Quantum Electronics
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Module No. # 05
Lecture No. # 24
Review of Quantum Mechanics

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Today, I will just briefly review some principles of Quantum Mechanics to recollect what you must have studied earlier. And before that do you have any questions on the portion that we have covered till now. So, we have done all classical pictures of non-linear interactions, do you have any questions?

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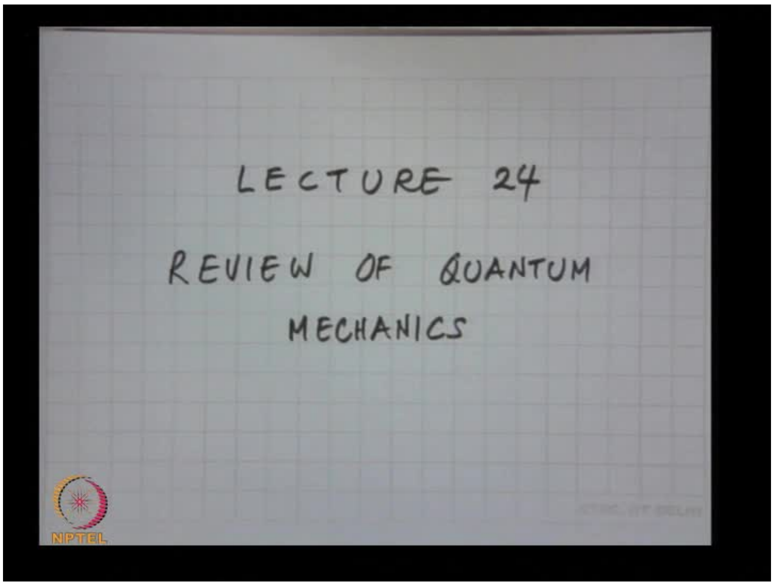

What I am going to do today is, a brief review it is not a detailed discussion on quantum mechanics but, I will just recall some of the basic principles and ideas because we would need them through the lectures. Because, our objective finally, is to quantize electromagnetic fields, obviously question arises why should I quantize electromagnetic fields. First thing is that there is a classical theory where you assume that everything is classical. Then there is a semi classical theory where you quantize atoms, you have energy levels of atoms, you represent the atoms by quantum mechanical principles but, you still consider light as a wave.

For example, applications of light can be treated using semi classical principles including lasers. You can understand most of lasers by using semi classical pictures. You do not need a fully quantum mechanical theory to understand the operation of lasers. Of course, you would have to introduce at some kind of phenomenological aspects like spontaneous emission because if you do a quantum analysis of atomic systems then you find energy eigenstates. Each energy eigenstate is supposed to be a stationary state which means if the atom is in this stationary state then the atom will remain in that stationary state forever. Because, stationary states are by definition those states for which the probability of finding the particle in that state is independent of time. So, if an atom goes to an excited state why should it jump down to lower energy state and emits spontaneously, this will need an explanation in terms of quantum mechanical principles. I would need to quantize light to understand the origin of spontaneous emission but, I can actually phenomenologically introduce spontaneous emission and understand most of the properties of lasers.

Similarly, photoelectric emission there is a semi classical theory which can explain photo electric emission. I do not need the concept of photons to explain photoelectric emission, there are many other aspects of light interacting with that on that where I can assume light to be an electromagnetic wave and analyze the problem in a semi classical picture. That means the light is a wave and the atoms or particles are distorted quantum mechanical.

Now, there are many applications where the semi classical theory cannot predict results of experiments. Spontaneous emission is one parametric down conversion that means a pump photons splitting spontaneously into 2 lower frequency photons. The moment I stuck to use photons, I am already in the quantum mechanical picture of light. The fact that when I shine light at a certain frequency into a crystal I can generate light at lower frequencies coming out of a crystal is not expandable, classically I would need a quantum mechanical picture to understand why there is spontaneous emission of light in the down conversion process.

Parametric amplification, as we discussed in the class can be explained through those classical equations the coupled equations of omega s, omega p and omega i will clearly explain to why there is amplification, there is phase sensitive amplification everything comes out there but, if I only input the pump into the crystal then there is no way to predict that. Therefore, down convert to generate lower frequency light that would need quantum mechanical aspects.

There are many other applications and especially with the advancement in technology as I was mentioning you can have sources of light which generates single photons. So, to understand some of the properties of this light which is called of nonclassical light I would need a completely quantum mechanical picture, I would need a quantized light, I would need a quantized Maxwell's equations and then use those quantum mechanical principles to explain my observed phenomenon. So, before we start to quantize light what I would like to do is to recall some of the basic postulates and principles in quantum mechanics just to refresh your memory and much of it we will start to use when we are going through some of them may be I will recall when we are going through, I may not be able to tell everything right now.

Whereas, we go through, we will start to analyze and look at some of these principles and to understand what is the meaning of quantization of light? What is the meaning quantum theory of light?

Now, you all know in classical pictures you represent for example, particles by position, momentum, and energy etcetera. These are classical variables and these can be precisely measure, and defined in classical picture. Whereas, quantum mechanics develop what was observed is that some of these quantities cannot be measured precisely simultaneously.

For example, positional momentum you cannot measure positional momentum simultaneously, precisely the act of observing, the act of measurement of some quantity of the particle of the object seems to disturb the corresponding other quantity. So, these are called conjugate variables, position momentum are a pair of conjugate variables. There could of course, the variables which are which can be simultaneously measured which means I can measure some quantity a and some quantity b in the given, again in a given system both of them simultaneously accurate. So, not all pairs of variables are having a situation where they cannot be measured precisely. So, I would have to introduce this concept somewhere in the mathematical structure of my analysis and that is what quantum mechanics does.

Quantum mechanics is a set of rules a set of principles tries to explain what we are observing. For example, you cannot measure positional momentum of a particle. Similarly, there are other variables in quantum mechanics what we do is we describe the state of a system or an object by what is called as a wave function or a state vector. Now, you see there are what we will do is very briefly discuss the abstract pictures of bracket notation which was introduced by Dirac. You will not going to the Schrodinger formulation of wave equation etcetera. Because, we would primarily be using the bracket notation in our quantization of light and that is what I want to recall.

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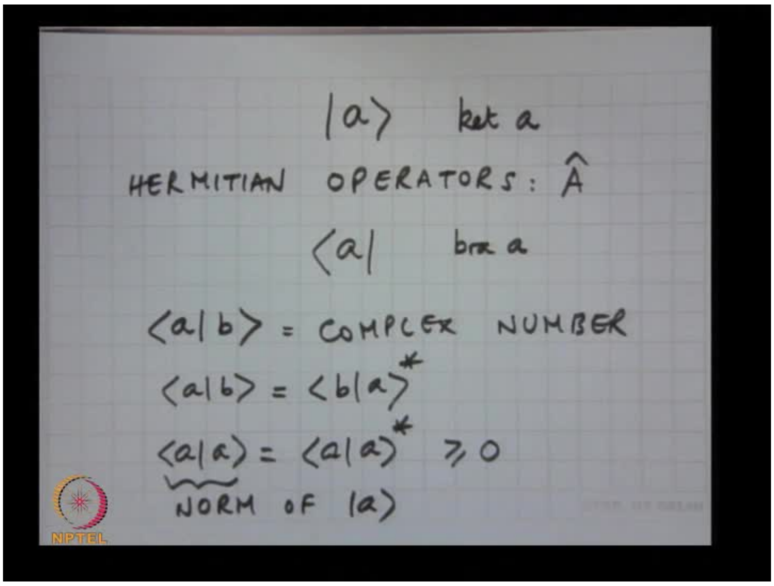

What we will do is we will represent the state of a system by a quantity like this is called ket a. Now, this is also called as the state vector. It is not a vector in the coordinate space it is a vector in some kind of a Hilbert space where it is an abstract space in which I will define. So, this vector or this quantity has all the information that is possible for you to know about the system. I would have to relate this to what is the meaning of my measuring position, what is the meaning of my measuring momentum, what is the meaning of my measuring energy etcetera. I would have to relate this, this state vector which is similar to the wave function in wave mechanics. Where, at the Schrodinger equation in terms of the differential operators and solve, there is a wave function there, here it is a ket vector.

This vector contains all information as for as the system is concerned and all observables will be represented by what are called as a Hermitian operators. For example, I will use a hat on the top of the quantity to define it is an operator. See, you are already familiar with operators differential is operator integration is an operator. So, there you already familiar with lot of operators, this is the kind of an operator this operator is Hermitian. I will define what Hermitian operator is, this operator actually operates on this vector state, vector and observables which means those thing that you can observe will be represented by a special kind of operators is called Hermitian operators.

We will, I will come back to this point. Now, for every ket a I can define another vector in another space called bra a. So, for every ket that you can define in a vector space which has called ket a, I define another corresponding vector in other space called bra a, this is from the word which is called bracket which Dirac introduced and I can define, for example, a function like this I have for example, a product a bracket a b. This is a function which is the product of this ket a and bra b, this is a complex number, we define a b is equal to b a star, star is complex conjugate. So, this number is a complex number and a b bracket is actually b a bracket star. So, by this you can see that a a must be equal to a a star.

So, a must be real quantity and we define this to be greater than equal to 0 and this is called the norm of this ket a. It is something like magnitude, now it happens that in quantum mechanics ket a and any multiple of ket a 2 times ket a 10 times ket a represent the same state. It is all like a vector in a 3 dimensional space where vector f and vector 2 times f are not the same vectors. But, in quantum mechanics the vectors are defined. The kets are defined that a or any multiple of a with complex or real whatever it is, exponential i 5 times ket a 10 times ket a all represent the same state. So, in some kind of a Hilbert space the direction of this vector if I can define a direction of this vector represents the state not the magnitude the direction of this vector.

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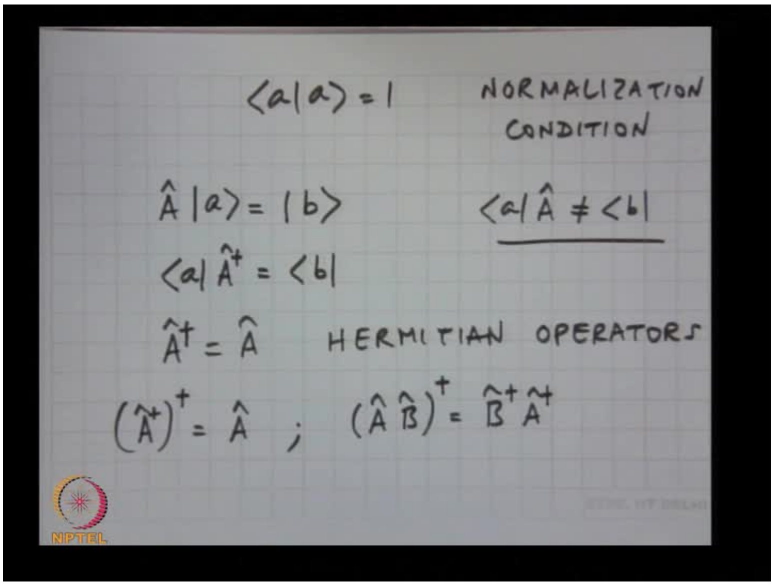

Usually, what one chooses is this a a is normalized to define this as so, this is called normalization condition. Now, I can operate on a ket to get another ket a is an operator which operates on a ket a that leads to another ket in the same space, same vector space. Now, I define for example, with respect to ket a there is also a bra a and with respect to ket b there is also bra b. So, what I defined is I define another operator which operating on bra a gives me bra b please note that a is not equal to b in general.

This operator operating on ket a gives me bra b, the same operator operating on ket a bra a may not give me bra b but, another operator a dagger operator operating on bra a gives me bra b and this is called the adjoint of this operator, a dagger is the adjoint of the operator a and operator satisfying this condition a dagger is equal to a, are called Hermitian. Please interrupt me, if you have any question in between and actually you can work out the algebra and show that a dagger of a dagger is actually a and if you have 2 operators the Hermitian conjugate of this is b dagger a dagger.

From these definitions you can actually work out these kinds of relationship between the operators. These operators and kets have very close similarity in the matrices this ket vector is like a column vector and these operators are like square matrices. So, if you multiply a column vector by a square matrix you will get another column vector that is what this equation is essentially.

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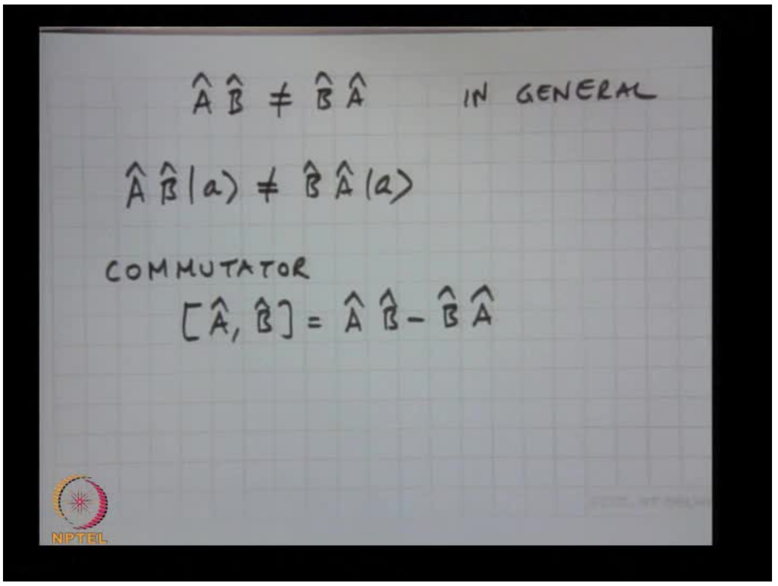

Please note that because, these are operators they do not in general commute which means a times b in general a b is not equal to b times a is not equal to b times a in general. So, that means they do not commute which means operating on a state this is not equal to b times a times a and such operators are called non commuting. You can define what is called as a commutator bracket a b minus b a.

If a and b commute then the commutator bracket is 0. In general for example, you know that a position momentum operator do not commutate with each other and is related to the physical principle that commuting operators observables which represent commuting operators can be measured simultaneously accurately. So, if 2 operators commute with each other it implies that the observables corresponding to these 2 operators can simultaneously be measured precisely.

See, if I have an operator a and operator b, and they commute that means I can measure the observable corresponding to operator a and the observable corresponding to operator b simultaneously absolutely accurate. So, operating on this kets is actually an atom observation, all it means is that for example, if you give me a system I measure the position of a particle first and then the momentum of the particle. I will get a certain set of results.

If you measure the momentum of the particle first and then the position of the particle this is also different because position of momentum do not commute with each other but, if I have 2 commuting operators I can do the measurement in either direction. In either way I can measure a and b, a first and b second or b first and a second and if a and b commute I will get a same set of results.

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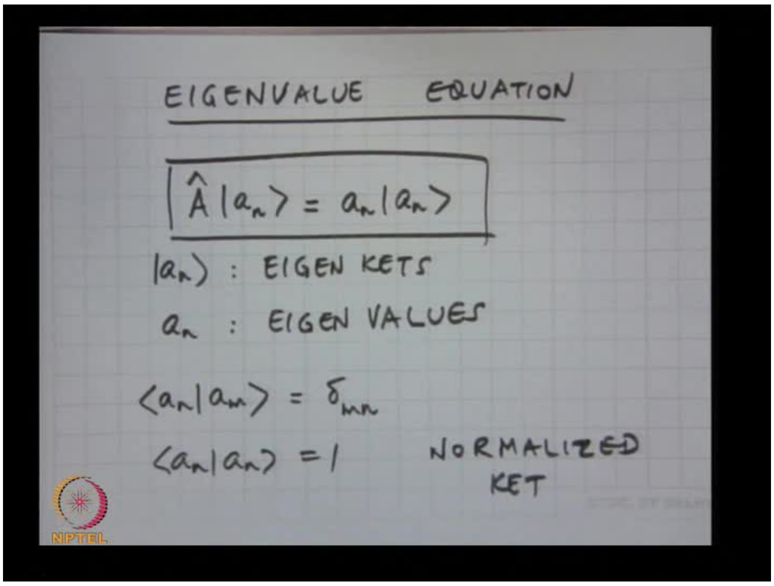

That is the physics behind this commutator and noncommutator operators, and actually this comes from this the introductions are commuted in a bracket it comes from the Hamiltonian formulation of classical mechanics. Now, I have to introduce what is called as an eigenvalue equation. You are all familiar with eigenvalue equations in matrices. So, here also we introduce an eigenvalue equation, if an operator a operates on a ket which I just denote by a n if it gives me the same ket with a number.

This is a number this operator operating on this ket vector gives me the same ket vector with some number outside. So, I am just using the same symbol for the state representing a n ket and the value a n this is called an eigenvalue equation, and this a n are called the eigen kets. These quantities a n are called the eigenvalues. This is similar again matrices if you have a column vector, if you have a square matrix you can actually calculate the eigenvalues of the matrix by this kind of an equation and you can show that this eigen vectors are such that when they operate, when you operate on eigen vector, you get the same eigen vector with an eigenvalue. Now, a could be any operator if it is an observable first a n are the values that you will measure of the observable of the of the quantity corresponding to the operator a.

Suppose, a represents momentum operator then if you are given a state of the system the measurement of the momentum. So, I will have to calculate what are the momentum, what are the equations corresponding to this, I will replace a by the momentum operator, and calculate the eigenvalues and eigen vectors. Any measurement of momentum of the system will give me one of the eigenvalues, the eigenvalues are the values that I will measure when I measure that quantity on the system and because all observables are real a has to be Hermitian operator because Hermitian operators have real eigenvalues. For a given state the eigenvalues are the values that I will measure, of the observable which means if you give me any state of the system when I do a measurement of the energy of the system I will find one of the eigenvalues..

Now, quantum mechanics has to explain things like interference that takes place in a Mach Zehnder Interferometer. You can have an interferometer for electrons and you have interference effects of electrons, you have interference of light, I would have to introduce the concept that there is interference. So, there has to be the concept of superposition, linear superposition in quantum mechanics I can have states which are superposition states, there is lot of physics behind the superposition state that if you have a state if the system can be in a state a or in a state b it can be in a state of superposition of state a and state b simultaneously.

Now, when we go through the analysis we will see that it creates some kind of a conceptual problem because especially in a beam splitter situation for example, light can be either reflected or transmitted. So, light actually is in a superposition of transmitted and reflected. Classically this not possible, you will see obviously the particle is either here or here but, quantum mechanics allows the particle to be present at 2 places simultaneously. So, for any observables are represented by Hermitian operators that is a dagger equal to a, if a is an observable, if a is a a operator corresponding to an observable a dagger is equal to a and I will write an equation like this. Solve this equation to find out what are the values of a n and what are the eigen kets a n, I will find that the eigenvalues are all real and for each eigenvalue there is an eigen ket.

Now, this eigenvalues could be a discrete set which means these are some numbers some values but, they can be infinite in number or I could have a continuous values range of values for example, our particle if it is a position operator the position can be any one anywhere it is all discretized. But, if it corresponds for example, as we will discuss in the Harmonic oscillator it corresponds to energy it has a discrete spectrum.

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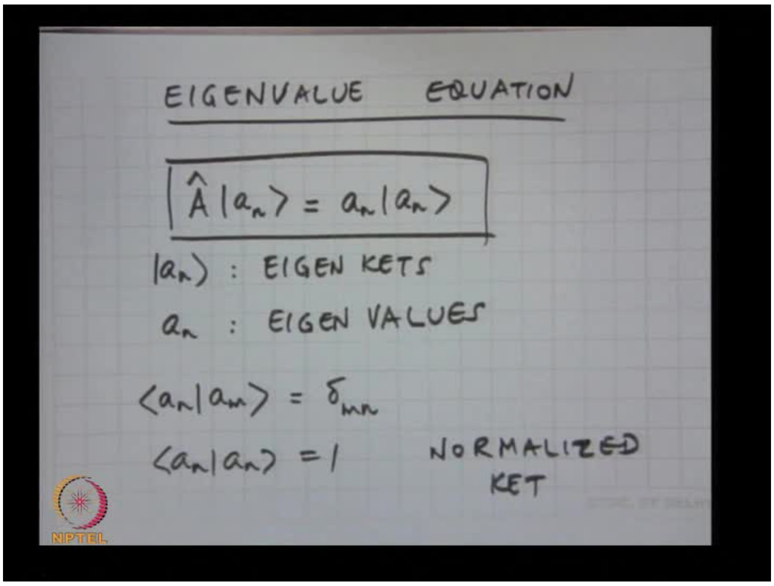

There only discrete values but, these are infinite in number that is separate problem. There are infinite possibilities but, each one of them is a particular value of the energy it is not a continuous spectrum, it is a discrete spectrum. Now, one can show that now it is possible in general that there may be more than one state corresponding to the same eigenvalue. So, if you look at matrices there is possibility that the eigenvalues repeated.

If you take a 3 by 3 matrix there will be 3 eigenvalues, 3 the eigen vectors. So, you have 2 of them which are the same which are equal, there is a possibility in a system in which you have different states corresponding the same eigenvalue. These are called degenerate states. So, this is the same eigenvalue for example, a could correspond to the energy operator also the Hamiltonian operator. If a is an energy operator then a ns will be the values of energy possible values of energy corresponding a n will be the corresponding kets that means the state of the system with that particular energy a n.

What it implies if I prepared a state in a ket a n or if I can destroyed my state which I have prepared by a ket a n it implies that it is in an eigenstate corresponding to an energy eigenvalue a n and any measurement that I do on the system will give me a value a n.

Yes mohith

Sir, corresponding to the eigenvalue separate eigenvalue a n we have (( ))

Yes

So,

But, as I said there may be more than one eigenvalue for which this the states are different the system can be 2 different states with the same eigenvalue.

Sir, Ket vector is not a state

Ket vector represent the state

So,

I could have 2 different values of the a n, 2 different kets a n and a n prime both of them having the same eigenvalue a n these are called degenerate states. They have the same energy. For example, if for every eigenvalue there is only one corresponding eigen ket these are nondegenerate states and as we will go through primarily looking at nondegenerate states and the different eigen kets are said to be orthonormal to each other, orthogonal to each other. So, this is delta m n but, delta is the chronicle delta function. So, if m is equal to n is equal to 1 otherwise it is 0. This is the meaning of orthogonality. It means independence it essentially implies that the state of a system, if the system is in a state ket a n I cannot write it as a superposition of the system state in other eigen kets. That means they are completely independent further modes of oscillation of the string if you have a mode oscillating in the first eigen, first excited state I cannot write it as a superposition of the other states of the oscillation of the of the string.

They are all independent orthonormal states, there are completely independent states. If you have a system in an eigen ket a n then it is orthonormal to all other eigen kets of the same system. That is this is in the nondegenerate case but, in degenerate case actually, I can write a linear combination of the states and make it satisfy this condition but, we will not discuss that and as I told you we usually choose, and that is why when m is equal to 1 n sorry m is equal to n. This becomes 1, this is normalization condition, and this is normalized.

If you have an eigenvalue equation like this the only values that you will ever get of a measurement of the observable corresponding to the operator a are one of the a ns. So, if you are given a system you represent the by ket and if you want to find out the eigenvalues corresponding to the energy. So, I will have the operator corresponding to energy multiplied by the ket is equal to some number multiplied by the same ket. I solve this equation and get the values of the eigen, the eigenvalues and the eigen kets. It implies that any measurement of energy if a represent the energy operator then any measurement of the energy can only give me one of the values a n that I have calculated from this equation.

Yes

Sir, basically a n is an Eigen ket

A n ket is an Eigen ket

So, the general state of the system will be like a which will be superposition of all that

I will come to it. Yeah but, first what I am saying is if you are given a system any arbitrary system corresponding to this description and if I measure the energy of that system I will always get one of the eigenvalues I cannot get anything else because that these are the only possible values of energy that this system can ever have.

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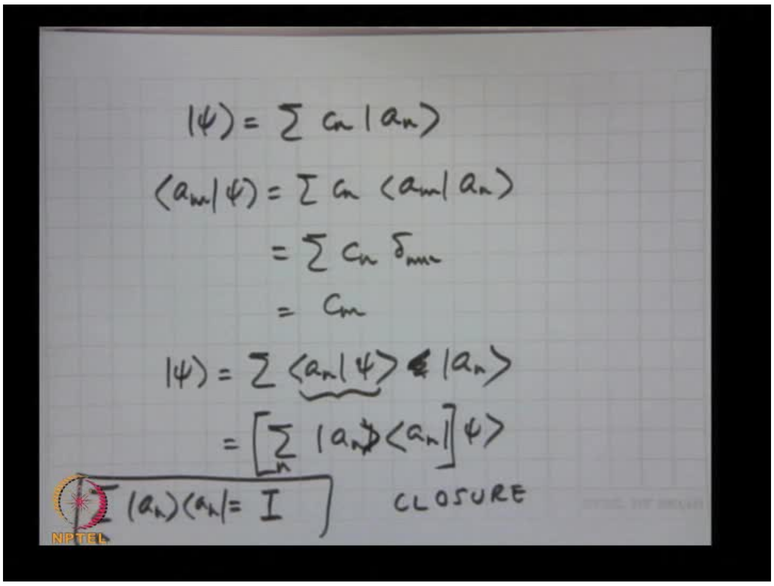

Now, this kets form a complete set of functions kets. So, any state of the system can be written as the superposition of this eigen kets here psi is a general state of the system a ns are the eigen kets of the operator. So, any state of the system and this is an important superposition principle this I am assuming superposition that means a n satisfy the equation and psi also satisfies the equations of quantum mechanics.

Now, you can actually use this condition to calculate the values of c n by multiplying by a m on the other side for example, so, I can multiply a m psi is equal to sigma c n, a m, a n and this is equal to sigma c n delta m n which is equal to c m. So, the expansion coefficients are actually given by this projection. So, I can write psi as sigma a n psi sorry a n. So, this is c n, this can also be written as sigma a n a n psi actually, this is the sigma the sigma is over n this must be identity because psi is equal to something into psi. So, this must be, this implies sigma n, a n, a n is equal to 1. Please remember, this is an operator this left product ket multiplied by bra is an operator this is not equal to bra multiplied by ket with this for different that is an operator because you can operate with this on a ket and you will get another ket this is called the closure property and this means essentially that these kets form a complete set of eigen functions or eigen kets and any ket any state of the system can be written as the superposition of this eigen kets.

This is some kind of a vector picture in terms of ket vectors etcetera. So, in quantum mechanics what quantum mechanics says is first thing is that the state of any system will be represented by a ket and that ket contains all the information that you can never have about the system. Now that is not sufficient for me I must for example, I need to know what if I give you a certain state with a certain ket what will I get if I measure the energy of the system. So, I define the expectation value.

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Now, you see what is happening is that although ket a ns are eigen kets of the system which means that if I had a system prepared in a state of a n ket if I measure for example, suppose a n ket corresponds to the to this eigenvalue and a happens to be the energy operator or the Hamiltonian operator then it means that if my state is prepared in a state a n ket when I measure the energy I will get the eigenvalue, I will get the value a n as the energy.

See, every time I measure in quantum mechanics because they act the measurement itself disturbs the system what I need to do is I need to prepare a large number of identical systems and I measure the energy of this, energy of this, energy of this, energy of this because if I measure the energy of this I already disturb the system. So, I cannot measure the energy again of that system to get a proper value.

So, what I do is I prepare an ensemble of systems, a large number of identical systems measure the energy here, energy here, energy here, energy here and then I may get the same value or I may get different values if all the measurements will give me the same value that means my system is prepared in one of the eigen kets corresponding to that operator.

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If all my ensemble system, if I measure the energy here, energy here, energy here I get the same value E 1 in all cases that means the state which I have prepared is in one of the eigen states of the Hamiltonian operator and that is why I am getting the same energy value every time. But, if I have a system prepared in a superposition state like this each measurement may give me different value.

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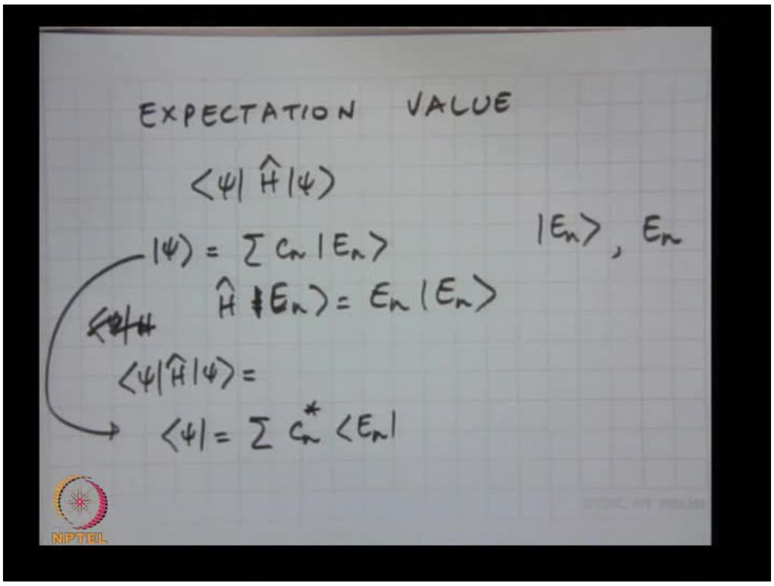

What will happen is the act of measurement mix the quantum system jump into one of the eigenstate of the system and if it jumps to the ket a n I will get a n, if it jumps to an eigen state a m I will get a m if it jumps to a 0 I get a zero eigenvalue etcetera. So, there is the finite probability of measuring each value of energy in the entire ensemble. Each measurement may give me different value, I define what is called as expectation value and I will define expectation value as for example, if an operator H, this is the expectation value it means that this is the value of the quantity average value of the quantity corresponding to the operator H that I will obtain on measurement of the systems.

Such postulates, I say that if you have an observable and H represents a corresponding operator of that observable. If you prepare a state of the system in a ket psi then this is the average value of the quantity corresponding to the observable, if H represents energy operator Hamiltonian operator then this is the average value of energy that I will measure among the ensemble states.

I measure the energy of each state and then I take an average, and I will get this expectation value. Let me for example, use this equation to simplify, let me assume that psi is generated in a state which is let me call this. Now, let me write because I am looking at energy instead of just a let me write E n. E n is the ket, E n is the eigen state of the Hamiltonian operator or the energy operator with an eigenvalue E n, ket E n represents the state of a system one of the eigen kets corresponding to an eigenvalue E n and because the state of the system can be written as a superposition of the states this is c n times E n. So, what happens to this H, this because E ns are the eigen kets this also implies that sorry, H into E n is equal to E n E n.

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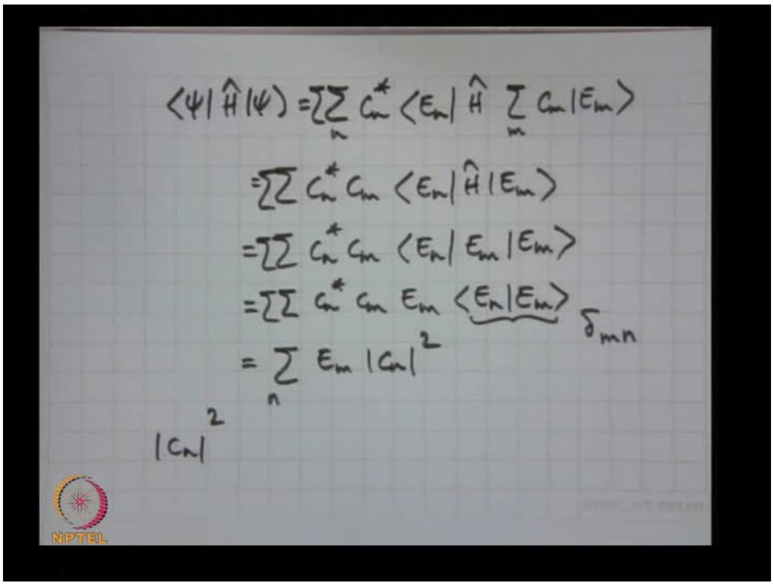

If H is a Hamiltonian operator or the energy operator then H E n is E n times E n. So, I will, if I calculate this quantity psi H psi, now please note from here psi is equal to sigma C n star E n. So, let me calculate this quantity psi H psi this is equal to sigma n, C n star, E n, H, sigma m, C m, E m the summation indices are different this is psi bra which is from here H vector H ket for the H operator here and then psi ket which is C m E m. So, this is equal to C n star C ns are just numbers I will just take it out here I get E n, H, E m which is equal to C n star, C m, E n. Now, H operating on E m simply gives me the eigenvalue E m E m and because this is the number sorry there are 2 sums here double summation n and m, C n star, C m, E m, E n, E m and what is this, this is delta m n. So, this is equal to, if I use the chronicle delta with this double sum this would be a single sum and I get E m, do mod.

That’s delta m n and one of the sums goes off with m replace by n and I get E m mod C n square. This implies because this is expectation value of energy, please note that the only values of energy that I can get on measurements or the values E ms, E 0, E 1, E 2, E 3 etcetera and because this is an average mod C n square must be the probability of finding the particle in the state corresponding to E m E n which this is equal to what is C n, can you tell me what is C n from this equation.

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Weightage of weightage given to

Yeah but, weightage give me a mathematical symbol for C n

Ok. That is E n psi

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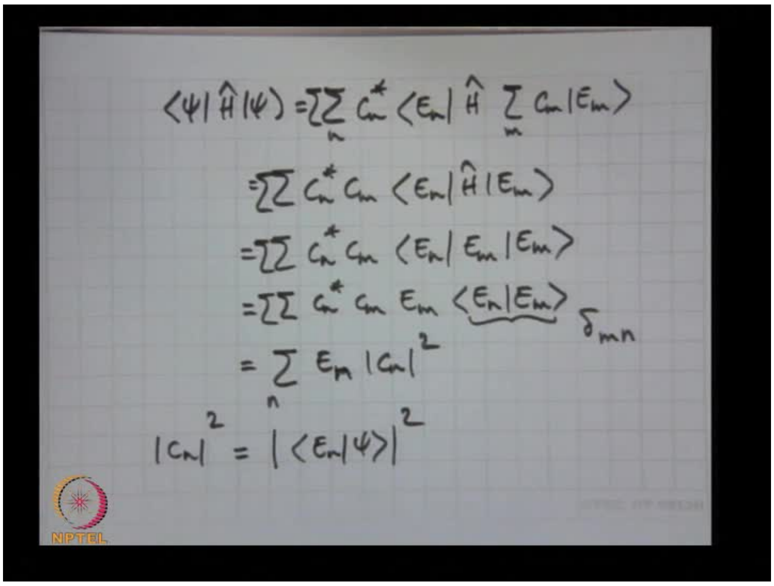

E n psi, this is mod, this is the probability of finding the system in the state corresponding to energy E n. The system can be in a state where it is in a multiple state of various energy simultaneously because classically this is not possible the system has some energy. What is the energy of the system, I can only calculate an average energy, the energy is the state when the system is in the state psi given by this equation. It is in a superposition state of various energy levels and that is a very strong statement in quantum mechanics because you allow, you see in waves there is a problem you can have a wave in which you can have a string oscillating in the fundamental and the first excited mode simultaneously.

But, for particles, for system like that the particle must have some energy, some definite energy but, this one says that the system is in a superposition of different energy values whenever you measure the energy of that system you will get one of the eigenvalues. The probability of getting E n as the measured value is given by E n bracket E n psi mod square. And because the probability of finding energy, any energy value, any one of the energy values is 1 mod C n square must be sum over it must be equal to 1. So, that is essentially the normalization condition again.

Sir

Yeah

(( ))

No. For example, the point is whenever you measure the energy you will find at one of the energy eigenvalues but, to be able to explain interference effects you have to have system in superposition states for example, in yesterday's class when I told about the Mach Zehnder Interferometer. The light seems to be, if I quantize light and talk all in terms of photons, it seems to be or if I can build a similar interferometer for the electrons for example, the electrons seems to be in both path simultaneously.

They have build double slit interference pattern for huge atoms like C 60 or this is a you cannot imagine C 60 as a wave, right? It contains 60 carbon atoms, it is a huge molecule people have performed interference experiments, double slit interference experiments which is 60 and u c interference.

So, it looks as if the C 60 particle or C 60 object is simultaneously passing through both slits if I can use this terminology the problem is we imagine in terms of particles and then we trying to understand what is happening. So, these are quantum objects, interference effects cannot be explained if you do not have superposition or diffraction effects they are all coming superposition effects. So, please remember I have prepared all the states in an identical state all the ensemble I have say a million such identical states all identical but, each measurement gives me different eigenvalue, each measurement gives me different value of energy it is like that Venice water experiment I send photon after photon, I am sending identical photons the system I am not changed anything but, sometimes the photon is picked up by detector sometimes it is picked up by a detector. There are 2 possibilities either the photon goes in this arm or the photon goes in this arm. Actually, the photon is going in both arms but, your act of measurement you are trying to detect and will appear here or here.

Neither the either state

Yes. Of the output, the fact that you are the system is in a superposition state of course,,, this is after all quantum mechanics, today explains all observed phenomenon is it right or wrong I do not know you come with a theory which explains all observed phenomenon which is not the same as quantum mechanics your theories as good.

Until somebody finds out that one of them does not explain something or one of them predict something and that is not verified experimentally. You see when I say there is an uncertainty in measurement of positional momentum of a particle does the particle have a position momentum or not is it that I am unable to measure it precisely or is it that I cannot even define a position momentum for the particle. There is nothing like position momentum you measure you get a position value, you measure you get a momentum value, the observer the act of observation creates a value for position and a value for momentum.

Before measurement is there a position or is there a momentum, right? So, we will continue in the next class what I will do is I will briefly review the harmonic oscillator problem which you must have discussed earlier but, because I will show you that the electromagnetic wave can be written can be considered as a superposition of an infinite number of harmonic oscillators.

Harmonic oscillator plays a very important role in the quantization of light and the photon that I will, that we will actually discuss are all states of harmonic oscillator but, with respect to the electromagnetic field and not with respect to particle which is vibrating. We will stop here, any questions?

So, I would urge you to go back and sort of revise the quantum mechanics because may be there may be some things which we will use later on which I may not be covering right now but, as we go through at those stages wherever we get some different concepts we will have to use them. Thank you.