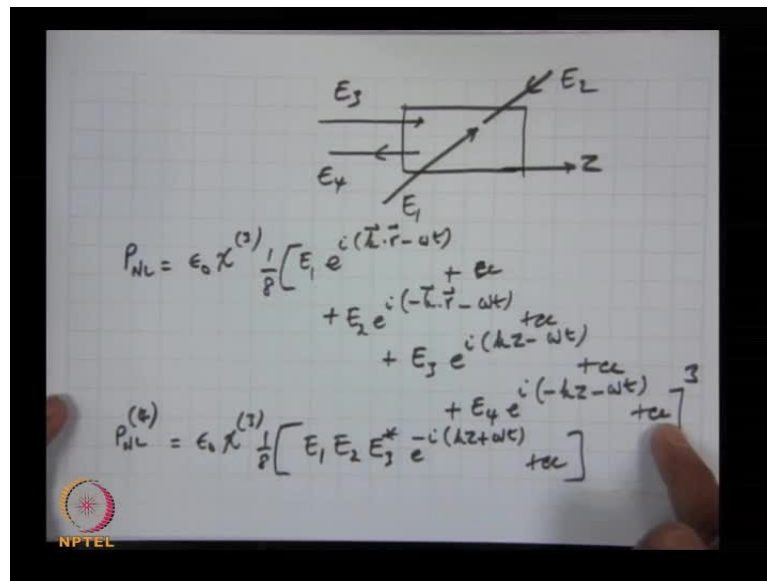


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**Module No. # 04**  
**Third Order Effects**  
**Lecture No. # 23**  
**Third Order Non - Linear Effects (Contd.)**

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So, today we will finish our discussion on phase conjugation, which we started discussing in the last class. **So**, let us recall that what I showed you is if you had a medium possessing a finite  $\chi^3$  and you send 2 beams in exactly opposite directions  $E_1$  and  $E_2$  waves and then you send another  $E_3$  in this some other direction then there is a non-linear polarization generated in the medium which can generate a wave going in the reverse direction to  $E_3$ .

The phase depends of  $E_4$  is exactly a phase conjugate of the phase depends of  $E_3$ . **So**, that  $E_4$  is called a phase conjugate wave and this process in which I use the  $\chi^3$  nonlinearity to generate a phase conjugate wave is called phase conjugation here.

Now there other techniques to develop to generate phase conjugated waves but, this is one interesting technique so, let us try to find out what is the condition under which I will generate  $E_4$  and what kind of amplitudes where can I get for  $E_4$ .

What we need to do is look at the wave equation satisfied by  $E_4$ . We should know the non-linear polarization generated for the  $E_4$  of electric field and substitute in the wave equation. **So**, the total non-linear polarization is given by  $\epsilon_0 \chi^{(3)} E_1^2 E_4$  of say  $E_1$  exponential  $i \mathbf{k} \cdot \mathbf{r} - \omega t$ .

I am assuming all fields to have the same frequency plus complex conjugate plus  $E_2 E_3$  to the power  $i \mathbf{k} \cdot \mathbf{r} - \omega t$  plus complex conjugate plus  $E_3$  exponential  $i k_z z - \omega t$  plus complex conjugate  $y$  this is  $z$  direction.

Please note that I showed you there is non-linear polarization that will try to generate a wave in the reverse direction. So I also add another electric field which is  $\mathbf{k} z - \omega t$  plus complex conjugate whole cube.

This is the wave which is going in this direction the second term is the wave going in the reverse direction exactly opposite to  $E_1$ ,  $E_3$  is the wave which is coming in some direction which I call as  $z$  direction and  $E_4$  is the phase conjugated wave.

Yesterday I showed you that the presence of these three terms generates a non-linear polarization at the frequency  $\omega$  and that polarization will try to generate a wave in the direction of exactly opposite to  $E_3$ .

**So**, assuming that such a wave can be generated I have added that also into this total electric field expression so from here what I need to get is an expression for non-linear polarization at  $E_3$  corresponding to  $E_3$  and corresponding to  $E_4$  fields.

That is the what is the non-linear polarization that will generate the wave going along the  $E_3$  direction and what will be the non-linear polarization corresponding to the  $E_4$  electric field direction.

**So**, now if you take a cube of this for example; if I want to calculate the non-linear polarization corresponding to the fourth wave this is equal to  $\epsilon_0 \chi^{(3)} E_1^2 E_4$  so, I will if I multiply this first term this term and the complex conjugate of this term I get the term corresponding to the exponential  $i \mathbf{k} \cdot \mathbf{r} - \omega t$ .

If I multiply this term this term and the complex conjugate of this term. So, I will have  $E_1 E_2 E_3^*$  into exponential minus  $i k z$  minus  $\omega t$  plus  $\omega t$  plus complex conjugate.

What will be the non-linear polarization at the for the wave corresponding to  $E_3$  so, that will be obtained by multiplying this and complex conjugate of  $E_4$  term because, the complex conjugate of this will have plus  $i k z$ .

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Handwritten equation on a grid background:

$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} [ \quad ]$$

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Handwritten diagram and equations on a grid background:

The diagram shows a rectangular box with a horizontal  $z$ -axis. Four electric field vectors are shown:  $E_1$  (diagonal),  $E_2$  (diagonal),  $E_3$  (horizontal), and  $E_4$  (horizontal).

Below the diagram, the following equations are written:

$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 e^{i(\vec{k} \cdot \vec{r} - \omega t)} + E_2 e^{i(-\vec{k} \cdot \vec{r} - \omega t)} + E_3 e^{i(kz - \omega t)} + E_4 e^{i(-kz - \omega t)} \right]^3$$

$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 E_2 E_3^* e^{-i(kz + \omega t)} + E_4 \right]$$

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And so I will have essentially the non-linear polarization corresponding to the third term is actually  $\epsilon_0 \chi^{(3)} \frac{1}{8}$  of again or there will be factor of 6 here I miss the factor of 6 here.

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A handwritten equation on a grid background: 
$$P_{NL}^{(2)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ 6 E_1 E_2 E_4^* e^{i(kz - \omega t) + \text{cc}} \right]$$

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A diagram showing a coordinate system with a z-axis. Four electric field vectors are shown:  $E_1$  (diagonal),  $E_2$  (diagonal),  $E_3$  (horizontal), and  $E_4$  (horizontal).

Below the diagram, the following equations are written:

$$P_{NL} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 e^{i(\vec{k} \cdot \vec{r} - \omega t) + \text{cc}} + E_2 e^{i(-\vec{k} \cdot \vec{r} - \omega t) + \text{cc}} + E_3 e^{i(kz - \omega t) + \text{cc}} + E_4 e^{i(-kz - \omega t) + \text{cc}} \right]^3$$

$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ 6 E_1 E_2 E_3^* e^{i(kz + \omega t) + \text{cc}} \right]$$

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$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ 6 E_1 E_2 E_4^* e^{i(kz - \omega t) + cc} \right]$$

$$P_{NL} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 e^{i(\vec{k} \cdot \vec{r} - \omega t) + cc} + E_2 e^{i(-\vec{k} \cdot \vec{r} - \omega t) + cc} + E_3 e^{i(kz - \omega t) + cc} + E_4 e^{i(-kz - \omega t) + cc} \right]$$

This factor of 6 here in the cube so 6 times  $E_1 E_2 E_4^*$  exponential  $i k z$  minus  $\omega t$  plus complex conjugate. Please note that this non-linear polarization gets generated by the mixing of  $E_1 E_2$  and  $E_3$  this non-linear polarization gets generated by the mixing of  $E_1 E_2$  and  $E_4$ .

**So**, this non-linear polarization will be responsible for the generation of the  $E_4$  wave and, this non-linear polarization will be responsible for modification of the propagation of  $E_3$  wave. If this non-linear polarization does not exist  $E_3$  wave would have just gone straight and no  $E_4$  would have been generated because of the non-linear polarization I will generate an  $E_4$  wave because of this polarization and because of this non-linear polarization the  $E_3$  wave would also get modified correspondingly.

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$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 e^{i(\vec{k} \cdot \vec{r} - \omega t)} + E_2 e^{i(-\vec{k} \cdot \vec{r} - \omega t)} + E_3 e^{i(kz - \omega t)} \right]^3$$

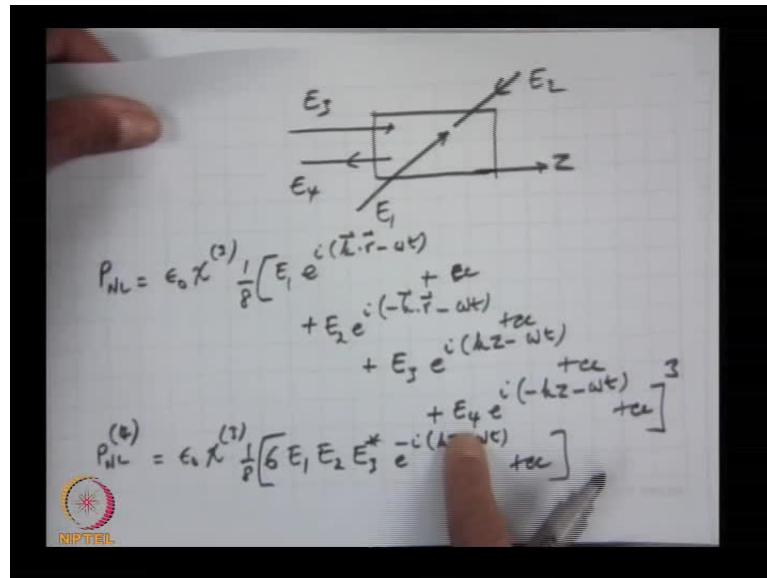
$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 E_2 E_3^* e^{i(kz - \omega t)} \right]$$

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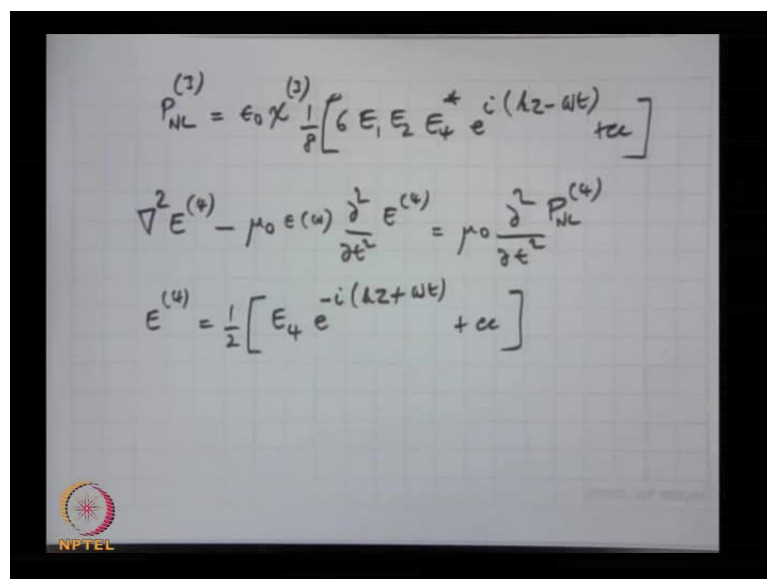
$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 E_2 E_4^* e^{i(kz - \omega t)} \right]$$

$$\nabla^2 E^{(4)}$$

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So, E 3 and E 4 gets coupled through interaction with these fields E 1 and E 2. So how do I write an equation for example; this electric field of the E 4 term so what remember we have this equation Del square let me call this I am calling let me call E 4 as the electric field this E 4 is the amplitude of electric field of this wave.

But let me call total electric field which contains a space and time dependence minus mu 0 epsilon of omega Del square by Del t square of E 4 is equal to mu naught Del square by Del t square of p non-linear corresponding to E 4.

We had use this similar equation in second harmonic generation, we had use the similar equation in parametric down conversion so, its electric field of the wave coming towards away from the medium this particular electric field wave will satisfy the total electric field will satisfy this equation where  $E_4$  is  $\frac{1}{2}$  amplitude exponential minus  $i k z$  plus  $\omega t$  plus complex conjugate.

$E_4$  is a wave going in the minus  $z$  direction  $E_3$  is a wave going at a plus  $z$  direction. So, my problem is now to substitute this on the left hand side and substitute this on the right hand side, neglect second differential of  $E_4$  with respect to  $z$  exactly like I did for second harmonic, then use the relationship between  $k$  and  $\epsilon \omega$  and simplify.

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$$P_{NL} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 e^{i(\vec{k} \cdot \vec{r} - \omega t)} + E_2 e^{i(-\vec{k} \cdot \vec{r} - \omega t) + cc} + E_3 e^{i(kz - \omega t) + cc} + E_4 e^{i(-kz - \omega t) + cc} \right]^3$$

$$P_{NL}^{(\omega)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ 6 E_1 E_2 E_3^* e^{-i(kz + \omega t) + cc} \right]$$



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$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} [6 E_1 E_2 E_4^* e^{i(k_2 - \omega t)} + cc]$$

$$\nabla^2 E^{(4)} - \mu_0 \epsilon^{(4)} \frac{\partial^2 E^{(4)}}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}^{(4)}}{\partial t^2}$$

$$E^{(4)} = \frac{1}{2} [E_4 e^{-i(k_2 + \omega t)} + cc]$$

$$-2ik \frac{dE_4}{dz} = \mu_0 (-\omega^2) \frac{3}{2} \epsilon_0 \chi^{(3)} E_1 E_2 E_3^*$$

The procedure is exactly identical to what we have done for the third the three wave mixing. **So** I substitute E 4 this expression for E 4 on the left hand side I substitute this expression for p non-linear corresponding to the electric field E 4 on the right hand side, neglect second differential of E 4 with respect to z, use an equation connecting k and epsilon omega and simplify the equation.

**So**, I will leave this substitution to you but, as you can already see the second differential I neglect, this second differential with respect to z actually gives me two terms, one of them will cancel off with this term and one coming from here **so**, what I will be left with is the following. **So** I will get minus 2 i k d E 4 by d z is equal to mu naught minus omega square into 3 by 2 epsilon 0 chi 3 E 1 E 2 E 3 star.

I leave this substitution to you it exactly the same procedure as we are done before. E 4 only depends on z **so**, del square is only z differential **so**, there will be second differential of this with respect to z which is neglected, second differential of this with respect to z which will cancel with this term and, then a product of first differential of this in first differential of this which is actually coming here.

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$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 e^{i(\vec{k} \cdot \vec{r} - \omega t)} + E_2 e^{i(-\vec{k} \cdot \vec{r} - \omega t)} + E_3 e^{i(kz - \omega t)} + E_4 e^{i(-kz - \omega t)} + cc \right]^3$$

$$P_{NL}^{(4)} = \epsilon_0 \chi^{(4)} \frac{1}{8} \left[ E_1 E_2 E_3^* e^{-i(kz + \omega t)} + cc \right]^3$$

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$$P_{NL}^{(4)} = \epsilon_0 \chi^{(4)} \frac{1}{8} \left[ E_1 E_2 E_4^* e^{i(kz - \omega t)} + cc \right]$$

$$\nabla^2 E^{(4)} - \mu_0 \epsilon^{(4)} \frac{\partial^2 E^{(4)}}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}^{(4)}}{\partial z^2}$$

$$E^{(4)} = \frac{1}{2} \left[ E_4 e^{-i(kz + \omega t)} + cc \right]$$

$$-2ik \frac{dE_4}{dz} = \mu_0 (-\omega^2) \frac{3}{2} \epsilon_0 \chi^{(4)} E_1 E_2 E_3^*$$

And of the right hand side it is simply a time differential of this term here. So what I am essentially doing is substituting the first term here substituting the first term here from here and equating on both sides.

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$$P_{NL} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 e^{i(\vec{k} \cdot \vec{r} - \omega t)} + E_2 e^{i(-\vec{k} \cdot \vec{r} - \omega t)} + E_3 e^{i(kz - \omega t)} + E_4 e^{i(-kz - \omega t)} \right]^3$$

$$= \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ 6 E_1 E_2 E_3^* e^{-i(kz + \omega t)} + \dots \right]$$

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$$P_{NL} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ 6 E_1 E_2 E_4^* e^{i(kz - \omega t)} + \dots \right]$$

$$\nabla^2 E^{(4)} - \mu_0 \epsilon^{(4)} \frac{\partial^2 E^{(4)}}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}^{(4)}}{\partial t^2}$$

$$E^{(4)} = \frac{1}{2} \left[ E_4 e^{-i(kz + \omega t)} + cc \right]$$

$$-2ik \frac{dE_4}{dz} = \mu_0 (-\omega^2) \frac{3}{2} \epsilon_0 \chi^{(3)} E_1 E_2 E_3^*$$

So, if I simplify this equation so let me simplify this equation as you can see here there is no phase mismatch term if the z dependence of non-linear polarization and the z dependence of the electromagnetic wave where different I would have got an extra exponential minus i something is here.

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$$P_{NL} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ E_1 e^{i(\vec{k}_1 \cdot \vec{r} - \omega t)} + E_2 e^{i(-\vec{k}_2 \cdot \vec{r} - \omega t)} + E_3 e^{i(kz - \omega t)} + E_4 e^{i(-kz - \omega t)} \right]^3$$

$$P_{NL}^{(4)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ 6 E_1 E_2 E_3^* e^{-i(kz + \omega t)} + \dots \right]$$

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$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ 6 E_1 E_2 E_4^* e^{i(kz - \omega t)} + \dots \right]$$

$$\nabla^2 E^{(4)} - \mu_0 \epsilon^{(4)} \frac{\partial^2 E^{(4)}}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}^{(4)}}{\partial t^2}$$

$$E^{(4)} = \frac{1}{2} \left[ E_4 e^{-i(kz + \omega t)} + cc \right]$$

$$-2ik \frac{dE_4}{dz} = \mu_0 (-\omega^2) \frac{3}{2} \epsilon_0 \chi^{(3)} E_1 E_2 E_3^*$$

But because both of them are exponential minus i k z. This is also exponential minus i k z polarization this is also exponential minus i k z so there is no phase mismatched term.

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$$\begin{aligned} \frac{dE_4}{dz} &= -i \frac{\omega^2}{c^2 2k} \frac{3}{2} \chi^{(3)} E_1 E_2 E_3^* \\ &= -i \frac{\omega^2}{c^2 \frac{\omega}{c} n} \frac{3}{4} \chi^{(3)} E_1 E_2 E_3^* \\ &= -i \frac{3\omega}{4cn} \chi^{(3)} E_1 E_2 E_3^* \\ &= -i \kappa E_3^* \\ \kappa &= \frac{3\omega}{4cn} \chi^{(3)} E_1 E_2 \end{aligned}$$

Now, I can simplify this and I get  $dE_4/dz$  is equal to minus  $i$  omega square by  $c$  square. **So**,  $2k$  into  $3$  by  $2$   $\chi^{(3)}$   $E_1 E_2 E_3^*$  **so**, this minus and plus **so**, we got a minus  $\omega \mu_0 \epsilon_0$  is one by  $c$  square so  $\omega$  square by  $c$  square and then a  $2k$ .

Therefore, this is equal to minus  $i$  omega square by  $c$  square  $k$  s  $\omega$  by  $c$  into refractive index into  $3$  by  $4$   $\chi^{(3)}$   $E_1 E_2 E_3^*$ , which is minus  $i$  omega  $3$  omega by  $4$   $c n$   $\chi^{(3)}$   $E_1 E_2 E_3^*$ . **So**, let me call this as minus  $i$  kappa  $E_3^*$ , that kappa is equal to  $3$  omega by  $4 c n$   $\chi^{(3)}$   $E_1 E_2$  in.

Here kappa is what is called as a coupling coefficient its coupling  $E_4$  to  $E_3$ . The rate of change of  $E_4$  depends on  $E_3$  through this term kappa **so**, if  $\chi^{(3)}$  is neglected if the power of very small if  $E_1$  and  $E_2$  are very small then this right hand side is very small and  $E_4$  does not get generated essentially that is a linear situation almost linear situation.

**So**, the coupling depends on the intensity of the light corresponding to the two waves which are going in opposite directions the stronger electric fields are stronger is the coupling between  $E_3$  and  $E_4$ .

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$$\frac{dE_3}{dz} = i\kappa E_4^*$$

$$P_{NL}^{(3)} = \epsilon_0 \chi^{(3)} \frac{1}{8} \left[ 6 E_1 E_2 E_4^* e^{i(k_2 - \omega t)} + cc \right]$$

$$\nabla^2 E^{(4)} - \mu_0 \epsilon^{(4)} \frac{\partial^2 E^{(4)}}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}^{(4)}}{\partial t^2}$$

$$E^{(4)} = \frac{1}{2} \left[ E_4 e^{-i(k_3 - \omega t)} + cc \right]$$

Now you can also derive the same similar equation for E 3 and you will get the following equation  $d E 3$  by  $d z$  is equal to  $i$  kappa  $E 4$  star.

**So**, I use the same procedure I use an equation for instead of E 4 equation I have an E 3 equation, I have a non-linear polarization at the corresponding to the electric field E 3 substitute simplify.

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$$\frac{dE_3}{dz} = i\kappa E_4^*$$

$$\frac{dE_4}{dz} = -i\kappa E_3^*$$

So, let me write both the equations here so  $dE_4/dz$  is equal to minus  $i$  kappa  $E_3$  star. Two equations which actually coupled  $E_3$  and  $E_4$  and the coupling coefficients contains the electric fields  $E_1$  and  $E_2$ .

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$$\begin{aligned} \frac{dE_4}{dz} &= -i \frac{\omega^2}{c^2 2k} \frac{3}{2} \chi^{(3)} E_1 E_2 E_3^* \\ &= -i \frac{\omega^2}{c^2 \frac{\omega}{c} n} \frac{3}{4} \chi^{(3)} E_1 E_2 E_3^* \\ &= -i \frac{3\omega}{4cn} \chi^{(3)} E_1 E_2 E_3^* \\ &= -i \chi E_3^* \\ \kappa &= \frac{3\omega}{4cn} \chi^{(3)} E_1 E_2 \end{aligned}$$

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$$\begin{aligned} \frac{dE_3}{dz} &= i \kappa E_4^* \\ \frac{dE_4}{dz} &= -i \kappa E_3^* \\ \frac{d^2E_4}{dz^2} &= -i \kappa \frac{dE_3^*}{dz} = -i \kappa (-i \kappa) E_4 \\ &= -\kappa^2 E_4 \end{aligned}$$

So, if I assume that  $E_1$  and  $E_2$  are constant that is do not I neglect the depletion of electric fields at the first and second frequencies the  $E_1$  and  $E_2$  waves then, I can assume kappa to be a constant and I can very quickly integrate this equation.

So, this is  $d^2 E_4$  by  $dz^2$  is equal to minus  $i \kappa d E_3^*$  by  $dz$  which is equal to minus  $i \kappa$  into minus  $i \kappa E_4$ , which is equal to minus  $\kappa^2 E_4$ .

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$$\begin{aligned} \frac{dE_4}{dz} &= -i \frac{\omega^2}{c^2 2k} \frac{3}{2} \kappa^{(3)} E_1 E_2 E_3^* \\ &= -i \frac{\omega^2}{c^2 \frac{\omega}{n}} \frac{3}{4} \kappa^{(3)} E_1 E_2 E_3^* \\ &= -i \frac{3\omega}{4cn} \kappa^{(3)} E_1 E_2 E_3^* \\ &= -i \kappa E_3^* \\ \kappa &= \frac{3\omega}{4cn} \kappa^{(3)} E_1 E_2 \end{aligned}$$

Let me write this as star because  $\kappa$  contains electric fields  $E_1$  and  $E_2$  which could have some phases in them. So I in general I can assume  $\kappa$  to be a complex because,  $E_1$  and  $E_2$  can have phase terms.

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$$\begin{aligned} \frac{dE_3}{dz} &= i \kappa E_4^* \\ \frac{dE_4}{dz} &= -i \kappa E_3^* \\ \frac{d^2 E_4}{dz^2} &= -i \kappa \frac{dE_3^*}{dz} = -i \kappa (-i \kappa^*) E_4 \\ &= -|\kappa|^2 E_4 \\ E_4(z) &= a \cos |\kappa|z + b \sin |\kappa|z \end{aligned}$$



So, when I substitute for  $E_3$  by  $E_4$  I get complex conjugate of this which is minus  $i$  into  $E_4$  so, this is  $\text{mod } \kappa^2$ . What your solution  $E_4$  of  $z$  is equal to  $a \cos \text{mod } \kappa z + b$ .

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$$E_3^*(z) = \frac{i}{\kappa} \cdot \frac{dE_4}{dz}$$

$$= \frac{i}{\kappa} \kappa (-a \sin \kappa z + b \cos \kappa z)$$

$$E_4(L) = 0$$

$$E_4(0) = E_{70}$$

$$a \cos \kappa L + b \sin \kappa L = 0$$

$$\Rightarrow a = -b \tan \kappa L$$

I can substitute this into this second equation and get the solution of  $E_3$ . So, let me write in a separate paper so,  $E_3$  of  $z$  will be  $i$  by  $\kappa$  into  $E_4$  by  $d z$  which is equal to  $i$  by  $\kappa$  into  $\text{mod } \kappa$  minus  $a \sin \text{mod } \kappa z + b \cos \text{mod } \kappa z$ .

I have just differentiated this  $E_4$  and substituted here so, that is  $E_3$  of  $z$ . Now let me try to find out the boundary conditions I need to calculate the constants  $a$  and  $b$  so, here is  $E_1$  here is  $E_2$  here is  $E_3$  coming from here and  $E_4$  is coming from here this is  $z$  is equal to  $0$  this is  $z$  is equal to  $l$ .

Now what are the boundary conditions I use.  $E_4$  what is the boundary condition on  $E_4$

$E_4$  at  $l$  is equal to Yes because beyond this point there is no medium. So any phase conjugated wave can only develop from this point onwards in the forward direction. So there is no  $E_4$  beyond this point there is a finite  $E_4$  here there is no  $E_4$  at all the backward wave get generated at the point less than  $L$  so, at  $l$  any wave at  $L$  should have come from beyond  $l$  and there is no medium beyond  $l$  and so there is no electric field  $E_4$  at  $l$ .

And of course,  $E_3$  of 0 which is wave which I am sending say  $E_{30}$ .  $E_{30}$  is the electric field of the incident wave at this plane and so, I can substitute this boundary conditions. So,  $E_3$   $E_4$  of 1 is equal to 0 implies  $a \cos \text{mod } \kappa L$  plus  $b \sin \text{mod } \kappa L$  is equal to 0 this implies  $a$  is equal to minus  $b \tan \text{mod } \kappa L$ .

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$$E_{30} = \frac{i|x|}{\kappa} b \Rightarrow b = -\frac{i\kappa}{|x|} E_{30}^*$$

$$E_3(z) = \frac{i}{\kappa} \frac{dE_4}{dz}$$

$$= \frac{i}{\kappa} |x| (-a \sin |x|z + b \cos |x|z)$$

$$E_4(L) = 0$$

$$E_3(0) = E_{30}^*$$

Now I substitute this second boundary condition in this equation and use this fact that  $a$  is minus  $b$  time  $\kappa L$  so, I will get  $E_3$  star  $E_3$  star 0 is equal to  $i \text{ mod } \kappa L$  by  $\kappa$ . So,  $z$  is equal to 0 so the first term first term goes off here. So, I will simply get  $b$  times so, this implies  $b$  is equal to minus  $i \kappa$  by  $\text{mod } \kappa L$  into  $E_{30}$  star.

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$$E_{30}^* = i \frac{|\kappa|}{\kappa} b \quad \Rightarrow \quad b = -i \frac{\kappa}{|\kappa|} E_{30}^*$$

$$a = i \frac{\kappa}{|\kappa|} E_{30}^* \tan(\alpha L)$$

$$E_4(z) =$$

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$$E_3(z) = \frac{i}{\kappa} \frac{dE_4}{dz}$$

$$= \frac{i}{\kappa} |\kappa| (-a \sin(\alpha z) + b \cos(\alpha z))$$

$$E_4(L) = 0$$

$$E_3(0) = E_{30}$$

$$a \cos(\alpha L) + b \sin(\alpha L) = 0$$

$$\Rightarrow a = -b \tan(\alpha L)$$

So, let me get the final solution for  $E_4$  of  $z$ .  $E_4$  of  $z$  is equal to now  $a$  is minus  $b \tan(\alpha L)$ . So, this implies  $a$  is equal to  $i \kappa / |\kappa| E_{30}^* \tan(\alpha L)$ . I have substituted the value of  $b$  in this equation to get the value of  $a$  in terms of  $E_{30}^*$ .

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$$E_{30}^* = i \frac{|\kappa|}{\kappa} \left( b \right) \Rightarrow b = -i \frac{\kappa}{|\kappa|} E_{30}^*$$

$$a = i \frac{\kappa}{|\kappa|} E_{30}^* \tan(\kappa L)$$

$$E_4(z) =$$

$$E_4(z) = i \frac{\kappa}{|\kappa|} E_{30}^* \left( \tan(\kappa L) \cos(\kappa z) \mp \sin(\kappa z) \right)$$

NPTEL

So,  $E_4$  of  $z$  is equal to now  $i \kappa$  by mod  $\kappa$   $E_{30}^* \tan \text{ mod } \kappa L \cos \text{ mod } \kappa z$  plus so minus  $\sin$ . I have substituted the value of  $a$  and  $b$  here this  $i$  times  $\kappa$  by mod  $\kappa$   $E_{30}^*$  is common and I get a  $\tan \text{ mod } \kappa L$  in the first term and a minus one is the second term because of minus sign.

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$$E_{30}^* = i \frac{|\kappa|}{\kappa} \left( b \right) \Rightarrow b = -i \frac{\kappa}{|\kappa|} E_{30}^*$$

$$E_3^*(z) = \frac{i}{\kappa} \cdot \frac{dE_4}{dz}$$

$$= \frac{i}{\kappa} |\kappa| (-a \sin(\kappa z) + b \cos(\kappa z))$$

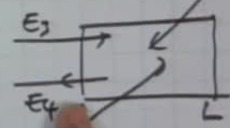
$$E_4(L) = 0$$

$$E_3(0) = E_{30}^*$$

$$a \cos(\kappa L) + b \sin(\kappa L) = 0$$

$$\Rightarrow a = -b \tan(\kappa L)$$

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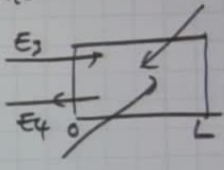
$$E_3^*(z) = \frac{i}{\kappa} \frac{dE_4}{dz}$$

$$= \frac{i}{\kappa} |\kappa| (-a \sin(\kappa z) + b \cos(\kappa z))$$

$$E_4(L) = 0$$

$$E_3(0) = E_{30}^*$$

$$a \cos(\kappa L) + b \sin(\kappa L) = 0$$

$$\Rightarrow a = -b \tan(\kappa L)$$


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$$E_{30}^* = \frac{i |\kappa|}{\kappa} b \Rightarrow b = -\frac{i \kappa}{|\kappa|} E_{30}^*$$

$$a = \frac{i \kappa}{|\kappa|} E_{30}^* \tan(\kappa L)$$

$$E_4(z) =$$

$$E_4(z) = \frac{i \kappa}{|\kappa|} E_{30}^* (\tan(\kappa L) \cos(\kappa z) + \sin(\kappa z))$$

$$E_4(0) = \frac{i \kappa}{|\kappa|} E_{30}^* \tan(\kappa L)$$

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What I am interested in is what is the value of E 4 at this point because beyond this once E 4 comes out it just going to propagate backwards there is no more generation of E 4 or modification of E 4 so, what is in its E 4 of 0. E 4 of 0 is equal to i kappa by mod kappa E 3 0 star z is equal to 0 that is 0.

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$$E_3^*(z) = \frac{i}{\kappa} \cdot \frac{dE_4}{dz}$$

$$= \frac{i}{\kappa} |\alpha| (-a \sin(\alpha z) + b \cos(\alpha z))$$

$E_4(L) = 0$   
 $E_3(0) = E_{30}$

$a \cos(\alpha L) + b \sin(\alpha L) = 0$   
 $\Rightarrow a = -b \tan(\alpha L)$

The diagram shows a transmission line of length  $L$ . The incident wave is labeled  $E_3$  and the reflected wave is labeled  $E_4$ .

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$$E_{30}^* = \frac{i|\alpha|}{\kappa} b \Rightarrow b = -\frac{i\kappa}{|\alpha|} E_{30}^*$$

$$a = \frac{i\kappa}{|\alpha|} E_{30}^* \tan(\alpha L)$$

$E_4(z) =$

$$E_4(z) = \frac{i\kappa}{|\alpha|} E_{30}^* (\tan(\alpha L) \cos(\alpha z) + \sin(\alpha z))$$

$$E_4(0) = \frac{i\kappa}{|\alpha|} E_{30}^* \tan(\alpha L)$$

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$$R = \left| \frac{E_4(z)}{E_3(z)} \right|^2 = \tan^2 \kappa L$$

$$E_3^*(L) = i \frac{\kappa}{\kappa_0} \left( -i \frac{\kappa}{\kappa_0} E_{30}^* \tan \kappa L \sin \kappa L \right) = i \frac{\kappa}{\kappa_0} E_{30}^* \cos \kappa L$$

So, this is the suppose I were to interrupted this as a mirror. How will I define the reflection coefficient  $E_3(0)$  is incident and  $E_4(0)$  is coming out. So, the reflection coefficient the energy reflection coefficient of this mirror would be  $r$  is equal to mod of  $E_4(0)$  by  $E_3(0)$  mod square which is equal to.

So, if  $\tan \kappa L$  is more than 1 this is more than 1. We add an approximation what approximation can be data involved.

I have not put any approximation of  $\cos \kappa L$ . I have only put an approximation  $E_1$  and  $E_2$  are come I am not differentiate  $E_1$  and  $E_2$  with respect to  $z$  that is no pump depletion.

First thing is you notice that if  $\cos \kappa L$  is more than 1 the reflection coefficient is more than 1. So it is an amplifying mirror it is just like phase conjugates and amplifies so it is a very interesting mirror because it amplifies.

And secondly what happens if  $\kappa L$  is equal to  $\pi/2$ , this becomes infinite. What is it mean, you can get a reflected signal without putting an input signal. This is the condition for oscillation, this medium starts to oscillates its starts to generate wave on its own because even if there is a slightest noise which enters as  $E_3$ .

Because the reflection coefficient is very high as  $\kappa L$  tends to  $\pi/2$  that is not  $\pi$  because the reflection coefficient is very high the forward going wave which is a very small wave generates a very strong backward wave which is then actually also generates strong forward wave because we have not rotated the second solution yet.

**So**, what is going to happen is as  $\kappa L$  tends to  $\pi/2$  this medium starts to resonate and start to generate waves. Now these waves are obviously coming from those two waves the energy that is generated here is from these two waves. Now let me look at the other solution  $E_3$  of  $z$ .

**So** I am interested in  $p$  of  $L$  which is what is coming out. So,  $E_3$  star of  $E_3$   $E_3$  star of  $E_3$  is actually  $i \kappa L$  by  $\kappa$  into minus  $a$  so, minus  $a$  is let me substitute from here minus  $i \kappa L$  by  $\kappa$   $E_3(0)$   $\tan \kappa L$  into  $\sin \kappa L$ , plus  $b$  which is which is minus  $i \kappa L$  by  $\kappa$   $E_3(0)$  into  $\cos \kappa L$ .

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$$P = |E_4(z)|^2$$

$$E_3^*(z) = \frac{i}{\kappa} \frac{dE_4}{dz}$$

$$= \frac{i}{\kappa} (\kappa) (-a \sin \kappa(z+b) \cos \kappa(z))$$

$E_4(L) = 0$   
 $E_3(0) = E_{30}$   
 $a \cos \kappa(L+b)$   
 $a = -b$

The diagram shows a transmission line of length  $L$ . An incident wave  $E_3$  is shown with an arrow pointing right. A reflected wave  $E_4$  is shown with an arrow pointing left. The total field is  $E_3 + E_4$ .



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$$R = \left| \frac{E_4(z)}{E_3(z)} \right|^2 = \tan^2 \kappa L$$

$$E_3^*(L) = i \frac{\kappa}{\kappa} \left( -i \frac{\kappa}{\kappa} E_{30}^* \tan \kappa L \sin \kappa L + \cos \kappa L \right)$$

$$= \frac{E_{30}^*}{\cos \kappa L}$$

I am just substituting into this equation which is the expression for  $E_3^*$  of  $z$  the values  $a$  and  $b$  which we have found out and at  $z$  is equal to  $L$  that is a transmission coefficient. The mirror is the region between  $z$  is equal to  $0$  and  $L$  so, wave essential from here it is generates a wave in this direction reflection coefficient and the transmission coefficient here.

**So**, let me simplify this. So, this is  $\cos \kappa L$  by  $\kappa$  cancels off with the minus  $i$  coming out that become plus  $1$  so,  $E_{30}^* \tan \kappa L \sin \kappa L + \cos \kappa L$  is one by  $\cos \kappa L$ .

Because you have  $\tan$  is  $\sin$  by  $\cos$  to  $\sin^2 + \cos^2$  by  $\cos \kappa L$  and that is simply  $E_{30}^* \sin^2 \kappa L + \cos^2 \kappa L$ . And  $E_3$  at the output is always bigger than  $E_3$  at the input it is an amplifier, it is like a it is like a parametric amplifier the energy for this is coming from the two pump waves which are  $E_1$  and  $E_2$ .

**So**, what is the actually happening is this  $E_1$  and  $E_2$  waves are mixing together with the  $E_3$  wave and the generating waves correspond to  $E_3$  and  $E_4$ .

And so, every time you generate at an  $E_3$  and  $E_4$  please remember all of them have same frequencies. So,  $1$  photon at  $E_1$   $E_1$  from this wave  $E_1$  photon  $1$  photon from here will dissipate and generate  $E_1$  photon here  $1$  photon here.

So, this is a very interesting mirror it is a very interesting device which it is called a phase conjugator and if you satisfy certain conditions you can actually make it resonate it starts to it start to it becomes an oscillator because, if you satisfy  $\text{mod } \kappa L$  it of the order of  $\pi$  by 2 then you can have this medium generating waves in two directions.

So, this is one interesting aspect of 3 wave mixing that the 4 wave mixing that we will discussing and this has applications in many areas like as I have mentioning yesterday on in correcting distortions of waves as they pass through distorting media because, this one is a phase conjugating wave medium.

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$$E_3^*(z) = \frac{i}{\kappa} \cdot \frac{dE_4}{dz}$$

$$= \frac{i}{\kappa} (\alpha) (-a \sin(\alpha z) + b \cos(\alpha z))$$

$$E_4(L) = 0$$

$$E_3(0) = E_{T0}$$

$$a \cos(\alpha L) + b \sin(\alpha L) = 0$$

$$\Rightarrow a = -b \tan(\alpha L)$$

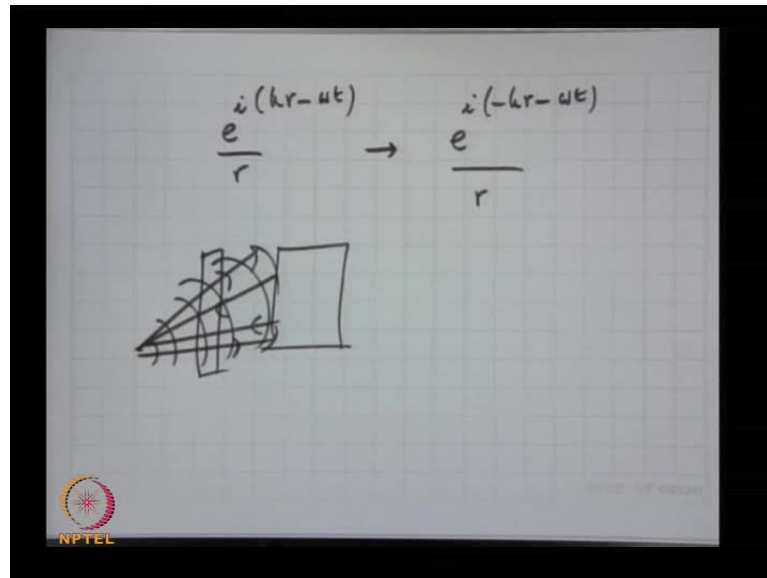
The diagram shows a rectangular medium of length  $L$ . An incident wave  $E_3$  is shown entering from the left, and a reflected wave  $E_4$  is shown exiting to the left. A diagonal line represents a phase conjugator interface.

**So**, as I mentioned to you what I have done here in the class is essentially, showing that a plane wave incident generates a phase conjugate at plane wave but, please note that any wave can be written as a superposition of plane waves propagating in different directions.

**So**, this phase conjugate mirror will phase conjugate each one of the plane waves and, you can show that for an arbitrary input field into this medium you will exactly generate the phase conjugated wave of that wave irrespective of what the wave is.

**So**, if it is the divergence spherical wave it hits and become it is a converging spherical wave. A divergence spherical wave is how do I write a expression for divergence spherical wave exponential.

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$i k r$  minus  $\omega t$  by  $r$ . So, this will become exponential  $i$  minus  $k r$  minus  $\omega t$  by  $r$ . A diverging spherical wave becomes a converging spherical wave so, if you had this medium like this and if you had wave coming here it reflects and come back to the same point irrespective where it is.

Therefore, you can actually use this to compensate for distortions if you have a distortion. Here what will happen is if there is distorting medium in between in the forward direction the wave gets distorted because a phase conjugate comes back that distortion exactly gets cancelled by propagating through the medium again its, if it was a normal mirror the distortion will add this is a phase conjugate mirror.

Because it is a phase conjugated wave returning the distortion in the forward direction gets cancelled by the reverse propagation direction. **So**, what you get back here is exactly the point source or point image again the entire wave focus us here if it was a normal mirror you would have got double the distortion.

**So**, where I am not discussing more details of this but, there are very interesting applications of this and there are different methods to produce this conjugation and what I have shown here is how to use  $\chi^3$  in a medium to generate a phase conjugation phase conjugate situation.

Now, with that we complete the discussion on classical aspects of non-linear optics. **So**, you have essentially seen second harmonic generation we have seen parametric down conversion parametric amplification parametric oscillation and with the chi 3 effects we started look at self-wave modulation cross wave modulation 4 wave mixing and so on.

In the 3 and in the 4 wave mixing we did not discuss too much but, this gives you a flavor of the effects of nonlinearity generated by the chi 3 in the medium. **So**, next what we will like to I would like to do is to get into the quantum aspects of light.

What we will do is first we will look at quantizing the electromagnetic field and studying some other properties of the quantum nature of light.

The question arises why should I quantize like as I yes, we have already seen for example, parametric down conversion classical picture does not predict the existence of this classical picture does not allow you to generate spontaneous emission. And plus there are many other experiments which people started to do some time back which have no classical explanation.

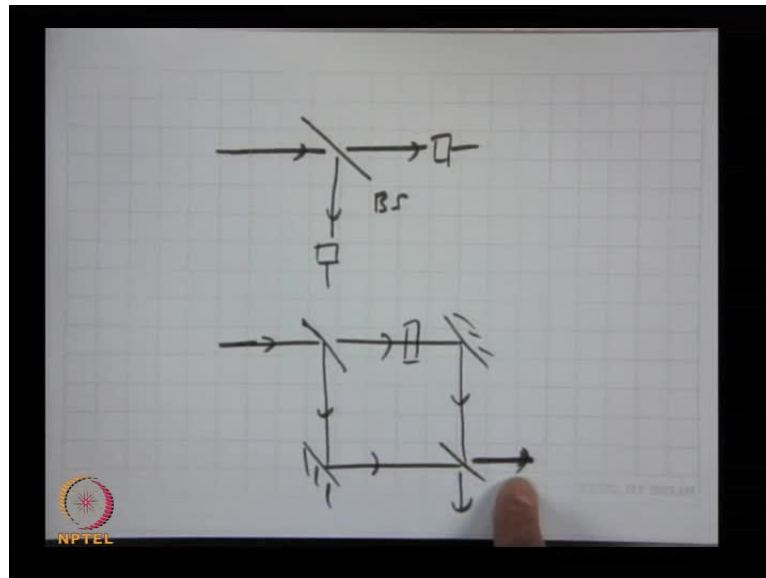
There is what is called as a semi classical theory where you treat material as a quantized atom are quantized but, light as a classical field it is called semi classical. A full quantum theory implies that your media quantized the atoms are quantized and light is also quantized.

And this complete quantum mechanical picture explains all experimentally observed dissolves. And so if when you do an experiment what are the results you are getting from their experiment are explainable from the picture of complete quantization.

**So**, there are wherever semi classical analysis phase different results it is a quantum theory which is the correct result. In most situations may be semi classical and quantum predict very similar results.

**So**, that is fine but, then if I need to get a correct interpretation I need to use the quantum nature of light which is essentially I need to quantize light. Now before we start to look at the quantum quantization of electromagnetic field, let me let us start looking at some simple experiment.

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Here I have example; I take a beam splitter. This is a this a device in which you send light from here it partly transmits and partly reflects. So depending on the transmission coefficient and reflection coefficient you can have 50 percent 70 percent 30 percent etcetera.

Let me take a 50 percent beam splitter. **So**, if I send 1 milliwatt of light here if I assume there is no absorption there is no other effect I will have half a milliwatt coming here and half a milliwatt going here.

Now I continue to reduce the intensity of this light. **So**, I come to a situation where then testing becomes so small that there are single photons coming now. I still do not know what the photon is but, let me assume that the intensity has gone so low and in fact there are now sources are available with send single photons that means the energy is  $h \times \omega$ .

What will happen if I start to I send single photons from here. Half of detector here, **so** 50 percent of them will get reflected and 50 percent of them will get transmitted. This beam splitter cannot split the photon, the photon is either reflected or transmitted. I have no way of predicting whether the next photon is going to be transmitted or reflected.

If I try to correlate the measurements here, there will be completely I mean anti-correlated. Which means whenever either this will detect or this will detect or this will detect or this will detect. So, the simple picture that I get is either the photon gets reflected or the photon get transmitted, I want to see whether this is a right picture.

**So**, I do not stop the experiment here I continue and put two mirrors and bring them back into another beam splitter what is this and parameter.

**So**, normally classically light waves come from here, they split into two parts go through different arms come back and depending on the path difference between this and this arm you will have constructive and destructive interference.

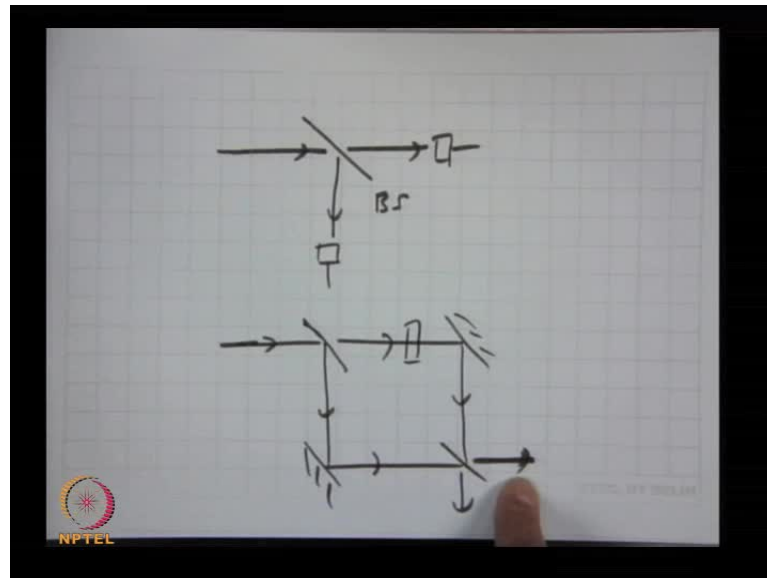
**So**, the if the arrangement is completely symmetric you can ensure that light coming from here only comes out from here. **If the if the if the** path difference is exactly 0 along both arms you see this light goes transmitted reflected this is reflected and transmitted they are identical paths.

If all elements are identical they will be in phase here this wave is transmitted and transmitted this wave is reflected and reflected so, they are different.

One is going through two transmissions the other is going through two reflections, while this in this arm transmission reflection transmission assuming these two beams splitter are identical I expect all the light come from here.

Now I send single photons I still see them coming here. If my earlier picture that light when it arrived here either goes here or here is true I should have got some photons here because, the photons could have 50 percent chance of coming here then out of that 50 percent another 50 percent of going here so, 25 percent.

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But this is also a 50 percent chance have coming like this and 25 percent so, make it 50 percent 50 percent. By whenever observe this I never see a photon coming here all the photons arrive only here.

**So**, it looks as safe the photon is taking both the paths. If there is to be interference there is no way other than that the photon in a in a classical way I will say the photon is going through both arms.

If I have to put a detector here. I will always see a full photon or low photon, if I remove this detector here the photon always arrives here. So it looks as if the photon is actually sensing both arms it is a both places simultaneously. **So**, the picture of a particle which is going here and there is not a very correct picture for photons.

**So**, somebody said may be the photon knows that this interferometer is there so it will take both arms both paths and if it knows you are putting a detector it will go through one of the paths.

**So**, let me so people said I will be this argument. **So**, what the people did is they decided suppose I remove this beam splitter what will happen it is exactly in this experiment if I remove this beam splitter there is no interference there is reflection there is transmission here there is reflection so, I have all I have done is to split the two arms that is all.

So, what I do is I allow the photon to come at rate of cross this beam splitter I decide to insert the beam splitter or do not insert the beam splitter to fool the photon. I still get the same experiment I still find if the beam splitter is there irrespective of when you have put all the photons arrive here if there was no beam splitter there are 50 percent chance of coming both the photons here and here.

Interference is a purely classical phenomenon well this is a purely particle phenomenon. I need a theory which can explain both results, see my analysis my mathematical theory should explain this as well as this.

And that quantum mechanics does. The quantum theory of radiation when I quantize the electromagnetic field and introduce the concept of the photons I will find that all these experiments are explainable. The problem is conceptual and conceptually understanding what exactly is happening in the system.

So, there are a large number of experiments in which they have measured and found that there is complete anti-correlation between these two detectors and in this experiment as you change for example; the phase of one of the arms you can change at the interference here which is standard classical experiment.

But the problem is that start to think of single photons then, changing this if it had that if it had occupied one of the arms and changing should not have so, much effect but, there is interference taking in this set up and the quantum mechanical analysis will predict all the experimental results.

So, there are a number of experiments this is just a sample which I will explain to you here, which need an explanation in terms of the mathematical analysis and what I need to do is to quantize Maxwell's equations.

Now you know in quantum mechanics I will we have we represent this state of a system by a ket vector and all observables are represented by hermitian operators. So I would archive to go back and read a bit of quantum mechanics back again to recall the quantum mechanical principles because, we will use the harmonic oscillator problem here very deeply because actually I will show you that electromagnetic field can be written as a superposition of harmonic oscillators of various frequencies.

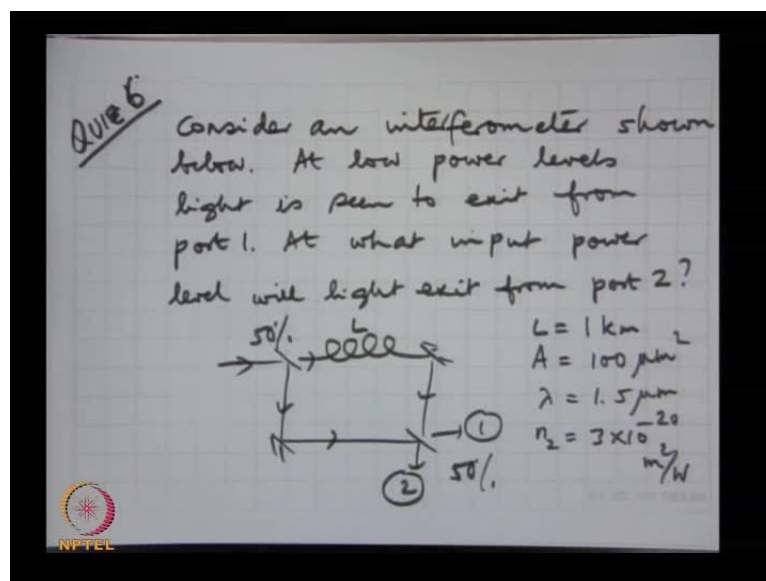


So, the quantization of electromagnetic field is exactly the same as a quantizing of the harmonic oscillator. And you recall that in a harmonic oscillator it has discrete energies  $H \times N + \text{half } H \times \omega$ .

So I get the same kind of a feature here and that is I have introduced a concept of photons.

So, when I have an energy which is  $n + \text{half } h \times \omega$  I will say that this field has  $n$  photons because, energy is  $n + \text{half } h \times \omega$  the level of expectation is corresponding to  $n$  photon excitation.

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So, we will stop here we have a quiz now. So, the next class I will start the quantum mechanical principles, we will start a quantization and radiation field and discuss the concept of photons etcetera. This is the classical picture here the interferometer this is an optical fiber and the length of this fiber is one kilometer. I am giving you the area of cross section of the fiber is 100 micron square  $\lambda$  is 1.5 micrometer and  $n_2$  is  $3 \times 10^{-20}$  meter square per watt.

So, when the input powers are low light comes out from port 1, as I increase the power I find that at some power level light comes out from port 2 not port 1. So, what is the power then I must input here for the light to switch from 1 to 2.

We have discussed the basic phases behind this problem.