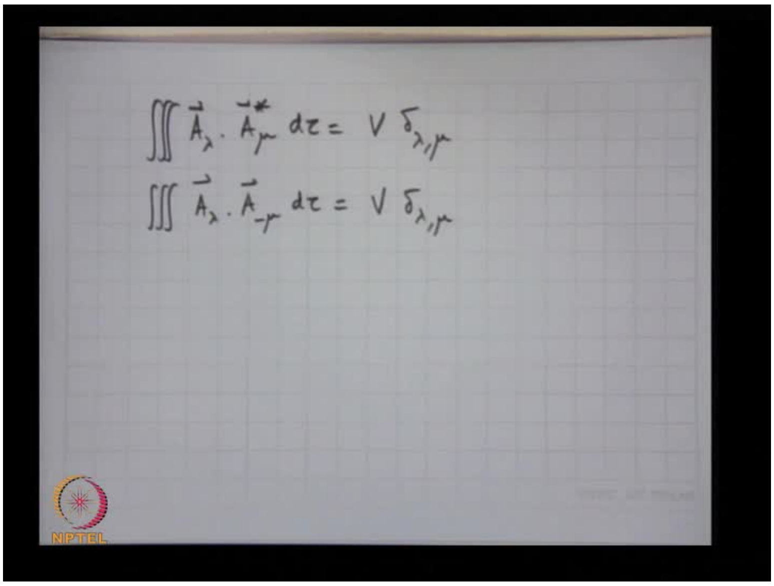

So, what i get is essentially? The normal that means the functions are normalized, so triple integral A lambda dot a mu star d tau is equal to volume into delta lambda mu and actually you can also show A lambda dot A minus mu, d tau is equal to V times this is a

Kronecker delta function, which is 1, if the indices are equal and a 0, if the indices are not equal.

So, all it says is the modes are this is an this is orthonormality condition. In quantum mechanics, if you take two wave function they satisfy integral psi 1 dot psi 2, psi 1 and psi 2 d tau is equal to 0 that is these are two independent solutions, so any two independent this means they are independent they are orthonormal, they are independent solutions and the way we have disturbed the derivation they are independent and this is the orthonormality condition because, we will need this conditions when we calculate the energy of the electromagnetic field. So, let me write again the expressions for the vector potential and from the vector potential, i can calculate the electric and magnetic field this is vector potential.

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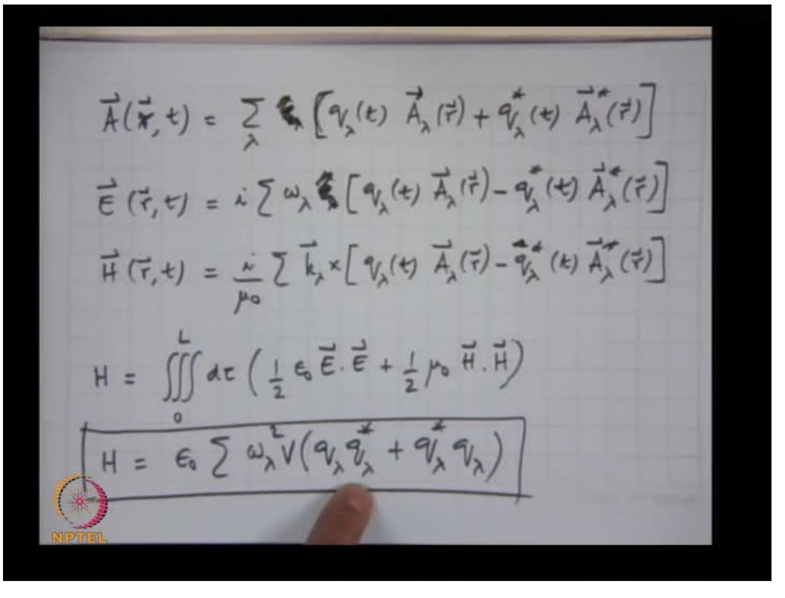

So, let me write it again here so, A of r t is equal to sigma lambda, let me write this as E lambda, q lambda of time, A lambda of r plus q lambda star of time, A lambda star of r. Where q lambda of time is q lambda exponential minus i omega t and a lambda of r is exponential i k dot r.

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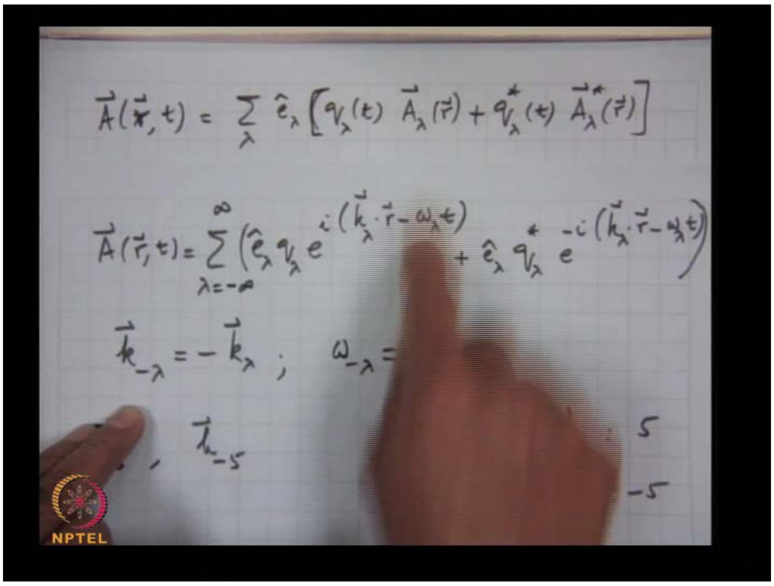

Here it is this q lambda of time is q lambda into exponential minus i omega t, A lambda of r is exponential i k lambda dot r and that is a complex conjugate of this this is just to contract and simplify the expression here on the on the paper.

Now, can you tell me what is the electric field corresponding to this vector potential? minus del a by del t, so if i differentiate what will i get, the time differential will be only of the exponential minus i omega lambda t, so E of r t becomes there is a minus del a del t that is the minus coming from here.

(Refer Slide Time: 38:31). So i will get i times sigma omega lambda, E lambda, q lambda of t, A lambda of r and then plane of plus sign or a minus sign yes, actually this i have taken out otherwise, i must write i here and that must be it is complex conjugate.

So, i have taken out the i you see the i will appear here and the second term is the complex conjugate of the first term, so there will be minus sign, because i have taken out i, this will be minus q lambda star of time and A lambda star of r and the magnetic field H vector is 1 by mu 0 del cross A, b is del cross A, so H is so H is given by actually, i should not write this here please remove this because this is the vector, already i am writing as a vector here this polarization is contained in the vector here okay, so please remove this e lambda. (Refer Slide Time: 38:31).

This A lambda vector actually contains a vector the polarization state of the field is contained in A lambda vector, If i write it in exponential then i must write a separate unit vector, but otherwise the vector is contained here, so what i will get is? i by mu 0, sigma, k lambda cross, q lambda of time, A lambda of r minus q lambda star sorry time A lambda star of r, this comes from del cross. Because it is exponential i k dot r, the del cross will be i k cross the function okay, so it is just the vector potential the electric field and the magnetic field corresponding to this plane waves in free in corresponding to any electromagnetic wave in free space. This is the total solution under the periodic boundary condition the vector potential can always be written as a sum of these terms and similarly, the electric field and the magnetic field.

Now, my problem is to find out the energy contained within this volume V of the electromagnetic field, which is the Hamiltonian. So, i must find out the Hamiltonian then i must find out i will show you that one the Hamiltonian looks like a sum of harmonic oscillator terms, so what will be the Hamiltonian? i am using the same H but, this is vector this is a Hamiltonian H please differentiate this is a vector H, H vector field this is Hamiltonian. So, what will be the energy of the electromagnetic field in the volume V? Integral over the volume V, d tau or half epsilon 0, E dot E plus half mu 0, H dot H. (Refer Slide Time: 38:31).

So, now the problem is to substitute this E vector into this term. Use the fact that the functions are A lambda star and A lambda (Refer Slide Time: 36:50) satisfy these orthonormality conditions and get an expression for the Hamiltonian.

I think we will stop here and we will start from this expression in the next class, because it is now substituting this electric field expression here and simplifying it using the relationship, but let me do one thing. Let me give you the final expression that you will get, which we will derive in the class, so what happens is? This Hamiltonian comes out to be epsilon 0 sigma, omega**,** lambda square into V times q lambda, q lambda star plus. (Refer Slide Time: 38:31).

I have to substitute the electric field expression here integrate use orthonormality conditions of A vectors, i have to substitute the magnetic field here again use the orthonormality condition and finally, i will land up with the expression for the

Hamiltonian which we will derive after the next class it looks like the harmonic oscillator problem already.

Yes.

Sir if we move a little back?

Yes.

If we try to find out normality condition with a with respect to

Yes.

At that point we got the integral of exponential i of k lambda, x minus d mu x, minus d x integral of this?

Now, if we integrate this we get a term exponential i k lambda x minus k 2 x L minus 1 (( )).

Yes this is what i did [FT].

So, sir we already know that this k lambda x into L should satisfy this 2 by n x.

Yes.

So, it automatically becomes 1.

(Refer Slide Time: 32:48). No, no, no, no but, there is also a denominator which becomes zeros because i am substituting only at the end because if i substitute here itself it becomes 1 minus 1 by 1 minus 1 what is the value? see k lambda x is 2 pi by L into sum integer k mu x is 2 pi L n into sum into integer i know that, so the integers are equal or the integers are different and the numbers are different.

(Refer Slide Time: 32:48). So, if any one of the n x, n y, or n z is different then the integral is 0. If all of them are equal then it becomes L V V the total integral, which means it only is a mathematical expression of the fact that plane wave is going in different directions are orthonormal to each other in the sense that they are independent solutions to my wave equation and they form a complete set of functions.

So, any wave can be written as the superposition of these plane waves going in different directions and for every direction and every frequency, there is there are two independent solutions which are degenerate and have orthogonal polarization states.

(Refer Slide Time: 38:31). So, one value of lambda, if i take one of the solutions, a particular value of lambda it will give me a particular value of electric field and a particular value of magnetic field that will correspond to one mode of propagation. i have written purposely like this q lambda, q lambda star is equal to q lambda star q lambda in classical picture they commute, but i have written like this because it will happen it will become a lambda, a lambda dagger plus a lambda dagger a lambda into something else.

(Refer Slide Time: 32:48). So, i just expressed it like this so, what i will do is? next class we will derive this from this from here and then i will show i will put the Hamiltonian in the form in which i will get variables which are conjugate variables then i will use the condition that q comma P is equal to i h cross and that is where i quantize the electromagnetic field and then we will come to the various pictures of i mean we will look at the solutions and understand try to understand what a photon is, if we can understand okay.

Thank you.