Quantum Electronics

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Module No. # 05

Lecture No. # 27

Quantization of EM Field

Today, what I want to do is to start the issue of quantization electromagnetic fields. So, just as I start this, I would like to discuss one interesting example of a system, which is not a classical harmonic oscillator, but which, whose equations look like harmonic oscillator equations, and hence I can quantize it; because when I quantize electromagnetic fields, the way I will approach is, I will calculate the total energy of the electromagnetic fields in terms of electric and magnetic fields. And then, show that a total energy can be written as a sum of an infinite number of terms; each one of those terms looks like the energy of a harmonic oscillator; it looks like p square by 2 n plus half m omega square x square, where x is something, p is something, need not necessary to be position momentum, but two conjugate variables.

At the moment I put the Hamiltonian in that form, immediately I can use the same principles as I have used for harmonic oscillators by writing x comma p operators is equal to I H cross, and then quantize it. So, the quantization essentially implies I, put this commutation relation, and then I assume that all observables are operators, this system is described by a ket, and then I can solve the problem. So, before I do that, for electromagnetic fields, I want to discuss a a classical problem, where I can use a similar procedure, and quantize the problem; and that is an L C circuit.

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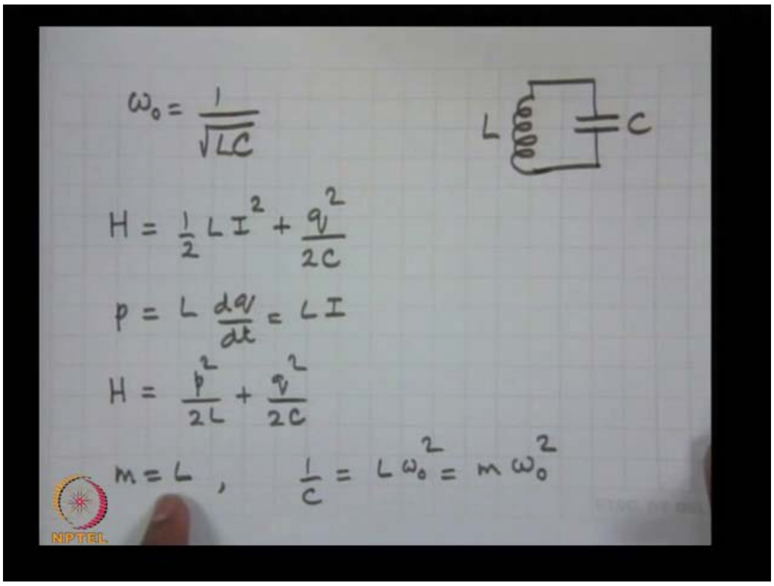

So, we all know that this L C circuit behaves like a an oscillator; and if you calculate the current in the circuit of the charge on the capacitor, it varies sinusoidal. And if I assume no resistance in the circuit, this will oscillates forever, just like harmonic oscillator, and on that harmonic oscillator; what is the frequency of oscillation? Omega 0 is equal to 1 by square root of L times C. Now, first I want to write the classical equation, and put that classical equation in a form, which looks like harmonic oscillator equation, and then I can quantize the L C circuit. So, what is the energy of this energy contained in the L C circuit? The energy is present in the inductance in the form of current variation and also charges on the capacitor. So, what is energy? Half L I square plus if I try it in terms of charge…

q square over 2 C

Now, let me define a let me define a quantity p, which is L d q by d t L I. So this, the Hamiltonian or the energy is simply p square by 2 L plus q square by 2 C. Please remember, in a harmonic oscillator, the energy is half m omega square x square plus half m (( )). But when I wrote the Hamiltonian, I wrote it as half m omega square x square plus p square by 2 m, because x and p are conjugant variables; they satisfied the Hamilton’s equations, which are the classical equations

So similarly, I defined a quantity p, which is L d q by d t; and I will show you that p and q are conjugate variables, which means that if this is the Hamiltonian of the system, and if I write the Hamilton's equations of motion for this system, they will satisfy the classical equations. Now, let me to make it more apparent, let me define a quantity m, which is equal to L; this will correspond to the mass of the corresponding harmonic oscillator. So so, what about 1 by C? 1 by C becomes L omega naught square, which is equal m omega naught square m is nothing but inductance here.

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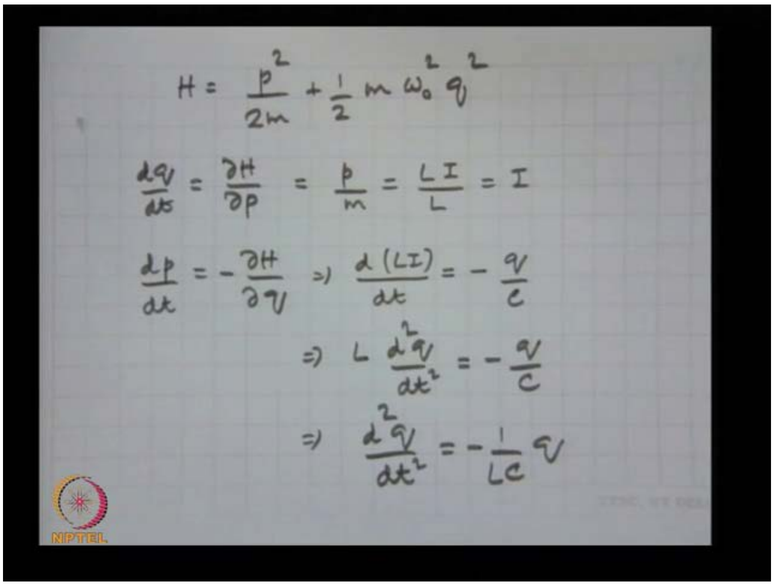

So, substituting the value of m and 1 by C here, the Hamiltonian simply becomes p square by 2 m plus half m omega naught square q square; this very similar to the harmonic oscillator. Now, let me just check whether p and q correspond to conjugate variables; so I need to calculate the if this is Hamiltonian, and p and q are conjugate variables, then you know that d q by d t must be equal to del H by del p, this is one of the Hamilton's equations; and the other equation is d p by d t is equal to minus del H by del q. So, what is del H by del p? It is p by m; if I differentiate del H by del p, becomes p by m; and remember p is L times I, and m is l; so this is equal to I, which is true; I is d q by d t, where q is the charge.

And the second equation implies d times L I by d t is equal to minus let me use this equation, this H for q square by 2 C; so, it becomes minus q by C. So, this implies L d square q by d t square is equal to minus q by C, which implies d square q by d t square is equal to minus 1 by L C times q, which is nothing but the equation for the charge variation in the L C circuit, because this will give you a solution, q of time is equal q 0 exponential cos omega naught t and sin omega naught the solutions; so this, taking q as one variable, and p as the conjugate variable, where p is L I, which is L d q by d t. Just like in the other harmonic oscillator, p was m d x by d t; so here it is m is L, and d x by d t is d q by d t. So, position in the harmonic oscillator is replaced by charge here, and the velocity or momentum is replaced by L d I by d q by d t.

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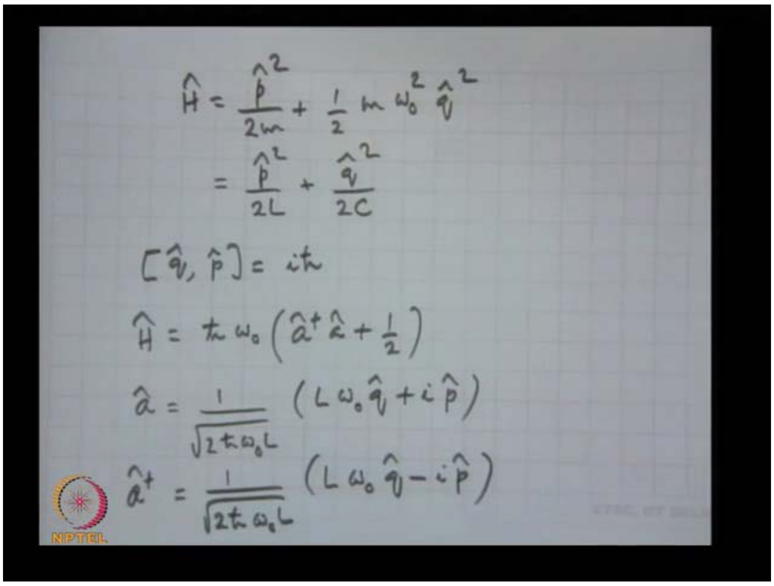

So, this tells me that this is a correct form of Hamiltonian of this L C oscillator. And so, now to quantize this, what I do is, I now assume that H is an operator, and given by p square by 2 m plus half m omega naught square q square, which is actually I can write it in terms of which is also p square by 2 L plus… And to quantize I say that q and p satisfy its commutation relation; this is the crucial point, when I am quantizing the L C oscillator. By saying that, q and p satisfy this commutation relation, I have transform the problem from classical to quantum; and the Hamiltonian of the system is given by p square by 2 L plus q square by 2 C

Sir, this expression for the commutators, this we are saying as per the comparison of this equation with the…

Yes, if I assume q as a variable and p as a conjugate variable, which is what I have shown you that because that satisfy the Hamilton's equations, which are consistent with the classical equations satisfied by L C circuit; p and q are conjugate variables, and q,p operator is equal I H cross, that is now the quantization; I have quantize the L C circuit by putting this condition. So, first what have done is I have written the Hamiltonian, in terms in in in the form of a harmonic oscillator problem, and then I quantize. So, what will I get? I can use the same, exactly the same principles as I have done for harmonic oscillator; and I will find energy Eigen states. I can I can find the Eigen states of the harmonic of the Hamilton operator.

And I will find that for example, H will be H cross omega naught a dagger a plus half, where a will be defined as 1 by under root 2 H cross omega naught L into L omega naught q plus I p. Remember 1 by 2 m H cross omega naught m omega q plus I p, this is what I did for harmonic oscillator; m is replaced by L; and q and p are not now position of omega, which is some other variable, it is a it is a variable correspond to my L C circuit. And similarly I can calculate from here a dagger.

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Sir, over here we are doing this analysis; let say we have system, which you do not know that it is upon it is harmonic oscillator or not. So, for solving the item, how will be approach (( )) over here we are saying that you know, we can take the charges one variable, and its conjugate variable is this quantity L I. So, how will be get to knowing the first place that they will be conjugate variable, I mean this (( )).

So, the first problem is to find out what is the Hamiltonian of the system?

(( ))

In terms of where is a conjugate variables?

How will we know that which is the conjugate (( ))?

So, I have to write a form of Hamiltonian, calculate the Hamilton's equations of motion; and check whether they are consistent with the classical equations.

For example, if you go back to the standard harmonic oscillator problem, and if you write the Hamiltonian as half m omega square x square plus half m v square, and assume that x and v are conjugate variables. And you calculate the Hamilton's equations, they will not be correct equations; they will be inconsistent with the original classical harmonic oscillator equation.

So, v is not the conjugate variable to x, it is p; because if you calculate del H by del q is equal to d q by d t is equal to del del H by del p, and the other simultaneous equation, these equations are be consistent with the actual dynamically equations of the system. So, I think Hamiltonian calculation of Hamiltonian is not a severe problem in more complex situations. But I need to form a Hamiltonian; in fact, one starts with the Lagrangian formulation, write the Lagrangian system, then from there define the conjugate moment, then write the Hamiltonian, then write the Hamilton equations of motion, and then quantize it; that is the standard procedure for a general problem.

But here, I am looking at a simplified problem, because the problem which I am tackling is like a harmonic oscillator.

Sir, for example, if we have some another problem; in which we know that you know the variation is as per the harmonic oscillator; we know before had it. Now, if we need to formulated in terms of this quantum mechanical analysis, so how when the how will be proceed to find the conjugate variable, this is that the question I have asking you. So, if you do, we have to follow the older procedure.

I think so. I do not think it is just straight forward to write…

This is fine that you are telling the this is about conjugate variable, we are saying that fine.

Because I have I have shown you by substituting into the Hamilton's equations, I have shown you there is consistent with the classical equations.

But when if we had…

I follow; in general, what do I do? I think in general, I have to try out different forms of Hamiltonian, and calculate, which are the corresponding conjugate variables? There may be a standard procedure for this, but it is not obvious I think. One has to find out the pair is a conjugate variables, and then impose the condition - commutation condition on those conjugated variables, and which will then quantize the problem immediately. So, in general I am not discussing here, because finally, our electromagnetic field also will be written as a harmonic oscillator; and harmonic oscillator has very nice solutions.

Sir,

Yeah.

Sir, how do we come to know that the commutation between p and q is going to be (( )). See remember, when I did harmonic oscillator, in the harmonic oscillator problem, the commutator between q and p, the plus of bracket between q and p is 1; I showed you that day.

x and p.

x and x and p, q is a equation… x and p; x and p is 1, because del x by del x into del x by del del p by del p minus this thing. So, there is a definition of poser bracket. So, poser bracket is equal to 1 x p. I say that the value of poser bracket remains the same numerically, and I replace the poser bracket by 1 I H cross commutated the bracket and that is the quantization now. I cannot derive this, because quantum mechanics is much broader than classical mechanics; I cannot derive quantum mechanics from classical mechanics, because classical mechanics is a subset of quantum mechanics.

So, I postulate some some postulates, I put some postulates; and using that postulates, I derive the solution to my problem. If my solution is consistent with what I find experimentally, my procedure is correct; I cannot derive, I cannot derive Schrodinger equation; that was just written by Schrodinger from his from his from his own mind.

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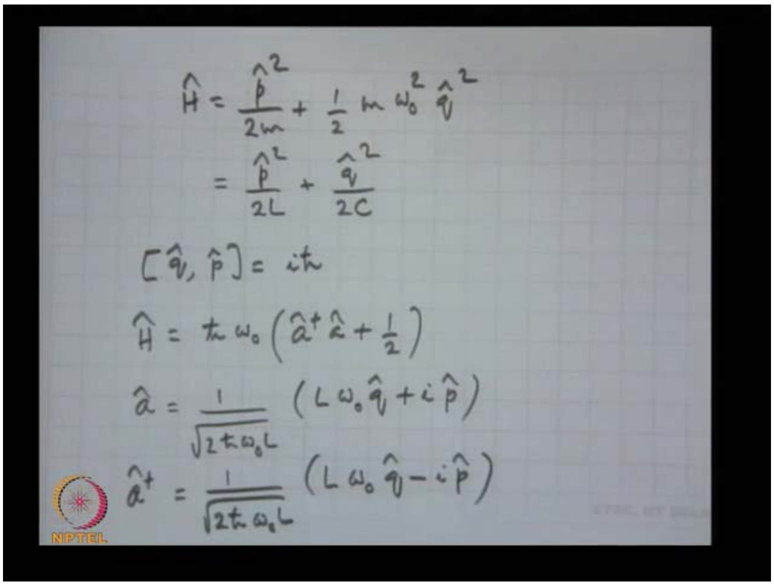

So, this particular procedure was proposed by Dirac, if you look at Dirac's book. A quantization means essentially, after I write Hamiltonian in terms of the conjugate variables, then I put q p is equal to I H cross that is quantization now. And if I quantize the problem, and please remember there is a correspondence principle that classical classical description should be also possible in terms of quantum mechanical there, because I see the harmonic oscillator oscillating back and forth.

So, I should be able to describe that quantum mechanically; please remember I told you that the the oscillator the mass spring system, which is oscillating like this, is not in one of the energy Eigen states, because if it was in one of the energy Eigen states, the expectation value position would be 0. And the short word it is I know, I am seeing the particle moving back and forth like this, which means the expectation value of the position of the particle, should be oscillating sinusoidal.

So, since the expectation values follow the classical laws, so that is why we said that whatever we observe the position that is the…

Most probable position, which is expectation all (( )) yes yes. Similarly, when I quantize the electromagnetic field, I will find that the the the standard the laser beam, which is coming out the sinusoidal oscillating field, which is coming out is not represented by a energy Eigen ket. And there, I will introduce what I have called as coherent states; I can do the same thing for the harmonic oscillator, particle oscillating particle, but then I assist the same. So, I do not want to do it for the harmonic oscillator problem, but I want to do it in the electromagnetic field problem. So, when I write this condition, I have quantize the problem; and I am assuming that q and p are operators, H is an operator, and I will describe the system by a ket, which contains all the information that I can have regarding the system.

Sir, in this case, I mean they wrote just this commutative relation and I H cross. So, I mean whenever there are two conjugate variables, we can write there are commutative relation (( ))

I think so; two conjugate variables will always satisfy the equation…

Value can (Audio not clear. Refer Time: 18:25)

No, because if they are conjugate variables, I think it will be I H cross; I would have to verify this, but I think it will be I H cross, two conjugate variables. But in general, I can have a commutator, which is not a number like this, it could be another operator; for example, between a and H for example, the commutator is not a number, it is another operator; so, it is in principle possible. But what I need to do is in principle, if I calculate the poser bracket, whatever the value I get? I replace the poser bracket by 1 by I H cross commutator bracket right.

So, in our case, what I am doing is, because the poser bracket of q and p, not for this problem, because I now reduce the problem to in equivalent harmonic oscillator problem; and I exactly borrow all the results of the harmonic oscillator input at here. I could have actually derived this and continued with all the calculation of Eigen states, instead of doing the harmonic oscillator, I could have done the L C circuit first; in a quantum mechanics course, you would have done an L C circuit.

So, these are all the same problem; mathematically they have the same problem, interpretation of q and p are different here; in harmonic oscillator, it is different and in electromagnetic field, it is different; q corresponds to something in the oscillator, here its corresponds to something else; in the electromagnetic field, it corresponds to some field variable.

Sir, (( ))

Yeah

Sir, in suppose we are write down there not sure that the commutator in which (( )) 2 conjugated (( )) then what in this case what is (( )) there is commutator (( ))

Because poser bracket to be in q and p is 1.

Sir.

Because the problem is related equivalent to harmonic oscillator problem.

Sir, but we can also say that since the expression that we have used, they reduce the Hamiltonian to the same expression…

As the harmonic oscillator.

As the harmonic oscillator except that x is placed by q and…

So, the relation should be I H …

Because those equations those Hamilton's equations are much more general than for a classical mechanical system. So, that is a much more general formulation; and I can write Hamiltonian of a system in terms of two conjugate variables, and those conjugate variables will satisfy this commutation relation immediately, because the corresponding harmonic oscillator the corresponding particle harmonic oscillator satisfies these equations, and this problem is exactly equivalent to a harmonic oscillator problem.

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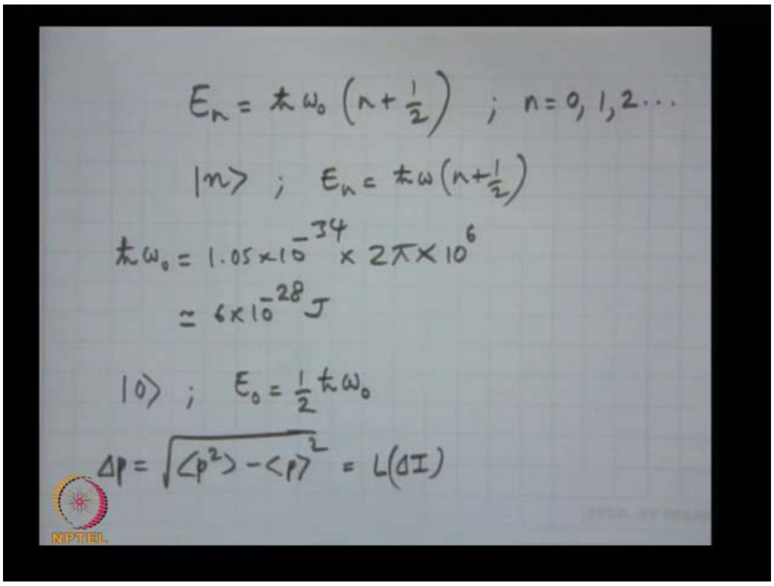

So, I can a I can invert this equation to calculate q and p; and the energy of this oscillator, energy states of the oscillator will be E n is equal to H cross omega naught and plus half, where n is equal to 0, 1, 2; and you can describe the what I have called as the number kets. This is a is a state of the system of the L C circuit, in which the energy corresponds to into n plus 1 half. So, you see this tells me that the energy of the L C circuit is quantized; now let me calculate what is the quantum of energy in this; which means what this means is that L C circuit can have only energies, which are difference by H cross omega naught, half H cross naught, 3 by 2 H cross omega naught, 5 by 2 omega naught etcetera. So, what is the value of H cross omega naught? Let me calculate what is the value of H cross H bar not h

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About 1 point

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0 5 or something, so let me assume 1.0 5 into 10 to the minus 34, and what is omega naught I assume for an L C circuit typically? This is nu omega naught naught nu naught, this not this first frequency, this is a radiance per second frequency. So, what is a value? 2 pi times… What is the oscillation frequency of a typical L C circuit in the lab? Is it in

hertz, kilohertz, megahertz, terahertz? What is the value? Capacitance you take you typically, can you can you estimate?

Pico farad

Pico farad and inductance.

Millihenry.

Millihenry. So, what is the frequency? Can you calculate? Omega naught 1 by root L C what is the value?

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10 power 6 yes. About 2 pi 10 power 6 yes per megahertz. So, what is this value? This is about 6 times 10 to the minus… This is hardly measurable by you 10 to the minus 28 joules. So, for your classical system, when you look at the L C circuit, it will have almost continues energy values, because the difference between the two energy states is so little, you cannot measure this difference; we will not be able to observed, but it is quantized.

Just like the harmonic oscillator, if you take the L C circuit in one state some state for example, let me… So, also note that the lowest possible state is a vacuum state called vacuum state, in which n is equal to 0. So, this is the lowest energy state, the energy is half H cross omega naught. This is again comes from the uncertainty principle, because I have put this commutated relation between q and p, q and p have a commutation relation, the charge, and the current in the circuit. So, I can actually, that means, what it means is that you cannot have an L C circuit, in which there is no current flowing. There is always fluctuation current, there is a fluctuating current, there are fluctuation fluctuating charges on the capacitor plates continuously; and those fluctuations there is a minimum amount of fluctuations and that is determined by quantum mechanics.

Of course, for this problem for the L C circuit, because omega is so small; the classical description is perfectly fine; just like for a pendulum, classical description is perfectly fine. You do not you you need to use quantum mechanics for that. For light, again you will find the same kind of frequency difference; this is 10 to the power 14 now. So, that may comes about 10 to the minus 19 joules that is typical energy of a photon. So, this I will say that if you are state is in this system, it is in the n th excited state. Just like for atoms, you have a different energy levels; here also there is a sequence of energy levels, starting from half H cross omega naught, 3 by 2 H cross omega naught, 5 by 2 H cross omega naught, there is a ladder, complete ladder up to infinity, there are infinite number of modes here.

So, what I have try to show here is simply the fact that for in L C circuit, I can write a Hamiltonian, in terms of pairs of conjugate variables; and when I quantize, I put q p is equal to I H cross, and then I employ my procedure and; that means, I have quantize the L C circuit. And I can calculate for example, I leave it to you to calculate, what is the current - fluctuating current in the L C circuit, when the L C circuit is in the lowest energy state? How will I calculate? All you need to do is you need to calculate delta p p square average minus p average square, because p is related to current, p is L times I. So, expectation value of p in the state will be 0, remember the harmonic oscillator, expectation value of p square will be finite. So, I will get some value for this, and this is nothing but L time delta I.

So, I can calculate delta I from here, and you will get typical values about 10 to the 13 amperes or something like that; 0.1 pico amperes of current, which is for this kind of oscillator, 1 megahertz oscillator, you will have that current fluctuation will continuously will be there all the time; you cannot get rid of their (( )), it is a quantum fluctuation. So, my objective to do do this problem was to sort of show you that if you are given a system, and if you can reduce the problem, harmonic oscillator to the Hamiltonian of the of the problem to an Hamiltonian, which looks like a harmonic oscillator Hamiltonian. Then I can use all the calculations of the harmonic oscillator problem to do this, and to calculate what are the energy Eigen states; what are the expectation values; what are the how does the system behave dynamically etcetera, etcetera; everything I can calculate.

Now, so what I want to do now is, so I am going to use a similar procedure for electromagnetic fields. So, for electromagnetic fields, I will have to calculate the total energy of the electromagnetic waves. And I will show you that the energy of the electromagnetic waves can be written as a sum, an infinite sum of terms, which look like harmonic oscillator terms. And So, I will show you that the electromagnetic wave consists of an infinite number of independent harmonic oscillators; each harmonic oscillator corresponds to one particular frequency. So, we need to start from classical Maxwell's equations, and we will do this quantization in free space.

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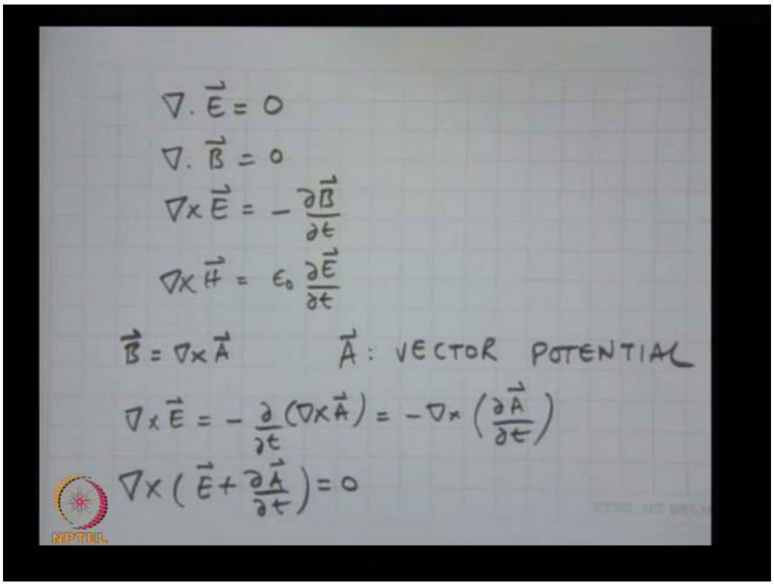

So, what are the Maxwell's equations in the free space? Divergence E is equal to 0, then divergence B is equal to 0, curl E is equal to minus del B by del t, and curl H is equal to epsilon 0, free space no charges, no free currents, no free charges. This del t by del t E is epsilon 0, E in free space. This equation divergence B is equal to tells me B can be written as in fact, that is a very general equation, whether it is materials or no material free space etcetera; and what is A called? The vector potential. So, if I substitute this into the third equation, I get del cross E is equal to minus del by del t of del cross A is equal to del cross… So, this tells me del cross E plus del A by del t is equal to 0. So, what is this imply? E plus del A by del t must be gradient of a scalar.

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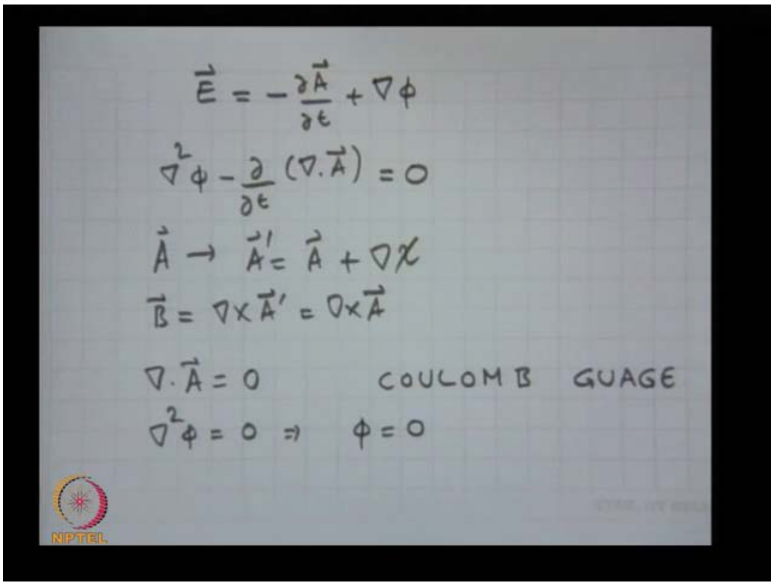

So, E must be equal to minus del A by del t plus gradient of a scalar function.

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I can substitute this into the first equation, and I will get del square phi minus del by del t of divergence A is equal to 0. If I substitute this electric field expression in the first equation, I will get del square phi minus del by del t divergence a is equal to 0. Now, please note that there is a bit of flexibility in my defining the vector potential, because if I add the scalar of any function to A, B will remain the same, the magnetic field does not change by replacing A by A plus gradient of some function chi, for example.

So, if I replace A by A prime is equal to A plus gradient of some chi, then B is still given by del cross A prime is equal to del cross A; B does not change by redefining the vector potential. So, I have a choice; it defining a vector potential; so the way I will choose is to make divergence A is equal to 0. I will choose suppose the A, which I get, does not satisfy divergence A is equal to 0. I will add the gradient of some scalar function to make sure that the final vector potential that I get satisfies this condition divergence A is equal to 0, and this is called the coulomb gauge.

And if divergence A is 0, del square phi satisfies, phi satisfy this equation, and I also choose phi is equal to 0; that means, phi is determine primarily by static charges that is a constant feel at the background, and I just choose the origin, I just just choose the scalar potential as 0 everywhere.

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Because of A arbitrary analysis arbitrary analysis defining the vector potential; I will define my vector potential such that divergence A is equal to 0. Suppose, I you give me an A, which is does not satisfy divergence A is equal to 0, I will add the gradient of a scalar function to the given A value, so that the new A, which I get satisfies divergence a is equal to 0. And if divergence a is equal to 0 del square phi is 0, and I choose the solution phi is equal to 0 as that means, the background there is no electrostatic potential. So, what I get is this is called coulomb gauge, there are other gauges which people use, there is another gauge called the Lorentz gauge, where the condition satisfied by A is different, and this is used in quantum electrodynamics.

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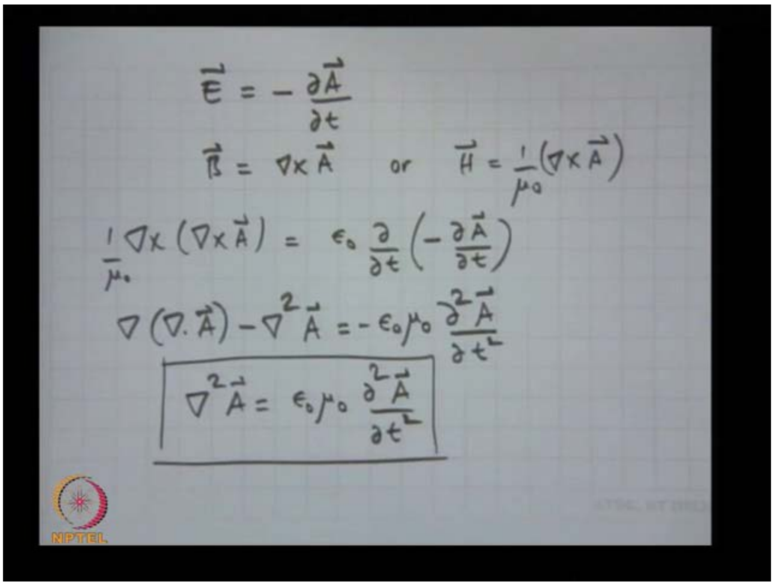

So, what I get now is that because phi is equal to 0, the electric field is given by minus del A by del t, and the magnetic field is given by… or H is equal to 1 by mu naught del cross A. So, if I get the value of vector potential A, I can find out both the electric field and the magnetic field. So, what is the equation satisfied by A? I substitute into the 4th equation now, the value for H, and the value for E. So, H is 1 by mu naught curl A, and E is minus del A by del t; so I will get curl of curl of A, with the 1 by mu naught is equal to epsilon 0 del by del t of minus del A by del t. This implies that, gradient of divergence A minus del square A is equal minus epsilon mu 0 del square A by del t square. And divergence A has been chosen to be 0. So, this tells me the equation satisfied by A, which is the wave equation, three-dimensional wave equation satisfied by the vector potential A.

So, if I solve this equation, and get the value of A, I can calculate the corresponding electric field and the magnetic field from these equations. So, instead of solving the electric and magnetic fields, I have reduced the problem to solving the problem for vector potential; and calculating vector potential for a given problem, I can immediately calculate the electric and magnetic fields. So, how do I solve this equation? Separation variables technique, for example.

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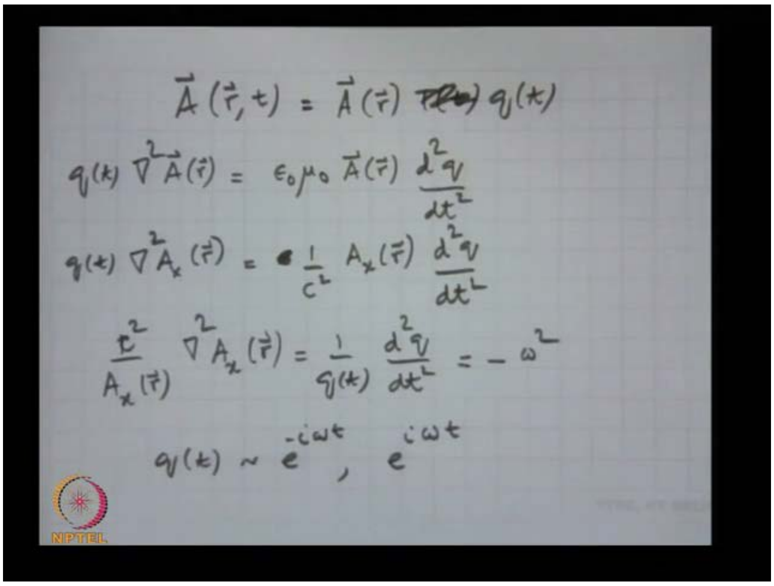

So, I can write the vector potential is actually function of position and time.

Excuse me.

This is this is there is the vector potential also vary as electromagnetic field I mean, because it satisfy the same wave equation. So…

Exactly, because A will have a time and space variation, which will give me corresponding electric fields and the magnetic fields; we will calculate we will calculate we will fall this equation, calculate the vector potential, and then use these equations to calculate the electric and magnetic fields in terms of the solution of the vector potential.

So, I can write this as some some function of r times some function of time. So, let me write this q of t. So, this equation becomes del square A into q of t is equal to epsilon 0 mu 0 r.

So, this is essentially, so let me write for example, one of the cartesian components of this A vector, so I will have del square A x of r into q of t is equal to epsilon 0 mu 0 which is 1 by C square A x of r. So, this tells me 1 by A x of r del square A x of r, let me take this C square here, C square times this thing is equal to 1 by q of t. So, this is one of the cartesian components of the vector potentials, satisfies this equation, and which I separate into a space dependent part on one side, and a time dependent part on the other side. So, the space and time dependences have separated out; and because this equation has to be valid for all space points and all times, this must to be equal to constant; and that constant I choose as minus omega square; minus omega square, so that the time variation is harmonic, and not exponentially amplifying or decreasing. So, the solution for q of t immediately I get, exponential I omega minus I omega t and exponential I omega t, two types of solutions.

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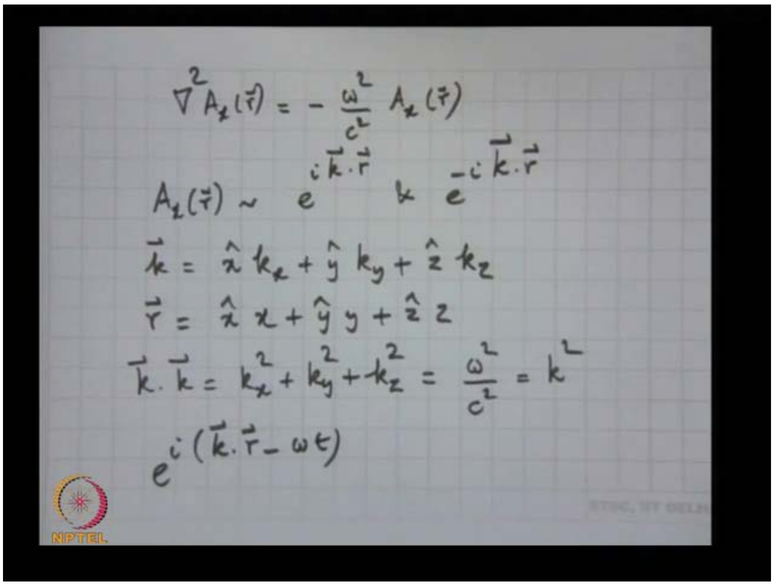

So, A x of r, r vector will now satisfy an equation, which I can write from the first this term, and equal to minus omega square, and what do I get? I get del square of A x of r is equal to minus omega square by C square. How do I solve this equation? For example, again I can choose separation variables technique, write this as a product of an x dependence, y dependence and z dependence; and what is the kind of solutions I will get? Plane wave solutions for example; so what will be the solution like? ...

(No audio from 41:16 to 41:26)

Where k vector is equal to x cap k x plus y cap k y plus z cap k z and r vector is a position vector x cap x plus y cap y plus z cap z.

(No audio from 41:46 to 41:57)

And k dot k is equal to k x square plus k y square plus k z square must be equal to how much?

If you substitute this a solution into this equation, I will get minus k x square, minus k y square, minus k z square is equal to omega square by C square, which is essentially in the propagation constant of the wave; so that is known. This is the frequency of the wave that is the propagation constant of the wave, and this is a wave, this solution is a wave corresponding to wave propagating in some direction defined by the k vector with components k x, k y, k z. So, time dependence is of the form exponential, plus I omega t or minus i omega t and the space dependence is of this form; it is completely classical; I am not do any quantum right now; it is completely classical solution.

Now, I need to consider an infinite space; problem in infinite space is all omegas are allowed out continuous spectrum of omega I can have any frequency of the electromagnetic field. So, when I continue with the problem to calculate. So, what is happening is the electromagnetic field consist of all frequencies that are possible, from omega is equal to 0 to infinity, corresponding k vector magnitude is omega by C right. Each solution corresponds to for example, if I multiply this time dependence, and this space dependence, what is it gives me? Something like exponential i k dot r minus omega t; what is that? What kind of wave is this? It is a plane wave; exponential i exponential i k dot r minus omega t is a plane wave going along the direction defined, where the k vector.

Similarly, exponential i k dot r plus omega t is (( )) wave going in the minus k direction. So, these gives me give me a solutions corresponding to, and because of this arbitrary values of k x, k y, k z the only condition is this must be satisfied. So, these these correspond to all kinds of plane waves, going in all directions, and having an arbitrary frequency. These are all the independent modes of propagation in free space, which means if you take a plane wave going like this at a certain frequency, it will propagate unchanged; if you take another plane wave going in this direction, will go unchanged; it will not get coupled to the other one, these are independent solutions. So, the total solution will be written as a integral of all the possible frequencies with all the possible directions of propagation.

Now this makes the problem little more complex to analyze. So, there are methods to simplify the problem, and one of them uses what is called as box normalization. Now, I do not know whether you have done lattice vibrations or phonon spectrum etcetera. So, for example, what I will do is, I will assume a finite volume of space, a cubic volume of space of side L, so L by L by L. And I will impose the following condition called the periodic boundary condition, which means I will say that the field on this plane x is equal to 0, must be equal to the field on the plane x equal to L; the field on the plane y is equal 0 must be equal to the field on the plane y is equal to L; the field on the plane z is equal to 0 must be equal to the field on the plane z is equal to l.

So, the moment I put this condition, I will discretize the I frequencies, because I have to satisfies the boundary condition that the field here, and the field here must be the same; I will find that not all values of k x are allowed; for example, if this value of this should be the same at x is equal to 0, and x is equal to L; I will I can show you that k x is must be an integral multiple of 2 pi by L. So, what will happen is, I will discretize the frequency spectrum, and instead of working with integrals, I can work with sums; and then of course, I can let L tend to infinity, because actually there are no boundaries in free space.

So, I start with a boundary condition employ boundary condition to get the solutions in terms of discretized set of modes, instead of a continuum of set of modes. And the once I get a discrete set of modes, I will solve the problem and later on finally, the solutions of problem must be independent of the volume. Unless, I actually take a cavity; see there are people work, who work with electromagnetic fields with in cavity, febriferous cavity, there is an actual cavity; and that actual cavity will only support a discrete frequency spectrum, then there is no problem. But if I am looking at free space, wave is coming and hitting a beam splitter, getting reflected etcetera, then I have to worry about the fact I am using free space propagation.

And So, I will use this procedure; in the next class, what we will do is, we will apply boundary conditions - periodic boundary conditions, and get the solutions in terms of propagating waves. So, once I get the solution for A vector, I will get the solution for E vector and H vector; and the energy of the electromagnetic field is the energy density if half epsilon 0 E dot E plus half mu 0 H dot h. So, we will substitute that, calculate the total Hamiltonian and finally, put Hamiltonian in terms of a sum of terms, which look like harmonic oscillator terms, which look like p square by 2 m plus half m omega square x square. So, and then the quantization becomes obvious. So, we will stop here.